NORTH
SYDNEY
GIRLS HIGH
SCHOOL

2019 | **Preliminary Mathematics Assessment Task 2**

Preliminary Mathematics Advanced

SOLUTIONS

General Instructions

- Reading Time – 2 minutes
- Working Time – 55 minutes
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided
- In Questions 6 – 9, show relevant mathematical reasoning and/or calculations

Total marks – 46

Section I – 5 marks (pages 2 – 4)

- Attempt Questions 1 – 5
- Allow about 6 minutes for this section

Section II – 41 marks (pages 5 – 8)

- Attempt Questions 6 – 9
- Allow about 49 minutes for this section

NAME:______________________________ **TEACHER:**___________________

Question	1 – 5	6	7	8	9	Total
Algebra and Arithmetic	/2	/11				**/13**
Functions	/2		/9	/3	/3	**/17**
Trigonometry	/1		/2	/7	/6	**/16**
	/5	**/11**	**/11**	**/10**	**/9**	**/46**

Section I

5 marks
Attempt Questions 1-5
Allow about 6 minutes for this section

Use the multiple choice answer sheet for Questions 1-5.

---

1 What is the value of $\frac{\sin 82.3^\circ}{e^3}$, correct to 3 significant figures?

A. 0.0289

B. 0.029

C. 0.049

D. 0.0493

2 Let $a = 4^x$. Which expression is equal to $\log_4\left(a^2\right)$?

A. x^2

B. 4^{x^2}

C. 4^{2^x}

D. $2x$

3 What is the range of the function $y = |x| - x$?

A. All real y

B. $y \geq 0$

C. $y \leq 0$

D. $y = 0$

4 If $f(x)=x^2-2x$, what is the simplest expression for $f(x+2)-f(2)$?

A. x^2+2x

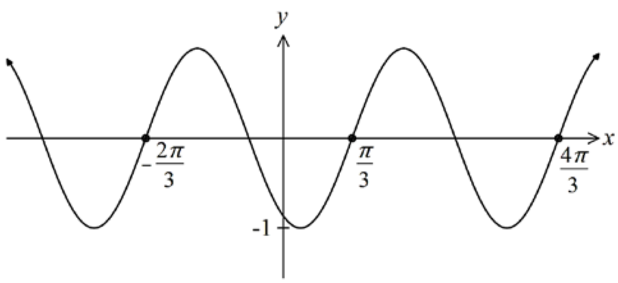
B. x^2+6x

C. x^2+2x-8

D. $x^2+6x-16$

5 Which diagram shows the graph of $y=\sin\left(2x-\dfrac{\pi}{3}\right)$?

A.



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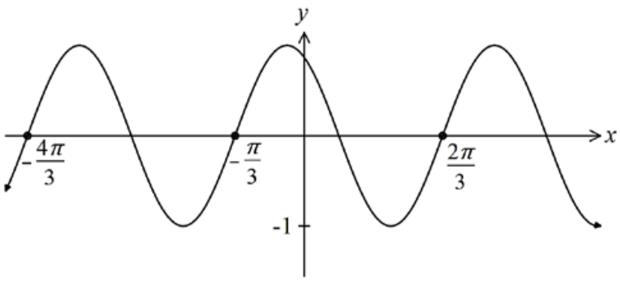

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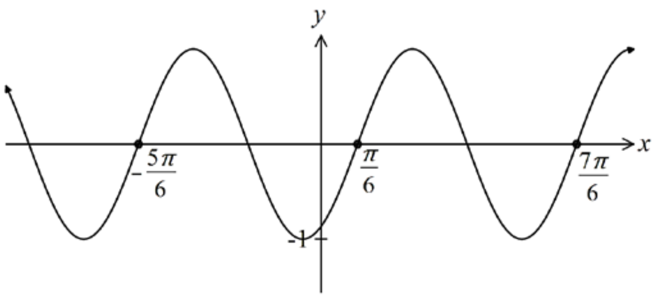

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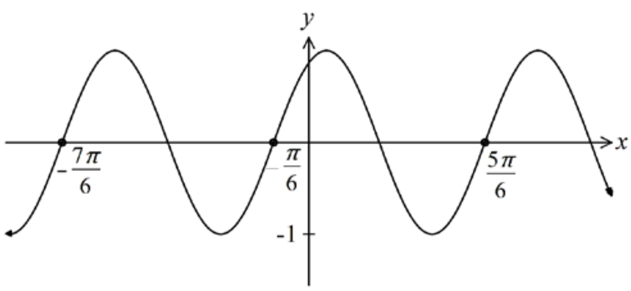


Section II

41 marks
Attempt Questions 6 to 9.
Allow about 49 minutes for this section.

