NORTH SYDNEY BOYS
HIGH SCHOOL

PRELIMINARY MATHEMATICS

2015 Preliminary HSC Course Assessment Task 2
Thursday June 4, 2015

General instructions

- Working time – 50 minutes. (plus 5 minutes reading time)
- Write using blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- Board approved calculators may be used.
- Attempt **all** questions.
- At the conclusion of the examination, bundle the booklets + answer sheet used in the correct order within this paper and hand to examination supervisors.
- Commence each new question on a new page. Write on both sides of the paper.
- All necessary working should be shown in every question. Marks may be deducted for illegible or incomplete working.

STUDENT NUMBER: .... **# BOOKLETS USED:** .....

Class (please ✔)

○ 11M2A – Mr Zuber

Marker's use only.

QUESTION	1	2	3	4	Total	%
MARKS	$\overline{14}$	$\overline{8}$	$\overline{9}$	$\overline{14}$	$\overline{45}$	$\overline{100}$

45 marks
Attempt Questions 1 to 4

Write your answers in the writing booklets supplied. Additional writing booklets are available. Your responses should include relevant mathematical reasoning and/or calculations.

Question 1 (14 Marks) Commence a NEW page. **Marks**

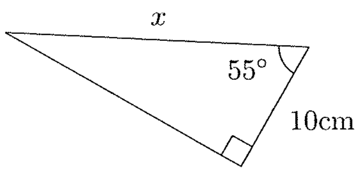
(a) If $\sin\alpha = 0.72$ Find the value of the acute angle α, rounding correct to the nearest minute. **1**

(b) Find the exact value of

i. $\cos 210°$ **1**

ii. $\dfrac{2\tan 60°}{1-\tan^2 60°}$ **2**

(c) Find the value of x, correct to 1 decimal place **2**

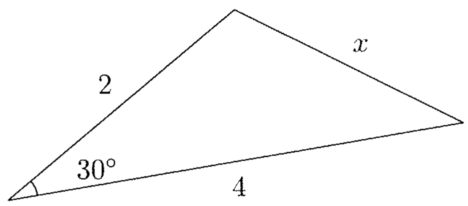


(d) If $\cos\theta = -\dfrac{3}{7}$ for $0 \leq \theta \leq 360°$ Find the value of

i. $\sec\theta$ **1**

ii. $\sin\theta$, given $\sin\theta$ is negative. **3**

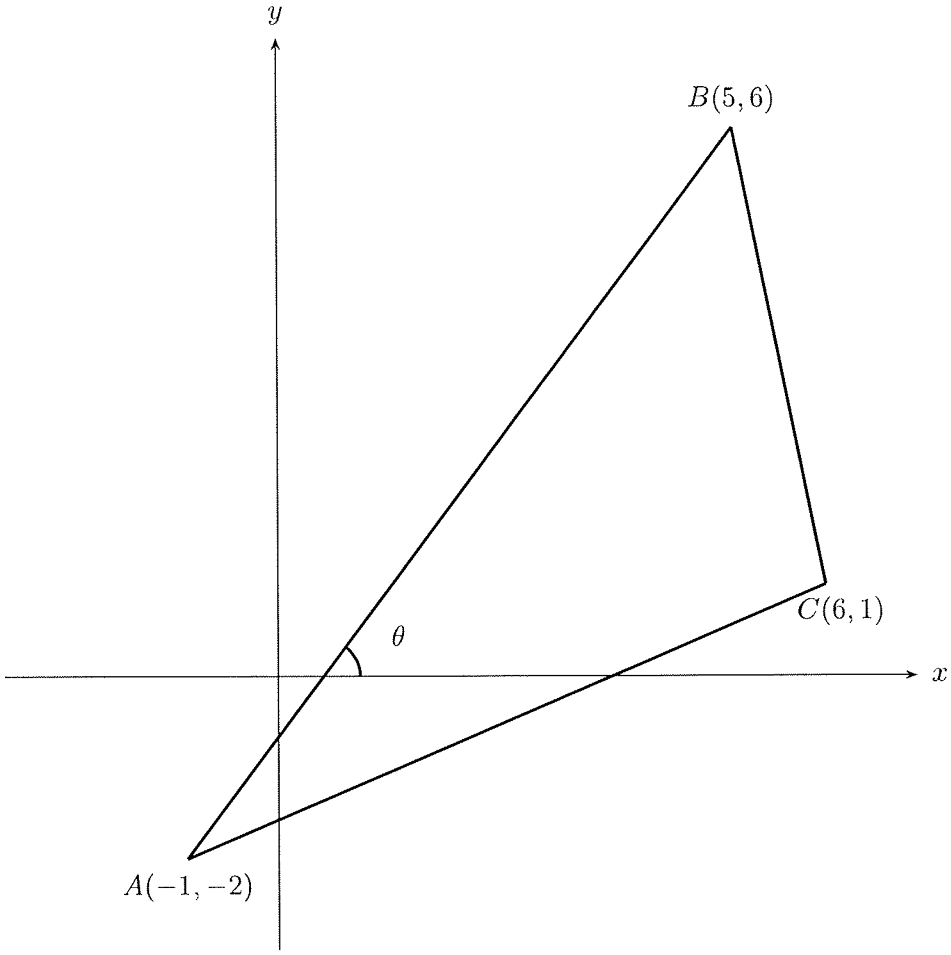
(e)



