



NORTH SYDNEY BOYS HIGH SCHOOL

2016 HSC ASSESSMENT TASK 4

Mathematics

General Instructions

- Working time – 50 minutes
(Plus 5 minutes reading time)
- Write on the lined paper in the booklet provided
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working should be shown in every question
- Each new question is to be started on a **new page**.
- Attempt all questions

Class Teacher:

(Please tick or highlight)

- ☐ Ms Lee
- ☐ Mr Berry
- ☐ Ms Ziazaris
- ☐ Mr Zuber
- ☐ Mr Hwang

Student Number: _____

(To be used by the exam markers only.)

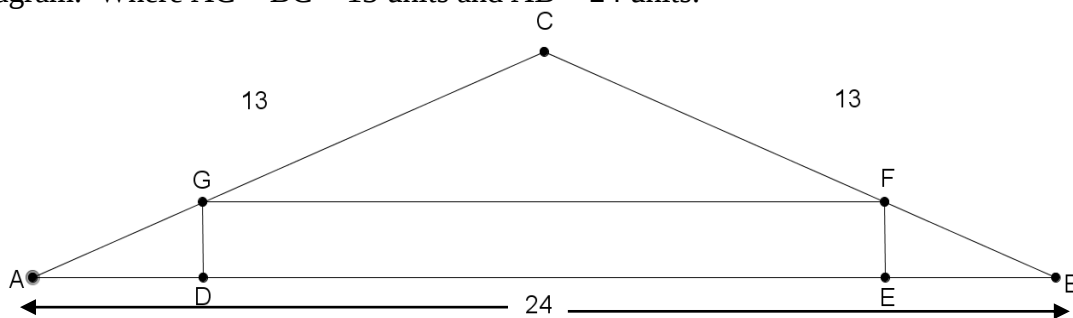
Question No	1	2	3	4	5	Total	Total %
Mark	$\frac{7}{7}$	$\frac{6}{6}$	$\frac{8}{8}$	$\frac{8}{8}$	$\frac{6}{6}$	$\frac{35}{35}$	$\frac{100}{100}$

Question 1 (7 marks) Start on a new page.

- (a) For the parabola $x = 3y^2 - 12y + 4$, find
- (i) The co-ordinates of its vertex. 2
 - (ii) The co-ordinates of the focus. 1
- (b) The vertex of a parabola is $V(2,5)$ and its directrix is $x = -3$
- (i) Find the focal length and the coordinates of the focus of the parabola. 2
 - (ii) Write down the equation of the parabola. 1
 - (iii) Sketch the parabola. 1

Question 2 (6 marks) Start on a new page.

A rectangle EDGF is constructed inside an isosceles triangle ABC as shown in the diagram. Where $AC = BC = 13$ units and $AB = 24$ units.



- (i) If $DE = x$ cm and $EF = y$ cm, show that $\frac{x}{24} = \frac{5-y}{5}$. 2
- (ii) Show that the area of the rectangle is $\frac{24}{5}(5y - y^2)$. 1
- (iii) Hence show that the maximum area of the rectangle is equal to half the area of the triangle. 3

Question 3 (8 marks) Start on a new page.

- (a) Given that the first three terms of a geometric series are 1, $(x + 3)$ and $(x + 3)^2$
- (i) Find the value of x for which the series converges. 2
 - (ii) Find the value of x when the limiting sum is x . 2

Question 3 continues

- (b) Books are stacked in a pile so that there are 14 books on the top row, 15 on the next row, 16 on the next and so on. If there are a total of 689 books
- (i) How many rows are there? 2
- (ii) How many books are on the bottom row? 2

Question 4 (8 marks) Start on a new page.

James deposits \$40 000 into an account which pays 2% interest per month on the balance of his account at the time. Immediately after each interest payment is made, James withdraws \$1000.

Using P to signify the initial deposit and A_n to represent the value of the investment after n withdrawals:

- (i) Write an expression for the value of his investment immediately after the first withdrawal. 1
- (ii) Show that after the third withdrawal, the balance of the account is
 $A_3 = P(1.02)^3 - 1000(1 + 1.02 + 1.02^2)$ 2
- (iii) Hence, find the expression for A_n 2
- (iv) For how many months can he withdraw \$1000? 3

Question 5 (6 marks) Start on a new page.

