NORTH SYDNEY GIRLS HIGH SCHOOL

2020

Preliminary Mathematics Assessment Task 2

Preliminary Mathematics Advanced

General Instructions

- Reading Time – 2 minutes
- Working Time – 55 minutes
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided
- In Questions 6 – 9, show relevant mathematical reasoning and/or calculations

Total marks – 50

Section I – 5 marks (pages 2 – 3)

- Attempt Questions 1 – 5
- Allow about 6 minutes for this section

Section II – 45 marks (pages 5 – 20)

- Attempt Questions 6 – 9
- Allow about 49 minutes for this section

NAME:_____ **TEACHER:**_____

Question	1 – 5	6	7	8	9	Total
Algebra and Arithmetic	/2	/11	/2			**/15**
Functions	/3	/2	/9		/2	**/16**
Trigonometry				/11	/8	**/19**
	/5	**/13**	**/11**	**/11**	**/10**	**/50**

Section I

5 marks
Attempt Questions 1-5
Allow about 6 minutes for this section

Use the multiple choice answer sheet for Questions 1-5.

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1 What is the value of $\frac{5\sqrt{\pi}}{\cos 28^0}$, correct to 2 significant figures?

A. 10.04

B. 10.03

C. 1.04×10^1

D. 1.0×10^1

2 What is the simplest expression for $5^x\times5^{2x}$?

A. 5^{2x^2}

B. 5^{3x}

C. 25^{2x^2}

D. 25^{3x}

3 What is the domain of $y=\sqrt{x-3}-\sqrt{7-x}$?

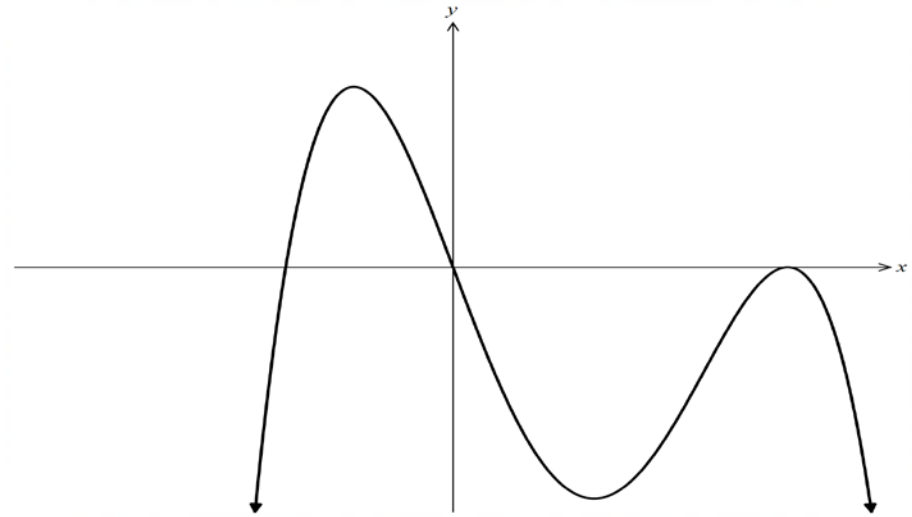
A. $(-\infty,3)\cup(7,\infty)$

B. $(-\infty,3]\cup[7,\infty)$

C. $[3,7]$

D. $(3,7)$

4 Consider the graph below.



Which of the following equations could represent the graph?

A. $y = -3x(x-4)^2(x+2)$

B. $y = 3x(x-4)^2(x+2)$

C. $y = -3x(x+4)^2(x-2)$

D. $y = 3x(x+4)^2(x-2)$

5 If $f(x-1) = x^2 - 2x + 3$, then which of the following is an expression for $f(x)$?

A. $x^2 - 2$

B. $x^2 + 2$

C. $x^2 - 2x + 2$

D. $x^2 - 2x + 4$

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Section II

50 marks
Attempt Questions 6 to 9.
Allow about 49 minutes for this section.

In questions 6 to 9, your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (13 marks**)**

(a) Given that $\log_a b = 4.25$ and $\log_a c = 0.75$, find the value of:

(i) $\log_a\left(\frac{c}{b}\right)$ **1**

(ii) $\log_a\left(bc\right)^3$. **1**

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(b) Solve $|4-3x|=7$ **2**

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Question 6 continues on Page 6

(c) Write $\dfrac{1+\sqrt{xy}}{x\sqrt{x}}$ in simplest form with a rational denominator, given $x, y > 0$ **2**

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(d) Solve $2x^2 - 2x - 1 = 0$. **2**

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Question 6 continues on Page 7

Question 6 (continued)

(e) Solve the following pair of simultaneous equations: **3**

$$7^{y-2x} = \frac{1}{7}$$
$$x^2 - 2x + y = 0$$

Question 6 continues on Page 8

Question 6 (continued)

(f) The resistance to the motion of a train R is the sum of two parts, one is a constant of 300 Newtons and the other is directly proportional to square of its speed, v m/s. If the total resistance is 350 Newtons when the speed of the train is $30\ \text{m/s}$, calculate the total resistance when the speed is $36\ \text{m/s}$. **2**

Question 7 (11 marks)

(a) Solve $12 + 5x - 2x^2 \leq 0$ **2**

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(b) The equation of a circle is $x^2 + y^2 - 2x + 4y - 20 = 0$. Find the circle's centre and radius. **2**

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Question 7 continues on Page 10

(c) A function is defined as:

$$f(x)=\begin{cases}5x-4 & \text{when } x\le 0\\ -x^2 & \text{when } x>0\end{cases}$$

