NORTH SYDNEY GIRLS HIGH SCHOOL

Preliminary Mathematics

Assessment Task 2

Term 2, 2017

Name: ________________________________ **Mathematics Class:**__________

Time Allowed: 55 minutes + 2 minutes reading time

Total Marks:

Section I (5 marks) - **Pages 2 - 3**
Attempt Questions 1- 5
Answer on Multiple Choice Answer Sheet

Section II (43 marks) **- Pages 4 - 7**
Attempt Questions 6 - 9
Show all necessary working

	MC		**Q6**		**Q7**		**Q8**		**Q9**		**Total**
Functions	**4, 5**	**/2**	**a-d**	**/10**	**a**	**/1**					**13**
Trigonometry	**2, 3**	**/2**			**b, c**	**/10**	**a, b**	**/4**	**a, b**	**/4**	**20**
Plane Geometry	**1**	**/1**					**c**	**/6**	**c**	**/6**	**13**
		/5		**/10**		**/11**		**/10**		**/10**	**/46**

Section I

5 Marks

Attempt Questions 1 – 5

Allow about 7 minutes.

Use the multiple choice answer sheet for questions 1 – 5.

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1. A polygon has an interior angle sum of $3960°$. How many sides does it have?

   (A) 11
   (B) 20
   (C) 22
   (D) 24

2. Simplify $\dfrac{\cos(90-\theta)}{\sin(90-\theta)}$.

   (A) 1
   (B) $\cot\theta$
   (C) $\tan\theta$
   (D) $-\tan\theta$

3. $\dfrac{2-\tan 60°}{\sec^2 45°}$ in exact form is

   (A) $2\sqrt{3}$
   (B) $1-\dfrac{\sqrt{3}}{2}$
   (C) $-\dfrac{1}{\sqrt{3}}$
   (D) $\dfrac{2-\sqrt{3}}{\sqrt{2}}$

4. What is the domain and range of the function $f(x)=\sqrt{1-x^2}$?

(A) Domain: $0 \leq x \leq 1$, Range: $-1 \leq y \leq 1$

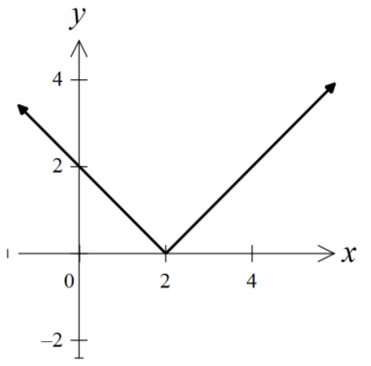
(B) Domain: $-1 \leq x \leq 1$, Range: $-1 \leq y \leq 1$

(C) Domain: $-1 \leq x \leq 1$, Range: $0 \leq y \leq 1$

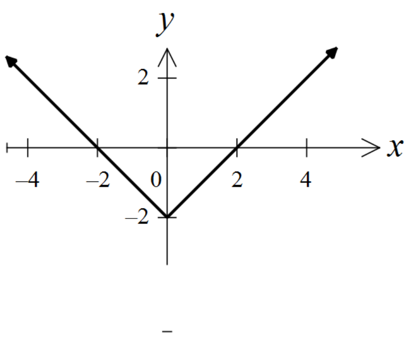
(D) Domain: $0 \leq x \leq 1$, Range: $0 \leq y \leq 1$

5. Which graph best represents $y=|x|-2$?

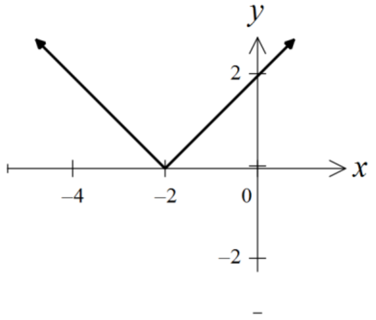
(A)



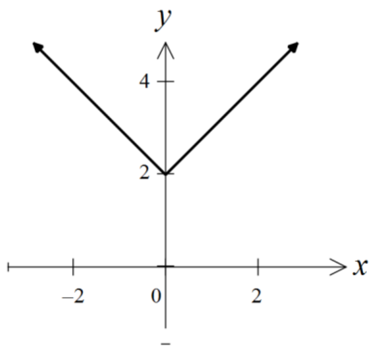
(B)



(C)



(D)



End of Section I

Section II 43 Marks

Attempt Questions 6 – 9

Allow about 48 minutes.