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In questions 6 to 9, your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (11 marks**)** Use a SEPARATE writing booklet

(a)	Solve $3(2x+7)=\dfrac{x+10}{4}+7$.	**2**

$$3(2x+7)=\frac{x+10}{4}+7$$
$$6x+21=\frac{x+10}{4}+7$$
$$6x+14=\frac{x+10}{4}$$
$$24x+56=x+10$$
$$23x=-46$$
$$x=-2$$

(b)	Express $6\sqrt{5}-\dfrac{1}{\sqrt{5}-2}$ in the form $a+b\sqrt{5}$, where a and b are integers.	**2**

$$6\sqrt{5}-\frac{1}{\sqrt{5}-2}=6\sqrt{5}-\frac{1}{\sqrt{5}-2}\times\frac{\sqrt{5}+2}{\sqrt{5}+2}$$
$$=6\sqrt{5}-\left(\frac{\sqrt{5}+2}{5-4}\right)$$
$$=6\sqrt{5}-\left(\sqrt{5}+2\right)$$
$$=6\sqrt{5}-\sqrt{5}-2$$
$$=5\sqrt{5}-2$$

which is in the form $a+b\sqrt{5}$

(c)	Solve $\lvert 2y-3\rvert=8$.	**2**

$\lvert 2y-3\rvert=8$

$2y-3=8$ or $2y-3=-8$

$2y=11$ $\qquad$ $2y=-5$

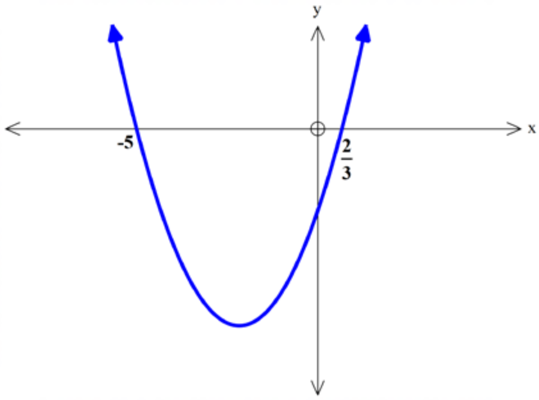
$y=5.5$ or $y=-2.5$

(d) Solve $10-13x-3x^2<0$. **2**

$$10-13x-3x^2<0$$
$$3x^2+13x-10>0$$
$$(3x-2)(x+5)>0$$

Therefore,

$$x<-5 \text{ or } x>\frac{2}{3}$$



(e) Solve for x: $\log_2(x+1)+\log_2(x+3)=3$ **3**

$$\log_2(x+1)+\log_2(x+3)=3$$
$$\log_2(x+1)(x+3)=3$$
$$\therefore\ 2^3=(x+1)(x+3)$$
$$8=x^2+4x+3$$
$$x^2+4x-5=0$$
$$(x+5)(x-1)=0$$
$$x=-5 \text{ or } x=1$$

but $x\neq-5$ since $\log_2(-5+1)$ is undefined

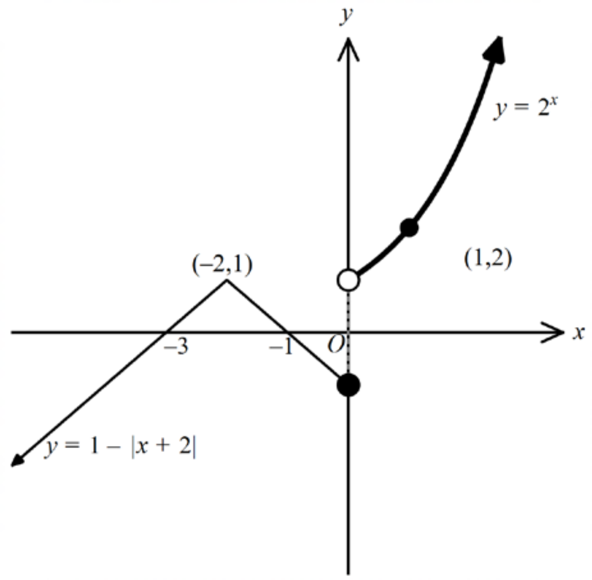
$$\therefore\ x=1$$

Question 7 (11 marks) Use a SEPARATE writing booklet

(a) A function is defined as **2**

$$f(x)=\begin{cases}1-|x+2| & \text{for } x\le 0\\ 2^x & \text{for } x>0\end{cases}$$

Sketch $y=f(x)$ clearly marking any intercepts.



Some students could draw the exponential section but couldn't draw the absolute value function. Students are reminded to label the intercepts and pay close attention as to whether a closed or open circle should be drawn.