(i) Evaluate $f(-3)+f(2)-f(0)$. **2**

(ii) Write an expression for $f(a^2+2)$. **1**

Question 7 continues on Page 11

Question 7 (continued)

(d) Let $f(x) = 3 + \sqrt{4 - x}$ and $g(x) = x^2$

(i) find an expression for $f(g(x))$ **1**

(ii) find the domain of $f(g(x))$ **1**

(iii) sketch $y = f(g(x))$. **2**

EXTRA WRITING SPACE

Question 8 (11 marks)

(a) Express 495^o in radians as an exact value. **1**

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(b) Find the exact value of $\csc\left(-\frac{4\pi}{3}\right)$. **1**

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(c) If $\cos\alpha = \frac{2}{5}$ and α is a reflex angle, find the exact value of $\tan\alpha$. **2**

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Question 8 continues on Page 14

(d) Show that $\frac{1}{\sin\alpha} - \frac{1}{\sin^2\alpha} = \frac{\sin\alpha - 1}{(1+\cos\alpha)(1-\cos\alpha)}$. **2**

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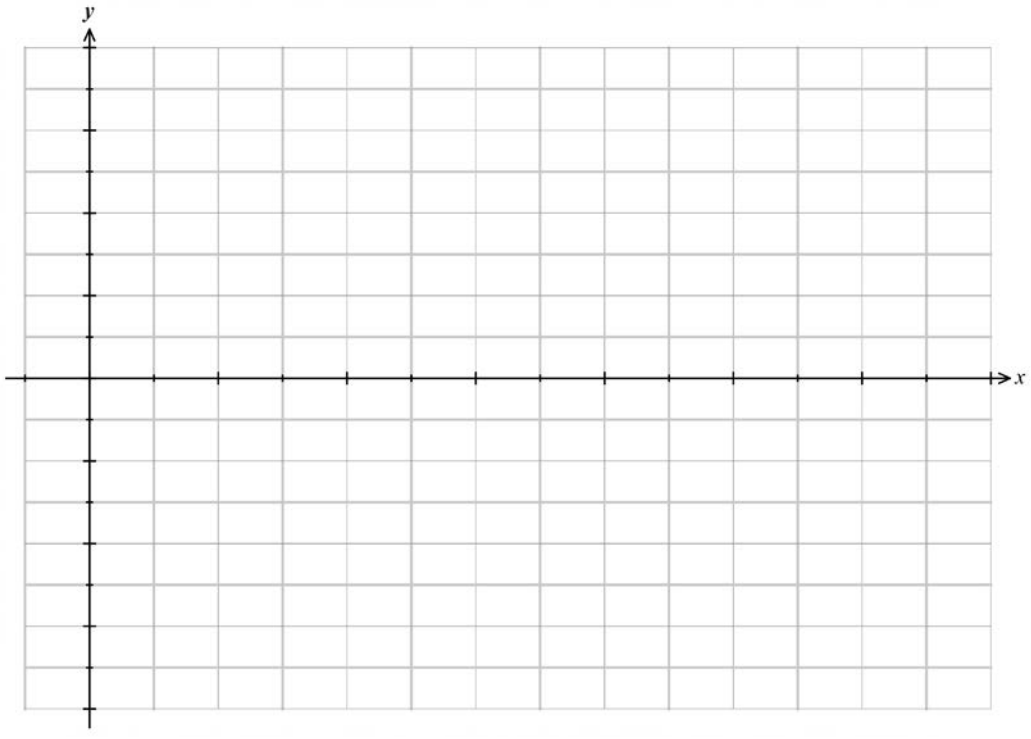
(e) Sketch $y = 2 + \sin\left(\frac{\pi x}{3}\right)$ for $0 \le x \le 12$, showing all important features. **2**



Question 8 continues on Page 15

(f) Solve $4\sin^2 x + 7\cos x = 4$, for $0^\circ \leq x \leq 360^\circ$. **3**

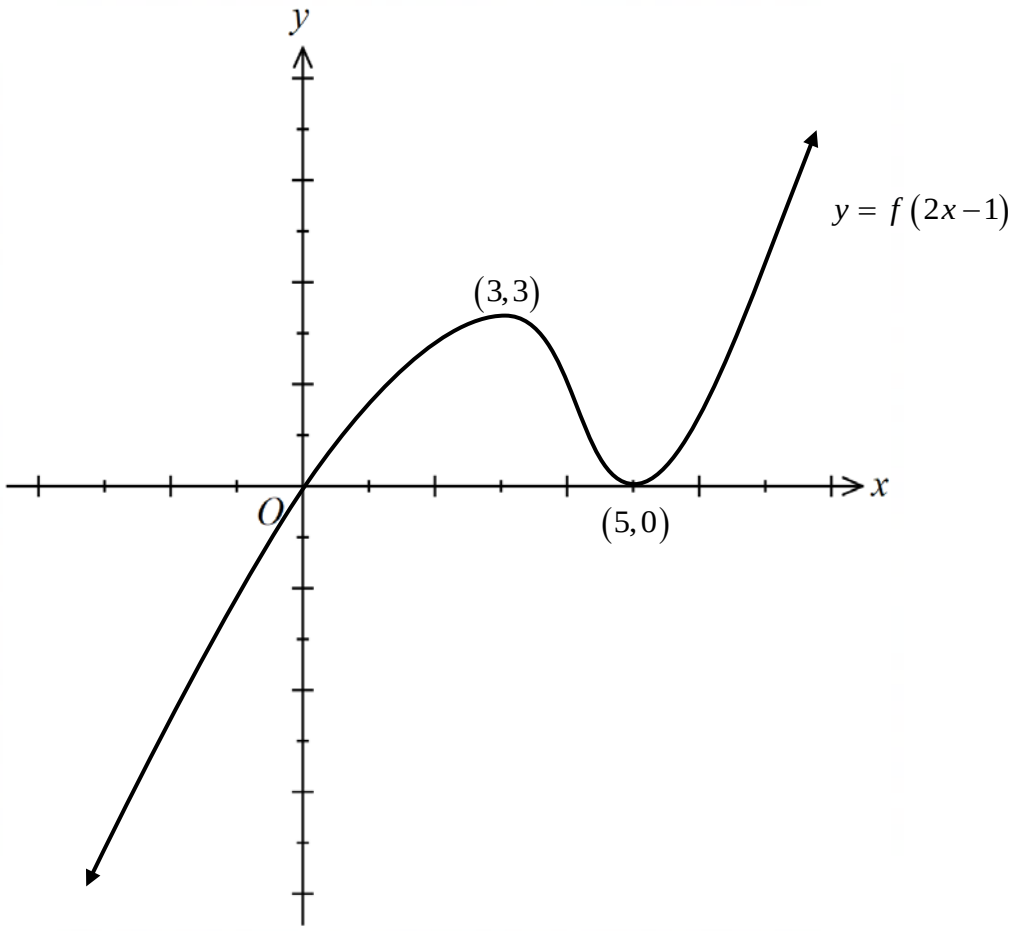
Give your answers correct to the nearest minute.

