NORTH
SYDNEY
GIRLS HIGH
SCHOOL

2021

Preliminary Mathematics Assessment Task 2

Preliminary Mathematics Advanced

General Instructions

- Reading Time – 2 minutes
- Working Time – 55 minutes
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided
- In Questions 6 – 9, show relevant mathematical reasoning and/or calculations

Total marks – 47

Section I – 5 marks (pages 2 – 3)

- Attempt Questions 1 – 5
- Allow about 6 minutes for this section

Section II – 42 marks (pages 4 – 17)

- Attempt Questions 6 – 9
- Allow about 49 minutes for this section

NAME:______________________________ TEACHER:__________________

Question	1 – 5	6	7	8	9	Total
Algebra and Arithmetic	/2	/5			/3	/10
Functions	/3	/5	/6			/14
Trigonometry		/3	/6	/9	/5	/23
	/5	/13	/12	/9	/8	/47

Section I

5 marks
Attempt Questions 1-5
Allow about 6 minutes for this section

Use the multiple choice answer sheet for Questions 1-5.

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1. What is the value of p so that $\dfrac{a^2 a^{-3}}{\sqrt{a}} = a^p$?

A -3

B $-\dfrac{3}{2}$

C $\dfrac{3}{2}$

D 2

2. Which interval gives the domain of $y = \log_3(7x-5)$?

A $\left(\dfrac{5}{7}, \infty\right)$

B $\left(-\dfrac{5}{7}, \infty\right)$

C $\left[\dfrac{5}{7}, \infty\right)$

D $\left[-\dfrac{5}{7}, \infty\right)$

3. Which interval gives the range of $y = 3 - 7\cos 2x$?

A $[-4, 10]$

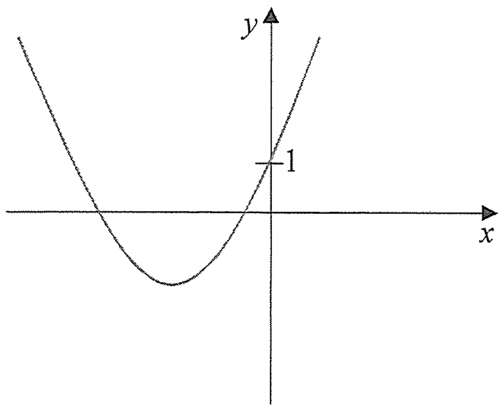
B $[1, 5]$

C $[-2, 5]$

D $[-10, -4]$

4. Which of the following could represent the graph of $y = -x^2 + bx + 1$, where $b > 0$?

A



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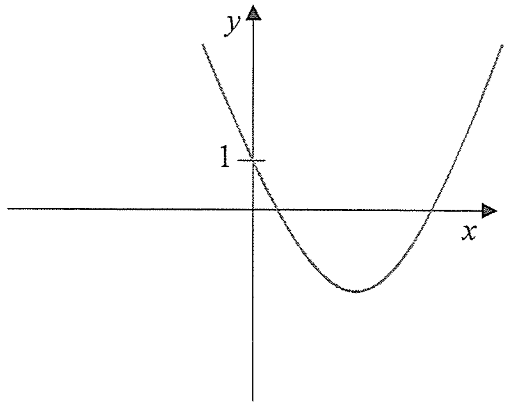

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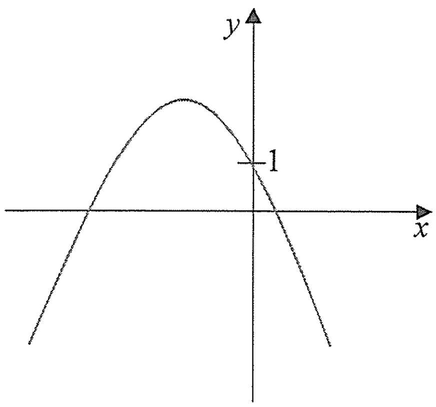
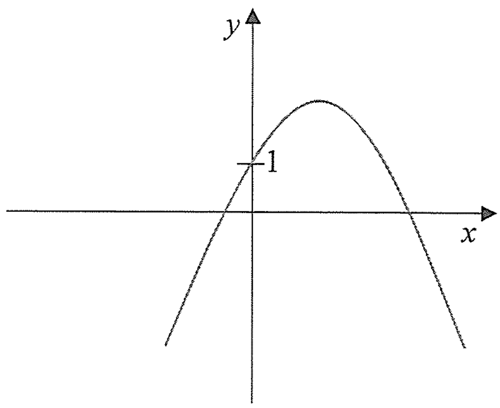


D



5. Given that $\log_x y = 5$ and $\log_x z = 8$.

What is the value of $\log_x \dfrac{xy}{\sqrt{z}}$?