Answer each question in a SEPARATE writing booklet. Extra writing paper is available.

In questions 6 – 9, you responses should include relevant mathematical reasoning and/or calculations.

Question 6 (10 marks) Use a separate writing booklet

(a) Is the function $f\left(x\right)=8x^3-7x$ even, odd or neither? **2**

Show working to support your answer.

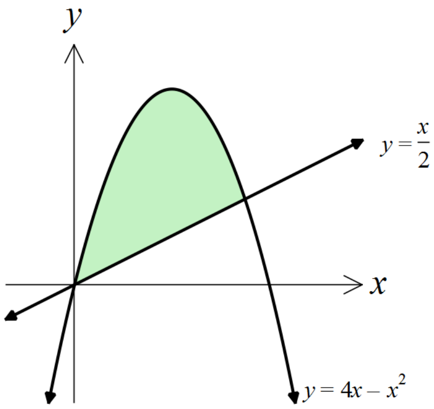
(b) Find the centre and radius of the circle with equation $x(x-2)+y(y+10)=23$. **2**

(c) On separate diagrams, sketch each of the following, showing all essential features.

(i) $y=\sqrt{x+7}$ **2**

(ii) $y=1-2^{-x}$ **2**

(d) The diagram shows the graphs of the functions $y=4x-x^2$ and $y=\dfrac{1}{2}x$.



Write down a pair of inequalities that specify the shaded region. **2**

End of Question 6

Question 7 (11 marks) Use a separate writing booklet

(a) The function $y = f(x)$ is defined as follows:

$$f(x) = \begin{cases} x-1 & \text{for } x \le -2 \\ -1 & \text{for } -2 < x \le 1 \\ x+1 & \text{for } x > 1 \end{cases}$$

Evaluate $f(-2) - f(17) + f(1)$. **1**

(b) Solve each equation for $0^\circ \le \theta \le 360^\circ$, correct to the nearest degree where necessary.

(i) $\cos^2\theta = \dfrac{3}{4}$ **2**

(ii) $\tan 2\theta = -5$ **2**

(c) A ship sails 50km from port *A* to port *B* on a bearing of 063° T and then sails 130km from port *B* to port *C* on a bearing of 296° T.

(i) Draw a diagram to represent this information. **1**

(ii) Explain why $\angle ABC = 53^\circ$ **1**

(iii) Find, correct to the nearest km, the distance from port *A* to port *C*. **2**

(iv) Find the bearing of port *A* from port *C*. Answer to the nearest degree. **2**

End of Question 7

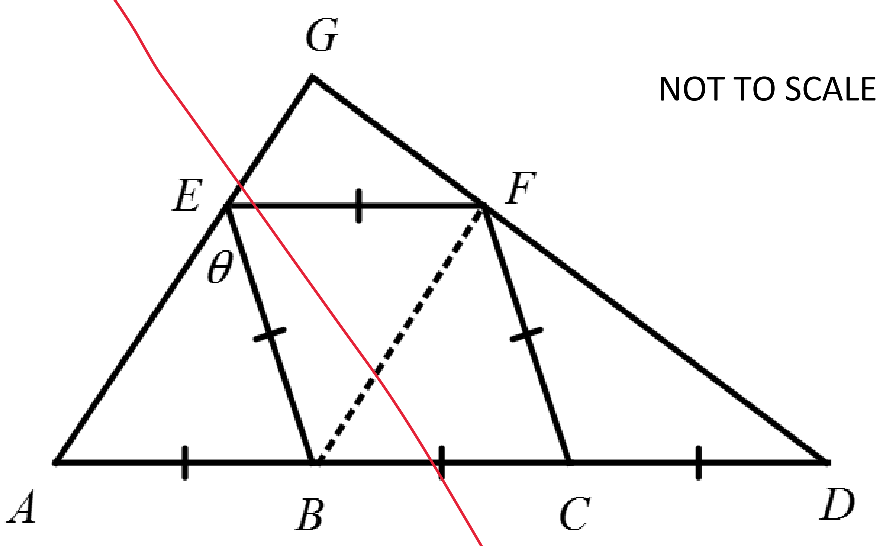
Question 8 (10 marks) Use a separate writing booklet

(a) Draw a neat sketch of $y = \operatorname{cosec} x$ for $0^\circ \le x \le 360^\circ$. **2**

(b) Prove that $\dfrac{\sin\beta}{1+\cos\beta} = \dfrac{1-\cos\beta}{\sin\beta}$. **2**

(c) $ABCD$ is a straight line such that $AB = BC = CD$.

