NORTH SYDNEY GIRLS HIGH SCHOOL

2015 Assessment Task 1
Term 4

Mathematics

General Instructions

- Reading Time – 2 minutes
- Working Time – 55 minutes
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- In Questions 6−8, show relevant mathematical reasoning and/or calculations

Total Marks – 46

Section I Pages 2−3

5 marks

- Attempt Questions 1−5
- Allow about 7 minutes for this section

Section II Pages 4−7

41 marks

- Attempt Questions 6−8
- Allow about 48 minutes for this section

Student Name:______________________________

Student Number:____________________________

Teacher:

- ○ Mr T
- ○ Ms Lee
- ○ Mr Moon
- ○ Mrs Juhn
- ○ Ms Everingham
- ○ Ms Narayanan
- ○ Mrs Kennedy
- ○ Mrs Thill

QUESTION	H5	H6	TOTAL
1−5	/4	/1	/5
6	/9	/5	/14
7	/12	/2	/14
8	/6	/7	/13
TOTAL	/31	/15	/46

Section I

5 marks
Attempt Questions 1–5
Allow about 7 minutes for this section

Use the multiple-choice answer sheet for Questions 1–5.

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1 What is the primitive function of $5\sqrt{x^3}$?

(A) $\frac{15}{2}\sqrt{x}+C$

(B) $2\sqrt{x^5}+C$

(C) $\frac{25}{2}\sqrt{x^5}+C$

(D) $5\sqrt{x^5}+C$

2 Which of the following is the 100$^{\text{th}}$ term of the series $24+17+10+3+...$?

(A) -669

(B) -645

(C) 717

(D) 741

3 What is the sum of all integers between 10 and 100 that are divisible by 3?

(A) 1540

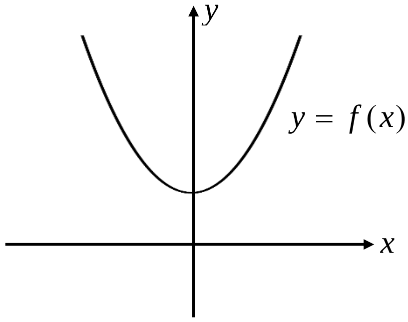
(B) 1554

(C) 1650

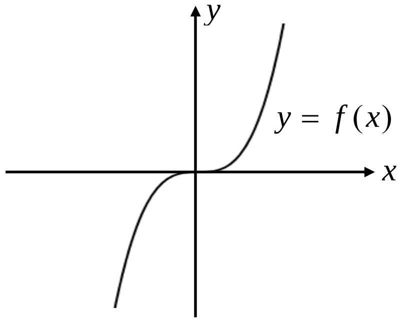
(D) 1665

4 In which of the following graphs is $f'(x) > 0$ for all x?

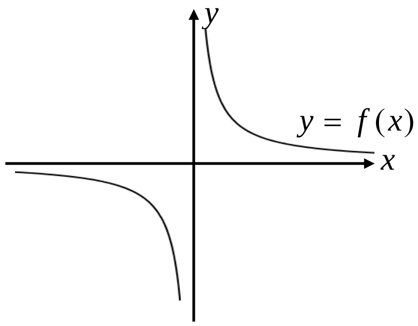
(A)



(B)



(C)



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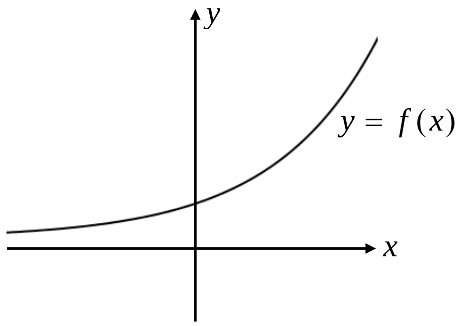


5 Which of the following statements must be true?

(A) All stationary points are turning points.

(B) If $f''(a) = 0$, there is a point of inflexion at $x = a$.

(C) If $f'(a) = 0$, there is a stationary point at $x = a$.

(D) If $f(x) > 0$, it is an increasing function.

Section II

41 marks
Attempt Questions 6–8
Allow about 48 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing papers are available.

In Questions 6–8, your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (14 marks) Use a SEPARATE writing booklet.

(a) Find a primitive function of $(1-3x)^7$. **2**

(b) The third term of an arithmetic series is 42 and the seventh term is 10.

(i) Find the first term and the common difference. **3**

(ii) Find the sum of the first twenty terms. **2**

(c) The sum of the first n terms of a certain series is given by $S_n = \dfrac{2n(n+1)}{3}$. **2**

Find an expression for the n^{th} term.

(d) For what values of x is the graph of the function $f(x) = 4x^3 - 5x^2 - 12x + 3$ concave up? **2**

(e) Consider the function $y = f(x)$, where $\dfrac{d^2y}{dx^2} = 2x + 1$. The curve has a stationary point **3**

at $(3, 2)$. Find y as a function of x.