A $6 - 2\sqrt{2}$

B $5 - 2\sqrt{2}$

C 2

D 1

End of Section I

Section II

42 marks
Attempt Questions 6 to 9.
Allow about 49 minutes for this section.

In questions 6 to 9, your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (13 marks)

(a) Solve $|5x-2|=13$. **2**

(b) Simplify $\dfrac{6^x-3^x}{2^x-1}$. **1**

(c) Solve $x^2-5x-6>0$. **2**

Question 6 continues on Page 5

(d) Consider $f(x)=x^2+6x-3$.

(i) Find $f(x-3)$. **1**

(ii) Show that $f(x-3)$ is even. **1**

e) Let $f\left(x\right)=\dfrac{1}{\sqrt{x}}$ and $g\left(x\right)=x^2-9$.

(i) Find an expression for $f\left(g\left(x\right)\right)$. **1**

(ii) By sketching the graph of $g\left(x\right)=x^2-9$ or otherwise find the domain of $y=f\left(g\left(x\right)\right)$. **2**

Question 6 continues on Page 6

(f) Write down the exact value of $\tan\left(\frac{5\pi}{3}\right)$. **1**

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(g) If $\frac{\pi}{2} < \beta < \pi$ and $\sin\beta = \frac{1}{7}$, find the exact value of $\tan\beta$. **2**

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End of Question 6

Section II extra writing space

If you use this space, clearly indicate which question you are answering

Question 7 (12 marks)

(a) Sketch the graph of $y = f(x)$, where $f(x)$ is defined below. **3**

$$f(x) = \begin{cases} \dfrac{1}{x} & \text{for } x < -2 \\ 4 - x^2 & \text{for } x \geq -2 \end{cases}$$

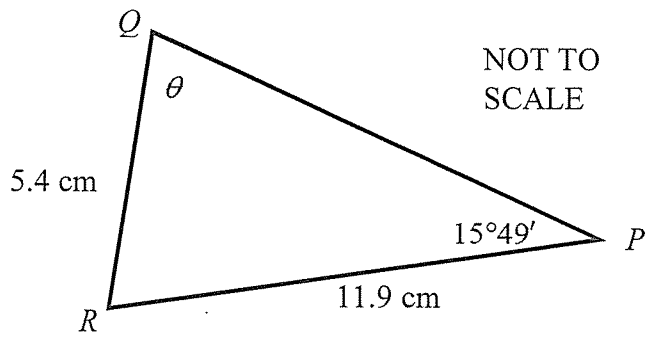
(b) The circle $x^2 + y^2 + 2x - 8y + 16 = 0$ is reflected in the y-axis. **3**

Sketch the reflected circle, showing the coordinates of the centre and the radius.

Question 7 continues on Page 9

(c) Solve $2\sin^2 x - 2 = \sqrt{3}\cos x$ for the domain $0° \le x \le 360°$. **3**

(d) In the triangle below $QR = 5.4$ cm, $RP = 11.9$ cm and $\angle QPR = 15°49'$



Find the value(s) of θ correct to the nearest minute. **3**

End of Question 7

Section II extra writing space

If you use this space, clearly indicate which question you are answering

Question 8 (9 marks)

(a) Solve $\tan 3\theta = -\frac{1}{\sqrt{3}}$ for $0 \leq \theta \leq \pi$. **3**

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(b) Prove that: **2**

$$\sec^2\theta + \sec\theta\tan\theta = \frac{1+\sin\theta}{\cos^2\theta}.$$

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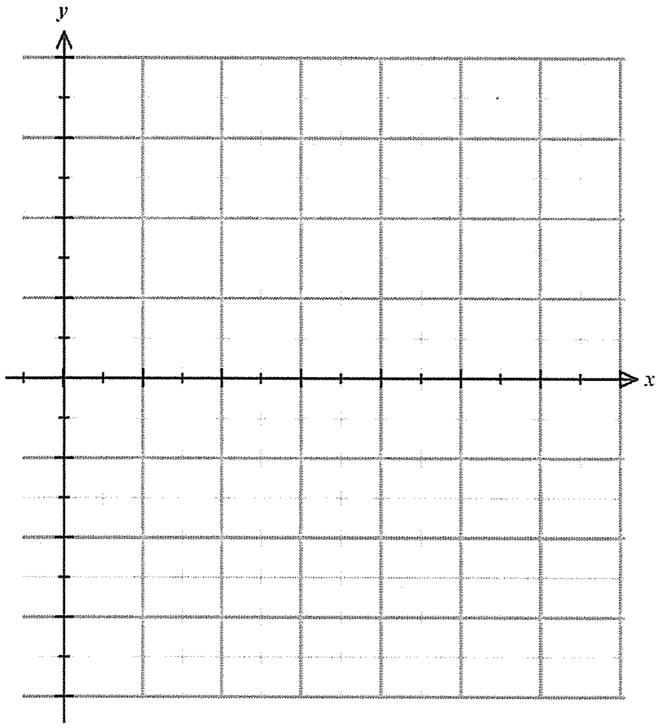
Question 8 continues on Page 12