$EFCB$ is a rhombus and $\angle AEB = \theta$.



(i) Show that $\angle EBC = 2\theta$. **2**

(ii) The diagonal BF is drawn in the rhombus $EFCB$. **2**

Show that $BF \parallel AE$.

(iii) Hence or otherwise, find the size of $\angle AGD$. **2**

Show all working.

End of Question 8

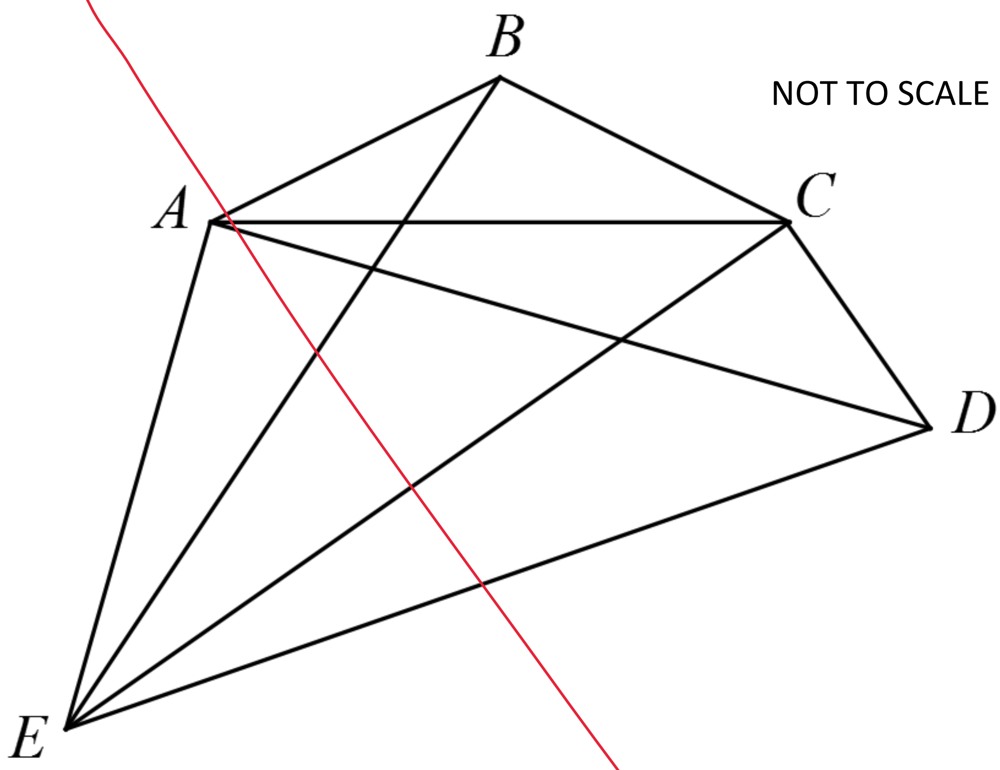
Question 9 (10 marks) Use a separate writing booklet

(a) If β is a reflex angle such that $\tan\beta = \dfrac{2}{3}$, find the exact value of $\cos\beta$. **1**

(b) Solve $6\sin^2 x + 7\cos x - 1 = 0$ for $0^\circ \le x \le 360^\circ$. **3**

Give answers to the nearest degree.

(c) In pentagon $ABCDE$, $AB = BC = CD$, $\angle AEC = \angle ACE$ and $\angle BAE + \angle BAC = \angle BCD$.



(i) Explain why $\angle BAC = \angle BCA$ **1**

(ii) Show that $\angle BAE = \angle ACD$. **2**

(iii) Hence, prove that $\Delta EAB \equiv \Delta ACD$ **3**

End of Examination

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Section I

5 Marks

Attempt Questions 1 – 5

Allow about 7 minutes.

Use the multiple choice answer sheet for questions 1 – 5.

