NORTH SYDNEY GIRLS HIGH SCHOOL

2018

Preliminary Mathematics Assessment Task 2

Preliminary Mathematics

General Instructions

- Reading Time – 2 minutes
- Working Time – 55 minutes
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided
- In Questions 6 – 9, show relevant mathematical reasoning and/or calculations

Total marks – 47

Section I – 5 marks (pages 2 – 3)

- Attempt Questions 1 – 5
- Allow about 7 minutes for this section

Section II – 42 marks (pages 5 – 7)

- Attempt Questions 6 – 9
- Allow about 48 minutes for this section

NAME:_____ **TEACHER:**_____

STUDENT NUMBER:_____

Question	1 – 5	6	7	8	9	Total
Functions	/3	/7	/2	/3	/4	**/19**
Trigonometry	/1	/6	/4	/2	/6	**/19**
Plane Geometry	/1		/4	/4		**/9**
	/5	**/13**	**/10**	**/9**	**/10**	**/47**

Section I

5 marks
Attempt Questions 1-5
Allow about 7 minutes for this section

Use the multiple choice answer sheet for Questions 1-5.

---

1 What is the equation of a circle with centre $(-4,8)$ and radius of 7 ?

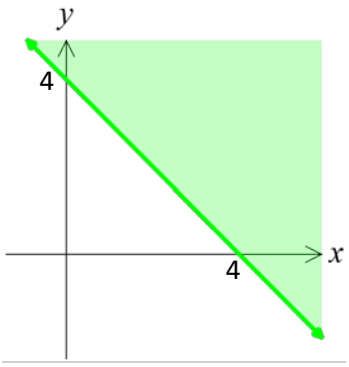
A. $(x-4)^2+(y+8)^2=49$

B. $(x+4)^2+(y+8)^2=49$

C. $(x+4)^2+(y-8)^2=49$

D. $(x+4)^2+(y+8)^2=49$

2 Consider the region below.



Which inequality represents the region?

A. $y \le x+4$

B. $x-y+4 \le 0$

C. $y \ge -x-4$

D. $y \ge 4-x$

3 Given that $g(x) = 2x^4 - x^2 - 4$, what is the value of $g(-3)$?

A. -3

B. 149

C. -149

D. 3

4 What does $(\cot x + \text{cosec} x)(\cot x - \text{cosec} x)$ simplify to?

A. 1

B. $\sin^2 x$

C. -1

D. $\cos^2 x$

5 Which of the following gives the number of sides of a regular polygon whose interior angle are $(a+2)°$?

A. $\dfrac{360}{a-178}$

B. $\dfrac{-360}{a-178}$

C. $\dfrac{180}{a-2}$

D. $180(a-2)$

Section II

42 marks
Attempt Questions 6-9
Allow about 48 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In questions 6-9, your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (13 marks**)** Use a SEPARATE writing booklet

(a) Sketch $f(x) = |x-2|-1$, showing all essential features. **2**

(b) State the domain and range of $y = \sqrt{9-x^2}$. **2**

(c) Given that $f(x) = x^3 - 4x$:

(i) Determine if $f(x)$ is odd, even or neither. **2**

(ii) If $f(a) = 10$, evaluate $f(a) - f(-a)$. **1**

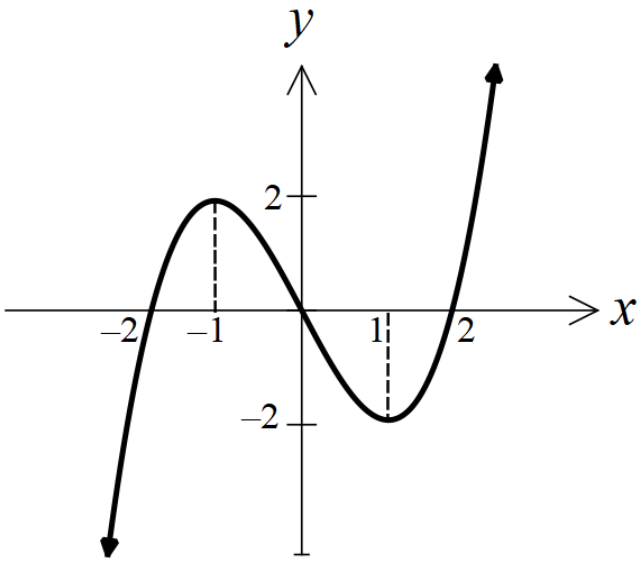
(d) From the top of a light house situated 100 metres above sea level, the angles of depression of boat *A* and boat *B* are $27°$ and $52°$ respectively. Calculate the distance between the boats to the nearest metre. **2**