(c) (i) Sketch $y = 2\sin\left(\frac{\pi x}{3}\right)$ for $0 \le x \le 6$, showing all important features. **2**



(ii) On the same set of axes above sketch $y = \frac{6-2x}{3}$ for $0 \le x \le 6$. **1**

(iii) Using your graphs, determine the number of solutions to the equation: **1**

$$\frac{6-2x}{3} = 2\sin\left(\frac{\pi x}{3}\right)$$

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End of Question 8

Section II extra writing space

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Section II extra writing space

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Question 9 (8 marks)

(a) It is known that y is directly proportional to the square of x and inversely proportional to the square root of z.

(i) Write a formula for y in terms of x and z. **1**

(ii) If $y = 6$ when $x = 2$ and $z = 4$, find y when $x = 3$ and $z = 16$. **2**

Question 9 continues on Page 16

(b) (i) By the grouping by pairs method or otherwise, express $x^3 - x^2 + x - 1$ as the product of linear and quadratic factors. **1**

(ii) Prove that $\dfrac{\operatorname{cosec}\theta - \sin\theta}{\sec\theta - \cos\theta} = \cot^3\theta$ **2**

(iii) Using your answers from part (i) and (ii) solve: **2**

$$\frac{\operatorname{cosec}\theta - \sin\theta}{\sec\theta - \cos\theta} + \cot\theta = \operatorname{cosec}^2\theta \text{ for } 0 \le \theta \le 2\pi .$$

End of Paper

Section II extra writing space

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Section I

5 marks
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---

1. What is the value of p so that $\dfrac{a^2a^{-3}}{\sqrt{a}} = a^p$?

$a^{2-3-\frac{1}{2}} = a^{\frac{-3}{2}}$

A -3

B $-\dfrac{3}{2}$

C $\dfrac{3}{2}$

D 2

2. Which interval gives the domain of $y = \log_3(7x-5)$?

$7x - 5 > 0$
$x > \frac{5}{7}$
$\therefore x \in \left(\frac{5}{7}, \infty\right)$

A $\left(\dfrac{5}{7}, \infty\right)$

B $\left(-\dfrac{5}{7}, \infty\right)$

C $\left[\dfrac{5}{7}, \infty\right)$

D $\left[-\dfrac{5}{7}, \infty\right)$

3. Which interval gives the range of $y = 3 - 7\cos 2x$?

$-1 \leq \cos 2x \leq 1$
$\cos 2x = -1$ then $y = 10$
$\cos 2x = 1$ then $y = -4$
So $-4 \leq y \leq 10$
$y \in [-4, 10]$

A $[-4,10]$

B $[1,5]$

C $[-2,5]$

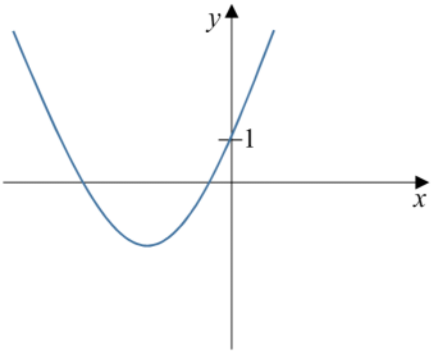
D $[-10,-4]$

4. Which of the following could represent the graph of $y = -x^2 + bx + 1$, where $b > 0$?

$-x^2$ so concave down

Axis of symmetry $= \frac{-b}{2(-1)} = \frac{b}{2} > 0$

A



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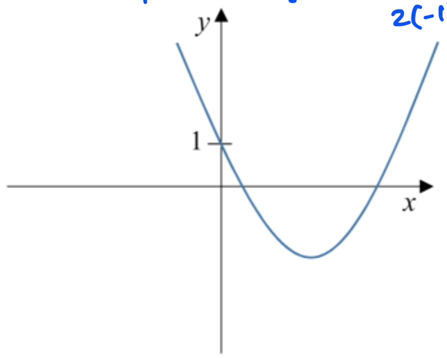

