# NORTH SYDNEY BOYS HIGH SCHOOL

## 2016 HSC ASSESSMENT TASK 1

# Mathematics

### General Instructions

- Reading time – **5 minutes**
- Working time – **55 minutes**
- Write in the booklet provided
- Write using blue or black pen
- Board approved calculators may be used
- Show all necessary working in every question
- Each new question is to be started on a **new page.**
- Use the **Reference Sheet** provided!

**Class Teacher:**
(Please tick or highlight)
- O Mr Berry
- O Mr Ireland
- O Mr Jomaa
- O Ms Lee
- O Mr Lin
- O Ms Ziaziaris
- O Mr Zuber

Student Number:________________________

(To be used by the exam markers only.)

| Question No | 1-5 | 6 | 7 | 8 | 9 | Total | Total % |
|---|---|---|---|---|---|---|---|
| Mark | /5 | /17 | /16 | /4 | /14 | /56 | /100 |

**Section I: Objective Response (5 marks)**

Mark your answers on the multiple choice box below. Marks

1 Evaluate $\sum_{n=2}^{5}(-1)^n n^2$ 1

(A) 54 (B) 14 (C) $-54$ (D) $-14$

2 For what values of $k$ does the equation $x^2-6x-3k=0$ have real roots? 1

(A) $k \geq -3$ (B) $k \leq -3$ (C) $k \geq 3$ (D) $k \leq 3$

3 If the 5th and 18th terms of an arithmetic sequence are 12 and 64 respectively, then the common difference of the sequence is: 1

(A) $-5$ (B) 4 (C) $-4$ (D) 5

4 A parabola has equation $x^2 = 12\,(8-y)$. Its directrix is: 1

(A) $y=-3$ (B) $y=3$ (C) $y=5$ (D) $y=11$

5 The roots of the quadratic equation $gx^2-x+h=0$ are $-1$ and $3$. 1

The value of $h$ is:

(A) $-6$ (B) $-3$ (C) $-\frac{3}{2}$ (D) 2

Select the alternative A, B, C or D that best answers the question above and indicate your choice by shading the appropriate letter.

| | | | | |
|---|---|---|---|---|
| 1. | A | B | C | D |
| 2. | A | B | C | D |
| 3. | A | B | C | D |
| 4. | A | B | C | D |
| 5. | A | B | C | D |

**Section I: Extended Responses**

Write your solutions in your exam booklet, starting a new page for each Question.

**Question 6 – Quadratics (17 marks)** Marks

(a) Find values of $a$, $b$, $c$ such that $3x^2 - 7x - 6 \equiv a(x-2)^2 + b(x-2) + c$ **2**

(b) Solve the inequality $2x^2 + x - 6 \geq 0$. **2**

(c) The quadratic equation $x^2 - 7x + 8 = 0$ has roots $\alpha$ and $\beta$.

Find:

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\dfrac{2}{\alpha} + \dfrac{2}{\beta}$ **1**

(iv) $\alpha^2 + \beta^2$ **2**

(d) Solve the equation $m^4 + 12m^2 - 64 = 0$. **3**

(e) The line $y = x + k$ is tangent to the parabola $y = x^2 - 3x - 5$.

If $P$ is the point where they meet,

(i) Show that the $x$-coordinate of $P$ satisfies $x^2 - 4x - 5 - k = 0$. **1**

(ii) Find the value of $k$. **2**

(iii) Find the coordinates of $P$. **2**

**Question 7 – Series (16 marks)** **Marks**

(a) The first 3 terms of an arithmetic progression are 7, 11, and 15.

(i) Is 111 a term in this sequence? Justify your answer. **2**

(ii) Find the sum of the first 26 terms. **2**

(b) Farmer Joe is trying to fill his 1200 litre water tank. **3**

On day 1, he puts 100 litres into the tank.

On day 2 he adds 90 litres to the tank.