EXTRA WRITING SPACE

Question 9 (10 marks)

(a) The graph of the curve $y = f\left(2x-1\right)$ is shown below. **2**

The graph passes through the origin and points $\left(3,3\right)$ and $\left(5,0\right)$ respectively as shown.



By using the graph, or otherwise:

(a) Find the number of solutions to the equation $f\left(2x-1\right)-\left|x\right|=0$. **1**

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(b) Solve $f\left(x\right)=0$ **1**

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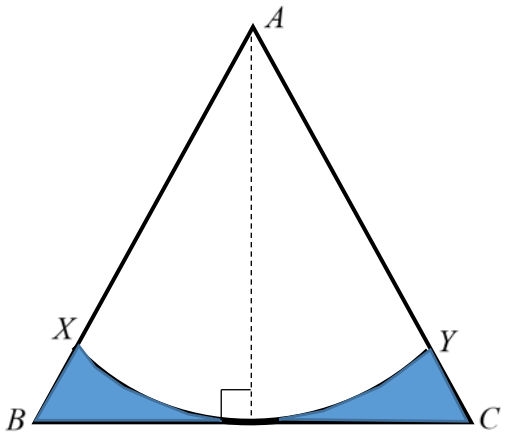
Question 9 continues on Page 18

(b) ΔABC is an equilateral triangle with sides of length 6 m.

An arc with centre A and BC as tangent cuts AB and AC at X and Y respectively.



(i) Show that the radius of the arc is $3\sqrt{3}$ m. **1**

(ii) Find the exact area of the shaded region. **3**

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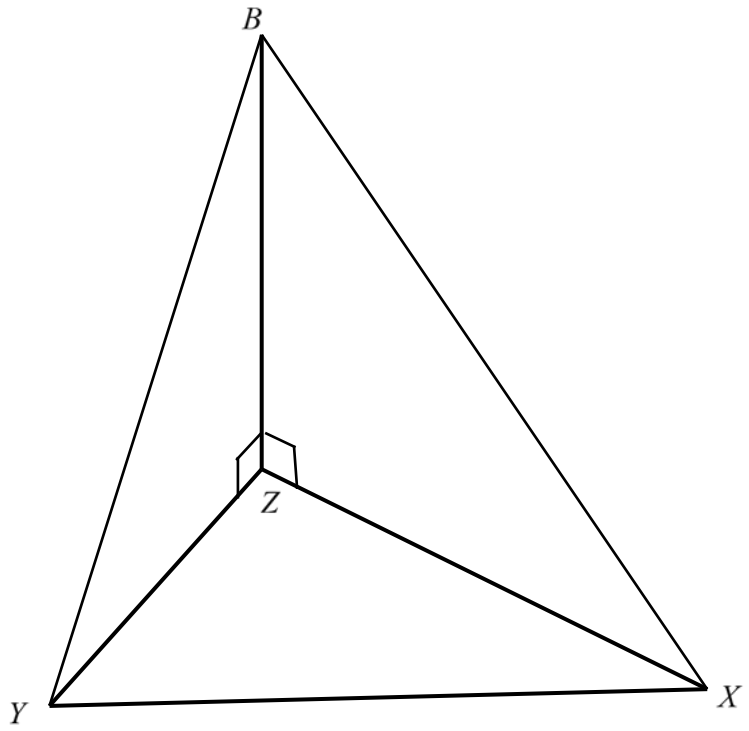
From point Y 4000 m due west of X, the bearing of the balloon is 049°.