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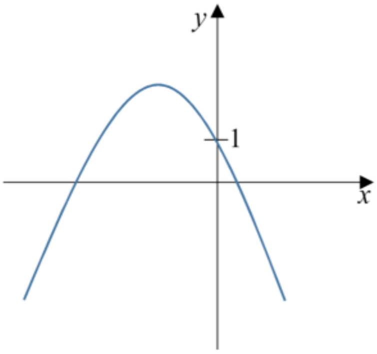

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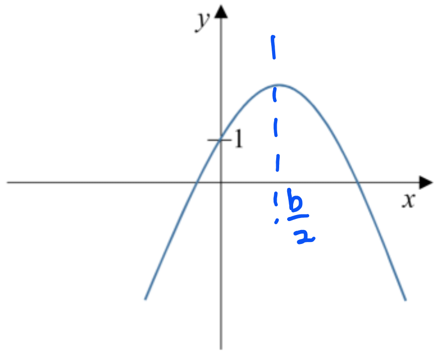


5. Given that $\log_x y = 5$ and $\log_x z = 8$.

What is the value of $\log_x \frac{xy}{\sqrt{z}}$?

A $6 - 2\sqrt{2}$

B $5 - 2\sqrt{2}$

C 2

D 1

$\log_x \frac{xy}{\sqrt{z}} = \log_x x + \log_x y - \frac{1}{2}\log_x z$

$= 1 + 5 - \frac{1}{2}(8)$

$= 1 + 5 - 4$

$= 2$

End of Section I

Section II

42 marks
Attempt Questions 6 to 9.
Allow about 49 minutes for this section.

In questions 6 to 9, your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (13 marks)

(a) Solve $|5x-2|=13$. **2**

$|5x-2|=13$

$5x-2=\pm 13$

$5x = 2 \pm 13$

$5x = 15$ or $5x = -11$

$x = 3$ or $x = -\frac{11}{5}$

(b) Simplify $\dfrac{6^x-3^x}{2^x-1}$. **1**

$\dfrac{6^x-3^x}{2^x-1} = \dfrac{(3\times 2)^x - 3^x}{2^x-1}$

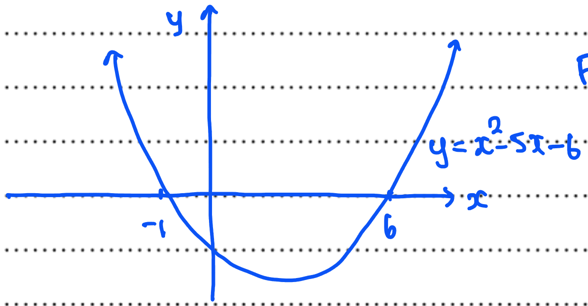
$= \dfrac{3^x\cancel{(2^x-1)}}{\cancel{2^x-1}}$

$= 3^x$

(c) Solve $x^2-5x-6>0$. **2**

$x^2-5x-6>0$

$(x-6)(x+1)>0$



From the graph,
$x^2-5x-6>0$
for $x<-1$ or $x>6$.

Question 6 continues on Page 5

(d) Consider $f(x) = x^2 + 6x - 3$.

(i) Find $f(x-3)$. **1**

$f(x-3) = (x-3)^2 + 6(x-3) - 3$

$= x^2 \cancel{-6x} + 9 + \cancel{6x} - 18 - 3$

$= x^2 - 12$

(ii) Show that $f(x-3)$ is even. **1**

Consider $g(x) = f(x-3)$ so $g(x) = x^2 - 12$ from (i)

Now $g(-x) = (-x)^2 - 12$

$= x^2 - 12$

$= g(x)$

$\therefore$ $g(x)$ is even. ie. $f(x-3)$ is even.

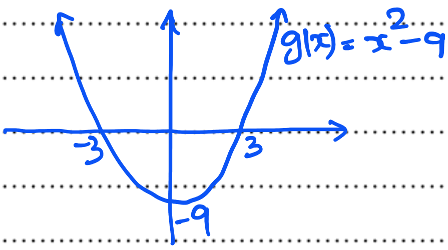
(e) Let $f(x) = \dfrac{1}{\sqrt{x}}$ and $g(x) = x^2 - 9$.

