NORTH SYDNEY BOYS HIGH SCHOOL

2015 HSC ASSESSMENT TASK 4

Mathematics

General Instructions

- Working time – 55 minutes
- Write on the lined paper in the booklet provided
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working should be shown in every question
- Each new question is to be started on a **new page.**
- Attempt all questions

Class Teacher:
(Please tick or highlight)

- O Ms Lee
- O Mr Berry
- O Ms Ziaziaris
- O Mr Zuber

Student Number:____________________

(To be used by the exam markers only.)

Question No	1-4	5	6	7	8	Total	Total %
Mark	—/4	—/7	—/14	—/11	—/9	—/45	—/100

SECTION A - MULTIPLE CHOICE *(4 Marks)*

Question 1

Which of the following is the sum of the first *n* terms of the series:

$(x+y)+(x+2y)+(x+3y)\ldots$

(a) $x+ny$

(b) $n(x+ny)$

(c) $\frac{(x+y)(y^{n-1}-1)}{y-1}$

(d) $\frac{n}{2}(2x+ny+y)$

Question 2

Consider the following sentence:

"House prices in Sydney are expected to rise over the next year however the they seem to be slowing and it is expected that prices will soon stabilise."

If *P* is the average Sydney house price and *t* is time, which of the following is true?

(a) $\frac{dP}{dt}>0\ \&\ \frac{d^2P}{dt^2}>0$

(b) $\frac{dP}{dt}>0\ \&\ \frac{d^2P}{dt^2}<0$

(c) $\frac{dP}{dt}<0\ \&\ \frac{d^2P}{dt^2}>0$

(d) $\frac{dP}{dt}<0\ \&\ \frac{d^2P}{dt^2}<0$

Question 3

If Timmy invests \$ 1200 on January 1 2008 into an account that pays 6% interest compounded monthly, when will he have \$2000?

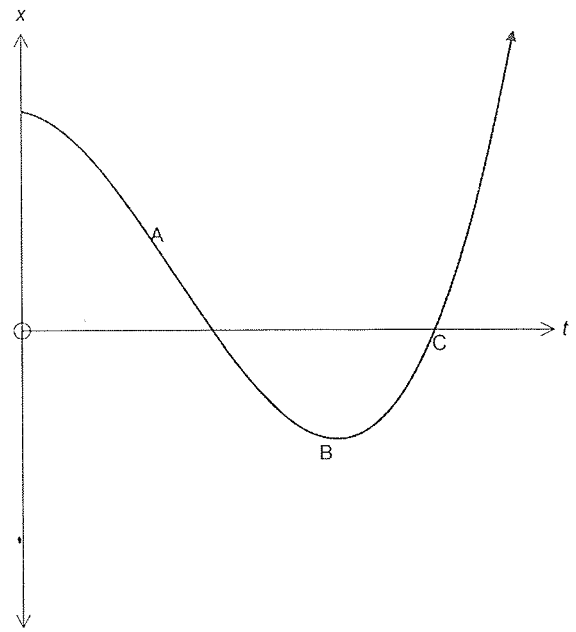
(a) July 2016

(b) May 2017

(c) April 2018

(d) February 2019

Question 4



The graph above is a displacement/time graph for a particle travelling in one dimension. At what point is the acceleration equal to zero?

(a) A

(b) B

(c) C

(d) This particle will never have an acceleration of zero.

SECTION B – EXTENDED RESPONSE

Question 5 – Sequences and Series *(7 Marks)*

			Marks
(a)		The third and seventh term of a geometric series are $1\frac{1}{4}$ and 20 respectively, Find:	
	(i)	The common ratio	2
	(ii)	The first term	1
	(iii)	The 15th term	1
(b)		Evaluate:	3

$$\sum_{n=1}^{14} 1 - 2n + 2^n$$

Question 6 – Rates of Change *(14 Marks)*

(a) The amount of water in a puddle over a four hour period is given by:

$W = 4t^3 - t^4 + 20$ where $0 \leq t \leq 4$

where W is the amount of water in litres and t is the time in hours.