(i) Draw ΔXYZ showing the bearings. **1**

(ii) Explain why $\angle YZX = 86^\circ$. **1**

(iii) Find the altitude of the balloon. **2**

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End of Paper

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	/5	/13	/11	/11	/10	/50

Section I

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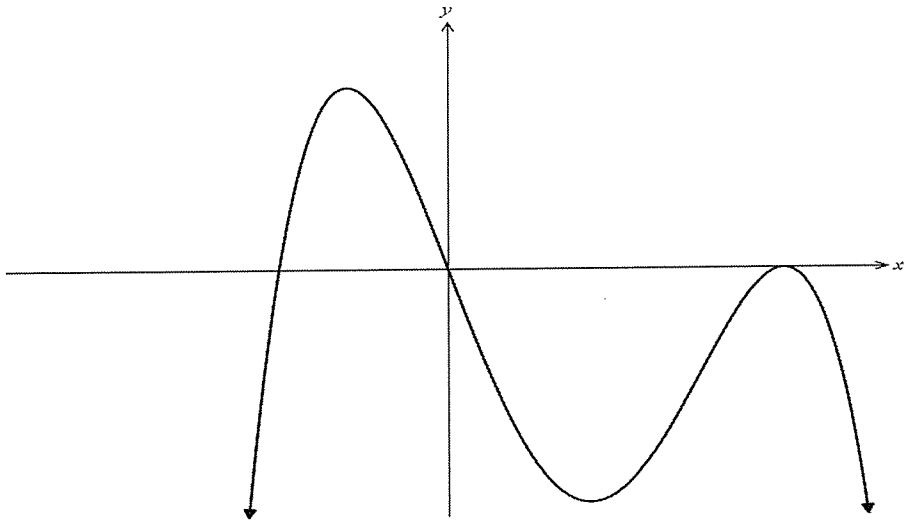
4 Consider the graph below.



Which of the following equations could represent the graph?

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B. $y=3x(x-4)^2(x+2)$

C. $y=-3x(x+4)^2(x-2)$

D. $y=3x(x+4)^2(x-2)$

5 If $f(x-1)=x^2-2x+3$, then which of the following is an expression for $f(x)$?

A. x^2-2

B. x^2+2

C. x^2-2x+2

D. x^2-2x+4

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Section II

50 marks
Attempt Questions 6 to 9.
Allow about 49 minutes for this section.

In questions 6 to 9, your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (13 marks)

(a) Given that $\log_a b = 4.25$ and $\log_a c = 0.75$, find the value of:

(i) $\log_a\left(\frac{c}{b}\right)$ **1**

(ii) $\log_a(bc)^3$. **1**

(i) $\log_a c - \log_a b = 0.75 - 4.25$

$= -3.5$

ii) $3\log_a bc = 3(\log_a b + \log_a c)$

$= 3(4.25 + 0.75)$

$= 15$

(b) Solve $|4-3x| = 7$ **2**

$4 - 3x = 7$ or $-4 + 3x = 7$

$3x = -3$ $3x = 11$

$x = -1$ $x = \frac{11}{3}$

Question 6 continues on Page 6

Question 6 (continued)

(c) Write $\dfrac{1+\sqrt{xy}}{x\sqrt{x}}$ in simplest form with a rational denominator, given $x, y > 0$ **2**

$$= \frac{(1+\sqrt{xy})}{x\sqrt{x}} \times \frac{\sqrt{x}}{\sqrt{x}}$$

$$= \frac{\sqrt{x} + \sqrt{x}\cdot\sqrt{xy}}{x\cdot x}$$

$$= \frac{\sqrt{x} + x\sqrt{y}}{x^2}$$

(d) Solve $2x^2 - 2x - 1 = 0$. **2**

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(2)(-1)}}{4}$$

$$= \frac{2 \pm \sqrt{12}}{4}$$

$$= \frac{2 \pm 2\sqrt{3}}{4}$$

$$= \frac{1 \pm \sqrt{3}}{2}$$

Question 6 continues on Page 7

Question 6 (continued)

(e) Solve the following pair of simultaneous equations: **3**

$$7^{y-2x} = \frac{1}{7} = 7^{-1} \quad \text{— ①}$$

$$x^2 - 2x + y = 0$$

From ① $\Rightarrow$ $y - 2x = -1$

$y = 2x - 1$

From ② $\Rightarrow$ $y = -x^2 + 2x$

$-x^2 + 2x = 2x - 1$

$x^2 = 1$

$x = 1$ or $x = -1$

$y = 2(1) - 1$ $y = 2(-1) - 1$

$y = 1$ $y = -3$

i.e. solutions are $x = 1, y = 1$ or $x = -1, y = -3$

Question 6 continues on Page 8

(f) The resistance to the motion of a train R is the sum of two parts, one is a constant of 300 Newtons and the other is directly proportional to square of its speed, v m/s. If the total resistance is 350 Newtons when the speed of the train is 30 m/s, calculate the total resistance when the speed is 36 m/s. **2**

$R = 300 + kV^2$

find k: when $R = 350$, $V = 30$

$350 = 300 + k \times 30^2$

$50 = 900k$

$k = \frac{50}{900} = \frac{1}{18}$

$\therefore R = 300 + \frac{V^2}{18}$

when $V = 36$, $R = 300 + \frac{36^2}{18}$

$= 372$ newtons

Question 7 (11 marks)

(a) Solve $12+5x-2x^2 \leq 0$ **2**

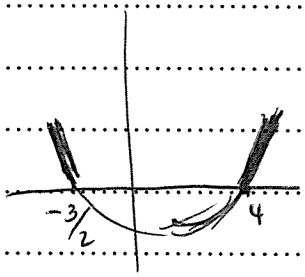
$2x^2 - 5x - 12 \geq 0$ or $(3+2x)(4-x) \leq 0$

$2x \quad -4$

$x \quad 3$

$(x-4)(2x+3) \geq 0$

$x \geqslant 4$ or $x \leqslant -\frac{3}{2}$



(b) The equation of a circle is $x^2+y^2-2x+4y-20=0$. Find the circle's centre and radius. **2**