On day 3 he puts in 81 litres.

If he continues to add water in this way, will he ever fill the tank?

Justify your answer.

(c) The sum of the first $n$ terms of a sequence is given by $S_n = n(38 - 2n)$

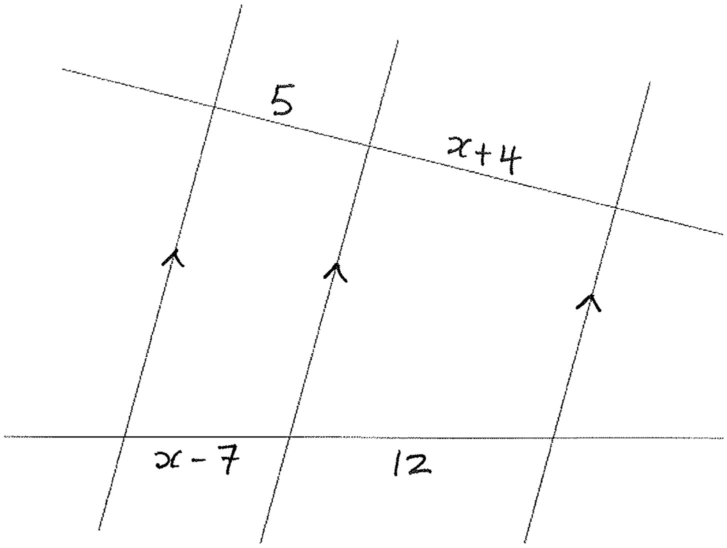
(i) Calculate $S_1$ and $S_2$. **2**

(ii) Find the first three terms of this series, $T_1$, $T_2$, and $T_3$. **2**

(iii) Find an expression for the $n^{th}$ term, $T_n$, giving your answer in simplified form. **2**

(d) The terms $x-1$, $x+1$, $x^2-1$ are in geometric progression. Find the three terms of the progression. **3**

**Question 8 – Geometry (4 marks)** **Marks**




Calculate the value of $x$, giving a geometrical reason. **4**

**Question 9 – Locus and Parabola (14 marks)** Marks

(a) Given a parabola with equation $x^2 - 6x - 7 = 8y$,

(i) Find the coordinates of the vertex **2**

(ii) Find the coordinates of the focus **1**

(iii) Find the length of the latus rectum chord **1**

(iv) Sketch the parabola, showing its focus, vertex and directrix. **2**

(b) A point $P(x, y)$ moves so that its distance from the point $A(6,1)$ is always twice its distance from the point $B(-3,4)$.

(i) Show that the locus of P is a circle. **2**

(ii) Write down the centre and radius of this circle **1**

(c) A point $P(x, y)$ moves so that it is equally distant from the point $A(-2,0)$ and the line $x = -6$. Write down the equation of the locus of $P$. **2**

(d) A sequence of parabolas is defined as follows:

$y = (ax-1)^2$, $y = (2ax-1)^2$, $y = (3ax-1)^2$, $y = (4ax-1)^2$, ...