---

1. A polygon has an interior angle sum of $3960°$. How many sides does it have?

(A) 11

(B) 20

(C) 22

(D) 24

$(n-2)180 = 3960$

$n-2 = 22$

$n = 24$

Answer: D

2. Simplify $\dfrac{\cos(90-\theta)}{\sin(90-\theta)}$.

(A) 1

(B) $\cot\theta$

(C) $\tan\theta$

(D) $-\tan\theta$

$= \cot(90-\theta)$

$= \tan\theta$

Answer: C

3. $\dfrac{2-\tan 60°}{\sec^2 45°}$ in exact form is

(A) $2\sqrt{3}$

(B) $1-\dfrac{\sqrt{3}}{2}$

(C) $-\dfrac{1}{\sqrt{3}}$

(D) $\dfrac{2-\sqrt{3}}{\sqrt{2}}$

$= \dfrac{2-\sqrt{3}}{(\sqrt{2})^2}$

$= \dfrac{2-\sqrt{3}}{2}$

Answer: B

4. What is the domain and range of the function $f(x)=\sqrt{1-x^2}$?

(A) Domain: $0 \le x \le 1$, Range: $-1 \le y \le 1$

(B) Domain: $-1 \le x \le 1$, Range: $-1 \le y \le 1$

(C) Domain: $-1 \le x \le 1$, Range: $0 \le y \le 1$

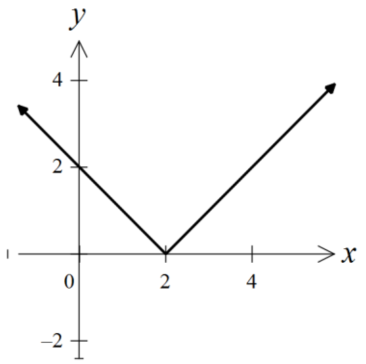
(D) Domain: $0 \le x \le 1$, Range: $0 \le y \le 1$

Answer: C

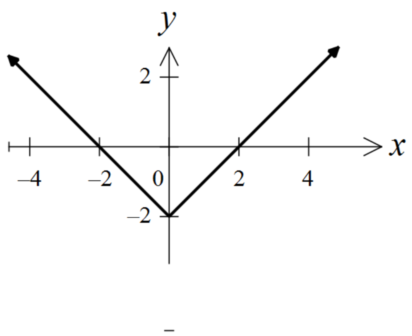
5. Which graph best represents $y=|x|-2$?

Answer: B

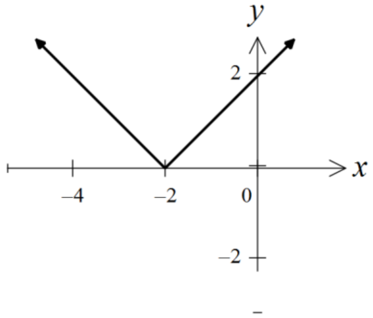
(A)



(B)



(C)



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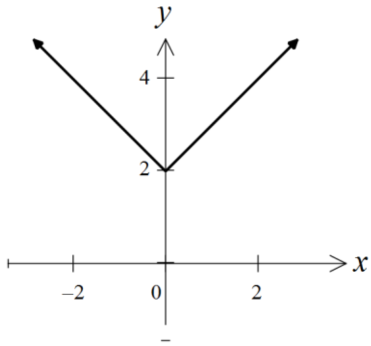


End of Section I

Section II 43 Marks

Attempt Questions 6 – 9

Allow about 48 minutes.

Answer each question in a SEPARATE writing booklet. Extra writing paper is available.

In questions 6 – 9, you responses should include relevant mathematical reasoning and/or calculations.

Question 6 (10 marks) Use a separate writing booklet

(a) Is the function $f\left(x\right)=8x^3-7x$ even, odd or neither? **2**

Show working to support your answer.

$f(-x)=8(-x)^3-7(-x)$

$= -8x^3+7x$ 1

$= -(8x^3-7x)$

$= -f(x)$

ODD function 1

(b) Find the centre and radius of the circle with equation $x(x-2)+y(y+10)=23$. **2**