(b) Determine whether the function $h(x)=\dfrac{x}{x^2-4}$ is even, odd or neither. **2**

Show working to support your answer.

$$h(x)=\frac{x}{x^2-4}$$

$$h(-x)=\frac{-x}{(-x)^2-4}$$ ← SHOW this substitution here

$$=\frac{-x}{x^2-4}$$

$$=-\frac{x}{x^2-4}$$

$$=-h(x)$$

Therefore $h(x)$ is odd.

Students MUST show the substitution of $-x$ into all terms containing a "x" when finding $h(-x)$

(c) The speed, s, of a particle is directly proportional to the square root of its kinetic energy, E. When $E = 225$, $s = 40$.

(i) Write a formula for s in terms of E. **2**

(ii) Find s when E= 900. **1**

(i)

$$s = k\sqrt{E}$$
$$40 = k\sqrt{225}$$
$$40 = k \times 15$$
$$k = \frac{40}{15} = \frac{8}{3}$$

$$s = \frac{8}{3}\sqrt{E}$$

(ii)

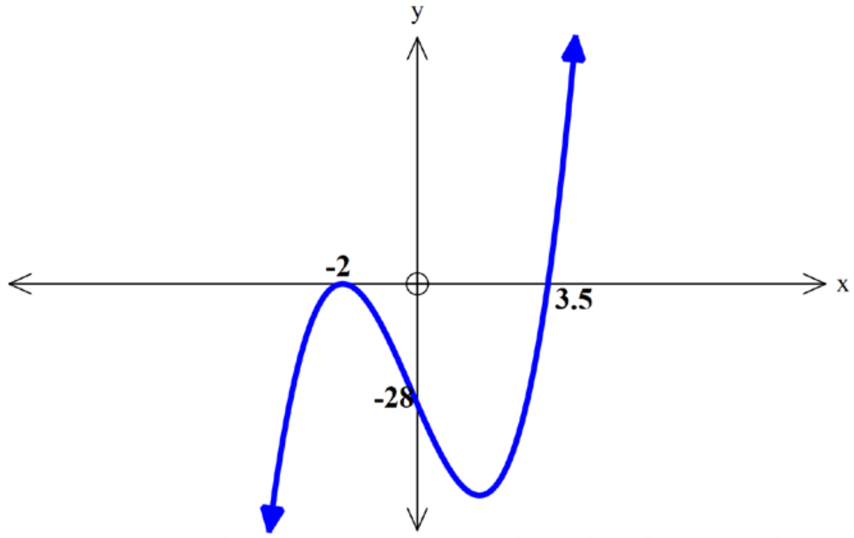
$$s = \frac{8}{3}\sqrt{900}$$
$$s = 80$$

(i) The most common error was students NOT writing the equation in part (i). They should look at the marks available and consider their response. 2 marks means you will have to show working.

Unfortunately, some students misread this question and either wrote an "inverse proportion" or simplified the question to s if proportional to E.

(ii) Most students could obtain this mark.

(d) Sketch the graph of $y=(x+2)^2(2x-7)$, showing all intercepts with the coordinate axes. **2**



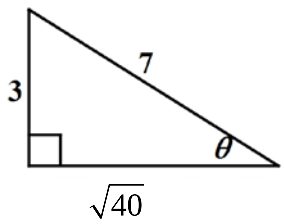
(e) State the amplitude and period of $y=-3\sin 4x$.

Amplitude = 3, Period = 90° or $\dfrac{\pi}{2}$

Some students wrote the amplitude was -3 and are reminded that amplitude is always positive.
For the period some students multiplied 360° by 4 instead of dividing it. Likewise with for students working in radians, some multiplied

Question 8 (10 marks) Use a SEPARATE writing booklet

(a) If $\sin\theta = -\frac{3}{7}$ and $\tan\theta \geq 0$ find the exact value of $\cos\theta$. **1**



Since $\cos\theta$ and $\sin\theta \leq 0$, θ lies in the 3rd quadrant.

$\therefore \cos\theta < 0$.

