



**NORTH
SYDNEY
GIRLS HIGH
SCHOOL**

2022

**Year 11
Mathematics Advanced
Assessment Task 2**

Year 11 Mathematics Advanced

General Instructions

- Reading Time – 2 minutes
- Working Time – 55 minutes
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided
- In Questions 6 – 9, show relevant mathematical reasoning and/or calculations

Total marks – 45

Section I – 5 marks (pages 2 – 4)

- Attempt Questions 1 – 5
- Allow about 6 minutes for this section

Section II – 40 marks (pages 5 – 17)

- Attempt Questions 6 – 9
- Allow about 49 minutes for this section

NAME: _____

TEACHER: _____

Question	1 – 5	6	7	8	9	Total
Algebra and Arithmetic	/2	/4	/3			/9
Functions	/2	/6		/6		/14
Trigonometry	/1		/8	/4	/9	/22
	/5	/10	/11	/10	/9	/45

Section I

5 marks

Attempt Questions 1-5

Allow about 6 minutes for this section

Use the multiple choice answer sheet for Questions 1-5.

1. Which of the following is the value of $\frac{\log_7 3}{\log_7 10}$, correct to 3 decimal places?

A. 0.300

B. 0.429

C. 0.477

D. 1.429

2. Which of the following is the solution to the inequation $x^2 - 2x + 1 \leq 4$?

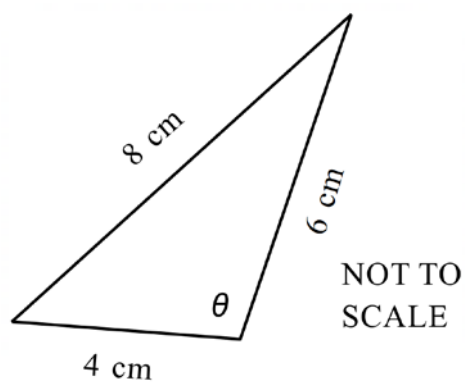
A. $x \leq 3$

B. $x \geq 1$

C. $-1 \leq x \leq 3$

D. $-3 \leq x \leq 1$

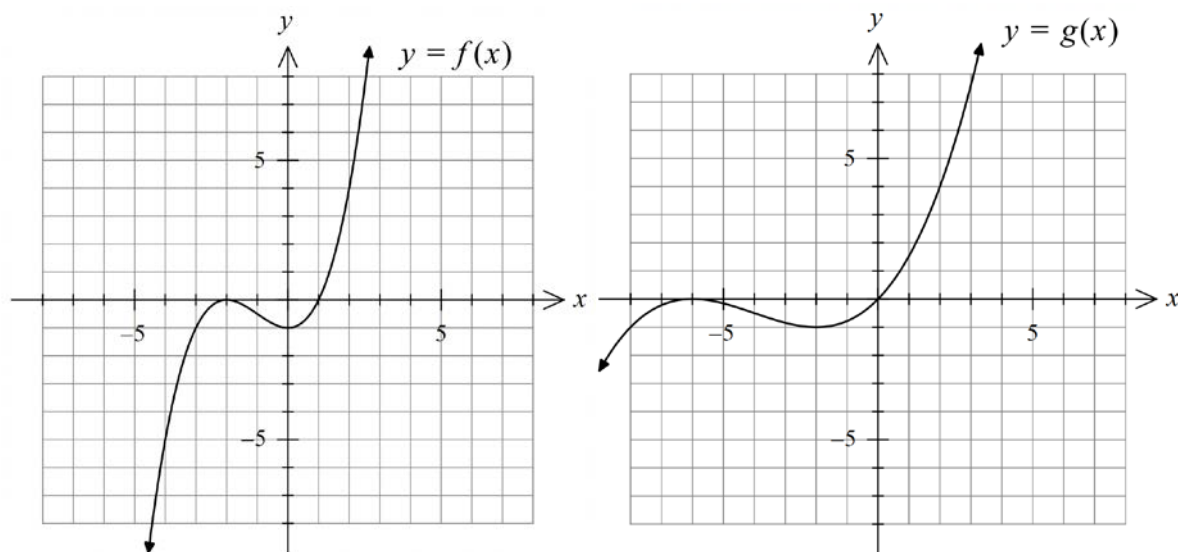
3. Consider the triangle given below.



Which of the following is correct?

- A. $\cos \theta = -\frac{1}{4}$
- B. $\cos \theta = -\frac{11}{16}$
- C. $\cos \theta = \frac{7}{8}$
- D. $\cos \theta = \frac{1}{2}$
4. Which interval gives the range of $y = 2 - 3^x$?
- A. $(-\infty, 2)$
- B. $(-\infty, 2]$
- C. $(2, \infty)$
- D. $[2, \infty)$

5. Consider the functions $y = f(x)$ and $y = g(x)$ graphed below. The graph of $y = g(x)$ is obtained by transforming the graph of $y = f(x)$.