Question 7 (14 marks) Use a SEPARATE writing booklet.

(a) Simone decides to train for an upcoming race. On the first day of training, she runs 100 metres. On each of the following days, she runs 10 metres more than on the previous day.

(i) How many days would it take for Simone to run a total of 14.3 km? **3**

(ii) How far does she run on the last day? **1**

(b) Consider the function $f(x) = \dfrac{x^3}{3} - 4x + 1$.

(i) Find the coordinates of the stationary points and determine their nature. **3**

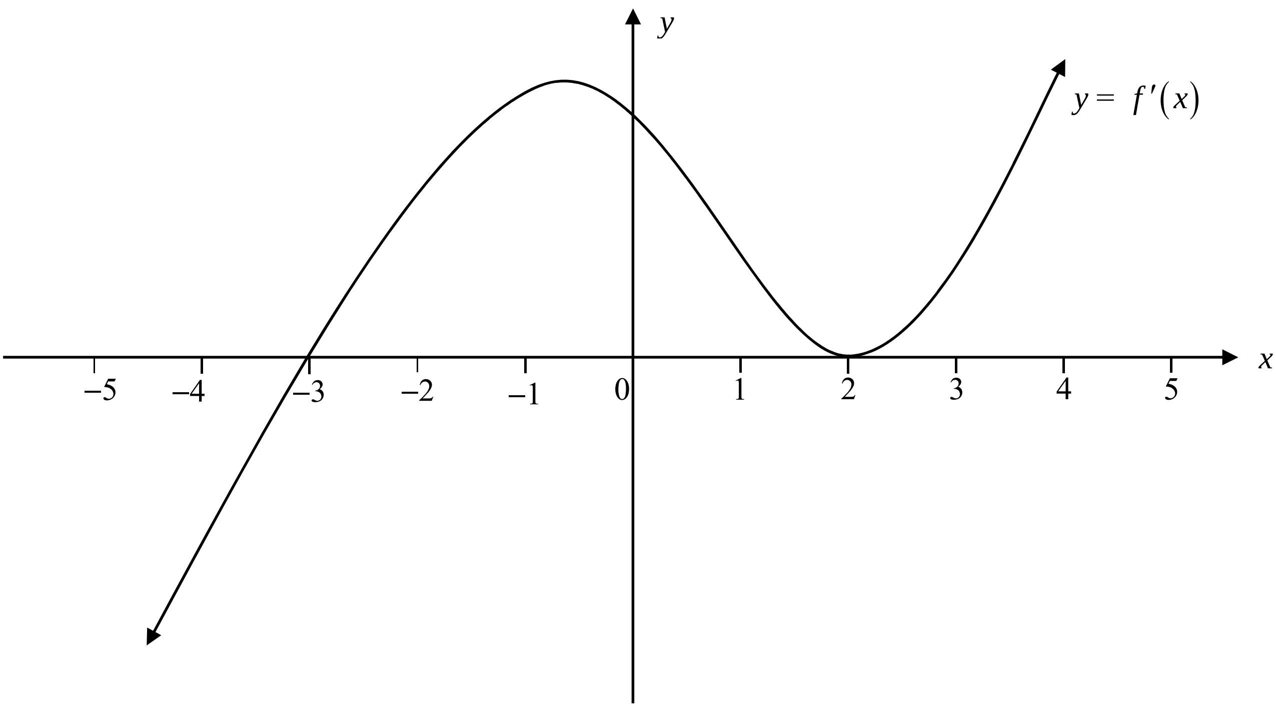
(ii) Find the coordinates of any point(s) of inflexion. **2**

(iii) Hence sketch the graph of $y = f(x)$ in the domain $-5 \le x \le 5$, showing the above information. You do not need to find the x-intercepts. **2**

(iv) What is the minimum value of $f(x)$ for $-5 \le x \le 5$? **1**

(c) The graph of the gradient function $y = f'(x)$ is shown below. **2**

Sketch a possible graph of $y = f(x)$.



Question 8 (13 marks) Use a SEPARATE writing booklet.

(a) Sketch the graph of a continuous curve $y = f(x)$ with the following properties. **3**

- $f'(0) = f'(3) = f''(3) = f''(1) = 0$
- $f''(x) > 0$ for $x < 1$ and $x > 3$
- $f''(x) < 0$ for $1 < x < 3$

(b) The number of unemployed people p at time t was studied over a period of time. At the start of the period, there were 750 000 unemployed people.