(e) Find the exact value of $\cos 420° + \tan(-30°)$ leaving your answer as a simplified fraction with a rational denominator. **2**

(f) Solve $\tan^2 \theta = 3$ for $0° \le \theta \le 360°$. **2**

Question 7 (10 marks) Use a SEPARATE writing booklet

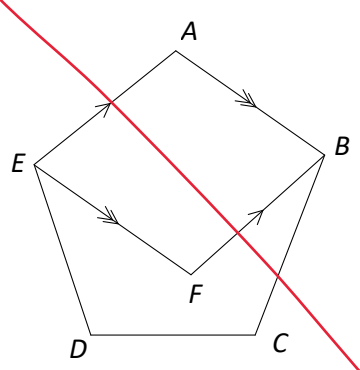
(a) Using $y = f(x)$ shown in the diagram below, sketch $y = -f(x-1)$. **2**



(b) If $\tan A = \frac{13}{5}$ and, $\cos A < 0$ find the exact value of $\sin A$. **2**

(c) Solve $\sin(\theta + 90°) = \frac{\sqrt{3}}{2}$ for $0° \le \theta \le 360°$. **2**

(d) The diagram below shows a regular pentagon *ABCDE*. *EF* is drawn so that $EF \| AB$ and $BF \| AE$.



(i) Show that $\angle EAB = 108°$. **1**

(ii) What kind of quadrilateral is $ABFE$? Justify your answer. **1**

(iii) Show that $\angle ABE = \angle EBF = \angle FBC = 36°$. **2**

Question 8 (9 marks) Use a SEPARATE writing booklet

(a) The function $f(x)$ is defined as

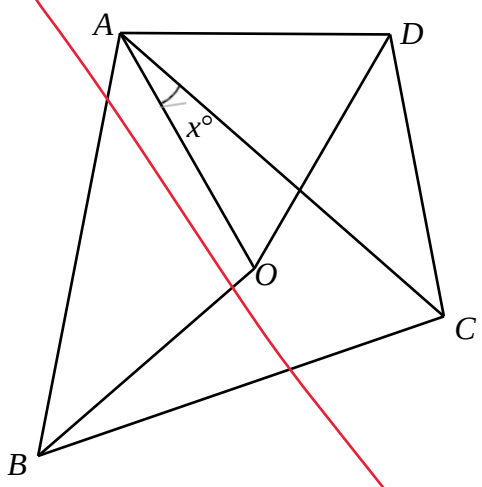
$$f(x) = \begin{cases} x^2 - 4 & \text{for } -3 \le x < 0 \\ 2x - 1 & \text{for } 0 \le x \le 2 \end{cases}$$

(i) Find $f(|-1|)$. **1**

(ii) Sketch $y = f(x)$. **2**

(b) Prove that $\cot\theta + \tan\theta = \sec\theta \operatorname{cosec}\theta$. **2**

(c) In the diagram below, triangles ABC and ADO are equilateral. $\angle OAC = x°$.



(i) Show that $\angle BAO = \angle DAC$. **1**

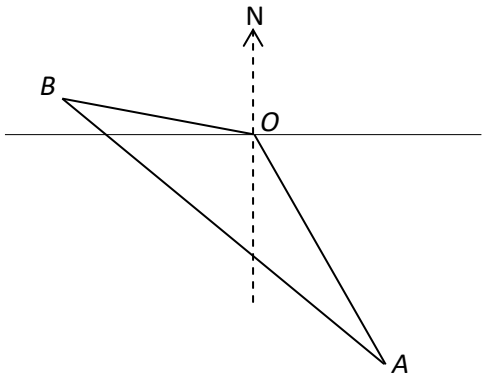
(ii) Prove that $\Delta AOB \equiv \Delta ADC$. **3**

Question 9 (10 marks) Use a SEPARATE writing booklet

(a) (i) If $f(x) = x$ and $g(x) = \frac{1}{x}$, sketch $y = f(x)$ and $y = g(x)$ on the same number plane showing the point(s) of intersection. **2**

(ii) From the diagram, solve $g(x) \geq f(x)$. **2**

(b) Two ships, *A* and *B*, depart simultaneously from a port *O*.
A sails twice as fast as *B*. Ship *A* is on a course of 160° T while ship *B* is on a course of 280° T. The diagram below shows the position of *A* and *B* after 1 hour.