(i) What is the focal length of the 20th parabola in this sequence? **2**

(ii) What point do the vertices of these parabolas approach? **1**

***END OF EXAM***

# 2016 - 2U Mathematics Task 1 {Suggested Solutions}

## Section I - Objective response

**1** $\sum_{n=2}^{5}(-1)^n n^2 = (-1)^2\cdot 2^2 + (-1)^3\cdot 3^2 + (-1)^4\cdot 4^2 + (-1)^5\cdot 5^2$

$= 4 - 9 + 16 - 25$

$= -14$ $\therefore$ D

**2** Real roots $\Rightarrow \Delta \geq 0$

$\therefore\ 36 - 4(1)(-3k) \geq 0$

$36 + 12k \geq 0$

$k \geq -3$ $\therefore$ A

**3** $a + 4d = 12$

$a + 17d = 64$

$\therefore\ 13d = 52$ $\therefore\ d = 4$ $\therefore$ B

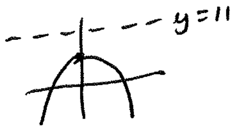
**4** $x^2 = 12(8-y)$

$\therefore\ x^2 = -12(y-8)$

$x^2 = -4\cdot 3\cdot(y-8)$

$\therefore\ V = (0,8)$, $a = 3$, concave down $\therefore$ D

$\therefore$ directrix is $y = 11$




**5** Sum of roots $\Rightarrow -1 + 3 = \frac{-(-1)}{g}$

$2 = \frac{1}{g}$ $\therefore\ g = \frac{1}{2}$

Product of roots $\Rightarrow (-1)(3) = \frac{h}{g}$

$-3 = \frac{h}{g}$

$\therefore\ h = -3g = -\frac{3}{2}$ $\therefore$ C

$\boxed{6}$ (a) $3x^2 - 7x - 6 \equiv a(x-2)^2 + b(x-2) + c$

sub $x=2 \Rightarrow$ $3\cdot 4 - 7\cdot 2 - 6 = 0 + 0 + c$

$-8 = c$

coeff. of $x^2$ term $\Rightarrow$ $a = 3$

sub. $x=0 \Rightarrow$

$0 - 0 - 6 = 4a - 2b + c$

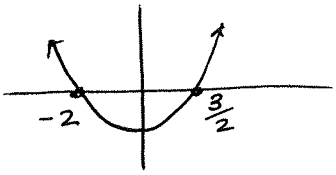
$-6 = 12 - 2b - 8$

$-10 = -2b \quad \therefore b = 5$ ✓ significant progress

Thus $a=3, \quad b=5, \quad c=-8.$ ✓ correct solution

(b) $2x^2 + x - 6 \geq 0$

$(2x-3)(x+2) \geq 0$




$\therefore x \leq -2$

or $x \geq \frac{3}{2}$ ✓✓

(c) $x^2 - 7x + 8 = 0$

(i) $\alpha + \beta = 7$ ✓

(ii) $\alpha\beta = 8$ ✓

(iii) $\frac{2}{\alpha} + \frac{2}{\beta} = \frac{2\alpha + 2\beta}{\alpha\beta} = \frac{2(\alpha+\beta)}{\alpha\beta} = \frac{14}{8}$

$= \frac{7}{4}$ ✓

(iv) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ ✓

$= 7^2 - 2(8)$

$= 49 - 16$

$= 33$ ✓

$\boxed{6}$ - continued

(d) $m^4 + 12m^2 - 64 = 0$

$\therefore (m^2 + 16)(m^2 - 4) = 0$ ✓

$\therefore m^2 + 16 = 0 \rightarrow$ no solution

or $m^2 - 4 = 0 \rightarrow m = \pm 2$. ✓✓

(e) (i) $y = x + k$ meets $y = x^2 - 3x - 5$

when $x + k = x^2 - 3x - 5$

$\therefore x^2 - 3x - 5 - x - k = 0$

i.e. $x^2 - 4x - 5 - k = 0$ (*) ✓

(ii) Tangent $\Rightarrow \Delta = 0$

$\therefore 4^2 - 4 \cdot 1(-5 - k) = 0$ ✓

$4 - (-5 - k) = 0$ ✓

$9 + k = 0 \quad \therefore k = -9$

(iii) Equation (*) becomes

$x^2 - 4x - 5 + 9 = 0$

$x^2 - 4x + 4 = 0$

$(x - 2)^2 = 0$

$\therefore x = 2,$

$y = 2 - 9 = -7$

$\therefore P = (2, -7)$ ✓✓

17

$\boxed{7}$

(a) $T_1 = 7, \quad T_2 = 11, \quad T_3 = 15$

(i) $\therefore \; a = 7$
$d = 4$

Then $111 = 7 + (n-1)4$

$\therefore \; n - 1 = \frac{104}{4} = 26, \quad n = 27.$

Integer, $\therefore$ 111 is a term.

