NORTH SYDNEY BOYS HIGH SCHOOL

MATHEMATICS

2017 HSC Course Assessment Task 1
December 6, 2017

General Instructions

- 60 mins working time
  plus 5 minutes reading time.
- Write using blue or black pen.
  Diagrams may be sketched in pencil.
- Board approved calculators may be used.
- Attempt all questions.

Section I - 50 marks

- Commence each new question on a new page.
- Show all necessary working.
- **Illegible or scrappy work will not be marked.** Rough working needs to be written as a proper solution to be marked.

STUDENT NUMBER: …… **# BOOKLETS USED:** …….

Class (please ✓)

☐ Mr Lin ☐ Mr Berry

☐ Mr Zuber ☐ Mr Hwang

☐ Ms Ziaziaris

Question	**1** Quadratic	**2** Locus	**3** Series	**4** Challenge	**Total**	**%**
Marks	13	10	12	7	47	

Section I – Short Answer

Question 1 (13 marks) Commence on a NEW page. *Marks*

a) Find the value of k in $x^2 + 6x + k = 0$ if one root is double the other. *2*

b) Solve $x^6 - 28x^3 + 27 = 0$. *3*

c) Let α and β be the solutions of $x^2 + 12x + 3 = 0$.

Find:

i) $\alpha + \beta$ *1*

ii) $\alpha\beta$ *1*

iii) $\frac{1}{\alpha} + \frac{1}{\beta}$ *1*

iv) $\alpha + \frac{3}{\alpha}$ *1*

d) For what values of q is the expression $3qx^2 - 5x + 3q$ negative definite? *4*

End of Question 1

Question 2 (10 marks) Commence on a NEW page. *Marks*

a) Write down the equation of the parabola with the x-axis as the axis of symmetry, and the vertex at the origin, and passing through the point $(-2,6)$. *2*

b) Find the vertex, focal length, focus and directrix of $x^2 - 6x + 4y + 29 = 0$. *4*

c) Describe in geometric terms the locus of a point P which moves so that its distance from $A(1,2)$ is twice its distance from point $B(4,5)$. *4*

Question 3 (12 marks) Commence on a NEW page.

a) Express the following series using sigma notation (**do not evaluate**):

$$1 \times 4 + 2 \times 5 + 3 \times 6 + \cdots \ \ldots + 10 \times 13.$$ *2*

b) For what value(s) of x does $(x-1), (x+3), (5x+3)$ form a geometric sequence? *2*

c) The fifth term of a geometric series is 2, and the eighth term is $\frac{2}{27}$.

i) Find the first term and the common ratio. *2*

ii) Find the limiting sum. *1*

d) A market gardener plants cabbages in rows. Owing to the wedge shape of his field, the first row has 43 cabbages, the second row has 47 cabbages and each succeeding row has four more cabbages than the previous row.

i) Which row is the first to contain more than 200 cabbages? *2*

ii) The farmer planted a total of 1065 cabbages. How many rows did he plant? *3*

End of Question 3. **Exam continues on the next page ⇒**

Question 4 (7 marks) Commence on a NEW page. *Marks*

a) Let α and β be the roots of the equation *3*

$$x^2 - px + q = 0$$

If the equation

$$y^2 - (\alpha + \beta + 1\ )y + (\alpha^2 + \beta^2) = 0$$

has equal roots, find the value of q in terms of p.

b) From 1 to 500 inclusive, find the sum of all the numbers which are divisible by 6 or 9. *4*

END OF THE EXAMINATION.

(1) a) $x^2 + 6x + k = 0$.

$\alpha + 2\alpha = -6$

$3\alpha = -6$

$\alpha = -2$. 1

Sub $x = -2$.

$4 - 12 + k = 0$

$k = 8$. 1

b) $x^6 - 28x^3 + 27 = 0$.

Let $a = x^3$

$a^2 - 28a + 27 = 0$

$(a-27)(a-1) = 0$ 1

$a = 27$, $a = 1$

$x^3 = 27$ $x^3 = 1$

$x = 3$ $x = 1$ 2

d) $x^2 + 12x + 3 = 0$

(i) $\alpha + \beta = -12$ 1

(ii) $\alpha\beta = 3$ 1

(iii) $\frac{\beta + \alpha}{\alpha\beta} = \frac{-12}{3} = -4$ 1

(iv) $\frac{\alpha^2 + 3}{\alpha}$ $-12\alpha = -12$. 1

e) $3q < 0$.

