NORTH SYDNEY BOYS
HIGH SCHOOL

MATHEMATICS (2U)

2017 HSC Assessment Task 1
Wednesday 7th December

General Instructions

- Working time – 55 minutes
(plus 5 minutes reading time).
- Use a blue or black pen.
Diagrams may be sketched in pencil.
- Board approved calculators may be used.
- All necessary working should be shown in every question.
- Attempt all questions.
- Commence each question on a new page.
- Write on both sides of the paper.
- Show all necessary working for every question. Marks may be deducted for illegible or incomplete working.

STUDENT NUMBER: …4………………………………… **# BOOKLETS USED:**

Class (please ✓)

☐	Mr Berry	☐	Ms Lee
☐	Mr Hwang	☐	Mr Lin
☐	Mr Ireland	☐	Ms Ziaziaris
☐	Dr Jomaa	☐	Mr Zuber

Question	**1** Series & Sequences	**2** Locus and the Parabola	**3** Quadratic Theory	**4** Challenge	**Total**	**%**
Marks	$\overline{}$ 14	9	13	3	39	100

Question 1 (14 marks) **Marks**

(a) Find the 47th term in the arithmetic sequence -4,-1,2,5… **2**

(b) Find the sum of the first 10 terms of the following geometric series: **2**

$$16 - 48 + 144 - 432 + \cdots$$

(c) The 8th term of an arithmetic series is 16 and the sum of the first 10 terms is 210. **4**

Find the sum of the first 15 terms.

(d) Determine the value of n if **3**

$$\sum_{p=1}^{n} 2p - 3 = 80$$

(e) The following is a geometric series:

$$1 + \frac{1}{1+\sqrt{2}} + \left(3 - 2\sqrt{2}\right) + \cdots$$

i) Explain why this geometric series has a limiting sum. **1**

ii) Find the limiting sum in the form $a + b\sqrt{c}$. **2**

Examination continues on the next page.

Question 2 (9 marks)

(a) For the equation $12x + y^2 + 4y = 44$ find:

i) The vertex. **2**

ii) The focus. **1**

iii) The equation of the directrix. **1**

(b) Given the points $A(4, 1)$ and $B(2, -3)$ and the point $P(x, y)$

i) Write an expression for the gradient PA. **1**

ii) Hence or otherwise find the equation of the locus such that $\angle APB$ is a right angle. **2**

iii) Describe the locus geometrically. **2**

Examination continues on the next page ➡

Question 3 (13 marks)

(a) Find the values of k for which equation $2x^2 - 5x - k = 0$ has real roots. **3**

(b) If α and β are the roots of the quadratic equation $x^2 - 3x - 5 = 0$, without solving the equation, find the values of :

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $(\alpha - 2)(\beta - 2)$ **1**

(iv) $\alpha^2 + \beta^2$ **1**

(v) $|\alpha - \beta|$ **1**

(c) Given the quadratic equation $x^2 - (2 + m)x + 3m = 0$, find the value of m if the product of the roots is four times the sum of the roots. **2**

(d) Solve for x: **3**

$$(x + 8)^4 = (2x + 16)^2$$

Question 4 (3 marks)

(a) Find the sum of the roots of the equation **3**

$$(x + 1)(x + 2) + (x + 3)(x + 4) + \cdots + (x + 99)(x + 100) = 0$$

End of examination.

Solutions NSBHS Mathematics (2U) 2017 Task 1

Q1 a) $T_1 = -4,\ T_2 = -1,\ T_3 = 2$

AP with $a = -4,\ d = 3$

$T_{47} = -4 + (47-1)\times 3$

$= 134.$

b) $S_n = 16 - 48 + 144 - 432 + \ldots$

is a GP with $a = 16,\ r = -3$

$\therefore S_{10} = \dfrac{a(r^{10}-1)}{r-1}$

$= \dfrac{16((-3)^{10}-1)}{(-3)-1}$

$= -236{,}192$

c) AP $T_8 = 16 = a + 7d \quad (1)$

$S_{10} = 210 = \frac{10}{2}(2a+9d) \quad (2)$

$$\begin{cases} a + 7d = 16 & (1) \\ 2a + 14d = 42 & (2) \end{cases}$$

$2a + 7d = 32 \quad (1)\times 2$

$2a + 14d = 42$

$5d = -10$

$d = -2$

$\therefore a = 16 - 7(-2)$

$= 30$

$\therefore S_{15} = \frac{15}{2}\left(2(30) + 14(-2)\right) = 240$

Q1 d. $\sum_{p=1}^{n} 2p-3=80$

This is an AP, $T_1=2(1)-3=-1$ $\xrightarrow{+2}$
$T_2=2(2)-3=1$ $\xrightarrow{+2}$
$T_3=2(3)-3=3$

$\therefore S_n=\frac{n}{2}(a+l)$

$=\frac{n}{2}((-1)+(2n-3))$

$\therefore \frac{n}{2}(2n-4)=80$

$n^2-2n-80=0$

$(n+8)(n-10)=0$

$n=10 \quad (n>10)$

e) GP: $1+\frac{1}{1+\sqrt{2}}+(3-2\sqrt{2})+\ldots$

(i) We are <u>told</u> this is a GP $\therefore r=\frac{\frac{1}{1+\sqrt{2}}}{1}=\frac{1}{1+\sqrt{2}}$

$1+\sqrt{2}>1$

$\therefore \frac{1}{1+\sqrt{2}}<1$ and $\frac{1}{1+\sqrt{2}}>0$

$\therefore |r|<1$ so we have a limiting sum.