A couple deposits \$2000 into a university fund paying 6% p.a. compounded monthly at the birth of their child and at every birthday after . They increase the deposit by 5% each year as they earn more money each year.

- (i) What is the value of the second deposit? 1
- (ii) Write an expression for the value of fund right after the fourth deposit. 2
- (iii) They will withdraw the money on the child's 18th birthday (with no deposit on 18th birthday). What is the value of the fund at 18th birthday? 3

End of paper

2016 HSC ASSESSMENT TASK 4 – suggested solutions

Question 1

(a) $x - 4 = 3(y^2 - 4y)$
 $x - 4 + 12 = 3(y^2 - 4y + 4)$
 $x + 8 = 3(y - 2)^2$
 $\frac{1}{3}(x + 8) = (y - 2)^2$ ✓

(i) V: $(-8, 2)$ ✓

(ii) $4a = -\frac{1}{3}$

$a = \frac{1}{12}$ (concave right)

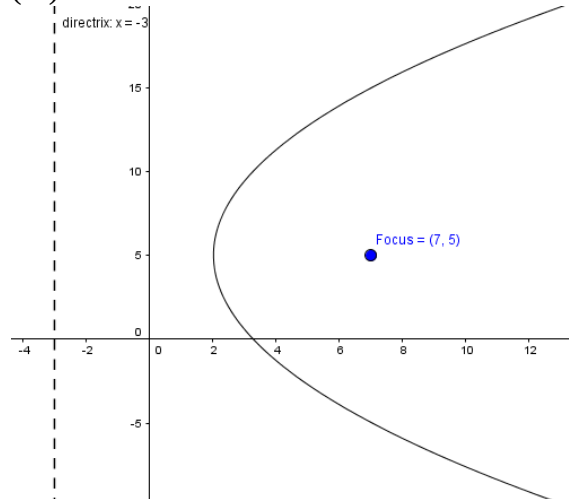
S: $(-7\frac{11}{12}, 2)$ ✓

(b) V: $(2, 5)$ and $x = -3$

(i) Focal length = 5 ✓ and focus $(7, 5)$ ✓

(ii) $(y - 5)^2 = 20(x - 2)$
 $y^2 - 10y + 25 = 20x - 40$ ✓

(iii)

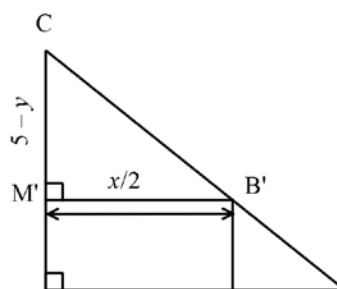
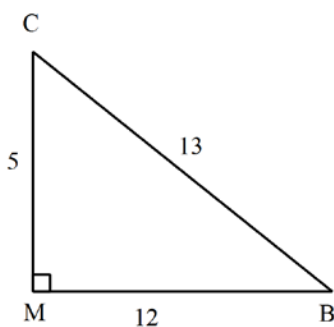


✓

Question 2

- (i) Construct a perpendicular line from C to point M, the midpoint of AB.
 Right triangle CMB has height 5 ($13^2 - 12^2 = 5^2$)

Two similar right triangles $\triangle CMB, \triangle CM'B'$ (sharing common angle C) have corresponding sides in proportion.



$$\frac{\frac{x}{2}}{12} = \frac{5-y}{5}$$

$$\frac{x}{24} = \frac{5-y}{5}$$

✓

2016 HSC ASSESSMENT TASK 4 – suggested solutions

(ii)	Area of the rectangle $= \frac{24}{5}(5 - y) \times y$ $= \frac{24}{5}(5y - y^2)$ ✓
(iii)	Area is a quadratic in y, $a < 0$, concave down, so maximum value is at vertex. $A = -\frac{24}{5y^2} + 24y$ Maximum when $y = -\frac{b}{2a} = \frac{24 \times 5}{2(24)} = \frac{5}{2}, A = 30 \text{ u}^2 \quad \checkmark$ Area of triangle = $24 \times 5 \times \frac{1}{2} = 60\text{u}^2 \checkmark$ $\frac{\text{Max area of rectangle}}{\text{area of the triangle}} = \frac{30}{60} = \frac{1}{2} \checkmark$