(i) Find an expression for $f(g(x))$. **1**

$f(g(x)) = f(x^2 - 9)$

$= \dfrac{1}{\sqrt{x^2 - 9}}$

(ii) By sketching the graph of $g(x) = x^2 - 9$ or otherwise find the domain of $y = f(g(x))$. **2**



$f(g(x)) = \dfrac{1}{\sqrt{x^2 - 9}}$

we need $x^2 - 9 > 0$

From the graph,

$x < -3$, $x > 3$

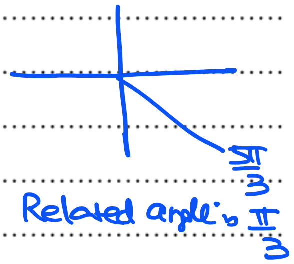
$\therefore$ Domain of $f(g(x))$ is $(-\infty, -3) \cup (3, \infty)$

Question 6 continues on Page 6

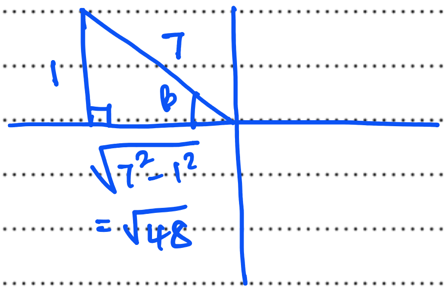
(f) Write down the exact value of $\tan\left(\frac{5\pi}{3}\right)$. **1**

$\tan\left(\frac{5\pi}{3}\right) = -\tan\frac{\pi}{3}$

$= -\sqrt{3}$



(g) If $\frac{\pi}{2} < \beta < \pi$ and $\sin\beta = \frac{1}{7}$, find the exact value of $\tan\beta$. **2**



$\tan\beta = -\frac{1}{\sqrt{48}}$

$= -\frac{1}{4\sqrt{3}}$

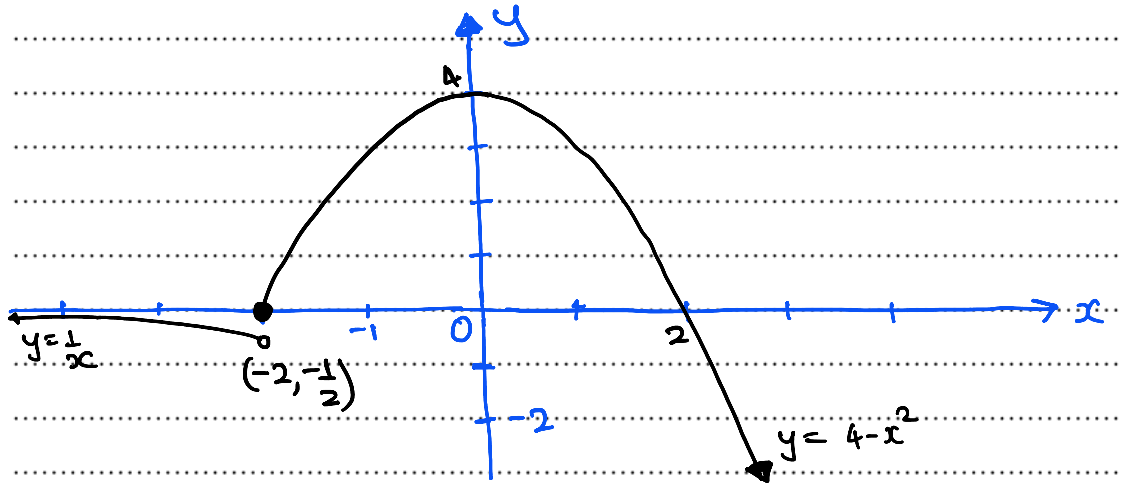
$= -\frac{\sqrt{3}}{12}$

End of Question 6

Question 7 (12 marks)

(a) Sketch the graph of $y = f(x)$, where $f(x)$ is defined below. **3**

$$f(x) = \begin{cases} \dfrac{1}{x} & \text{for } \; x < -2 \\ 4 - x^2 & \text{for } \; x \geq -2 \end{cases}$$