$x^2 - 2x + 1 + y^2 + 4y + 4 = 20 + 1 + 4$

$(x-1)^2 + (y+2)^2 = 25$

radius = 5

centre = $(1, -2)$

Question 7 continues on Page 10

(c) A function is defined as:

$$f(x)=\begin{cases}5x-4 & \text{when } x\le 0\\ -x^2 & \text{when } x>0\end{cases}$$

(i) Evaluate $f(-3)+f(2)-f(0)$. **2**

(ii) Write an expression for $f(a^2+2)$. **1**

(i) $f(-3) = 5(-3) - 4 = -19$

$f(2) = -(2)^2 = -4$

$f(0) = -4$

$f(-3) + f(2) - f(0) = -19 - 4 + 4$

$= -19$

(ii) $f(a^2+2) = -(a^2+2)^2$ or $-a^4 - 4a^2 - 4$

as $a^2+2 \geq 0$

Question 7 continues on Page 11

Question 7 (continued)

(d) Let $f(x)=3+\sqrt{4-x}$ and $g(x)=x^2$

(i) find an expression for $f(g(x))$ **1**

(ii) find the domain of $f(g(x))$ **1**

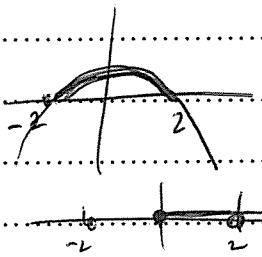
(iii) sketch $y=f(g(x))$. **2**

(i) $f(g(x)) = f(x^2)$

$= 3 + \sqrt{4-x^2}$

(ii) $4 - x^2 \geq 0$

$(2-x)(2+x) \geq 0$

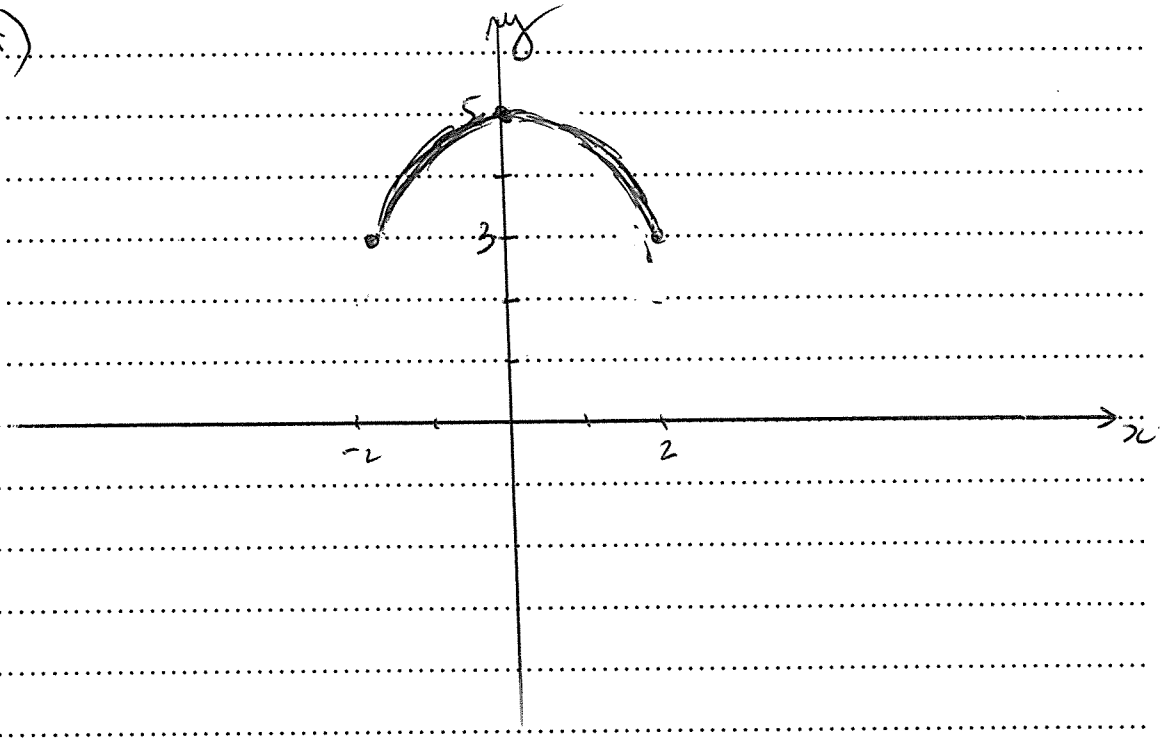


$-2 \leq x \leq 2$

and Domain of $g(x) = x^2$: $-\infty < x < \infty$

$\therefore$ Domain of $f(g(x))$: $-2 \leq x \leq 2$

(iii)



EXTRA WRITING SPACE

Question 8 (11 marks)

(a) Express $495°$ in radians as an exact value. **1**

$495° \times \frac{\pi}{180} = \frac{11\pi}{4}$

(b) Find the exact value of $\csc\left(-\frac{4\pi}{3}\right)$. **1**

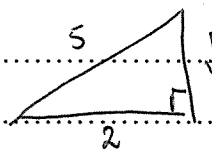
$= \csc\left(\frac{2\pi}{3}\right)$

$= \csc\left(\pi - \frac{\pi}{3}\right)$

$= \csc\frac{\pi}{3} = \frac{2}{\sqrt{3}}$

(c) If $\cos\alpha = \frac{2}{5}$ and α is a reflex angle, find the exact value of $\tan\alpha$. **2**

$\pm$ ✗



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$\tan\alpha = -\frac{\sqrt{21}}{2}$