(ii) $S_{26} = \frac{26}{2}(a + l)$ where $l = 111 - 4$
$= 107$

$\therefore \; S_{26} = \frac{26}{2}(7 + 107)$

$= 1482$

(b) Amount added to tank is

$S = 100 + 90 + 81 + \ldots$

This is a G.P. with $a = 100, \quad r = 0 \cdot 9$

$S_\infty = \frac{a}{1-r} = \frac{100}{1 - 0 \cdot 9} = 1000$

Since the limiting sum 1000 L is less than the tank's capacity, it will not be filled.

7 - continued

(c) $S_n = n(38-2n)$

(i) $S_1 = 1(38-2) = 36$

$S_2 = 2(38-4) = 68$ So $S_1 = 36$, $S_2 = 68$ ✓✓

(ii) $T_1 = S_1 = 36$

$T_2 = S_2 - S_1 = 68 - 36 = 32$

Since $T_2 - T_1 = 32 - 36 = -4$

$\therefore T_3 = 32 - 4 = 28$

So $T_1 = 36$, $T_2 = 32$, $T_3 = 28$ ✓✓

(iii) $T_n = S_n - S_{n-1}$ ✓ uses relation

$= n(38-2n) - (n-1)[38 - 2(n-1)]$

$= 38n - 2n^2 - [(n-1)(40-2n)]$

$= 38n - 2n^2 - [40n - 2n^2 - 40 + 2n]$

$= 38n - \cancel{2n^2} - 40n + \cancel{2n^2} + 40 - 2n$ ✓

$\therefore T_n = 40 - 4n$

(d) $x-1,\ x+1,\ x^2-1$ are in G.P.

$\therefore \dfrac{x+1}{x-1} = \dfrac{x^2-1}{x+1}$ ✓ uses definition

$\therefore \dfrac{x+1}{x-1} = \dfrac{\cancel{(x+1)}(x-1)}{\cancel{(x+1)}}$

$\therefore x+1 = x^2 - 2x + 1$

$x^2 - 3x = 0$

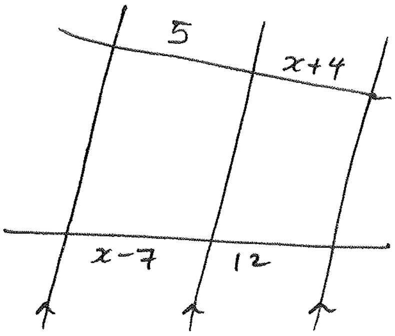
$\therefore x = 0$ or $x = 3$ ✓

So the 3 terms are $-1, 1, -1$ ✓

OR $2, 4, 8.$

16

8




$$\frac{x+4}{5} = \frac{12}{x-7}$$ (Ratio of intercepts made by transversals on parallel lines)