$\cos\theta = -\frac{\sqrt{40}}{7}$ or $\cos\theta = -\frac{2\sqrt{10}}{7}$

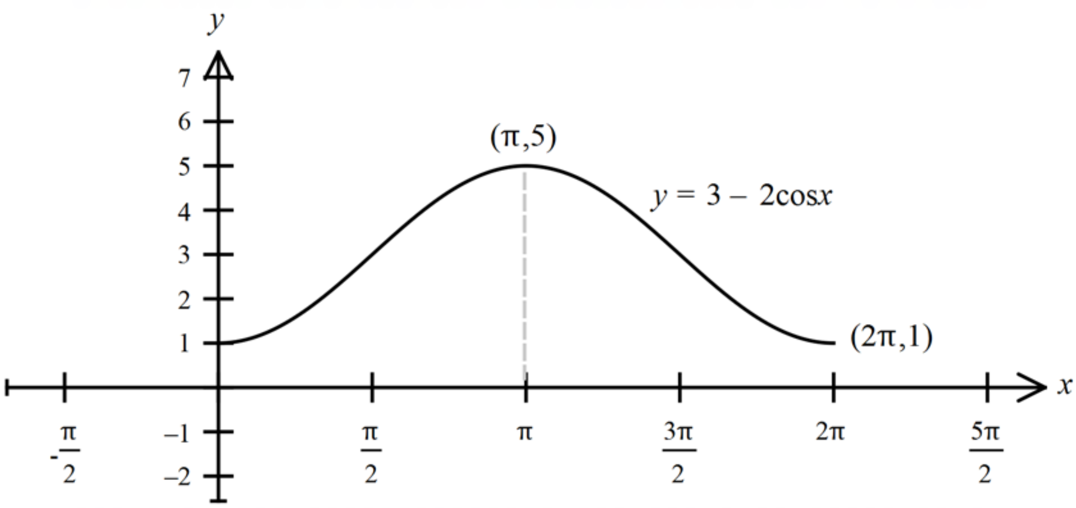
Many students did not account for the fact that θ is in the 3rd quadrant, so $\cos\theta$ is negativ

(b) Sketch $y = 3 - 2\cos x$ for $0 \leq x \leq 2\pi$ showing all important features. **2**

Range:

$$\begin{array}{lrcccl} & -1 & \leq & \cos x & \leq & 1 \\ [\times -2] & 2 & \geq & -2\cos x & \geq & -2 \\ \text{ie.} & -2 & \leq & -2\cos x & \leq & 2 \\ [+3] & 1 & \leq & 3-2\cos x & \leq & 5 \end{array}$$

Period $= \frac{2\pi}{1} = 2\pi$



Issues:	Many students:	(1)	could not get the correct range
		(2)	drew a graph with a period of π instead of 2π.
		(3)	ignored the specified domain
		(4)	drew the graph upside down
		(5)	used degrees instead of radians on the x-axis
		(6)	could not get the shape correct. The curve must be horizontal at the extreme values.

(c)	(i)	Show that $2\sin^2 x = 1 - 5\cos x$ can be written as $2\cos^2 x - 5\cos x - 1 = 0$.	**1**
	(ii)	Hence solve $2\sin^2 x = 1 - 5\cos x$ for $0 \le \theta \le 360^\circ$. Give answers to the nearest degree.	**3**

(i)

$$2\sin^2 x = 1 - 5\cos x$$
$$2\sin^2 x + 5\cos x - 1 = 0$$
$$2(1 - \cos^2 x) + 5\cos x - 1 = 0$$
$$2 - 2\cos^2 x + 5\cos x - 1 = 0$$
$$-2\cos^2 x + 5\cos x + 1 = 0$$
$$2\cos^2 x - 5\cos x - 1 = 0$$

(ii)

$$2\cos^2 x - 5\cos x - 1 = 0$$
$$\cos x = \frac{-(-5) \pm \sqrt{5^2 - 4 \times 2 - 1}}{2 \times 2}$$
$$\cos x = \frac{5 \pm \sqrt{33}}{4}$$

$\cos x = 2.686$ or $\cos x = -0.186...$

no solution rel $\angle = 79^\circ$

cos is negative in the 2nd and 3rd quadrants

$\therefore x = 180 - 79$ or $x = 180 + 79$

$x = 101^\circ$ or $x = 259^\circ$

(i) Well done. Some students wrote ‘LHS’ and followed it with the entire result, both LHS and RHS (not penalised, but not meaningful)

(ii) A disappointing number of students did not know the correct quadratic formula or could not substitute correctly into it. You need to justify why the positive solution was dropped. I was very lenient, and looked only for some indication that this was addressed, even if you just wrote ‘rejected’).

Far too many students could not get the correct angles from the value of $\cos\theta$. The ‘related angle’ must be in the FIRST quadrant. Ignore the negative when calculating the related angle.

(d) Given $f(x)=\sqrt{9-x^2}$ and $g(x)=2x+3$,

(i) State the domain and range of $f(x)$. **1**

(ii) Hence, or otherwise, find the domain and range of $f(g(x))$. **2**

(i)	D: $-3\le x\le 3$ R: $0\le y\le 3$
(ii)	D: $-3\le 2x+3\le 3$ $-6\le 2x\le 0$ $\therefore\quad -3\le x\le 0$ or $[-3,0]$ R: $0\le y\le 3$ or $[0,3]$

(d) (i) A number of students did not get the range correct. No algebra is necessary – it is a semi-circle.