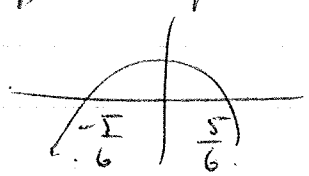
$kq < 0$

$b^2 - 4ac < 0$

$25 - 4(3q)(3q) < 0$

$25 - 36q^2 < 0$.

$(5-6q)(5+6q) < 0$



$q < -\frac{5}{6}$ or $q > \frac{5}{6}$.

$\therefore$ Soln $q < -5/6$ as $q < 0$.

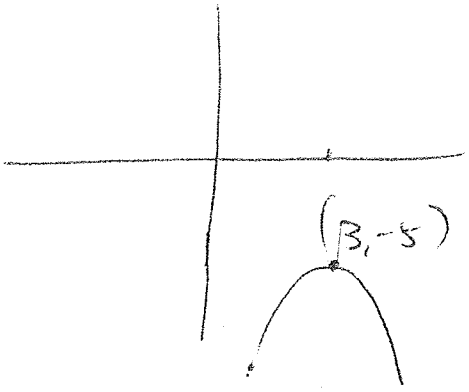
2b) $(x-3)^2 = -4y - 29 + 9$.

$(x-3)^2 = -4y - 20$

$(x-3)^2 = -4(y+5)$

$\boxed{V(3,-5)}$ $\quad 4a = 4$

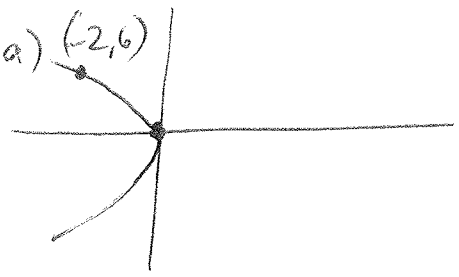
$\boxed{a = 1}$



$\therefore \boxed{S(3,-6)}$

$\boxed{Dir: y = -4}$

(2) a) $(-2,6)$



$y^2 = -4ax$.

$36 = 8a$

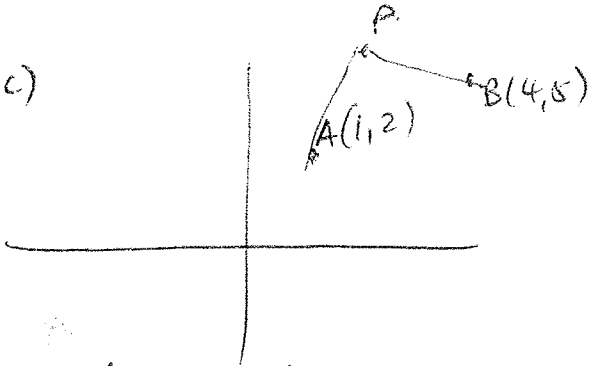
$a = \frac{9}{2}$.

$\therefore y^2 = -4\left(\frac{9}{2}\right)x$

$y^2 = -\frac{36}{2}x$.

2a: $\boxed{y^2 = -18x}$.

2c)



$PA = 2PB$

$PA^2 = 4PB^2$

$(x-1)^2 + (y-2)^2 = 4\left[(x-4)^2 + (y-5)^2\right]$ 1

$x^2 - 2x + 1 + y^2 - 4y + 4 = 4\left[x^2 - 8x + 16 + y^2 - 10y + 25\right]$

$x^2 - 2x + y^2 - 4y + 5 = 4x^2 + 4y^2 - 32x - 40y + 164$

$0 = 3x^2 + 3y^2 + 30x - 36y + 159$.