(b) The circle $x^2 + y^2 + 2x - 8y + 16 = 0$ is reflected in the y-axis. **3**

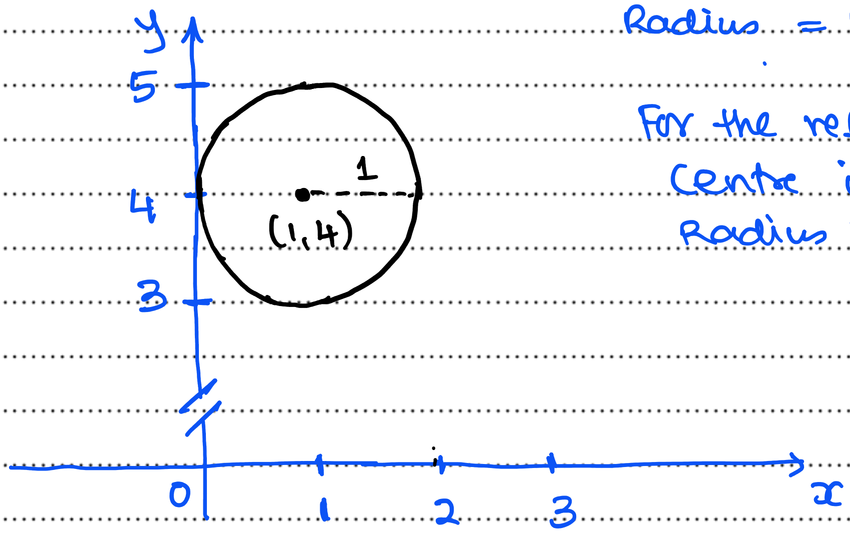
Sketch the reflected circle, showing the coordinates of the centre and the radius.

Completing the square in x and y

$x^2 + 2x + 1 + y^2 - 8y + 16 = -16 + 16 + 1$

$(x+1)^2 + (y-4)^2 = 1$ centre is $(-1, 4)$

Radius $= 1$



For the reflected circle

centre is $(1, 4)$

Radius is 1

Question 7 continues on Page 9

(c) Solve $2\sin^2 x - 2 = \sqrt{3}\cos x$ for the domain $0° \le x \le 360°$. **3**

$2\sin^2 x - 2 = \sqrt{3}\cos x$

$2(1-\cos^2 x) - 2 = \sqrt{3}\cos x$

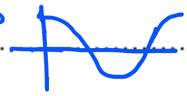
$\cancel{2} - 2\cos^2 x \cancel{-2} = \sqrt{3}\cos x$

$2\cos^2 x + \sqrt{3}\cos x = 0$

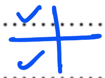
$\cos x\,(2\cos x + \sqrt{3}) = 0$

$\cos x = 0$ or $2\cos x + \sqrt{3} = 0$

$x = 90°, 270°$



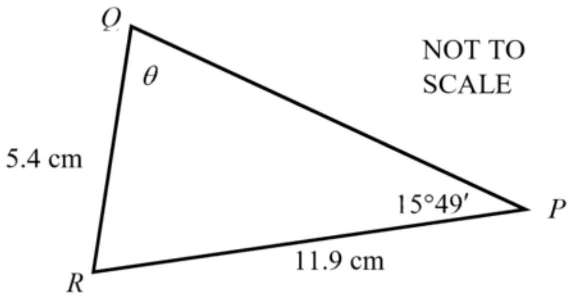
$\cos x = -\frac{\sqrt{3}}{2}$



$x = 150°, 210°$

$\therefore$ solutions are $x = 90°, 150°, 210°, 270°$

(d) In the triangle below $QR = 5.4$ cm, $RP = 11.9$ cm and $\angle QPR = 15°49'$



Find the value(s) of θ correct to the nearest minute. **3**

$\frac{\sin\theta}{11.9} = \frac{\sin(15°49')}{5.4}$ Using the Sine Rule

$\sin\theta = \frac{11.9 \times \sin(15°49')}{5.4}$

$= 0.6006\ldots$

$\theta = 36°54'57''$ or $180° - 36°54'57''$

$= 36°55'$ or $143°5'$ (nearest minute)

Check: $143°5' + 15°49' = 158°54' < 180°$

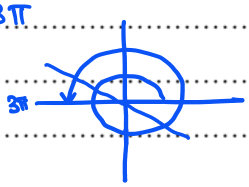
$\therefore$ this is a valid solution for this triangle

End of Question 7

Question 8 (9 marks)

(a) Solve $\tan 3\theta = -\frac{1}{\sqrt{3}}$ for $0 \leq \theta \leq \pi$. **3**

$\tan 3\theta = -\frac{1}{\sqrt{3}}$ $\qquad 0 \leq 3\theta \leq 3\pi$



Related angle is $\frac{\pi}{6}$

$3\theta = \frac{5\pi}{6}, \frac{11\pi}{6}, \frac{17\pi}{6}$

$\theta = \frac{5\pi}{18}, \frac{11\pi}{18}, \frac{17\pi}{18}$

(b) Prove that: **2**

$$\sec^2\theta + \sec\theta\tan\theta = \frac{1+\sin\theta}{\cos^2\theta}.$$

LHS $= \sec^2\theta + \sec\theta\tan\theta$

$= \frac{1}{\cos^2\theta} + \frac{1}{\cos\theta} \times \frac{\sin\theta}{\cos\theta}$