(ii) See solutions for the best method here. When the inside function is linear, you don’t need a complicated method.

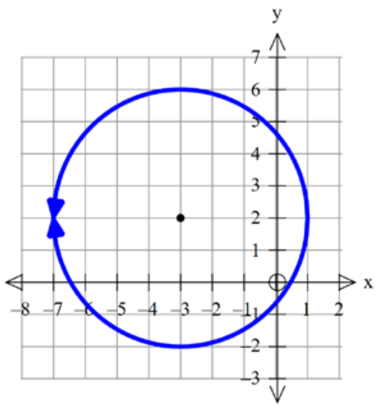
Question 9 (9 marks) Use a SEPARATE writing booklet

(a)	Determine the range of the relation $x(x+6)+y(y-4)=3$.	**3**

$$x(x+6)+y(y-4)=3$$
$$x^2+6x+y^2-4y=3$$
$$x^2+6x+9+y^2-4y+4=3+9+4$$
$$(x+3)^2+(y-2)^2=16$$

circle centre $(-3,2)$, radius $= 4$

R: $-2 \le y \le 6$



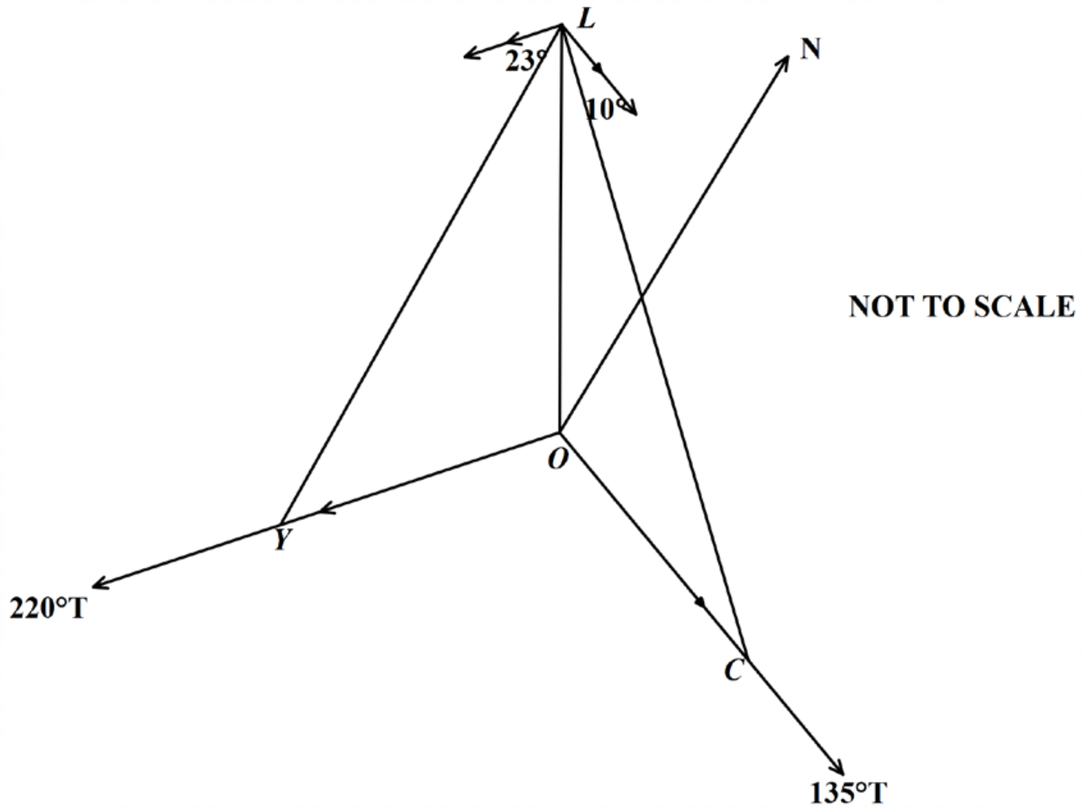
Overall well done. Some students made minor numerical errors.