$0 = x^2 + y^2 + 10x - 12y + 53$

$0 = (x-5)^2 + (y-6)^2 + 53 - 36 - 25$

$(x-5)^2 + (y-6)^2 = 8$ 2

$\therefore$ locus is a circle centre $(5,6)$ with radius $2\sqrt{2}$ units 1

(3) a) $(1\times4)+(2\times5)+(3\times6)+\ldots\ldots+(10\times13)$

$$\sum_{k=1}^{10} k(k+3)$$

3 b) GP when $\frac{T_2}{T_1} = \frac{T_3}{T_2}$

$$\frac{x+3}{x-1} = \frac{5x+3}{x+3}$$

$$(x+3)^2 = (5x+3)(x-1) \quad 1$$

$$x^2+6x+9 = 5x^2-2x-3.$$

$$0 = 4x^2-8x-12$$

$$x^2-2x-3=0.$$

$$(x-3)(x+1)=0$$

$$\therefore x = 3, -1. \quad 1$$

$x=3$	$x=-1$
$2, 6, 18$	$-2, 2, -2.$

3 c) $T_5 = 2 \quad ar^4 = 2. \quad -①$

$T_8 = \frac{2}{27} \quad ar^7 = \frac{2}{27} \quad -②$

② ÷ ① $\quad r^3 = \frac{2}{27} \times \frac{1}{2}$

(i) $\boxed{r = \frac{1}{3}}$

$$a\left(\frac{1}{3}\right)^4 = 2$$

$$a = 2\times81$$

$$\boxed{a = 162}$$

3 (ii) $162, \frac{162}{3}, \frac{169}{9}, \ldots$

$$S_\infty = \frac{a}{1-r}$$
$$= \frac{162}{\frac{2}{3}}$$
$$= 3 \times 81$$
$$= 243.$$ 1

3d)(i) $43, 47, 51, \ldots$

$T_n = 200$

$200 = a + (n-1)d$

$200 = 43 + (n-1) \cdot 4$ 1

$200 = 43 + 4n - 4$

$200 - 39 = 4n$

$n = 40.25$

$\therefore$ Row 41. 1

(ii) $1065 = \frac{n}{2}[2a + (n-1)d]$

$2130 = n(86 + 4n - 4)$ 1

$= n(82 + 4n)$

$2130 = 82n + 4n^2$

$1065 = 41n + 2n^2$

$2n^2 + 41n - 1065 = 0.$

$n = \frac{-41 \pm \sqrt{41^2 - 4(2)(-1065)}}{4}$ 1

$n = \frac{-41 \pm 101}{4}$

$n = 15, -35.5 \quad n > 0$

$\therefore n = 15$ only soln 15 rows 1

4(b) $x^2 - px + q = 0$.

$y^2 - (\alpha+\beta+1)y + (\alpha^2+\beta^2) = 0$.

$\alpha+\beta = -p$ $\quad y^2 - (-p+1)y + (-p)^2 - 2q = 0$.

$\alpha\beta = q$ $\quad y^2 - (1-p)y + p^2 - 2q = 0$

$\Delta = 0$

$(1-p)^2 - 4(p^2-2q) = 0$

$1 - 2p + p^2 - 4p^2 + 8q = 0$.

$-3p^2 + 8q + 1 = 2p$

$\frac{-3p^2 - 2p + 1}{-8} = q$.

$\frac{3p^2 + 2p - 1}{8} = q$.

$1 + 6p - 3p^2 = 0$

$3p^2 - 6p - 1 = 0$

$p = \frac{6 \pm \sqrt{36 - 4(3)(-1)}}{6}$

$p = \frac{6 \pm \sqrt{48}}{6}$

$p = \frac{6 \pm 4\sqrt{3}}{6}$

$$\boxed{q = \frac{3p^2+2p-1}{8}}$$

Both: 18, 36, 54, ... 486

(4) (a) 6: 6, 12, 18, 24, 30, 36, 42, 48, 54 . . . 498.

9: 9, 18, 27, 36, 45, 54, ... 495

$S_{\text{DIV BY 6}} = \frac{n}{2}(a+l)$

$= \frac{83}{2}(6+498)$

$= 20916.$

$n = \frac{498-6}{6} + 1 = 83$

$S_{\text{DIV BY 9}} = \frac{n}{2}(a+l)$

$= \frac{55}{2}(9+495)$

$= 13860$

$n = \frac{495-9}{9} + 1 = 55$

$S_{\text{DIV BY BOTH}} = \frac{n}{2}(a+l) = \frac{27}{2}(18+486) = 6804$

TOTAL = 27972