Question 3

- (a)(i) $a=1, r = (x + 3)$
 Series converges for $|r| < 1 \checkmark$
 $|x + 3| < 1$
 $-1 < x + 3 < 1,$
 $\therefore -4 < x < -2 \checkmark$

(ii) $S_{\infty} = x$
 $\therefore x = \frac{a}{1-r} = \frac{1}{1-(x+3)} = \frac{1}{-x-2}$

$$-x(x+2) = -x^2 - 2x = 1 \checkmark$$

$$x^2 + 2x + 1 = 0$$

$$(x+1)^2 = 0$$

$$\therefore x = -1 \checkmark$$

However this is outside of (i)

$\therefore$ no solution!

(b) $689 = \frac{n}{2}[28 + (n-1)] \checkmark$

(i) $1378 = 28n + n^2 - n$

$$n^2 + 27n - 1378 = 0$$

$$n = 26 \text{ rows, } n = -53 \checkmark$$

(ii) $T_{26} = 15 + (26-1)1 = 39 \checkmark \checkmark$

2016 HSC ASSESSMENT TASK 4 – suggested solutions

Question 4

(i) $A_1 = P(1.02) - 1000$ ✓

(ii) $A_2 = (P(1.02) - 1000)(1.02) - 1000$

$$A_3 = (A_2)(1.02) - 1000$$

$$A_3 = P(1.02)^3 - 1000(1.02)^2 - 1000(1.02) - 1000$$
 ✓

$$A_3 = P(1.02)^3 - 1000(1 + 1.02 + 1.02^2)$$
 ✓

(iii) $A_n = P(1.02)^n - 1000(1 + 1.02 + \dots + 1.02^{n-1})$

$$a = 1, \quad r = 1.02$$
 ✓

$$\therefore A_n = P(1.02)^n - 1000 \left(\frac{1.02^n - 1}{0.02} \right)$$
 ✓

(iv) $0 = 40\,000(1.02)^n - 1000 \left(\frac{1.02^n - 1}{0.02} \right)$ ✓

$$0 = 40\,000(1.02)^n - 50\,000(1.02^n - 1)$$

$$0 = (40\,000 - 50\,000)(1.02)^n + 50\,000$$

$$(10\,000)(1.02)^n = 50\,000$$

$$(1.02)^n = \frac{50\,000}{10\,000}$$

$$n \ln(1.02) = \ln(5)$$
 ✓

$$n = 81.27 \text{ months}$$

He can withdraw \$1000 for 81th months (and won't have enough on 82th month). ✓

Notes from the markers

Many students are not clearly showing how A_3 or A_n are derived. Recommended method for HSC questions of this style are to show the connections between each annual balance. At minimum, clearly indicate the origin of each term in the series expression, then show the elements of the GP and the summation.

Many students do not know how to efficiently rearrange the equation to solve when the reducing balance reduces zero. The best method is to immediately factor the $(1.02)^n$ term. The answer to the question should be expressed in a sentence to demonstrate clear understanding of the meaning of the result.

2016 HSC ASSESSMENT TASK 4 – suggested solutions

Question 5

(i) The second deposit is $2000(1.05)$ ✓

(ii) Interest is 6% p.a, compounded monthly.

$$r = 1 + \frac{6}{100} \times \frac{1}{12} = 1.005$$

Method 1: A_n is the amount at the beginning of the year, after the deposit paid
ie: Interest from the previous year has been paid, and the deposit made.

$$A_4 = 2000(1.005)^{36} + 2000(1.005)^{24}(1.05) + 2000(1.005)^{12}(1.05)^2 + 2000(1.05)^3$$

Method 2: A_n is the amount at the end of the year

Let A_n be the balance of the account at the end of the year.

$$A_4 = 2000(1.005)^{48} + 2000(1.005)^{36}(1.05) + 2000(1.005)^{24}(1.05)^2 + 2000(1.005)^{12}(1.05)^3$$

✓✓

1 mark for the compounding monthly interest, 1 mark for the deposit compounding.
Either answer accepted – the question was open to interpretation.