(b)	Prove the following identity: $$\frac{\sin A-\cos A}{\operatorname{cosec} A-\sin A}=\tan^2 A-\tan A$$	**2**

$$\text{LHS} = \frac{\sin A-\cos A}{\operatorname{cosec} A-\sin A}$$
$$= \frac{\sin A-\cos A}{\dfrac{1}{\sin A}-\sin A}$$
$$= \frac{\sin^2 A-\sin A\cos A}{1-\sin^2 A}$$
$$= \frac{\sin^2 A-\sin A\cos A}{\cos^2 A}$$
$$= \frac{\sin^2 A}{\cos^2 A} - \frac{\sin A\cos A}{\cos^2 A}$$
$$= \tan^2 A - \frac{\sin A}{\cos A}$$
$$= \tan^2 A - \tan A$$
$= \text{RHS}$

Generally well done. Some students lost 1/2 mark for forgetting to insert brackets. Also, students should be encouraged to show as much working as possible for a prove question. So not jumping from $\dfrac{\sin^2 A-\sin A\cos A}{\cos^2 A}$ to $\tan^2 A-\dfrac{\sin A}{\cos A}$ but need to show the intermediate step $\dfrac{\sin^2 A}{\cos^2 A}-\dfrac{\sin A\cos A}{\cos^2 A}$. This was not penalised this time.

(c)	From the top of a lighthouse, L, 115 metres above sea level, a container ship, C, is seen on a bearing of $135^{o}\mathrm{T}$, at an angle of depression of 10^{o}. A yacht, Y, is also sighted on a bearing of $220^{o}\mathrm{T}$, at an angle of depression of 23^{o}. This is illustrated in the diagram below.	



(i)	Show that the square of the distance between the two vessels, $(CY)^2$, is given by: $(CY)^2 = 115^2\left(\tan^2 67^{o} + \tan^2 80^{o} - 2\tan 67^{o}\tan 80^{o}\cos 85^{o}\right)$. Give clear reasons for each step in your calculation.	**3**

In ΔLOY, $\angle YLO = 90 - 23$

$= 67^{o}$

$\tan 67^{o} = \dfrac{OY}{115}$

$\therefore\ OY = 115\tan 67^{o}$

$\angle TOC = 220 - 135$

$= 85^{o}$

Similarly,

In ΔLOC, $\angle CLO = 90 - 10$

$= 80^{o}$

$\tan 80^{o} = \dfrac{OC}{115}$

$\therefore\ OY = 115\tan 80^{o}$

In ΔTOC,

$(CY)^2 = 115^2\tan^2 67^{o} + 115^2\tan^2 80^{o} - 2(115\tan 67 \times 115\tan 80 \times \cos 85)$

$= 115^2\tan^2 67^{o} + 115^2\tan^2 80^{o} - 2\times 115^2 \times \tan 67^{o}\tan 80^{o}\cos 85^{o}$

$= 115^2\left(\tan^2 67^{o} + \tan^2 80^{o} - 2\tan 67^{o}\tan 80^{o}\cos 85^{o}\right)$

(ii) Hence, evaluate CY correct to 1 decimal place. **1**

$CY = 684.076\ldots$
$= 684.1$ m

i) This question was not as well done. The question says **"Give clear reasons"** and this was even more important in this question as it was a 'show' question and students were given the answer.
- Therefore students were penalised (1/2 mark) for not showing clear reasoning as to how they got their angles for their expressions of OY and OC. Many students went straight to $\tan 23^\circ = \frac{OL}{OY}$. The question gives the angles of depression, hence students need to first explain why $\angle OYL = 23^\circ$ by using alternate angles, or at least show the angle in a diagram (note: students should be encouraged to draw diagrams in 3d trig questions such as this, a significant proportion of the grade did not have any diagrams at all).
- Also many students who used the right angle to correctly deduce that $\angle OLY = 90^\circ - 23^\circ = 67^\circ$ used 'complementary angles' as their reason, again students should be reminded that 'complementary angles' is never sufficient reasoning here, they need to state 'right angle'.
- Other ways of deducing this angle were by using angle sum of a triangle but again most just assumed $\angle OYL = 23^\circ$ whilst never explaining why first.
- If there was no reasoning as to how they found the 85° they were also penalised ½ (if the 85° was shown on a diagram, it was accepted).
- Also as it was a show question, students needed to either explain why $\cot 23 = \tan 67$ by stating that they used complementary angles or show $\cot 23^\circ = \tan(90^\circ - 23°) = \tan 67^\circ$

ii) Mostly well done. However a significant number of students cannot enter this expression correctly in their calculator to evaluate it.