$x^2-2x+y^2+10y=23$

$x^2-2x+1+y^2+10y+25=23+1+25$

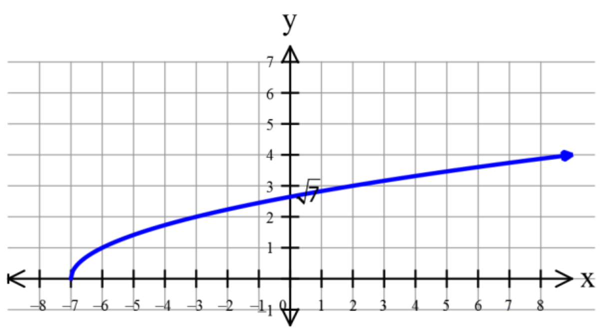
$(x-1)^2+(y+5)^2=49$

Centre (1, -7) 1

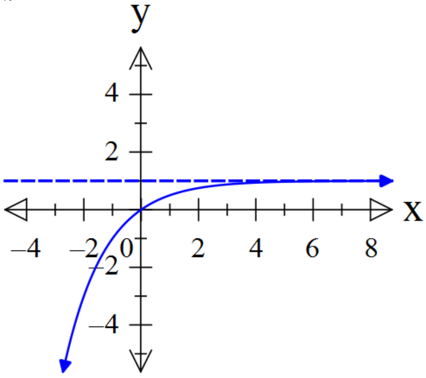
Radius r = 7 1

(c) On separate diagrams, sketch each of the following, showing all essential features.

(i) $y=\sqrt{x+7}$ **2**



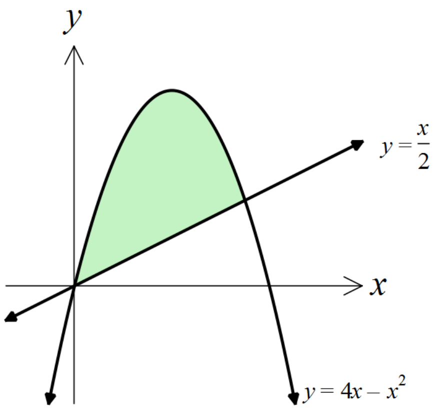
(ii) $y = 1 - 2^{-x}$ **2**



Students were penalised here (0.5 mk) if they didn't label a point (other than the origin)

Also it was necessary to draw the asymptote.

(d) The diagram shows the graphs of the functions $y = 4x - x^2$ and $y = \frac{1}{2}x$.



Write down a pair of inequalities that specify the shaded region. **2**

$y \le 4x - x^2$ 1

$y \ge \frac{1}{2}x$ 1

Students were penalised here (0.5 mk) if they used < or >.

End of Question 6

Question 7 (11 marks) Use a separate writing booklet

(a) The function $y = f(x)$ is defined as follows:

$$f(x) = \begin{cases} x-1 & \text{for } x \le -2 \\ -1 & \text{for } -2 < x \le 1 \\ x+1 & \text{for } x > 1 \end{cases}$$

Evaluate $f(-2) - f(17) + f(1)$. **1**

$f(-2) - f(17) + f(1) = (-2-1) - (17+1) + (-1)$

$= -3 - 18 - 1$

$= -22$ 1

(b) Solve each equation for $0° \le \theta \le 360°$, correct to the nearest degree where necessary.

(i) $\cos^2\theta = \dfrac{3}{4}$ **2**

$\cos^2\theta = \dfrac{3}{4}$

$\cos\theta = \pm\dfrac{\sqrt{3}}{2}$ 1

$\theta = 30°$

$\theta = 30°, 150°, 210°, 330°.$ 1

(ii) $\tan 2\theta = -5$ **2**

$\tan 2\theta = -5$

$ref\angle = 78°41'$ 1

tan negative in 2nd and 4th quadrants

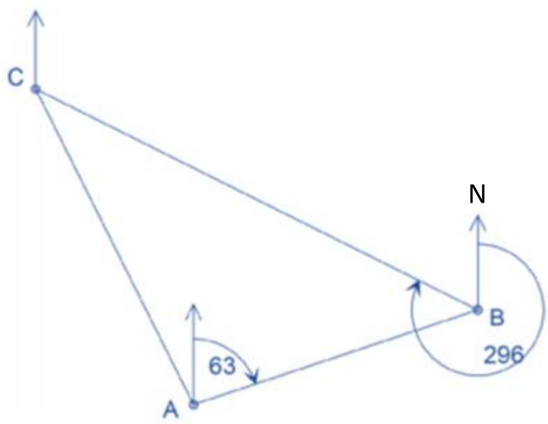
$2\theta = (180 - 78°41')$ or $2\theta = (360 - 78°41')$ or $2\theta = 360 + 101°19'$ or $2\theta = 360 + 281°19'$

$2\theta = 101°19'$ or $2\theta = 281°19'$ or $2\theta = 461°19'$ or $2\theta = 641°19'$

$\theta = 50°40'$ or $\theta = 140°40'$ or $\theta = 230°40'$ or $\theta = 320°40'$ 1

(c) A ship sails 50km from port A to port B on a bearing of $063°$T and then sails 130km from port B to port C on a bearing of $296°$T.