(i) How much water was initially in the puddle? **1**

(ii) What was the maximum amount of water in the puddle during this time? **3**

(iii) When was the amount of water increasing the most rapidly? **2**

(iv) Neatly sketch $W = 4t^3 - t^4 + 20$ for $0 \leq t \leq 4$, showing all relevant features **2**

(b) The percentage of Carbon 14 in an organism falls exponentially after the death of the organism. After 1845 years 80% of the original concentration of Carbon 14 remains.

Using the equation $C = C_0 e^{-kt}$ to represent the exponential fall of Carbon 14:

(i) Find the exact value of k **2**

(ii) An artefact containing this organism has 65% of the original concentration of Carbon 14. How long has this organism been dead? Give your answer to the nearest year. **2**

(iii) A sea sponge containing this organism has been dead for 12000 years. What percentage (to 1 decimal place) of the original Carbon 14 concentration does it have? **2**

Question 7 - Applications of Series *(11 Marks)*

(a) James received 3 tonnes of topsoil for his yard. He uses a wheelbarrow which can hold 150 kg to spread the soil.

(i) How many loads in the wheelbarrow will he need? **1**

He begins at the pile of topsoil and deposits the first load 3 metres form the pile. Each successive load is dumped half a metre further from the pile.

(ii) How far from the pile will he leave the final barrow load? **2**

(iii) What is the total distance that James will travel with the wheelbarrow if he finishes back at his starting point? **2**

(b) Pepe is saving for a new gaming computer. He opens an "Incentive Saver Account" which pays interest at the rate of 4.8% p.a compounded monthly at the end of each month.

Pepe decides to deposit $40 into the account on the first of each month. He makes his first deposit on the 1st January 2015 and his last on the 1st July 2017.

He withdraws the entire amount, plus interest, immediately after his final interest payment on 31st July 2017.

(i) How much did Pepe deposit into his "Incentive Saver Account"? **1**

(ii) How much did Pepe withdraw from his account on the 31st July 2017? (Show all relevant working) **3**

(iii) Pepe spends all of the money on his new computer. Five years later it is worth half of its original price. Calculate its annual rate of depreciation. **2**

Question 8 – Motion *(9 Marks)*

(a) A particle P is moving in a straight line so that its velocity v metres per second after t seconds is given by $v = 12 - 4t$. Initially P is 3 metres to the right of the origin O.

(i) Find the initial velocity and acceleration of P. **2**

(ii) If the displacement of P from O is x metres, find an expression for x in terms of t. **2**

(iii) Find when and where P is stationary **2**

(iv) Sketch the graph of $v = 12 - 4t$, for $0 \leq t \leq 5$ **1**

(v) Hence, or otherwise, find the total distance travelled by P during the first five seconds. **2**

End of Examination

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \ x \neq 0, \ \text{if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