(i) Throughout this period, $\frac{dp}{dt} > 0$. What does this say about the number of unemployed people during this period? **1**

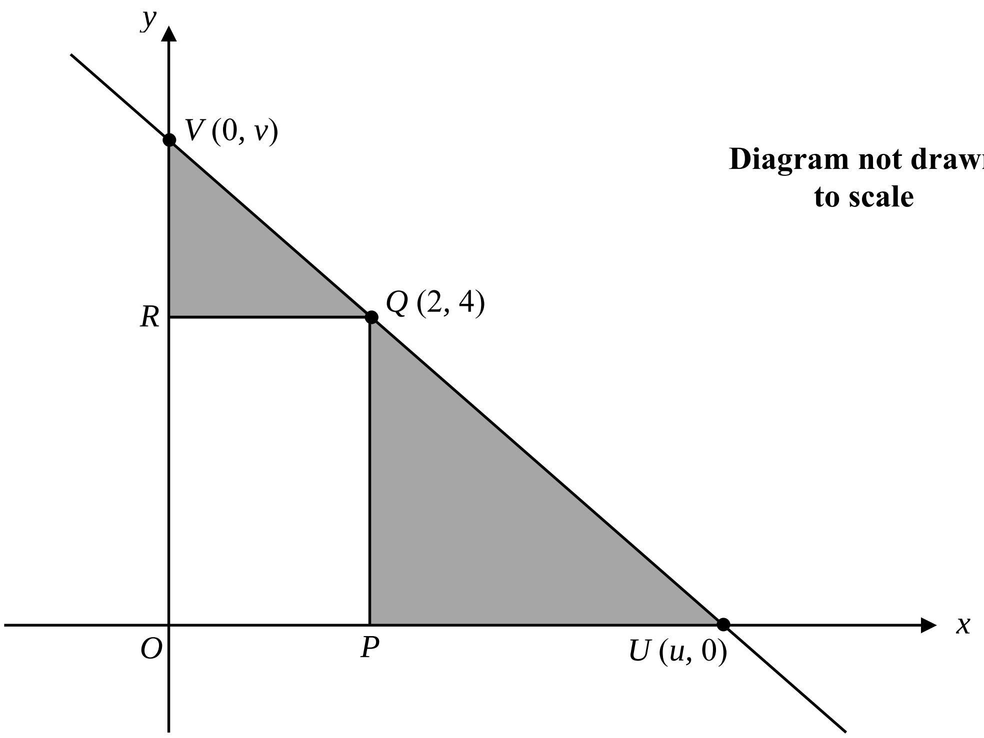
(ii) It is observed that, throughout this period, $\frac{d^2p}{dt^2} < 0$. What else does this say about the number of unemployed people during this period? **1**

(iii) Sketch a possible graph of p as a function of t. **1**

Question 8 continues on Page 7

Question 8 (continued)

(c) A line intersects the coordinate axes at U and V with coordinates $(u, 0)$ and $(0, v)$ respectively, where u and v are positive real numbers and $\frac{5}{2} \leq u \leq 6$. The rectangle $OPQR$ has a vertex at Q on the line. The coordinates of Q are $(2, 4)$, as shown.



(i) Explain, showing calculations, why $v = \dfrac{4u}{u-2}$. **2**

(ii) A is the total shaded area. Show that $A = \dfrac{2u^2}{u-2} - 8$. **1**

(iii) Find the minimum total shaded area and the value of u for which the area is a minimum. **4**

End of Paper

Section I

5 marks
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Allow about 7 minutes for this section

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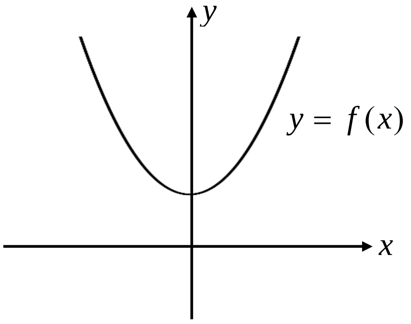

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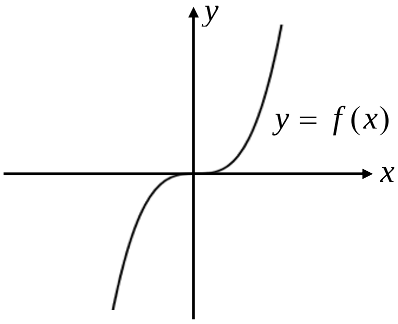

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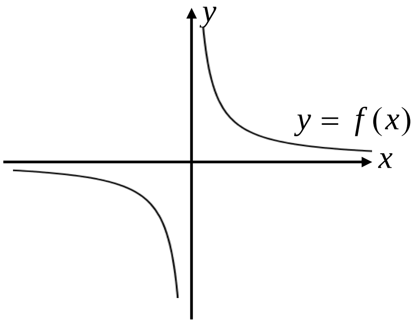
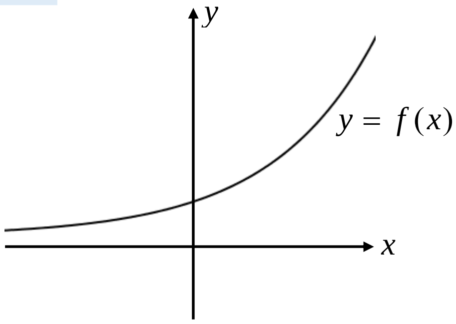


(D)



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(A) All stationary points are turning points.