Which of the following could be the equation for $g(x)$?

- A. $g(x) = f\left(2x + \frac{1}{2}\right)$
- B. $g(x) = f(2x + 1)$
- C. $g(x) = f\left(\frac{1}{2}x + \frac{1}{2}\right)$
- D. $g(x) = f\left(\frac{1}{2}x + 1\right)$

End of Section I



**NORTH
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Mathematics Advanced

Section II Answer Booklet 1

Section II

40 marks

Attempt Questions 6 to 9.

Allow about 49 minutes for this section.

Booklet 1 – Attempt Questions 6-7 (21 marks)

Booklet 2 – Attempt Questions 8-9 (19 marks)

Answer the questions in the space provided. These spaces provide guidance for the expected length of response.

In questions 6 to 9, your responses should include relevant mathematical reasoning and/or calculations.

Extra writing space is provided at the end of each booklet. If you use this space, clearly indicate which question you are answering.

Question 6 (10 marks)

(a) Solve $|2x - 3| = 4$.

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(b) Solve $3^{2x} = 16$. Leave your answer in exact form.

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Question 6 continues on Page 8

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(c) Consider $f(x) = \sqrt{2x+3}$.

(i) Find an expression for $f(3x+5)$. 1

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(ii) State the domain of $f(3x+5)$. 1

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(iii) Sketch the graph of $y = f(3x+5)$. 2
Clearly indicate the intercepts if they exist.

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Question 6 continues on Page 9

(iv) Hence solve the equation $f(3x+5) = x+1$.

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End of Question 6

Question 7 (11 marks)

- (a) Given that $\sec \alpha = \frac{\sqrt{6}}{2}$ and $\tan \alpha < 0$, find the exact value of $\sin \alpha$.

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- (b) Solve $\cos 2x = \frac{\sqrt{2}}{2}$ for $0 \leq x \leq 2\pi$.

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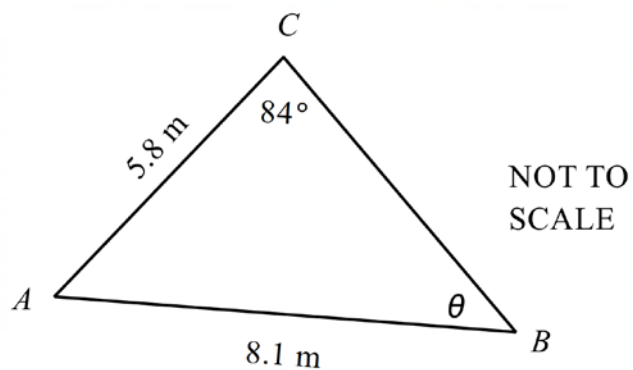
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Question 7 continues on Page 11

- (c) $\triangle ABC$ is given below. Use the information to find the area of the triangle.

4



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- (d) Solve the equation $\log_2(x-3) + \log_2(x+4) = 3$. Leave your answer(s) in exact value. 3

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End of Question 7

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**NORTH
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Mathematics Advanced

Section II Answer Booklet 2

Booklet 2 – Attempt Questions 8-9 (19 marks)

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In questions 6 to 9, your responses should include relevant mathematical reasoning and/or calculations.

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Question 8 (10 marks)

(a) Let $f(x) = \begin{cases} (x-1)^2, & x < 1 \\ \log_3 x, & x \geq 1 \end{cases}$.

(i) Sketch $y = f(x)$.

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(ii) Determine the value of $f(-3)$.

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(iii) Find the value(s) of x for which $f(x) = 2$. Leave your answer(s) in exact form.

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Question 8 continues on Page 15

(b) Sketch $y = \sec x$ for $0 \leq x \leq 2\pi$, showing all important features.

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(c) Prove that:

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$$\frac{2\sec^2 \theta - 2}{\sin \theta} = 2 \tan \theta \sec \theta .$$

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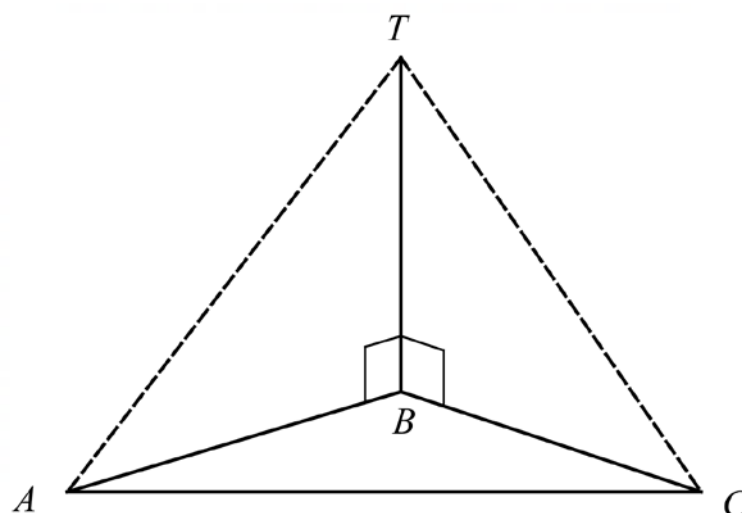
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End of Question 8

Question 9 (9 marks)

- (a) Sarah observes the base of a high-rise building, B , at a bearing of 067° from point A . The angle of elevation of the top of the building, T , is 32° . She then walks 168 m east to point C and observes T at the same angle of elevation of 32° . Let the height of the building, BT , be h metres.