Section A

1./ D

2./ B

3./ A

4./ A

Question 5

Marks

(a) $T_3 = 5/4$

$T_7 = 20$

(i) $\frac{T_7}{T_3} = \frac{ar^6}{ar^2}$

$\therefore r^4 = 20/(5/4)$ ✓

$\therefore r^4 = 16$

$r = \pm 2$ ✓

(ii) $20 = ar^6$

$a = \frac{20}{(\pm 2)^6}$

$= 5/16$ ✓

(iii) $T_{15} = (5/16)(\pm 2)^{14}$

$= 5120$ ✓

(b) $\sum_{n=1}^{14} (1-2n) + (2^n)$

$= \frac{14}{2}((-1) + (-27)) + \frac{2(2^{14}-1)}{2-1}$ ✓✓

$= -196 + 32766$

$= 32570$ ✓

8mm A4

Question 6

Marks

(a) (i) 20L ✓

(ii) $W = 4t^3 - t^4 + 20$

$\frac{dW}{dt} = 12t^2 - 4t^3$

when $\frac{dW}{dt} = 0$

$4t^2(3-t) = 0$

$\therefore$ test $t=0$ & $t=3$ ✓

t	-1	0	1	3	4
dW/dt	16	0	8	0	-64
W	/	—	/	—	\

✓

$\therefore$ maxima occurs when $t=3$

$W = 4 \times 3^3 - 3^4 + 20$

$= 47\text{ L}$ ✓

(iii) to maximise $\frac{dW}{dt}$, $\frac{d^2W}{dt^2} = 0$

$\frac{d^2W}{dt^2} = 24t - 12t^2$

$\therefore 12t(2-t) = 0$

$\therefore t = 2$ or $t = 0$ ✓

disregard $t=0$ as part (ii) established when $t=0$ $dW/dt = 0$

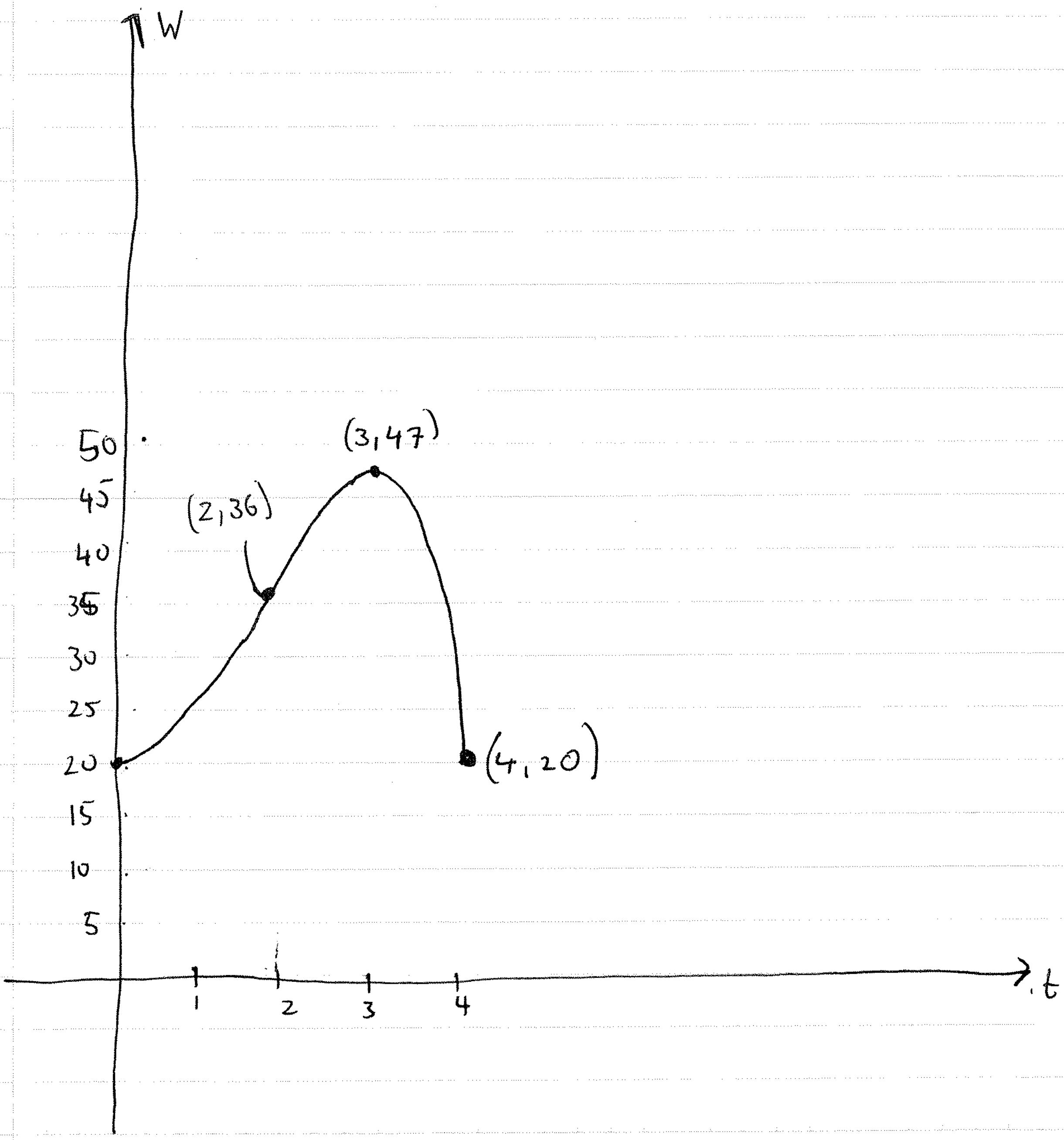
8mm A4

Marks.