NOT TO SCALE

Copy the diagram in to your answer booklet

(i) Show that $\angle BOA = 120°$. **1**

(ii) Let the speed of *B* be *x* km/h. **2**

By using the cosine rule show that $AB^2 = 7x^2$.

(iii) After one hour the ships are 70 km apart. Find the value of *x*. **1**

(iv) Determine the bearing of *B* from *A* at this time, **2**

as a true bearing correct to the nearest degree.

End of Paper

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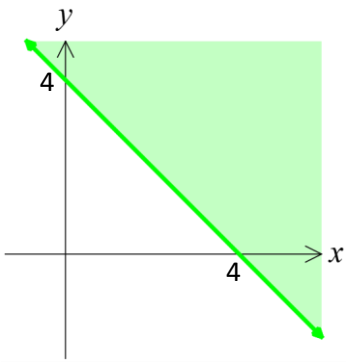

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3 Given that $g(x) = 2x^4 - x^2 - 4$, what is the value of $g(-3)$?

A. -3

B. 149

C. -149

D. 3

$g(x) = 2x^4 - x^2 - 4$

$g(-3) = 2(-3)^4 - (-3)^2 - 4$

$g(-3) = 149$

4 What does $(\cot x + \operatorname{cosec} x)(\cot x - \operatorname{cosec} x)$ simplify to?

A. 1

B. $\sin^2 x$

C. -1

D. $\cos^2 x$

$(\cot x + \operatorname{cosec} x)(\cot x - \operatorname{cosec} x)$

$= \cot^2 x - \operatorname{cosec}^2 x$

$= -1$ *Since* $\cot^2 x = \operatorname{cosec}^2 x - 1$

5 Which of the following gives the number of sides of a regular polygon whose interior angle are $(a+2)°$?

A. $\dfrac{360}{a-178}$

B. $\dfrac{-360}{a-178}$

C. $\dfrac{180}{a-2}$

D. $180(a-2)$

$n(a+2) = (n-2) \times 180$

$na + 2n = 180n - 360$

$na - 178n = -360$

$n(a-178) = -360$

$n = \dfrac{-360}{a-178}$

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Question 6 (13 marks) Use a SEPARATE writing booklet

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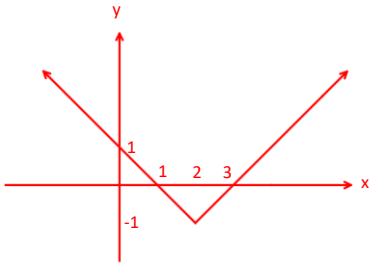


(b) State the domain and range of $y = \sqrt{9-x^2}$. **2**

Domain $= -3 \le x \le 3$

Range $= 0 \le y \le 3$

(c) Given that $f(x) = x^3 - 4x$:

(i) Determine if $f(x)$ is odd, even or neither. **2**

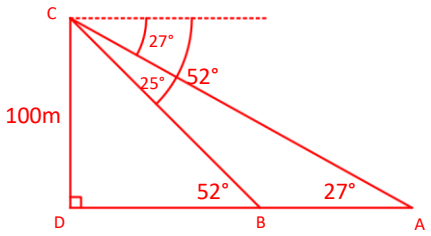
$f(-x) = (-x)^3 - 4(-x)$
$= -x^3 + 4x$
$= -(x^3 - 4x)$
$= -f(x)$

$\therefore$ the function is odd.

(ii) If $f(a) = 10$, evaluate $f(a) - f(-a)$. **1**

Since the function is odd.
$f(a) - f(-a) = 10 - (-10)$
$= 20$

(d) From the top of a light house situated 100 metres above sea level, the angles of depression of boat A and boat B are $27°$ and $52°$ respectively. Calculate the distance between the boats to the nearest metre. **2**



Label C and D on the diagram

$\angle CAB = 27°$ (*alternate angles on parallel lines*)
$\angle CBD = 52°$ (*alternate angles on parallel lines*)