(iii) **Method 1: A_n is the amount at the beginning of the year, after the deposit paid**

$$A_{18} = 2000(1.005)^{12 \times 17} + 2000(1.005)^{12 \times 16}(1.05) + 2000(1.005)^{12 \times 15}(1.05)^2 + \dots + 2000(1.005)^{12}(1.05)^{17} + 2000(1.05)^{18}$$

$n = 18$ (18 payments have been made)

$$a = 2000(1.005)^{12 \times 17}$$

$$r = \frac{1.05}{(1.005)^{12}}$$

$$A_{18} = 2000 (1.005)^{12 \times 17} \times \left(\frac{\left(\frac{1.05}{(1.005)^{12}} \right)^{18} - 1}{\left(\frac{1.05}{(1.005)^{12}} \right) - 1} \right)$$

But there is still one more interest period, so the amount at the end of year is:

$$A_{18} \times (1.005)^{12} = \$96395.63 \text{ (to the nearest cents) } \checkmark \checkmark \checkmark$$

2016 HSC ASSESSMENT TASK 4 – suggested solutions

Method 2: A_n is the amount at the end of the year

$$A_{18} = 2000(1.005)^{12 \times 18} + 2000(1.005)^{12 \times 17}(1.05) \\ + 2000(1.005)^{12 \times 16}(1.05)^2 + \dots + 2000(1.005)^{24}(1.05)^{17} \\ + 2000(1.005)^{12}(1.05)^{18}$$

$n = 18$ (18 payments have been made)

$$a = 2000(1.005)^{12 \times 18}$$

$$r = \frac{1.05}{(1.005)^{12}}$$

$$A_{18} = 2000 (1.005)^{216} \times \left(\frac{\left(\frac{1.05}{1.005^{12}} \right)^{18} - 1}{\left(\frac{1.05}{1.005^{12}} \right) - 1} \right) = \$96395.63$$

Notes from the markers A truly wicked problem. Probably the hardest series money problem ever written. If you got part marks, be proud. It had it all: monthly compounding but annual deposits, an increasing deposit, and numeric values easily confused or incorrectly put into the calculator (1.05 and 1.005), with powers of powers. Part marks were awarded throughout for any working that showed some comprehension of the skills required. Students who did not see the monthly compounding made the problem much easier and hence were not given as many ‘error carried forward’ marks.

	Interest Rate	6%		
	Effective Annual Compounding	1.0617		
End Year	Prev Balance	With Interest	Deposit	Balance
0			\$ 2,000.00	\$ 2,000.00
1	\$ 2,000.00	\$ 2,123.36	\$ 2,100.00	\$ 4,223.36
2	\$ 4,223.36	\$ 4,483.84	\$ 2,205.00	\$ 6,688.84
3	\$ 6,688.84	\$ 7,101.40	\$ 2,315.25	\$ 9,416.65
4	\$ 9,416.65	\$ 9,997.44	\$ 2,431.01	\$ 12,428.46
5	\$ 12,428.46	\$ 13,195.02	\$ 2,552.56	\$ 15,747.58
6	\$ 15,747.58	\$ 16,718.86	\$ 2,680.19	\$ 19,399.05
7	\$ 19,399.05	\$ 20,595.54	\$ 2,814.20	\$ 23,409.74
8	\$ 23,409.74	\$ 24,853.60	\$ 2,954.91	\$ 27,808.51
9	\$ 27,808.51	\$ 29,523.68	\$ 3,102.66	\$ 32,626.34
10	\$ 32,626.34	\$ 34,638.66	\$ 3,257.79	\$ 37,896.45
11	\$ 37,896.45	\$ 40,233.82	\$ 3,420.68	\$ 43,654.49
12	\$ 43,654.49	\$ 46,347.01	\$ 3,591.71	\$ 49,938.72
13	\$ 49,938.72	\$ 53,018.83	\$ 3,771.30	\$ 56,790.13
14	\$ 56,790.13	\$ 60,292.82	\$ 3,959.86	\$ 64,252.68
15	\$ 64,252.68	\$ 68,215.65	\$ 4,157.86	\$ 72,373.51
16	\$ 72,373.51	\$ 76,837.35	\$ 4,365.75	\$ 81,203.09
17	\$ 81,203.09	\$ 86,211.52	\$ 4,584.04	\$ 90,795.56
18	\$ 90,795.56	\$96,395.63		