(i) Draw a diagram to represent this information. **1**



(ii) Explain why $\angle ABC = 53^\circ$

$\angle CBN + 296 = 360$ (angles at a point)

$\angle CBN = 64°$

$\angle ABC + \angle CBN + 63° = 180°$ (cointerior angles on parallel lines)

$\therefore \angle ABC = 180° - 64 - 63$

$\angle ABC = 53°$

(or other solution - eg alternate angles)

(iii) Find, correct to the nearest km, the distance from port A to port C. **2**

$b^2 = 130^2 + 50^2 - 2 \times 130 \times 50 \times \cos 53°$ 1

$b^2 = 11576.4$ ....

$b = 107.59$

$b = 108$ km 1

(iv) Find the bearing of port A from port C. Answer to the nearest degree. **2**

$\frac{\sin C}{50} = \frac{\sin 53}{107.59}$

$C = 22°$ 1

bearing = 116 + 22

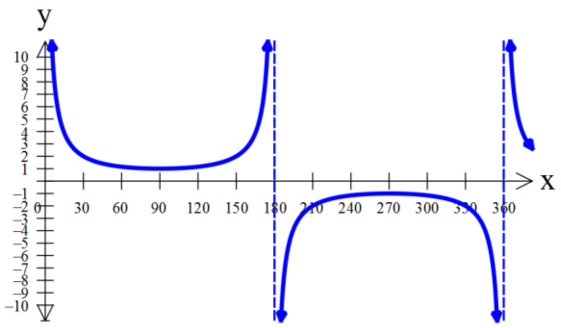
= 138° 1

Students who used the SINE rule are reminded to check the ambiguous case, which worked in this question.

End of Question 7

Question 8 (10 marks) Use a separate writing booklet

(a) Draw a neat sketch of $y = \operatorname{cosec} x$ for $0° \le x \le 360°$. **2**



(b) Prove that $\dfrac{\sin\beta}{1+\cos\beta} = \dfrac{1-\cos\beta}{\sin\beta}$. **2**

$$LHS = \frac{\sin\beta}{1+\cos\beta}$$

$$= \frac{\sin\beta}{1+\cos\beta}\times\frac{1-\cos\beta}{1-\cos\beta}$$

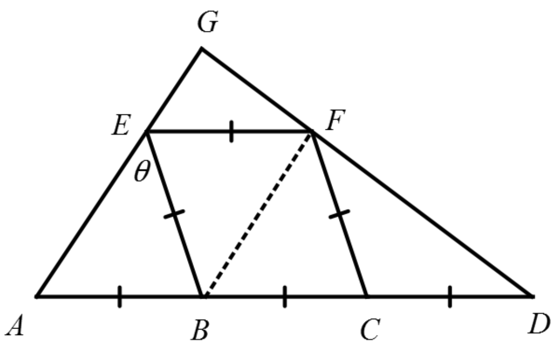
$$= \frac{\sin\beta(1-\cos\beta)}{1-\cos^2\beta}$$

$$= \frac{\sin\beta(1-\cos\beta)}{\sin^2\beta}$$

$$= \frac{1-\cos\beta}{\sin\beta}$$

$= \text{RHS}$

(c) $ABCD$ is a straight line such that $AB = BC = CD$. $EFCB$ is a rhombus and $\angle AEB = \theta$.



NOT TO SCALE

(i) Show that $\angle EBC = 2\theta$. **2**

$\angle EAB = \theta$ (equal angles opposite equal sides, EB=AB) 1

$\angle EBC = \angle BAE + \angle AEB$ (exterior angle of a triangle) 1

$= \theta + \theta$

$= 2\theta$

(ii) The diagonal BF is drawn in the rhombus $EFCB$. **2**

Show that $BF \parallel AE$.