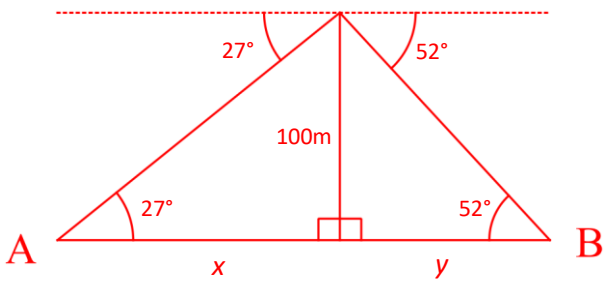
$$\sin 52° = \frac{100}{CB}$$
$$CB = 126.9018...$$

$$\angle BCA = 52° - 27°$$
$$= 25°$$

$$\frac{AB}{\sin 25°} = \frac{CB}{\sin 27°}$$
$$AB = 118.132...$$

$\therefore$ the distance between the boats is 118m (nearest metres).

Alternate solution



$$\tan 27° = \frac{100}{x}$$

$$\therefore x = \frac{100}{\tan 27°}$$

$$\tan 52° = \frac{100}{y}$$

$$\therefore y = \frac{100}{\tan 52°}$$

$$\therefore \text{the distance between AB is} = x + y$$

$$= \frac{100}{\tan 27°} + \frac{100}{\tan 52°}$$

$$= 274.38...$$

$$= 274\,m\,(\textit{nearest metre})$$

(e) Find the exact value of $\cos 420° + \tan(-30°)$ leaving your answer as a simplified fraction with a rational denominator. **2**

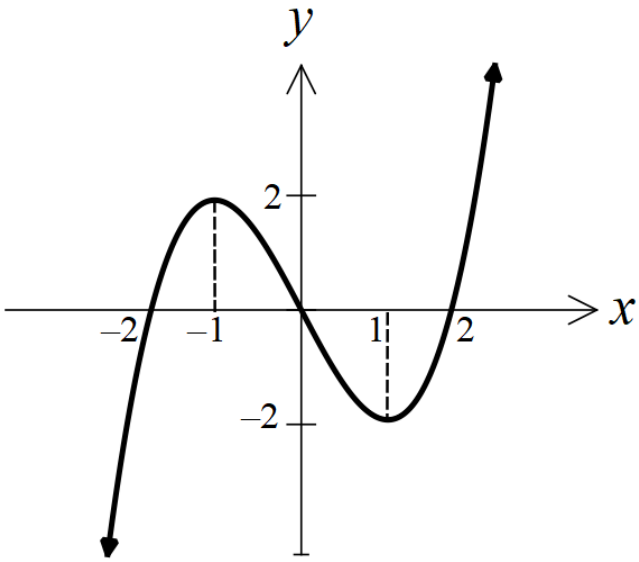
$$\cos 420° + \tan(-30°)$$
$$= \frac{1}{2} + -\left(\frac{1}{\sqrt{3}}\right)$$
$$= \frac{1}{2} - \frac{1}{\sqrt{3}}$$
$$= \frac{1}{2} - \frac{\sqrt{3}}{3}$$
$$= \frac{3-2\sqrt{3}}{6}$$

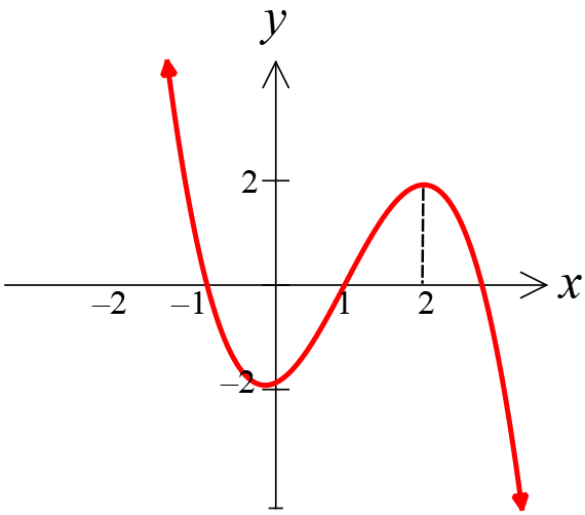
(f) Solve $\tan^2 \theta = 3$ for $0° \le \theta \le 360°$. **2**

$$\tan \theta = \pm\sqrt{3}$$
$$\theta = 60°, 120°, 240°, 300°$$

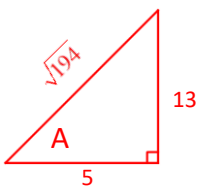
Question 7 (10 marks) Use a SEPARATE writing booklet

(a) Using $y = f(x)$ shown in the diagram below, sketch $y = -f(x-1)$. **2**





(b) If $\tan A = \frac{13}{5}$ and, $\cos A < 0$ find the exact value of $\sin A$. **2**



It's in the 3rd quadrant as tan is positive and cos is negative.