(B) If $f''(a) = 0$, there is a point of inflexion at $x = a$.

(C) If $f'(a) = 0$, there is a stationary point at $x = a$.

(D) If $f(x) > 0$, it is an increasing function.

Section II

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Allow about 48 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing papers are available.

In Questions 6–8, your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (14 marks) Use a SEPARATE writing booklet.

(a) Find a primitive function of $(1-3x)^7$. **2**

Let $y' = (1-3x)^7$

$$y = \frac{(1-3x)^8}{8\times(-3)} + C$$

$$\therefore \frac{(1-3x)^8}{-24} + C$$

(b) The third term of an arithmetic series is 42 and the seventh term is 10.

(i) Find the first term and the common difference. **3**

(ii) Find the sum of the first twenty terms. **2**

$T_3 = 42, T_7 = 10$

(i) $T_3 = a + 2d$

$\therefore a + 2d = 42$ .... (1)

$T_7 = a + 6d$

$\therefore a + 6d = 10$ .... (2)

$(2)-(1):$

$4d = -32$

$d = -8$

$\therefore a = 42 - 2\times(-8)$

$\therefore a = 58$

$\therefore$ first term = 58

common difference = -8

(ii) $S_{20} = 10\left(2\times 58 - 8\times 19\right)$

$\therefore S_{20} = -360$

(c) The sum of the first n terms of a certain series is given by $S_n = \dfrac{2n(n+1)}{3}$. **2**

Find an expression for the n^{th} term.

$$S_n = \frac{2n(n+1)}{3}$$
$$T_n = S_n - S_{n-1}$$
$$= \frac{2n(n+1)}{3} - \frac{2(n-1)(n)}{3}$$
$$= \frac{2n^2 + 2x - 2n^2 + 2n}{3}$$
$$\therefore T_n = \frac{4n}{3}$$

(d) For what values of x is the graph of the function $f(x) = 4x^3 - 5x^2 - 12x + 3$ concave up? **2**

$$f(x) = 4x^3 - 5x^2 - 12x + 3$$

concave up when $f''(x) > 0$

$$f'(x) = 12x^2 - 10x - 12$$
$$f''(x) = 24x - 10$$
$$= 2(12x - 5)$$
$$\therefore 2(12x - 5) > 0$$
$$12x > 5$$
$$\therefore x > \frac{5}{12}$$

(e) Consider the function $y = f(x)$, where $\frac{d^2y}{dx^2} = 2x+1$. The curve has a stationary point at $(3,2)$. Find y as a function of x. **3**

Stationary point at $(3,2)$

$\therefore \frac{dy}{dx} = 0$ when $x = 3$,

$\frac{d^2y}{dx^2} = 2x+1$

$\frac{dy}{dx} = x^2 + x + C_1$

$\frac{dy}{dx} = 0$ when $x = 3$

$9 + 3 + C_1 = 0$

$C_1 = -12$

$\therefore \frac{dy}{dx} = x^2 + x - 12$

$y = \frac{x^3}{3} + \frac{x^2}{2} - 12x + C_2$

when $x = 3, y = 2$

$2 = 9 + \frac{9}{2} - 36 + C_2$

$C_2 = \frac{49}{2}$

$\therefore y = \frac{x^3}{3} + \frac{x^2}{2} - 12x + \frac{49}{2}$

Question 7 (14 marks) Use a SEPARATE writing booklet.

(a) Simone decides to train for an upcoming race. On the first day of training, she runs 100 metres. On each of the following days, she runs 10 metres more than on the previous day.

(i) How many days would it take for Simone to run a total of 14.4 km? **3**

(ii) How far does she run on the last day? **1**

$T_1 = 100, d = 10$

$T_2 = 110$

$T_3 = 120$

(i) 14.4 km = 14400 m

$S_n = 14400$

$\therefore 14400 = \frac{n}{2}\left(100 \times 2 + 10(n-1)\right)$

$28800 = n(200 + 10n - 10)$

$10n^2 + 190n - 28800 = 0$

$n^2 + 19n - 2880 = 0$

$(n+64)(n-45) = 0$

$\therefore n = 45 \text{ or } -64$

but $n > 0$

$\therefore n = 45$

(ii) $T_{44} = 100 + 44 \times 10$

$= 540$

$\therefore$ 540 m

(b) Consider the function $f(x) = \frac{x^3}{3} - 4x + 1$.