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B. $y \geq 0$

C. $y \leq 0$

D. $y = 0$

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A. $x^2 + 2x$

B. $x^2 + 6x$

C. $x^2 + 2x - 8$

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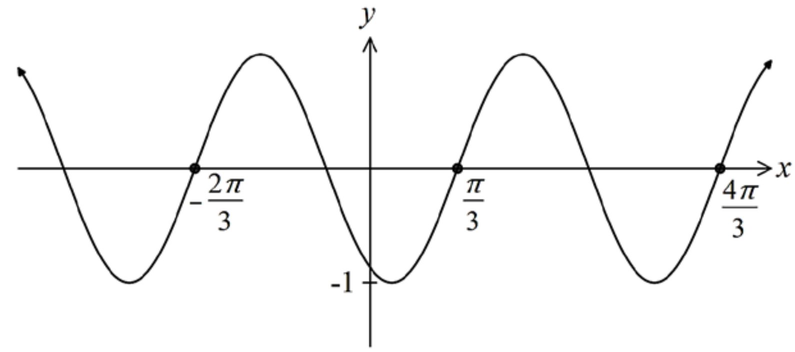

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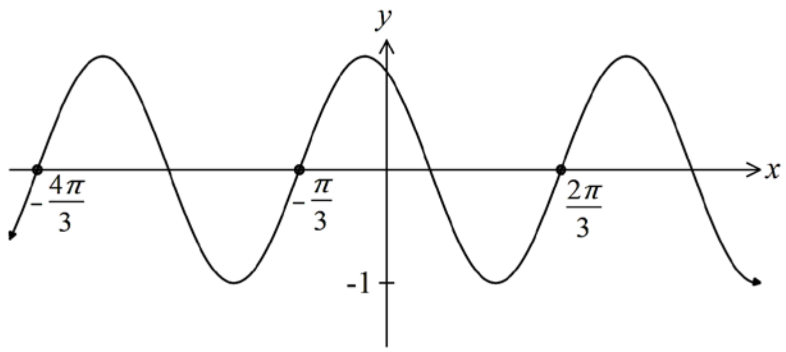
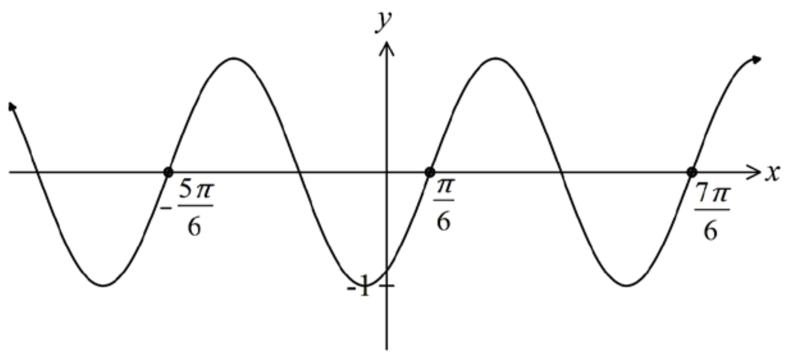
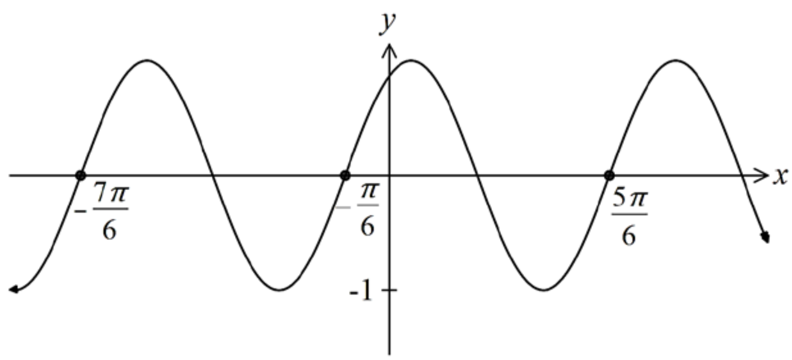


C.



D.



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(b) Express $6\sqrt{5}-\dfrac{1}{\sqrt{5}-2}$ in the form $a+b\sqrt{5}$, where *a* and *b* are integers. **2**

(c) Solve $|2y-3|=8$. **2**

(d) Solve $10-13x-3x^2<0$. **2**

(e) Solve for *x*: $\log_2(x+1)+\log_2(x+3)=3$ **3**

Question 7 (11 marks) Use a SEPARATE writing booklet

(a) A function is defined as **2**

$$f(x) = \begin{cases} 1 - |x+2| & \text{for} \quad x \le 0 \\ 2^x & \text{for} \quad x > 0 \end{cases}$$

Sketch $y = f(x)$ clearly marking any intercepts.

(b) Determine whether the function $h(x) = \dfrac{x}{x^2 - 4}$ is even, odd or neither. **2**

Show working to support your answer.

(c) The speed, s, of a particle is directly proportional to the square root of its kinetic energy, E. When $E = 225$, $s = 40$.