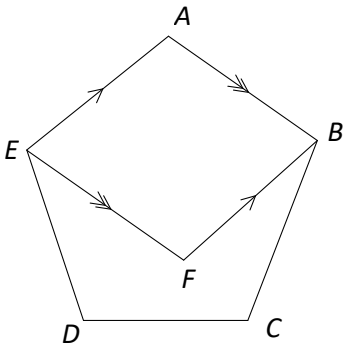
$\therefore \sin A = -\frac{13}{\sqrt{194}}$ *or* $-\frac{13\sqrt{194}}{194}$

(c) Solve $\sin(\theta+90°)=\frac{\sqrt{3}}{2}$ for $0°\le\theta\le360°$. **2**

$(\theta+90°)=120°$ *or* $420°$ *for* $90°\le\theta+90°\le450°$

$\theta=30°$ *or* $330°$

(d) The diagram below shows a regular pentagon *ABCDE*. *EF* is drawn so that $EF\|AB$ and $BF\|AE$.



(i) Show that $\angle EAB=108°$. **1**

$\angle EAB=\frac{180°(5-2)}{5}$

$=108°$

(ii) What kind of quadrilateral is *ABFE* ? Justify your answer. **1**

ABFE is a rhombus since it has two pairs of parallel lines and adjacent sides are equal

(iii) Show that $\angle ABE=\angle EBF=\angle FBC=36°$. **2**

$\angle EAB+\angle ABF=180°$ (co interior angles, $EA\|FB$)

$108°+\angle ABF=180°$

$\therefore\angle ABF=72°$

$\therefore\angle ABE=\angle EBF=36°$ (Diagonals of rhombus bisect angles)