i. Find the value of x, correct to 2 decimal place. **2**

ii. Find the area of the triangle. **2**

Question 2 (8 Marks) Commence a NEW page. **Marks**



(a) Find the gradient of AB **1**

(b) Find the size of θ to the nearest degree. **1**

(c) Show that the equation of AB is $4x - 3y - 2 = 0$ **2**

(d) Find the perpendicular distance of AB from the point C. **2**

(e) Hence, find the area of the triangle ABC. **2**

Question 3 (9 Marks) Commence a NEW page. **Marks**

(a) Solve the following equations:

i. $\cot x = -\sqrt{3}$ for $0° \leq x \leq 360°$ **2**

ii. $\sqrt{2}\sin 2x = 1$ for $-180° \leq x \leq 180°$ **3**

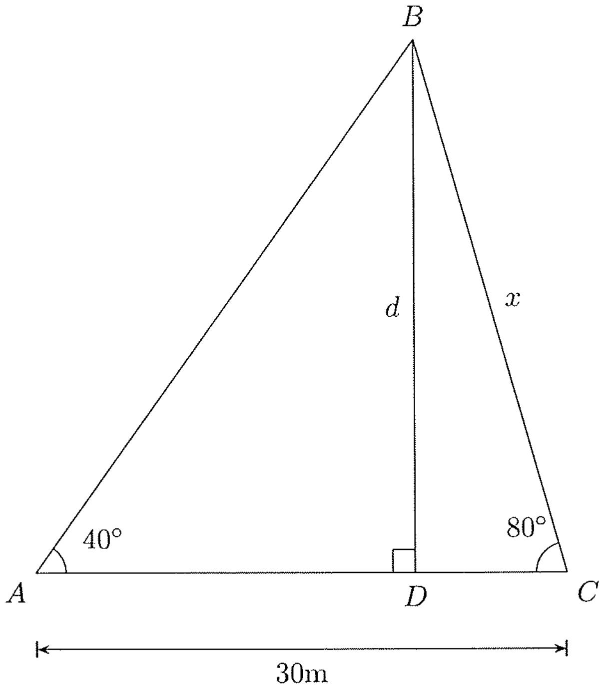
(b) Prove the following identity: **4**

$$\frac{\cos\theta}{1+\sin\theta} + \tan\theta = \sec\theta$$

Question 4 (14 Marks) Commence a NEW page. **Marks**

(a) Solve $2\cos^2\theta - 5\cos\theta = 2$, for $0 \leq \theta \leq 360°$. **4**

(b)



In the above triangle. $\angle ABC = 40°$ and $\angle ACB = 70°$. The base side, BC is 30m in length.

i. Find the length of x. **2**

ii. Hence, find its perpendicular height h. **1**

(c) i. Sketch the curve $y = 3\cos 2x$, for $0° \leq x \leq 360°$ **2**

ii. On the same diagram sketch the curve $y = 1 - \sin x$ **3**

iii. Hence find the number of solutions to the equation. **2**

$$1 - \sin x - 3\cos 2x = 0, \text{for } 0° \leq x \leq 360°$$

Explain your reasoning. (You are **not** required to solve the equation.)

End of paper.