(i) Write a formula for s in terms of E. **2**

(ii) Find s when $E = 900$. **1**

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(e) State the amplitude and period of $y = -3\sin 4x$. **2**

Question 8 (10 marks) Use a SEPARATE writing booklet

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(b) Sketch $y = 3 - 2\cos x$ for $0 \leq x \leq 2\pi$ showing all important features. **2**

(c) (i) Show that $2\sin^2 x = 1 - 5\cos x$ can be written as $2\cos^2 x - 5\cos x - 1 = 0$. **1**

(ii) Hence solve $2\sin^2 x = 1 - 5\cos x$ for $0 \leq \theta \leq 360^{\circ}$. **3**

(d) Given $f(x) = \sqrt{9 - x^2}$ and $g(x) = 2x + 3$,

(i) State the domain and range of $f(x)$. **1**

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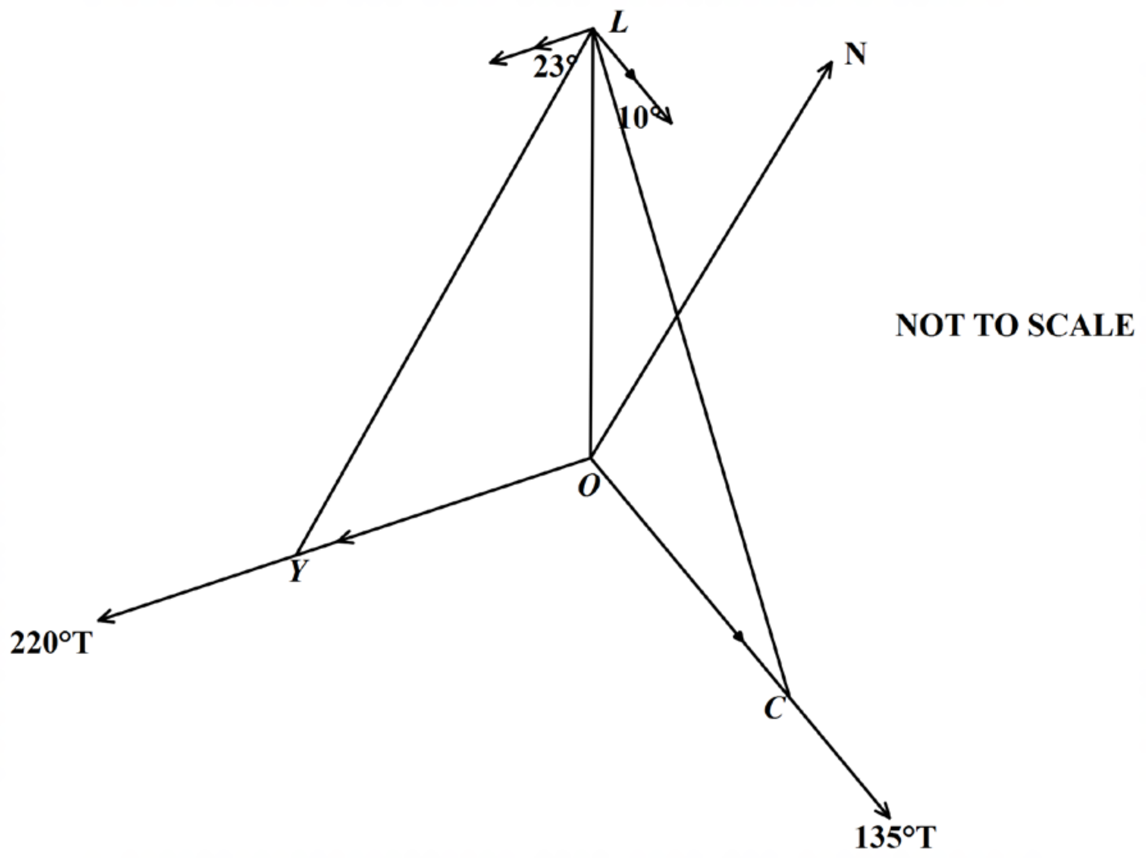
Question 9 (9 marks) Use a SEPARATE writing booklet

(a) Determine the range of the relation $x(x+6)+y(y-4)=3$. **3**

(b) Prove the following identity: **2**

$$\frac{\sin A - \cos A}{\operatorname{cosec} A - \sin A} = \tan^2 A - \tan A$$

(c) From the top of a lighthouse, L, 115 metres above sea level, a container ship, C, is seen on a bearing of $135^\circ\mathrm{T}$, at an angle of depression of 10°. A yacht, Y, is also sighted on a bearing of $220^\circ\mathrm{T}$, at an angle of depression of 23°. This is illustrated in the diagram below.



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(ii) Hence, evaluate CY correct to 1 decimal place. **1**

End of Paper