(i) Find the coordinates of the stationary points and determine their nature. **3**

(ii) Find the coordinates of any point(s) of inflexion. **2**

(iii) Hence sketch the graph of $y = f(x)$ in the domain $-5 \le x \le 5$, showing the above information. You do not need to find the x-intercepts. **2**

(iv) What is the minimum value of $f(x)$ for $-5 \le x \le 5$? **1**

$f(x) = \frac{x^3}{3} - 4x + 1$

(i) $f'(x) = x^2 - 4$

$f''(x) = 2x$

For stationary points, let $f'(x) = 0$,

$x^2 - 4 = 0$

$(x+2)(x-2) + 0$

$\therefore x = 2, -2$

For $x = 2$,

$f(2) = -\frac{13}{3}$

$f''(2) = 4$, which is >0

$\therefore$ there is a minimum turning point at $\left(2, -\frac{13}{3}\right)$

For $x = -2$,

$f(-2) = \frac{7}{3}$

$f''(-2) = -4$, which is < 0

$\therefore$ there is a maximum turning point at $\left(-2, \frac{19}{3}\right)$

(ii) $f''(x) = 2x$

To find points of inflexion, let $f''(x) = 0$

$2x = 0$

$\therefore x = 0,\ f(0) = 1$

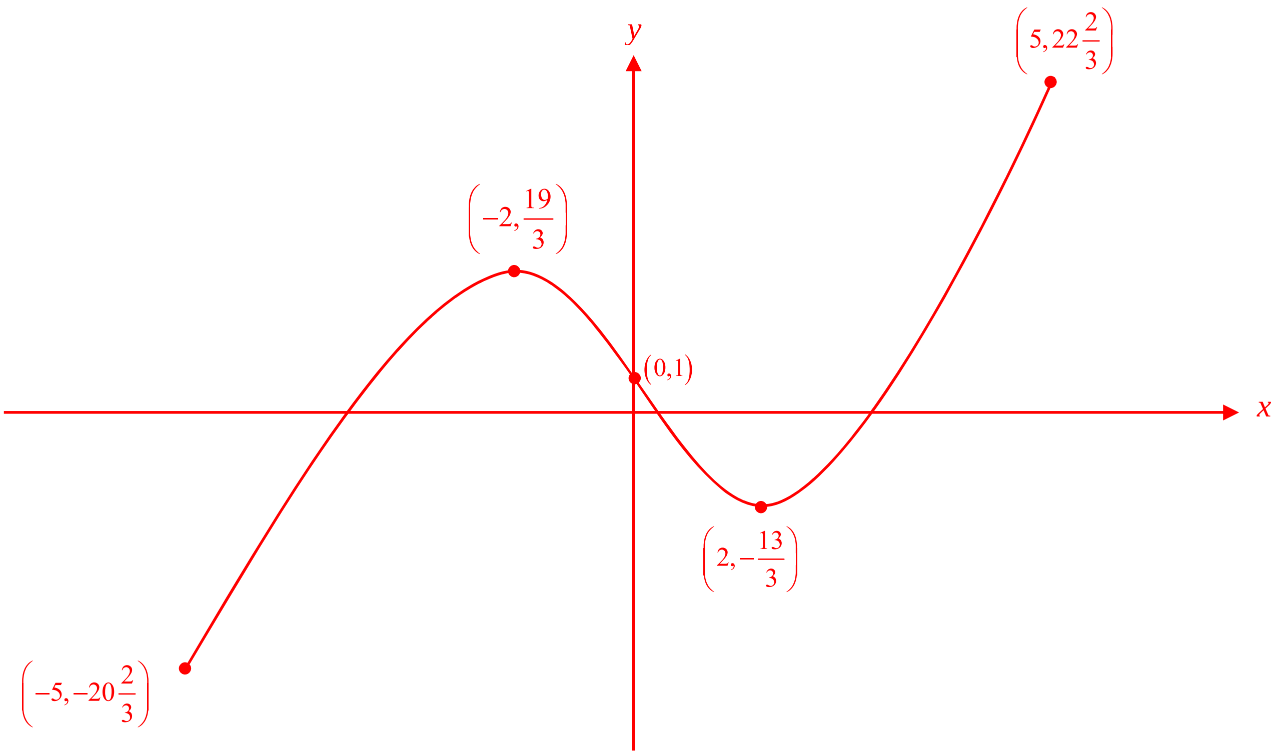
x	-1	0	1
$f''(x)$	-2	0	1

There is a change of concavity at $x = 0$

$\therefore$ point of inflexion at $(0,1)$

(iii) $f(-5) = -20\frac{2}{3}$

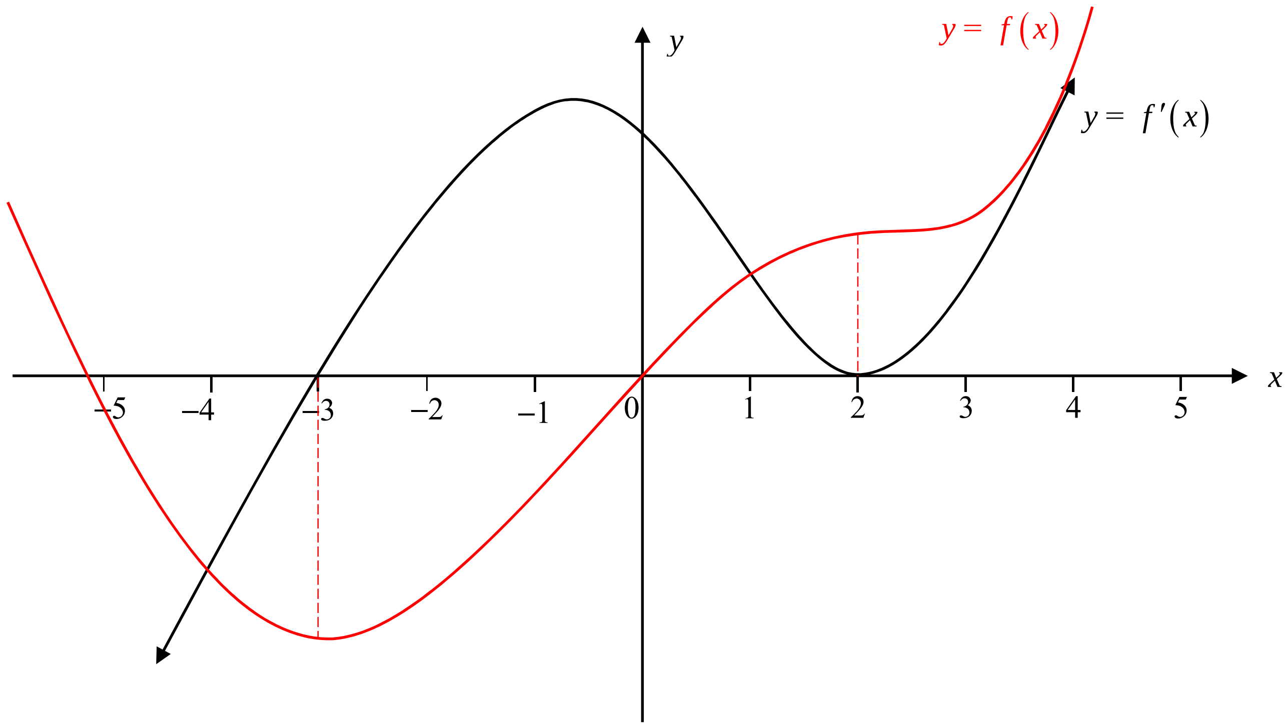
$f(5) = 22\frac{2}{3}$



(iv) $-20\frac{2}{3}$

(c) The graph of the gradient function $y = f'(x)$ is shown below. **2**

Sketch a possible graph of $y = f(x)$.

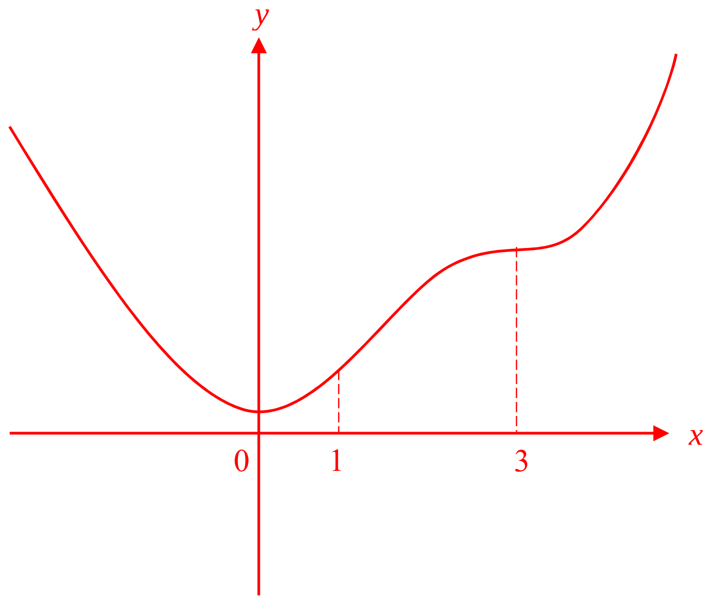


Question 8 (13 marks) Use a SEPARATE writing booklet.

(a) Sketch the graph of a continuous curve $y = f(x)$ with the following properties. **3**

- $f'(0) = f'(3) = f''(3) = f''(1) = 0$
- $f''(x) > 0$ for $x < 1$ and $x > 3$
- $f''(x) < 0$ for $1 < x < 3$

Minimum turning point at $x = 0$
Stationary point of inflexion at $x = 3$
Point of inflexion at $x = 1$



(b) The number of unemployed people p at time t was studied over a period of time. At the start of the period, there were 750 000 unemployed people.