✓ equation
✓ reason

$\therefore \quad (x-7)(x+4) = 60$

$x^2 - 3x - 28 - 60 = 0$

$x^2 - 3x - 88 = 0$

$(x-11)(x+8) = 0$

$\therefore \quad x = 11$ or $-8$

But $x$ is a length, $\therefore \quad x = 11.$

✓✓ 4

9 (a) (i) $x^2 - 6x - 7 = 8y$

$\therefore x^2 - 6x + 9 - 9 - 7 = 8y$

$(x-3)^2 = 8y + 16$

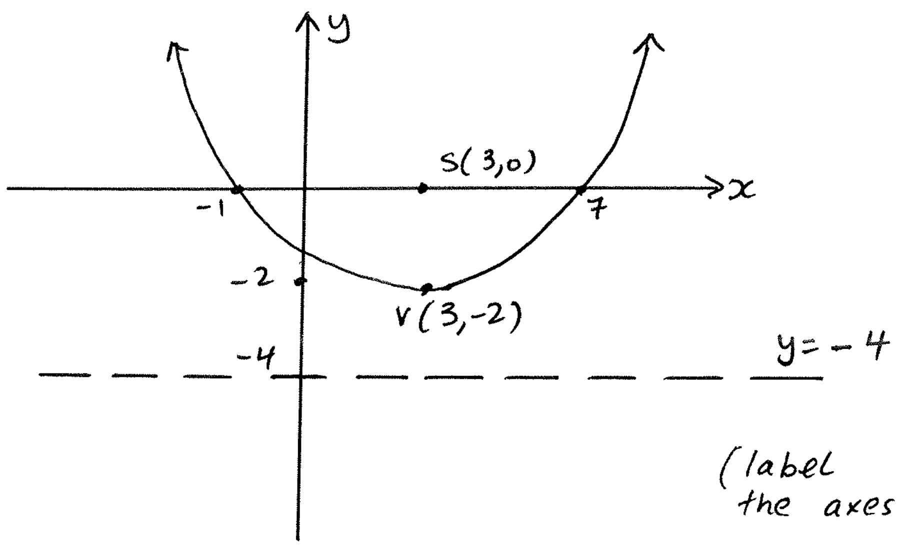
$\therefore \quad (x-3)^2 = 8(y+2)$

$\therefore$ Vertex $= (3, -2)$ ✓✓

(ii) $a = 2 \quad \therefore \quad S = (3, 0)$ ✓

(iii) Length of latus rectum $= 4a = 8$ ✓

(iv)




✓✓

(label the axes!!)

$\boxed{9}$ – continued

(b) $AP = 2 \times AB$ $\qquad \begin{cases} P = (x, y) \\ A = (6, 1) \\ B = (-3, 4) \end{cases}$

(i) $AP^2 = 4 \times AB^2$

$\therefore (x-6)^2 + (y-1)^2 = 4\left[(x+3)^2 + (y-4)^2\right]$

$= 4\left[x^2 + 6x + 9 + y^2 - 8y + 16\right]$

$x^2 - 12x + 36 + y^2 - 2y + 1 = 4x^2 + 24x + 36 + 4y^2 - 32y + 64$

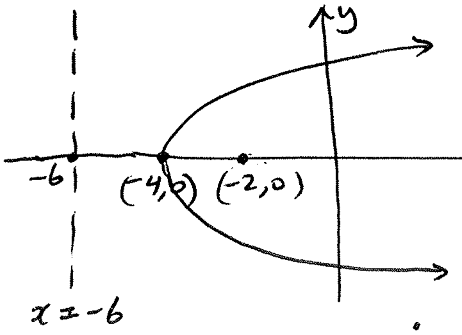
$3x^2 + 36x + 3y^2 - 30y + 63 = 0$

$x^2 + 12x + 36 + y^2 - 10y + 25 = -21 + 36 + 25$

$(x+6)^2 + (y-5)^2 = 40$

(ii) This is the equation of a circle with centre $(-6, 5)$, radius $= \sqrt{40}$
$= 2\sqrt{10}$

(c) By definition, this is a parabola with focus $(-2, 0)$ and directrix $x = -6$ :




i.e. focal length $a = 2$,
$V = (-4, 0)$

$\therefore \; y^2 = 8(x+4)$.

(d) (i) Following the pattern, $T_{20}$ is $y = (20ax - 1)^2$

i.e. $y = \left[20a\left(x - \frac{1}{20a}\right)\right]^2 = 400a^2\left(x - \frac{1}{20a}\right)^2$

Rearrange $\rightarrow$ $\left(x - \frac{1}{20a}\right)^2 = 4 \cdot \frac{1}{1600a^2} \cdot y$

So the focal length $= \dfrac{1}{1600a^2}$

(ii) The vertices approach $(0, 0)$.

/14