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100 Years
NORTH SYDNEY BOYS HIGH SCHOOL
1912 - 2012

MATHEMATICS

2014 HSC Course Assessment Task 4
August 13, 2014

General Instructions

- Working time – 55 minutes (plus 5 minutes reading time).
- Write using blue or black pen. Diagrams may be sketched in pencil.
- Board approved calculators may be used.
- All necessary working should be shown in every question.
- Attempt all questions.

This paper contains 4 questions.

- Commence each new question on a new page.
- Show all necessary working for every question.
- Marks may be deducted for illegible or incomplete working.

STUDENT NUMBER: ..... **# BOOKLETS USED:** .....

Class (please ✓)

☐ Mr Lin ☐ Mr Berry

☐ Mr Weiss ☐ Mr Lowe

☐ Mr Zuber

Question	**1**	**2**	**3**	**4**	**Total**
Marks	6	6	9	12	33

Question 1 (6 marks)

An object, initially at rest at the origin, moves in a straight line with velocity v ms^{-1} so that $v = 4t(5 - t)$ where t is the time elapsed in seconds.

Find:

(i) The acceleration of the object at the third second. **2**

(ii) An expression for the displacement (x) of the particle in terms of t. **2**

(iii) The position for the particle where it again comes to rest. **2**

Question 2 (6 marks)

There are three cards marked X, Y, and Z and three envelopes also marked X, Y, and Z respectively. One card is randomly placed in each envelope.

What is the probability that:

(i) All the cards are put into an envelope with the same letter as the card? **1**

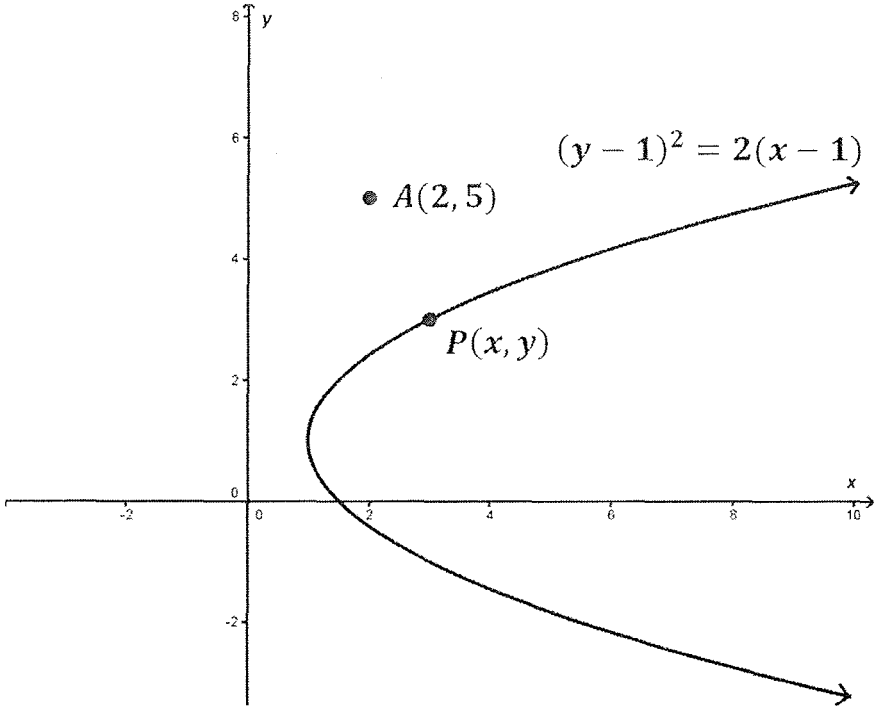
(ii) No cards are put into envelopes with the same letter as the card? **1**

(iii) Exactly one card is placed into an envelope with the same letter as the card? **2**

(iv) At least one of the cards is put into an envelope with the same letter as the card? **2**

Examination continues on the next page.

Question 3 (9 marks)



NOT TO SCALE

The diagram above shows the graph of the parabola $(y-1)^2 = 2(x-1)$. The point $A(2,5)$ is outside the parabola while the point $P(x,y)$ is on the parabola.

(i) If D is the distance between the points A and P, show that **3**

$$D^2 = \left[\frac{1}{2}(y-1)^2 - 1\right]^2 + (y-5)^2$$

(ii) Find $\dfrac{d(D^2)}{dy}$ and show that the value of D in the equation in part (i) is a **4**

minimum when $y = 3$.

(iii) Hence or otherwise find the distance between A and P and find the coordinate of **2**
the point P

Examination continues on the next page.

Question 4 (12 marks)

A fund is established to provide birthday presents for Bill. \$10 000 is placed in the fund when Bill was born. It is decided that \$450 will be drawn from the fund each year to purchase his birthday presents. The money in the fund is invested 3% p.a. compounded annually with the interest paid into the fund before each of his birthdays.

(i) Show the fund contains \$9695.50 after Bill's second birthday. **2**

(ii) If A is the amount in dollars remaining in the fund after n^{th} birthday, prove that $A_n = 5000(3 - 1.03^n)$. **3**

(iii) Find the amount of money in the fund after Bill's 25^{th} birthday. Give your answer correct to the nearest dollar. **1**

(iv) Find the maximum number of birthday presents that can be financed using this fund. **2**

(v) For the fund described above, after Bill's first birthday his parents decided to increase the amount of money withdrawn for each birthday by 2% each year.