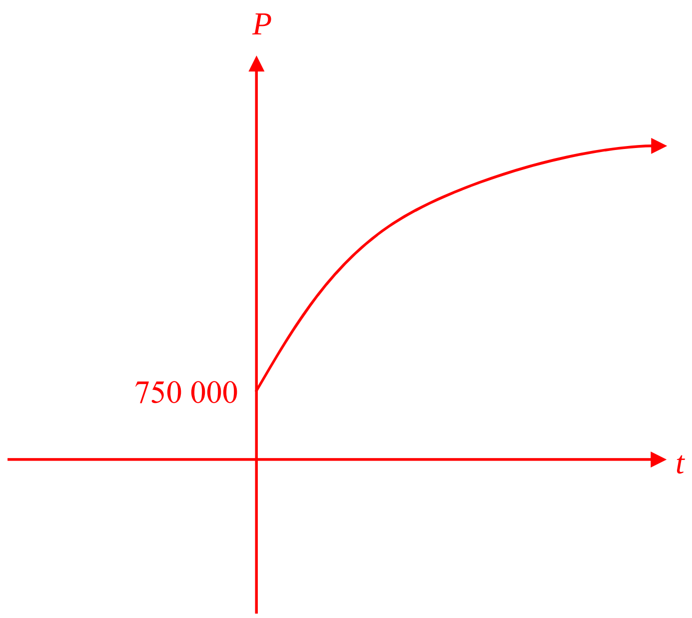
(i) Throughout this period, $\frac{dp}{dt} > 0$. What does this say about the number of unemployed people during this period? **1**

The number of unemployed increases during this period

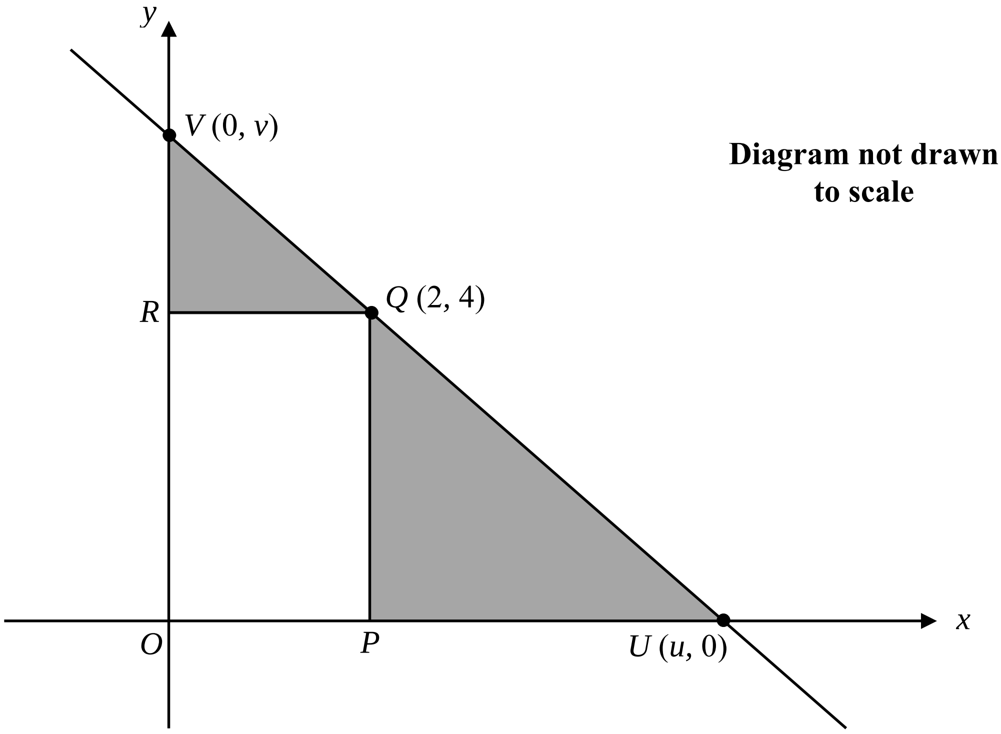
(ii) It is observed that, throughout this period, $\frac{d^2p}{dt^2} < 0$. What else does this say about the number of unemployed people during this period? **1**

The rate of change of unemployed decreases during this period

(iii) Sketch a possible graph of p as a function of t. **1**



(c) A line intersects the coordinate axes at U and V with coordinates $(u, 0)$ and $(0, v)$ respectively, where u and v are positive real numbers and $\frac{5}{2} \le u \le 6$. The rectangle $OPQR$ has a vertex at Q on the line. The coordinates of Q are $(2, 4)$, as shown.



(i) Explain, showing calculations, why $v = \dfrac{4u}{u-2}$. **2**

$m_{UV} = -\dfrac{v}{u}$

$m_{UV} = \dfrac{4}{2-u}$

$m_{QV} = m_{UQ}$

$-\dfrac{v}{u} = \dfrac{4}{2-u}$

multiplying both sides by -1

$\therefore v = \dfrac{4u}{u-2}$

(ii) A is the total shaded area. Show that $A = \dfrac{2u^2}{u-2} - 8$. **1**

$$A = \frac{1}{2} \times u \times v - 8$$
$$= \frac{uv}{2} - 8$$
$$= \frac{4u^2}{u-2} \times \frac{1}{2} - 8$$
$$\therefore A = \frac{2u^2}{u-2} - 8$$