$36°+36°+\angle FBC=108°$

$\therefore\angle FBC=36°$

Question 8 (9 marks) Use a SEPARATE writing booklet

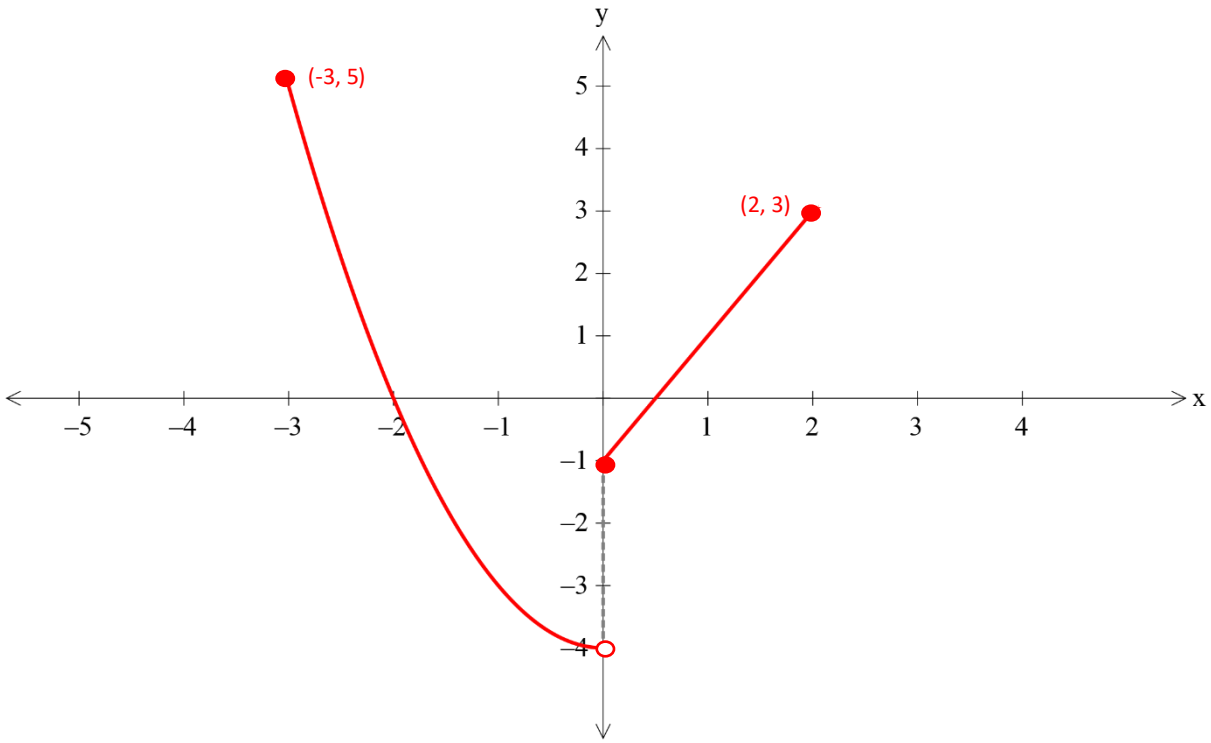
(a) The function $f(x)$ is defined as

$$f(x)=\begin{cases} x^2-4 & \text{for } -3\le x<0 \\ 2x-1 & \text{for } 0\le x\le 2 \end{cases}$$

(i) Find $f(|-1|)$. **1**

$f(|-1|)=f(1)$
$f(1)=2(1)-1$
$=1$

(ii) Sketch $y=f(x)$. **2**



(b) Prove that $\cot\theta + \tan\theta = \sec\theta\operatorname{cosec}\theta$. **2**

$LHS = \cot\theta + \tan\theta$

$$= \frac{1}{\tan\theta} + \tan\theta$$

$$= \frac{1}{\tan\theta} + \frac{\tan^2\theta}{\tan\theta}$$

$$= \frac{1+\tan^2\theta}{\tan\theta}$$

$$= \frac{\sec^2\theta}{\tan\theta}$$

$$= \frac{\frac{1}{\cos^2\theta}}{\frac{\sin\theta}{\cos\theta}}$$

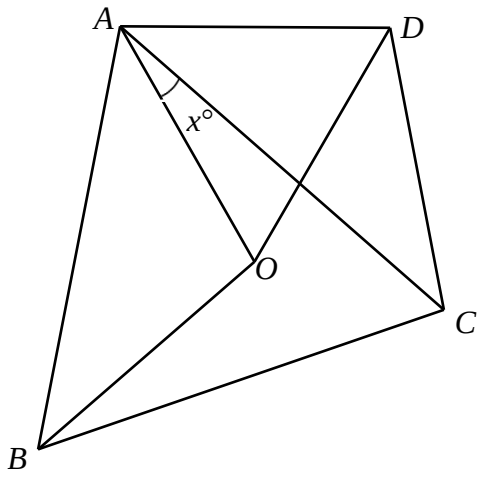
$$= \frac{1}{\cos^2\theta} \times \frac{\cos\theta}{\sin\theta}$$

$$= \frac{1}{\cos\theta} \times \frac{1}{\sin\theta}$$

$= \sec\theta\operatorname{cosec}\theta$

$= RHS$

(c) In the diagram below, triangles ABC and ADO are equilateral. $\angle OAC = x°$.



(i) Show that $\angle BAO = \angle DAC$. **1**

$\angle BAC = 60°$ (Equilateral triangle)

$\angle OAD = 60°$ (Equilateral triangle)