Question 8 continues on Page 14

(d) Show that $\dfrac{1}{\sin\alpha} - \dfrac{1}{\sin^2\alpha} = \dfrac{\sin\alpha - 1}{(1+\cos\alpha)(1-\cos\alpha)}$. **2**

LHS $= \dfrac{1}{\sin\alpha} \times \dfrac{\sin\alpha}{\sin\alpha} - \dfrac{1}{\sin^2\alpha}$

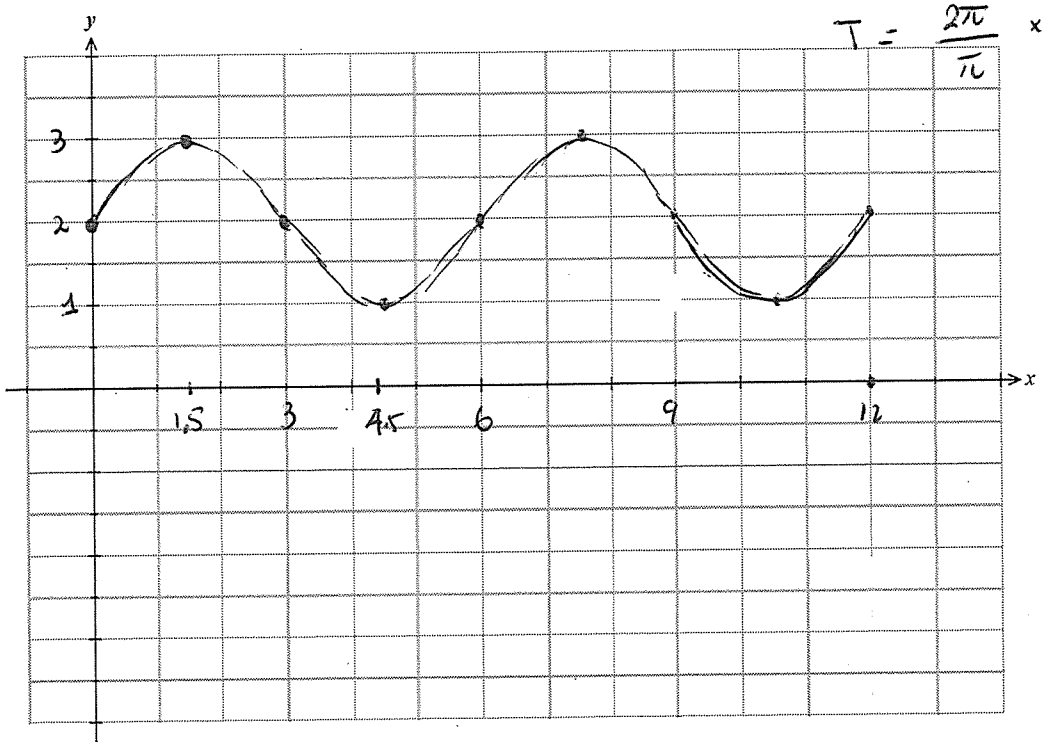
$= \dfrac{\sin\alpha - 1}{\sin^2\alpha}$

$= \dfrac{\sin\alpha - 1}{1 - \cos^2\alpha}$

$= \dfrac{\sin\alpha - 1}{(1-\cos\alpha)(1+\cos\alpha)}$

$=$ RHS

(e) Sketch $y = 2 + \sin\left(\dfrac{\pi x}{3}\right)$ for $0 \le x \le 12$, showing all important features. **2**



$T = \dfrac{2\pi}{\pi} \times 3 = 6$

Question 8 continues on Page 15

Question 8 (continued)

(f) Solve $4\sin^2 x + 7\cos x = 4$, for $0° \le x \le 360°$. **3**
Give your answers correct to the nearest minute.