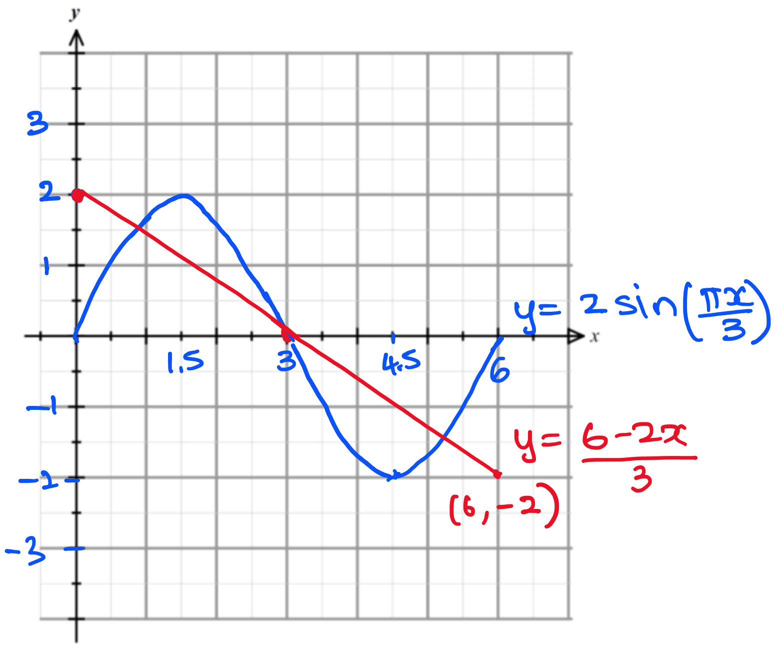
$= \frac{1}{\cos^2\theta} + \frac{\sin\theta}{\cos^2\theta}$

$= \frac{1+\sin\theta}{\cos^2\theta}$

$=$ RHS

Question 8 continues on Page 12

(c) (i) Sketch $y = 2\sin\left(\frac{\pi x}{3}\right)$ for $0 \le x \le 6$, showing all important features. **2**



(ii) On the same set of axes above sketch $y = \frac{6-2x}{3}$ for $0 \le x \le 6$. **1**

(iii) Using your graphs, determine the number of solutions to the equation: **1**

$$\frac{6-2x}{3} = 2\sin\left(\frac{\pi x}{3}\right)$$

From the graphs, we can see that there are 3 intersection points in this domain for $y = \frac{6-2x}{3}$ and $y = 2\sin\left(\frac{\pi x}{3}\right)$

The x-coordinates of these points of intersection are the solutions to $\frac{6-2x}{3} = 2\sin\left(\frac{\pi x}{3}\right)$

Hence, there are three solutions to this equation.

End of Question 8

Question 9 (8 marks)

(a) It is known that y is directly proportional to the square of x and inversely proportional to the square root of z.

(i) Write a formula for y in terms of x and z. **1**

$y \propto \frac{x^2}{\sqrt{z}}$

$\therefore \quad y = \frac{k x^2}{\sqrt{z}}$

(ii) If $y = 6$ when $x = 2$ and $z = 4$, find y when $x = 3$ and $z = 16$. **2**

Sub $y=6, \ x=2, \ z=4$

$6 = k \times \frac{2^2}{\sqrt{4}}$

$k = \frac{2 \times 6}{4}$

$k = 3$

$\therefore \quad y = \frac{3x^2}{\sqrt{z}}$

when $x=3$ and $z=16$

$y = \frac{3 \times 9}{\sqrt{16}}$

$= \frac{27}{4}$

Question 9 continues on Page 16

(b) (i) By the grouping by pairs method or otherwise, express $x^3 - x^2 + x - 1$ as the product of linear and quadratic factors. **1**

$$x^3 - x^2 + x - 1 = x^2(x-1) + 1(x-1)$$
$$= (x^2+1)(x-1)$$

(ii) Prove that $\dfrac{\operatorname{cosec}\theta - \sin\theta}{\sec\theta - \cos\theta} = \cot^3\theta$ **2**

$$LHS = \frac{\operatorname{cosec}\theta - \sin\theta}{\sec\theta - \cos\theta} = \frac{\frac{1}{\sin\theta} - \sin\theta}{\frac{1}{\cos\theta} - \cos\theta} \quad \begin{matrix}\times \sin\theta\cos\theta \\ \times \sin\theta\cos\theta\end{matrix}$$

$$= \frac{\cos\theta - \sin^2\theta\cos\theta}{\sin\theta - \sin\theta\cos^2\theta}$$