(iii) Find the minimum total shaded area and the value of u for which the area is a minimum. **4**

$$\frac{dA}{du} = \frac{4u(u-2) - 2u^2}{(u-2)^2}$$
$$= \frac{4u^2 - 8u - 2u^2}{(u-2)^2}$$
$$= \frac{2u^2 - 8u}{(u-2)^2}$$
$$= \frac{2u(u-4)}{(u-2)^2}$$

$\dfrac{dA}{du} = 0$ when $u = 0, 4$

but $u \neq 0$ as $\dfrac{5}{2} \leq u \leq 6$

$\therefore u = 4$

$$A = \frac{2 \times 16}{4-2} - 8 = 8$$

u	3	4	5
$\dfrac{dA}{du}$	-6	0	$\dfrac{10}{9}$

$\therefore$ minimum value occurs when $u = 4$

$\therefore$ minimum area is $8u^2$ and this occurs when $v = 8$

Question 6

a) This was completed well but many students either forgot to divide by the derivative of (1-3x) or multiplied by it.

b) i) This was completed very well. Students are encouraged to use the nth term formula and solve simultaneously.

ii) Very well done, however there were a few students who used incorrect formula for the sum of the first n terms

c) This question was completed very poorly. The overwhelming majority of students did not use notice that the best way to complete this method was to use the formula: $S_n - S_{n-1} = T_n$.

Many students just assumed that the series was an arithmetic series and used the nth term formula. These students were quite lucky in this particular case as the series was arithmetic, however many student made a number of errors as this method was much more complicated.

d) This question was completed quite well. Many students included a long analysis of turning points and their concavity which was completely unnecessary.

e) Extremely well done. The most common errors were:

- Substituting in $x = 3$ and $\frac{dy}{dx} = 2$ into the derivative instead of $\frac{dy}{dx} = 0$
- Not evaluating the constant of integration in the final equation for y.

Question 7

(a) (i) Students MUST know how to fluently solve quadratic equations. If you don't, REVISE NOW!

Students MUST know Series formula and when to use each one.

(ii) If you didn't get an answer in part i. you should still make an educated guess at an answer and use this result in part ii. to get a follow through mark.

(b) (i) Most students did this well but need to ensure they clearly finish the question off by writing the stationary points (as an ordered pair) and their nature at the end.

(ii) Must use a test in the second derivative to conclude a point of inflexion. When testing, it is not good practice to use / - \ symbols because you are testing for a change in concavity, not whether a function is increasing or decreasing.

If you use a non-standard method (ie third derivative) your answer must be fully justified.

(iii) Read the question carefully. Many students did not use the specified domain.
The point of inflexion was NOT a horizontal point of inflexion.
You are advised to work in pen (even for sketches). In the HSC your solutions will be scanned for marking so you need to ensure your solution will be seen. You might like to use pencil and then when you have your final answer go over it in pen.

(iv) minimum value means min. value of function i.e. give the minimum y-value.

(c) Not well done. Students need to think about where does the primitive function have stationary points and what does the graph tell you about the type of stationary points.
Where is the primitive function increasing or decreasing?

Question 8

(a) Not well done. Many students introduced an extra turning point between 1 and 3, while others didn't seem to understand the difference between stationary points and inflexions. Others introduced extra inflexions. A few chose not to continue the graph to the left of $x = 0$.

Once you have drawn your curve, check it off step by step against the description. Provided you understand the geometrical significance of each derivative, there is no reason that you should not be able to tell whether your completed curve is correct.

(b) (i) Generally well done.

(ii) Many thought that there was a maximum value of p. The curve is increasing throughout this period. Saying "concave down" does not answer the question. Also, the RATE is NOT necessarily "decelerating" – it is merely decreasing.

(iii) To get the full mark, you needed to mark 750 000 on the vertical axis. A number of students thought this value belonged on the horizontal axis. The curve has to be increasing at a decreasing rate, with no stationary points.

(c) (i) There were numerous ways of getting this result.

If you chose to use similar triangles, your ratio statement had to show pairs of matching sides. (Many students claimed that this is what they had, but they in fact had ratios of sides in the same triangle.)

Other valid methods include:
(1) equating gradients of line segments
(2) finding the equation of UV using two points, then substituting the third point
(3) equating two expressions for the area of the rectangle.

(ii) The easiest method was to use SUBTRACTION of areas. The minus 8 should have been the clue.

Students who did this by addition of areas were much more likely to make algebraic errors.

(iii) Many students inexplicably chose to differentiate using the product rule instead of the quotient rule. Many more errors arose this way.

Others chose to combine the two terms in the area expression into one fraction. Again, this led to more errors in the differentiation.

Many students did not answer the question. You were asked for the MINIMUM AREA, not just the corresponding value of u .

Others did not test for maximum/minimum. This must be done every time.

Technically you should be checking the endpoints of the supplied domain. But as this did not cause a problem in this particular question, there were no marks allocated to it.