<u>OR</u> $\frac{1}{1+\sqrt{2}}\approx 0.442 \Rightarrow |r|<1$

(ii) $S_\infty=\frac{a}{1-r}=\frac{1}{1-\frac{1}{1+\sqrt{2}}}=\frac{1}{\frac{1+\sqrt{2}-1}{1+\sqrt{2}}}=\frac{1+\sqrt{2}}{\sqrt{2}}=$

$=\frac{\sqrt{2}+2}{2}$

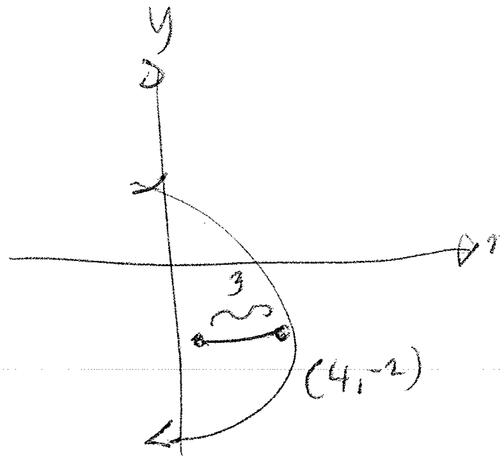
$=1+\frac{1}{2}\sqrt{2}$

Q2 (a) $12x + y^2 + 4y = 44$

$$y^2 + 4y = -12x + 44$$
$$(y+2)^2 - 4 = -12x + 44$$
$$(y+2)^2 = -12x + 48$$
$$(y+2)^2 = -12(x-4)$$

$\therefore$ Vertex is $(4,-2)$

$a = 3$

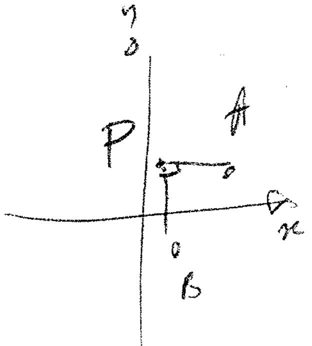


Focus is $(1,-2)$

Directrix $x = 7$

(b) Let $P(x,y)$ be on the locus.

$A(4,1)$
$B(2,-3)$



(i) $m_{PA} = \dfrac{y-1}{x-4}$

(ii) $m_{PB} = \dfrac{y+3}{x-2}$

Require $m_{PA} = -\dfrac{1}{m_{PB}}$

$$\therefore \frac{y-1}{x-4} = -\left(\frac{x-2}{y+3}\right)$$
$$(y-1)(y+3) = -(x-2)(x-4)$$
$$y^2 + 2y - 3 = -(x^2 - 6x + 8)$$
$$x^2 - 6x + 8 + y^2 + 2y - 3 = 0$$
$$(x-3)^2 - 9 + 8 + (y+1)^2 - 1 - 3 = 0$$
$$(x-3)^2 + (y+1)^2 = 5$$

Locus is a circle centre $(3,-1)$ radius $\sqrt{5}$

Q3a) $2x^2-5x-k=0$

has real roots when $\Delta \geq 0$

$$(-5)^2-4(2)(-k) \geq 0$$
$$25+8k \geq 0$$
$$k \geq -\frac{25}{8}$$

b) $x^2-3x-5=0$

i) $\alpha+\beta=3$

ii) $\alpha\beta=-5$

iii) $(\alpha-2)(\beta-2) = \alpha\beta-2(\alpha+\beta)+4$
$$= -5-2(3)+4$$
$$= -7$$

iv) $\alpha^2+\beta^2 = (\alpha+\beta)^2-2\alpha\beta$
$$= (3)^2-2(-5)$$
$$= 9+10$$
$$= 19$$

v) $|\alpha-\beta| = \sqrt{(\alpha-\beta)^2}$
$$= \sqrt{\alpha^2+\beta^2-2\alpha\beta}$$
$$= \sqrt{19-2(-5)}$$ from (iv)
$$= \sqrt{29}.$$

<u>OR</u> use $(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta$

<u>OR</u> $\alpha = \frac{-b+\sqrt{\Delta}}{2a}$

$\beta = \frac{-b-\sqrt{\Delta}}{2a}$ $\quad \therefore |\alpha-\beta| = \frac{2\sqrt{\Delta}}{2a} = \frac{\sqrt{\Delta}}{a}$

Q3 (c) $x^2-(2+m)x+3m=0$

$$\alpha\beta = 4(\alpha+\beta)$$
$$3m = 4(2+m)$$
$$3m = 8+4m$$
$$m=-8$$

(d) $(x+8)^4 = (2x+16)^2$

$$(x+8)^4 = (2(x+8))^2$$
$$(x+8)^4 = 4(x+8)^2$$

Let $u=(x+8)^2$

$$u^2 = 4u$$
$$u^2-4u=0$$
$$u(u-4)=0$$

$\therefore x+8=0$ or $(x+8)^2=4$

$x+8=\pm 2$

$x=-8$ $\quad\therefore x=-6$ or $x=-10$

$\therefore x = -10, -8, -6$

Q4. $(x+1)(x+2)+(x+3)(x+4)+\ldots(x+99)(x+100)=0$

$$x^2+3x+2+x^2+7x+12+\ldots+x^2+199x+9900=0$$
$$50x^2+\underbrace{(3+7+\ldots+199)}_{\text{AP: 50 terms}}x+K=0$$

b coefficient $= \frac{50}{2}(3+199) = 5050$

$$\therefore \alpha+\beta = -\frac{b}{a} = -\frac{5050}{50} = -101$$