$$= \frac{\cos\theta(1-\sin^2\theta)}{\sin\theta(1-\cos^2\theta)}$$

$$= \frac{\cos\theta \cdot \cos^2\theta}{\sin\theta \cdot \sin^2\theta}$$

$$= \frac{\cos^3\theta}{\sin^3\theta}$$

$$= \cot^3\theta = RHS$$

(iii) Using your answers from part (i) and (ii) solve: **2**

$$\frac{\operatorname{cosec}\theta - \sin\theta}{\sec\theta - \cos\theta} + \cot\theta = \operatorname{cosec}^2\theta \text{ for } 0 \le \theta \le 2\pi.$$

$$\frac{\operatorname{cosec}\theta - \sin\theta}{\sec\theta - \cos\theta} + \cot\theta = \operatorname{cosec}^2\theta$$

$\cot^3\theta + \cot\theta = \operatorname{cosec}^2\theta$ using (ii)

$\cot^3\theta + \cot\theta = 1 + \cot^2\theta$

$\cot^3\theta - \cot^2\theta + \cot\theta - 1 = 0$

$(\cot^2\theta + 1)(\cot\theta - 1) = 0$ using (i)

(continued)

Q9 b) iii) continued

$\cot^2\theta + 1 = 0$ or $\cot\theta - 1 = 0$

$\cot^2\theta = -1$ or $\cot\theta = 1$

no solutions $\theta = \frac{\pi}{4}, \frac{5\pi}{4}$

$\therefore$ solutions are $\theta = \frac{\pi}{4}, \frac{5\pi}{4}$

End of Paper

Year 11 Mathematics Advanced – Task 2 Markers Feedback

Question 6

a) Well done.

b) Students who recognised $6^x = 2^x \times 3^x$ simplified well. Without this step, it was not possible to achieve any marks.

c) Generally well done. Some students attempted to solve the inequality without sketching the associated parabola and frequently ran into problems.

d) (i) Well done.
(ii) Generally well done. Some responses had two unequal quantities but somehow managed to make them equal.

e) (i) Well done.
(ii) Mixed effort and results. A number of students had no idea how to approach this problem. Looking at the restrictions inherent in the equation of $f\left(g\left(x\right)\right)$ could have provided a starting point.

f & g) Both generally well done. Most errors were due to the omission of a negative sign, either by ignoring or incorrectly deducing the quadrant.

Question 7

a) Generally well done. Take care at the endpoints of each piece. At $x = 2$, it is an open circle on the hyperbola and a filled in circle on the parabola. The two pieces do not join up.

b) Well done. Students are advised to read the question carefully as some students forgot to reflect the circle. Students should note that when sketching a circle the scale on the *x* and *y* axes should be chosen to be the same – otherwise the sketch will not appear to be circular.

c) Students should be careful not to cancel a common factor of $\cos x$ on both sides as this results in the loss of a solution. Instead, the terms should be collected on one side and the expression factorised to yield two branches each of which yields two solutions.

d) When using the Sine Rule to find an unknown angle in a triangle, students should always consider both the acute angle and obtuse angle solutions unless the question tells you if the angle is acute or obtuse. Diagrams are generally not to scale and you cannot rely on the appearance of the angle to decide if both solutions are valid. Instead check the validity of the solutions using the angle sum of a triangle. Many students did not list the obtuse angle solution or failed to verify that it was a valid solution.

Question 8

a) Students who remembered to adjust the domain first were mostly successful in finding all 3 solutions. If they didn't do this, almost all made errors in their solutions. Other errors were mainly in not using radians or trivial errors in working with fractions.

b) Generally quite well done but setting out can be improved. **Do not fudge** a "show" question. Stick with what you have, if you have not got time to correct or can not identify your error. This work may have merit. **Never** just jump from incorrect to what is to be shown. Also, do not cross out working if not replacing with another response. Your incorrect response may still be awarded some credit if you are seen to be making progress towards the solution.

c) Too many errors here. Amplitude was generally well done but period was not. The period is calculated as $\frac{2\pi}{n}$, which was not known by a significant number of students and is vital in drawing all trig functions.

Question 9

a) Many students had difficulty interpreting the information in the question. Rather than interpreting this as y varying with x and z at the same time (joint variation) many students set up two different proportionality statements. ii) Students who interpreted part i) incorrectly then calculated two different values for y based on their two equations. These students were awarded partial credit.

b) (i) Generally well done.

(ii) Students are encouraged to simplify the compound fraction by multiplying both top and bottom of the fraction by the least common denominators of all the fractions.

(iii) Students who noticed the links to the previous parts could successfully reduce the equation to two simple equations in $\cot\theta$. Some students failed to note that θ is acute and consequently did not eliminate a solution.