(a) Show that the amount remaining after Bill's 2^{nd} birthday is \$9686.50 **2**

(b) Find the amount remaining in the fund after Bill's 25^{th} birthday. Give your answer correct to the nearest dollar. **2**

End of examination.

Question 1

$V = 4t(5-t)$

$= 20t - 4t^2$

a) $\frac{dv}{dt} = 20 - 8t$

when $t = 3$

$\frac{dv}{dt} = 20 - 8 \times 3$

$= 20 - 24$

$= -4$

b) $x = \int 20t - 4t^2 \, dt$

$= 10t^2 - \frac{4t^3}{3} + C$

when $t = 0$ $x = 0$

$\therefore C = 0$

$\therefore x = 10t^2 - \frac{4t^3}{3}$

c) comes to rest: $v = 0$

$\therefore t = 5$

when $t = 5$, $x = 10(5^2) - \frac{4}{3}5^3$

$= 2 \times 5^3 - \frac{4}{3} \times 5^3$

$= \frac{2}{3} \times 5^3 = \frac{250}{3}$ m.

Question 2

a) $P(\text{all cards go into correct envelopes})$

$= P(ABC)$

$= \frac{1}{3} \times \frac{1}{2} \times 1$

$= \frac{1}{6}$

b) $P(\text{no cards go into correct envelopes})$

$= P(\bar{A}\,\bar{B}\,\bar{C})$

$= \frac{2}{3} \times \frac{1}{2}$

$= \frac{1}{3}$

c) $P(\text{exactly one card})$

$= P(A\bar{B}\bar{C} \text{ or } B\bar{A}\bar{C} \text{ or } C\bar{A}\bar{B})$

$= \frac{1}{3} \times \frac{1}{2} \times 1 + \frac{1}{3} \times \frac{1}{2} \times 1 + \frac{1}{3} \times \frac{1}{2} \times 1$

$= \frac{1}{2}$.

d) $P(\text{at least 1 card})$

Method 1

$1 - P(\text{no cards correct})$

$= 1 - \frac{1}{3}$

$= \frac{2}{3}$

Method 2

$P(1 \text{ card}) + P(\text{all cards})$

$= \frac{1}{2} + \frac{1}{6}$

$= \frac{4}{6}$

$= \frac{2}{3}$

Question 3

$D^2 = (x-2)^2 + (y-5)^2$

$(y-1)^2 = 2(x-1)$

$(x-1) = \frac{(y-1)^2}{2}$

$x = \frac{(y-1)^2}{2} + 1$

$\therefore D^2 = \left[\frac{(y-1)^2}{2} + 1 - 2\right]^2 + (y-5)^2$

$= \left[\frac{(y-1)^2}{2} - 1\right]^2 + (y-5)^2$

$\frac{dD^2}{dy} = 2\left[\frac{(y-1)^2}{2} - 1\right] \cdot (y-1) + 2(y-5)$

$= (y-1)\left[\frac{(y-1)^2}{2} - 1\right] + 2(y-5)$

$= \frac{1}{2}(y-1)^3 - (y-1) + 2(y-5)$

$= \frac{1}{2}(y-1)^3 - 8$

$= \left[(y-1) - 2\right]\left[(y-1)^2 + 2(y-1) + 4\right]$

$= (y-3)\left[y^2 - 2y + 1 + 2y - 2 + 4\right]$

$= (y-3)\left[y^2 + 3\right]$

$y^3 + 3y - 3y^2 - 9$

$\frac{dD^2}{dy} = 0 \Rightarrow (y-3)(y^2+3) = 0$

$y = 3.$

$\frac{d^2(D^2)}{dy} = 3y^2 - 6y + 3$

when $y = 3$

$\frac{d^2D^2}{dy} = 27 - 18 + 3$

$= 12$

> 0

$\therefore$ minimum at $y = 3$

c) when $y = 3$.

$D^2 = \left[\frac{1}{2}(3-1)^2 - 1\right]^2 + (3-5)^2$

$= \left[1^2\right] + (-2)^2$

$= 5$

$\therefore D = \sqrt{5}$, $D > 0$.

Question 4.

1. a) $A_1 = \$10000 \times 1.03 - 450$

$A_2 = (\$10000 \times 1.03 - 450)1.03 - 450$

$= \$10000 \times 1.03^2 - 450(1 + 1.03)$

$= \$9695.50$

b) $A_3 = (\$10000 \times 1.03^2 - 450 \times 1.03 - 450)1.03 - 450$

$= 10000 \times 1.03^3 - 450 \times 1.03^2 - 450 \times 1.03 - 450$

$= 10000 \times 1.03^3 - 450(1 + 1.03 + 1.03^2)$

$\therefore A_n = 10000 \times 1.03^n - 450(\underbrace{1 + 1.03 + 1.03^2 + \ldots + 1.03^{n-1}})$

GP: $a = 1$ $r = 1.03$

$= 10000 \times 1.03^n - 450\left(\frac{1.03^n - 1}{0.03}\right)$

$= 10000 \times 1.03^n - 15000(1.03^n - 1)$

$= 10000 \times 1.03^n - 15000 \times 1.03^n + 15000$

$= 15000 - 5000 \times 1.03^n$

$= 5000(3 - 1.03^n)$

c) When $n = 25$

$A_{25} = 5000(3 - 1.03^{25})$

$= \$4531$ to the nearest dollar

d) $A_n = 0 \Rightarrow 5000(3 - 1.03^n) = 0$

$3 - 1.03^n = 0$

$1.03^n = 3$

$n = \frac{\log_e 3}{\log_e 1.03} \doteq 37.17$

37 birthdays.