Question 1

a) $\alpha = 46^\circ 3'$

b) i) $-\frac{\sqrt{3}}{2}$

ii) $\tan 60 = \sqrt{3}$

$\therefore \frac{2(\sqrt{3})}{1-(\sqrt{3})^2}$

$= \frac{2\sqrt{3}}{-2}$

$= -\sqrt{3}$

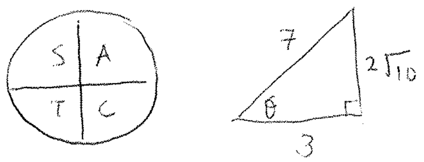
c) $\cos 55^\circ = \frac{10}{x}$

$x = \frac{10}{\cos 55^\circ}$

$\approx 17.4 \text{ cm}$

d) i) $\sec\theta = -\frac{7}{3}$

ii)



$\therefore \sin\theta = -\frac{2\sqrt{10}}{7}$

e) i) $x^2 = 2^2 + 4^2 - 2\times 2\times 4\cos 30^\circ$

$\doteqdot 6.14$

$x \doteqdot 2.48$ (to 2.d.p)

ii) $\text{Area} = \frac{1}{2}\times 2\times 4\times \sin 30^\circ$

$= 4\times\frac{1}{2}$

$= 2 \text{ units}^2$

Question 2

a) $m_{AB} = \dfrac{6-(-2)}{5-(-1)}$

$= \dfrac{4}{3}$ [1]

b) $\tan\theta = \dfrac{4}{3}$ $\therefore$ $\theta = 53°$ (to nearest degree) [1]

c) using the point gradient formula

$y - 6 = \frac{4}{3}(x-5)$ [1]

$3y - 18 = 4x - 20$ [1] for correct step to get into general form.

$4x - 3y - 2 = 0$

d) $a = 4$ $b = -3$ $c = -2$ $x_1 = 6$ $x_2 = 1$

$\therefore d = \dfrac{|4\times 6 + (-3)\times 1 + (-2)|}{\sqrt{4^2+(-3)^2}}$

$= \dfrac{|19|}{\sqrt{25}}$

$= \dfrac{19}{5}$ units.

e) $AB = \sqrt{(5-(-1))^2+(6-(-2))^2}$

$= \sqrt{36+64}$

$= \sqrt{100}$

$= 10$

$\therefore$ Area of triangle $ABC = \frac{1}{2}\times 10 \times \frac{19}{5}$

$= 19$ units^2

Question 1

a) $\sin\alpha = 0.72 \quad \therefore \quad \alpha = 46°3'$ [1]

b) i) $\cos 210° = \cos(180° + 30°)$

$= -\cos 30°$ [1]

$= -\frac{\sqrt{3}}{2}$ [1]

ii) $\tan 60° = \sqrt{3}$ [1]

$\therefore \frac{2\tan 60°}{1-\tan^2 60°} = \frac{2\times\sqrt{3}}{1-(\sqrt{3})^2}$

$= \frac{2\sqrt{3}}{-2}$

$= -\sqrt{3}$ [1]

c) $\cos 55° = \frac{10}{x}$ [1]

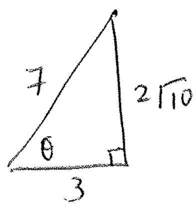
$\therefore x = \frac{10}{\cos 55°}$

$= 17.4\text{ cm}$ [1]

d) i) $\sec\theta = \frac{1}{\cos\theta}$ [1]

$= -\frac{7}{3}$

(ii)



$\therefore \sin\theta = -\frac{2\sqrt{10}}{7}$

[1] for correct triangle
[1] for $\sin\theta$ ratio
[1] for sign.

e) i) $x^2 = 2^2 + 4^2 - 2\times 2\times 4\times\cos 30°$ [1]

$= 20 - 16\cos 30°$

$\doteqdot 6.14$ (to 2 d.p.)