$4\sin^2 x + 7\cos x - 4 = 0$

$4(1 - \cos^2 x) + 7\cos x - 4 = 0$

$4 - 4\cos^2 x + 7\cos x - 4 = 0$

$-4\cos^2 x + 7\cos x = 0$

$\cos x(7 - 4\cos x) = 0$

$\cos x = 0$ | $4\cos x = 7$

$x = 90°, 270°$ | $\cos x = \frac{7}{4}$

↑

No solution as $-1 \le \cos x \le 1$

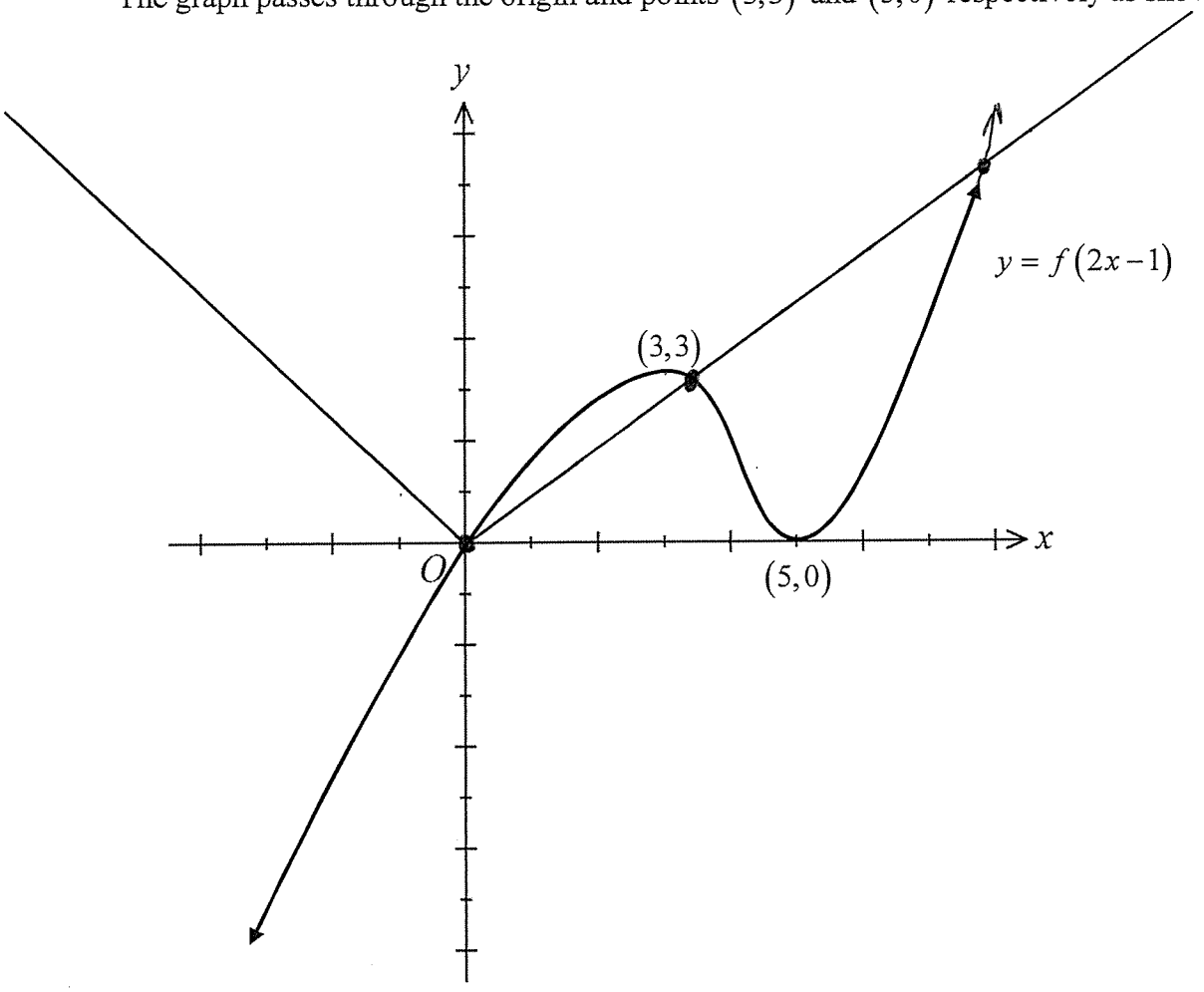
EXTRA WRITING SPACE

Question 9 (10 marks)

(a) The graph of the curve $y = f(2x-1)$ is shown below. **2**

$= f\left[2\left(x-\frac{1}{2}\right)\right]$

The graph passes through the origin and points $(3,3)$ and $(5,0)$ respectively as shown.



By using the graph, or otherwise:

(a) Find the number of solutions to the equation $f(2x-1)-|x|=0$. **1**

$f(2x-1) = |x|$

three solutions

(b) Solve $f(x)=0$ two solutions $f(2x-1)=0$ at $x=0,5$ **1**

$f(2\times 5-1) = f(9) = 0$

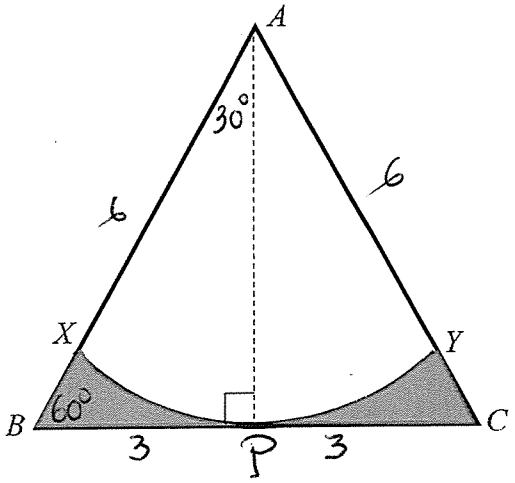
$f(2\times 0-1 = f(-1) = 0$

$x = -1, 9$

Question 9 continues on Page 18

(b) ΔABC is an equilateral triangle with sides of length 6 m.

An arc with centre A and BC as tangent cuts AB and AC at X and Y respectively.



(i) Show that the radius of the arc is $3\sqrt{3}$ m. **1**

(ii) Find the exact area of the shaded region. **3**

$\sin 60° = \frac{AP}{6}$

$\frac{\sqrt{3}}{2} = \frac{AP}{6} \Rightarrow AP = \frac{6\sqrt{3}}{2} = 3\sqrt{3}$ m

ii) shaded A = Area Δ − Area sector

$= \frac{6 \times 3\sqrt{3}}{2} - \frac{1}{2} \times \frac{\pi}{3} \times (3\sqrt{3})^2$

$= 9\sqrt{3} - \frac{1}{2} \times \frac{\pi}{3} \times 9 \times 3$

$= 9\sqrt{3} - \frac{9\pi}{2}$ or equivalent

OR shaded A $= \frac{6 \times 6 \times \sin\frac{\pi}{3}}{2} - \frac{1}{2}(3\sqrt{3})^2 \times \frac{\pi}{3}$