$\angle BAO = 60° - x°$

$\angle DAC = 60° - x°$

$\therefore \angle BAO = \angle DAC$

(ii) Prove that $\Delta AOB \equiv \Delta ADC$. **3**

$In\ \Delta AOB \equiv \Delta ADC$

$\angle BAO = \angle DAC\ (proven\ in\ part\ a)$

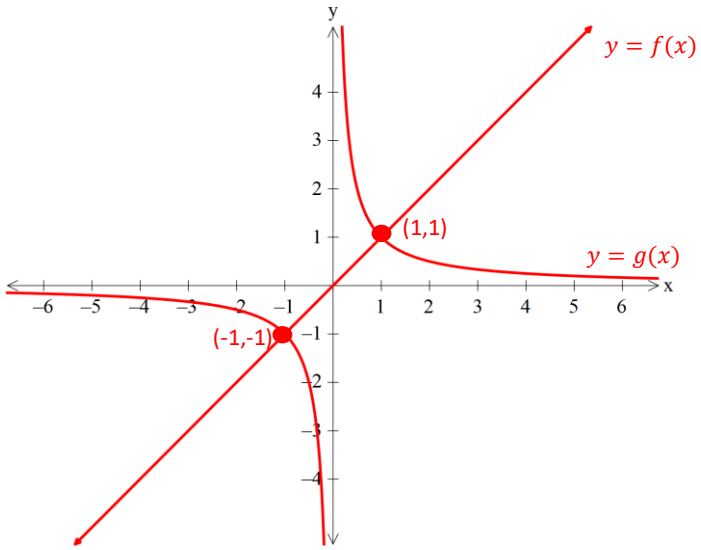
$AB = AC\ (Equaliteral\ triangle)$

$AO = AD(Equaliteral\ triangle)$

$\therefore \Delta AOB \equiv \Delta ADC\ (SAS)$

Question 9 (10 marks) Use a SEPARATE writing booklet

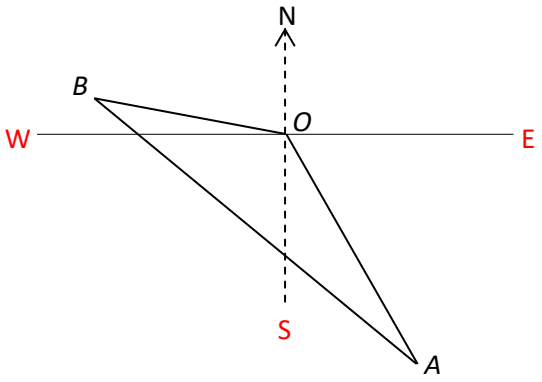
(a) (i) If $f(x) = x$ and $g(x) = \dfrac{1}{x}$, sketch $y = f(x)$ and $y = g(x)$ on the same number plane showing the point(s) of intersection. **2**



(ii) From the diagram, solve $g(x) \geq f(x)$. **2**

$x \leq -1, \quad 0 < x \leq 1$

(b) Two ships, *A* and *B*, depart simultaneously from a port *O*.
A sails twice as fast as *B*. Ship *A* is on a course of 160° T while ship *B* is on a course of 280° T. The diagram below shows the position of *A* and *B* after 1 hour.



NOT TO SCALE

Copy the diagram in to your answer booklet

(i) Show that $\angle BOA = 120°$. **1**

$\angle AOB = \text{Reflex}\angle NOB - \angle NOA$
$= 280° - 160°$
$= 120°$

(ii) Let the speed of B be x km/h. **2**

By using the cosine rule show that $AB^2 = 7x^2$.

Distance of $OA = 2x \times 1$
Distance of $OB = x \times 1$

$AB^2 = x^2 + (2x)^2 - 2(x)(2x)(\cos 120°)$

$AB^2 = 5x^2 - 4x^2\left(-\frac{1}{2}\right)$

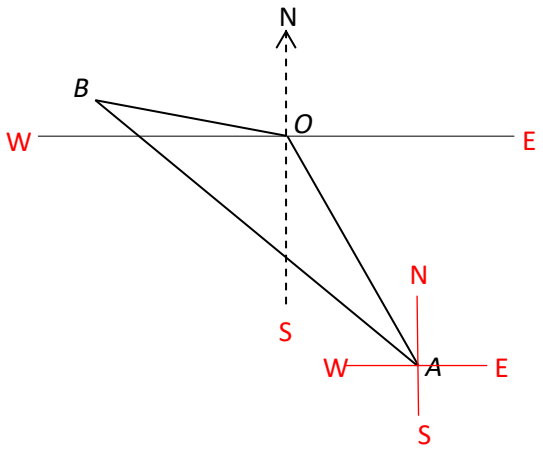
$AB^2 = 7x^2$

(iii) After one hour the ships are 70 km apart. Find the value of x. **1**

$70^2 = 7x^2$
$x = \sqrt{700}$ *or* $10\sqrt{7}$

(iv) Determine the bearing of B from A at this time, **2**

as a true bearing correct to the nearest degree.



NOT TO SCALE

$\angle OAN + 160° = 180°(\text{co interior } \textit{angles in parallel lines})$

$\angle OAN = 20°$

$$\frac{\sqrt{700}}{\sin \angle OAB} = \frac{70}{\sin 120°}$$

$\angle OAB = 19°6'23.78"$

$\therefore \angle BAN = 19°6'23.78" + 20°$

$= 39°9'23.78"$

$\therefore$ The bearing of B from A is $360° - 39°9'23.78"$

$= 320°54'(\textit{nearest}\ \text{minute})$

End of Paper

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