$\therefore x = \sqrt{6.14}$, as $x > 0$

$\doteqdot 2.48$ (to 2.d.p) [1]

ii) Area $= \frac{1}{2}\times 2\times 4\times\sin 30°$ [1]

$= 4\times\frac{1}{2}$

$= 2\text{ units}^2$ [1]

Question 3

a) (i) $\cot x = -\sqrt{3}$

$\tan x = -\frac{1}{\sqrt{3}}$. [1]

Since $\tan x < 0$, $90° < x < 180°$, $270° < x < 360°$

$x = 150°, 330°$ [1]

(ii) $\sin 2x = \frac{1}{\sqrt{2}}$ for $-180° \leq x \leq 180°$ [1] for change in domain

$\therefore -360° \leq 2x \leq 360°$

$2x = -315°, -225°, 45°, 135°$ [1]

$x = \frac{-315°}{2}, -\frac{225°}{2}, \frac{45°}{2}, \frac{135°}{2}$. [1]

b) RTP: $\frac{\cos\theta}{1+\sin\theta} + \tan\theta = \sec\theta$

$$LHS = \frac{\cos\theta}{1+\sin\theta} + \tan\theta$$

$$= \frac{\cos\theta}{1+\sin\theta} + \frac{\sin\theta}{\cos\theta}$$

$$= \frac{\cos^2\theta + \sin\theta + \sin^2\theta}{(1+\sin\theta)\cos\theta}$$

$$= \frac{1+\sin\theta}{(1+\sin\theta)\cos\theta}$$

$$= \frac{1}{\cos\theta}$$

$$= \sec\theta$$

$= RHS$.

Question 4

a) $2\cos^2\theta - 5\cos\theta - 2 = 0$ [1] for understanding it is quadratic in disguise.

$$\cos\theta = \frac{5 \pm \sqrt{41}}{4}$$

$$= \frac{5+\sqrt{41}}{4} \quad \text{or} \quad \frac{5-\sqrt{41}}{4} \quad [1]$$

$\therefore$ no solution for $\cos\theta = \frac{5+\sqrt{41}}{4}$ as $\cos\theta > 1$ [1]

for $\cos\theta = \frac{5-\sqrt{41}}{4}$

$\theta = 110°32'$ or $249°28'$ [1]

b) i) $\angle ABC = 60°$ (angle sum of triangle) [1]

$\therefore \frac{x}{\sin 40°} = \frac{30}{\sin 60°}$ (using the sine rule) [1]

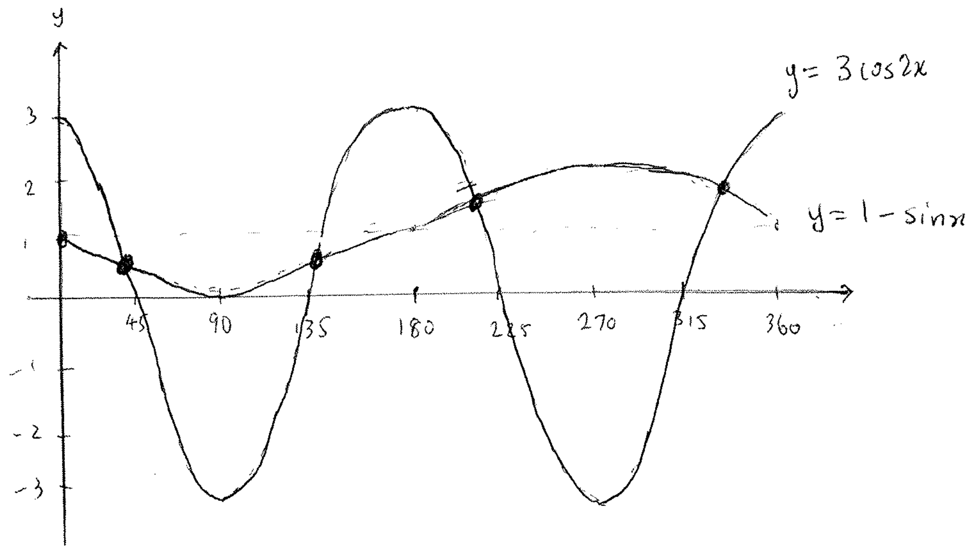
$\therefore x \doteqdot 22.3\text{m}$

ii) $\sin 80° = \frac{d}{22.3}$

$\therefore d = 22.3 \times \sin 80°$

$\doteqdot 21.9\text{m}$

c)



i) [1] for amplitude

[1] for period and correct shape

ii) [1] for flip of sin x

[1] for translation

[1] for correct sine shape.

$1 - \sin x - 3\cos 2x = 0$

$1 - \sin x = 3\cos 2x$

$\therefore$ the solutions are the x-coordinate of the intersections of the two curves [1] reasoning

$\therefore$ there are 4 intersection points

$\therefore$ 4 solutions [1]