$\angle EBF = \angle FBC$ (diagonals of a rhombus bisect the vertex angles)

$\angle EBF = \angle FBC = \theta$

$\angle AEB = \angle EBF$ (both θ)

AE is parallel to BF (alternate angles equal on parallel lines)

(iii) Hence or otherwise, find the size of $\angle AGD$. Show all working **2**

$\angle BFC = \angle FBC$ (equal angles opposite equal sides)

$\angle FCD = \angle BFC + \angle FBC$ (exterior angle of a triangle)

$\angle FCD = 2\theta$

OR $\angle FCD = \angle EBC$ (corresponding angles, FC || EB)

$\angle FCD = 2\theta$

$\angle FDC = \angle CFD$ (equal angles opposite equal sides)

$\angle FCD + \angle FDC + \angle CFD = 180$ (angle sum of triangle)

$\angle FDC = \dfrac{180 - 2\theta}{2}$

$= 90 - \theta$

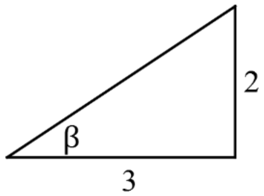
$\angle AGD + \theta + (90 - \theta) = 180$ (angle sum of triangle)

$\angle AGD = 90°$

End of Question 8

Question 9 (10 marks) Use a separate writing booklet

(a) If β is a reflex angle such that $\tan\beta = \frac{2}{3}$, find the exact value of $\cos\beta$. **1**



tan is +ive so B is in 3rd quadrant (cos –ive)

$$\cos\beta = -\frac{3}{\sqrt{13}}$$

(b) Solve $6\sin^2 x + 7\cos x - 1 = 0$ for $0° \le x \le 360°$. **3**

Give answers to the nearest degree.

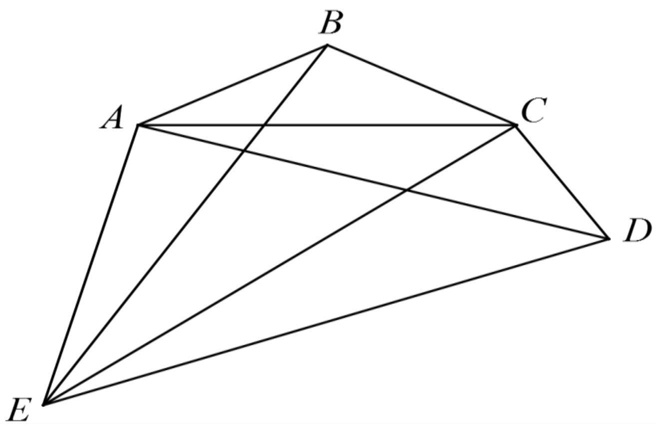
$$6(1-\cos^2\theta) + 7\cos\theta - 1 = 0$$
$$-6\cos^2\theta + 7\cos\theta + 5 = 0$$
$$6\cos^2\theta - 7\cos\theta - 5 = 0$$
$$(3\cos\theta - 5)(2\cos\theta + 1) = 0$$
$$\cos\theta = \frac{5}{3} \quad \text{or} \quad \cos\theta = -\frac{1}{2}$$

no solution

cos negative in 2nd / 3rd quadrants

$$\theta = 180 - 60°, \quad 180 + 60°$$
$$\therefore \theta = 120°, 240°$$

(c) In pentagon $ABCDE$, $AB = BC = CD$, $\angle AEC = \angle ACE$ and $\angle BAE + \angle BAC = \angle BCD$.



NOT TO SCALE

(i) Explain why $\angle BAC = \angle BCA$ **1**

$\angle BAC = \angle BCA$ (equal angles opposite equal sides, BA = BC)

(ii) Show that $\angle BAE = \angle ACD$. **2**

$\angle BAE + \angle BAC = \angle BCD$ given

$\angle BAE = \angle BCD - \angle BAC$

$\angle BAE = \angle BCD - \angle BCA$ (from i)

$\angle BAE = \angle ACD$ ($\angle BCA$ & $\angle ACD$ are adjacent angles)

(iii) Hence, prove that $\Delta EAB \equiv \Delta ACD$ **3**

In ΔEAB & ΔACD

$AB = CD$ (given)

$\angle BAE = \angle ACD$ (from above (ii))

$AE = CA$ (equal sides opposite equal angles, given $\angle ACE = \angle AEC$)

$\therefore\ \Delta EAB \equiv \Delta ACD$ (SAS)

End of Examination