t	1	2	3
$\frac{d^2w}{dt^2}$	12	0	-36
w	$\cup$	$\cdot$	$\cap$

$\therefore$ P.O.I occurs at $t = 2$

so most rapid increase at $t = 2$



Marks

(b) (i)

$$0.80 = e^{-1845k}$$ ✓

$$\log_e(0.80) = -1845k$$

$$\therefore k = -\frac{\log_e(0.80)}{1845}$$ ✓

(ii) $$0.65 = e^{-kt}$$ ✓

$$-kt = \log_e(0.65)$$

$$t = \frac{\log_e(0.65)}{-k}$$

$$= 3562 \text{ years (nearest yr)}$$ ✓

(iii) $$A = e^{-12000k}$$

$$= 0.234256\ldots$$ ✓

$\therefore$ 23.4% ✓

8mm A4

Question 7 | Marks

(a)(i) $\frac{3000}{150} = 20$ ✓

(ii) This is an AP where:

$a = 3$
$d = 0.5$
$n = 20$ ✓

$T_{20} = 3 + (20-1) \times 0.5$
$= 12.5$ m ✓

(iii) This is an AP where

$a = 6$
$d = 1$
$n = 20$ ✓

$S_{20} = \frac{20}{2}(2 \times 6 + (20-1) \times 1)$

$= 310$ m. ✓

(b) (i) $31 \times \$40 = \$1240$ ✓

(ii) $A_{31} = 40(1.004)^{31}$
$A_{30} = 40(1.004)^{30}$
$A_{29} = 40(1.004)^{29}$
and so on until

$A_1 = 40(1.004)^1$ ✓

Marks

$A = A_1 + A_2 + A_3 + \dots + A_{31}$

$= 40(1.004 + 1.004^2 + 1.004^3 + \dots + 1.004^{31})$

since $\frac{1.004^2}{1.004} = \frac{1.004^3}{1.004^2}$

$= 1.004$

$\therefore$ this is a GP where $r = 1.004$

$\therefore A = \frac{40 \times 1.004(1.004^{31} - 1)}{1.004 - 1}$ ✓

$= \$1322.63$ (2 d.p.). ✓

(iii). $0.5 = (1-r)^5$ ✓

$\therefore 1-r = \sqrt[5]{0.5}$

$\therefore r = 0.12944$

$r = 13\%$ (nearest %). ✓

8mm A4

Question 8.

Marks.

(a) (i). when $t=0$

$v = 12\ m/s$

$a = -4\ m/s^2$

(ii) $x = 12t - 2t^2 + c$

when $t = 0$ $x = 3$

$\therefore 3 = 12 \times 0 - 2 \times 0 + c$

$c = 3$

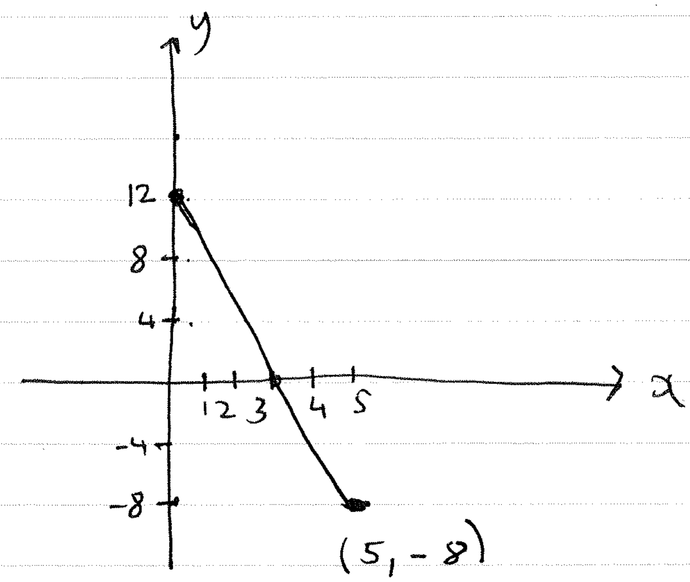
$\therefore x = \left(12t - 2t^2 + 3\right)\ m$

(iii) when $v = 0$

$12 - 4t = 0$

$t = 3,\quad x = 21m$

(iv).



(v) $d = \int_0^3 (12-4t)\,dt + \left|\int_3^5 (12-4t)\,dt\right|$

$= \left(\frac{1}{2} \times 3 \times 12\right) + \left(\frac{1}{2} \times 2 \times 8\right)$

$= 26\ m.$