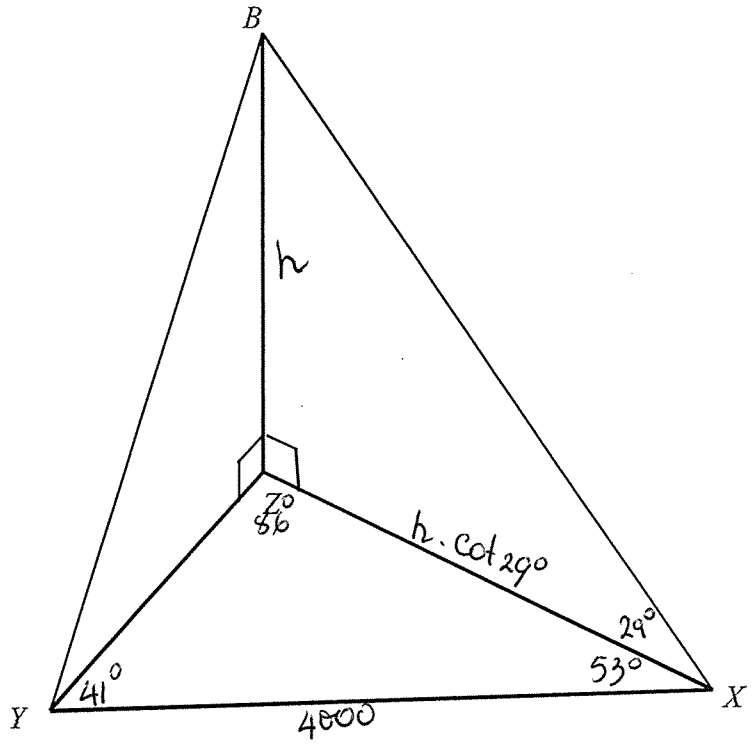
$= 18 \cdot \frac{\sqrt{3}}{2} - \frac{9\pi}{2}$

$= 9\sqrt{3} - \frac{9\pi}{2}$

Question 9 continues on Page 19

Question 9 (continued)

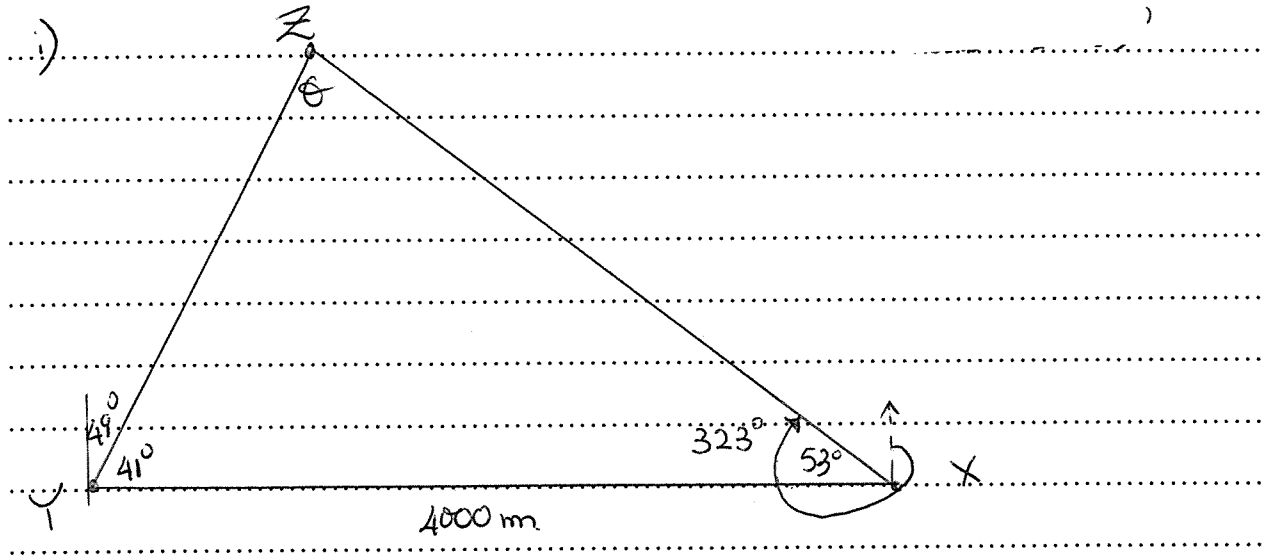
(c) The angle of elevation of a hot air balloon B from a point on the ground X is 29° and the bearing is 323°.
From point Y 4000 m due west of X, the bearing of the balloon is 049°.



(i) Draw ΔXYZ showing the bearings. **1**

(ii) Explain why $\angle YZX = 86^\circ$. **1**

(iii) Find the altitude of the balloon. **2**



iii) let BZ be h

In Δ BXZ

$$\tan 29^\circ = \frac{h}{XZ}$$

$$XZ = \frac{h}{\tan 29^\circ} = h \cot 29^\circ$$

In Δ XYZ :

Using Sine Rule

$$\frac{h \cot 29^\circ}{\sin 41^\circ} = \frac{4000}{\sin 86^\circ}$$

$$h = \frac{4000 \times \sin 41^\circ}{\sin 86^\circ \times \cot 29^\circ}$$

$$h = \frac{4000 \times \sin 41^\circ \times \tan 29^\circ}{\sin 86^\circ}$$

$$= 1458.2 \text{ m}$$

$$\simeq 1459 \text{ m}$$

OR

$$\frac{h / \tan 29^\circ}{\sin 41^\circ} = \frac{4000}{\sin 86^\circ}$$

$$\frac{h}{\tan 29^\circ \sin 41^\circ} = \frac{4000}{\sin 86^\circ}$$

$$h = \frac{4000 \cdot \sin 41^\circ \cdot \tan 29^\circ}{\sin 86}$$

$$= 1459 \text{ m}$$

End of Paper