- (i) Show that $AB = h \cot 32^\circ$.

1

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Question 9 continues on Page 17

(ii) Hence find h , correct to 3 significant figures.

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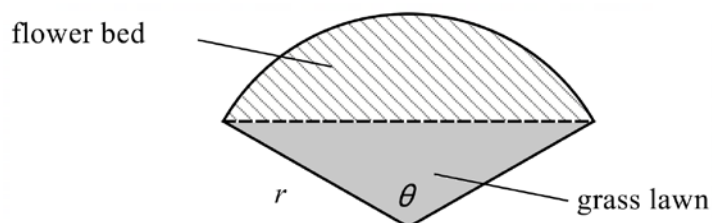
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Question 9 continues on Page 18



- (i) Find an expression for the area of the flower bed, A , in terms of r and θ . 1

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- (ii) Given that $A = 15\pi - 9 \text{ m}^2$, $r = 6 \text{ m}$ and the ratio of the areas of the flower bed to the grass lawn is $\frac{5\pi - 3}{3}$, find the exact value(s) of θ . 4

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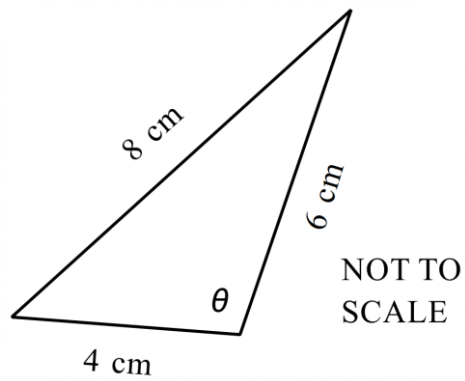
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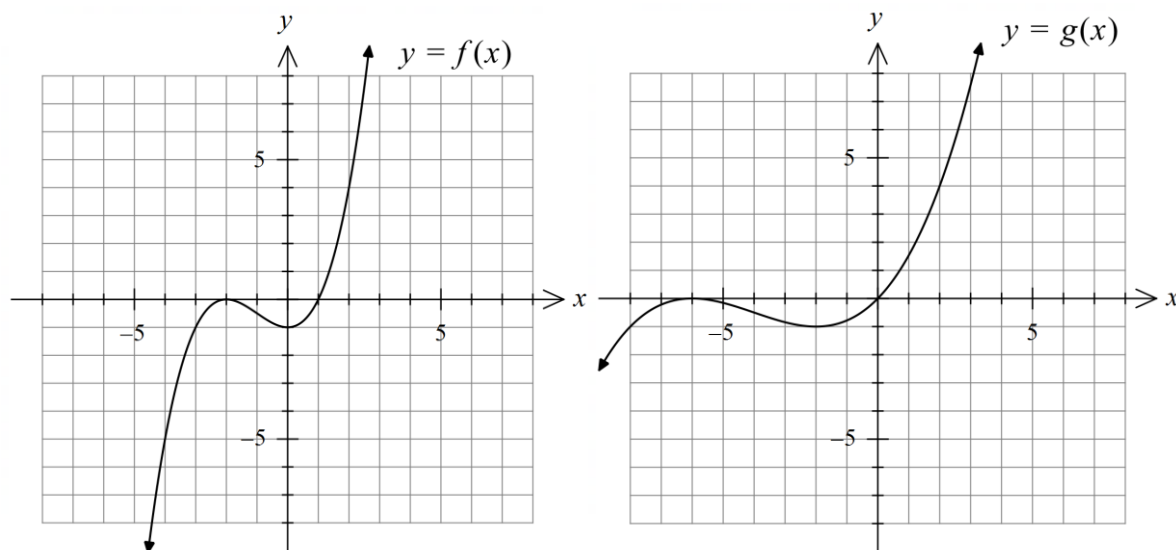


Which of the following is correct?

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End of Section I



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Section II Answer Booklet 1

Section II

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Question 6 (10 marks)

(a) Solve $|2x-3|=4$.

2

$$\begin{array}{ll} 2x-3=4 & -(2x-3)=4 \\ x=\frac{7}{2} & 2x-3=-4 \\ & x=-\frac{1}{2} \end{array}$$

Generally well done.

(b) Solve $3^{2x}=16$. Leave your answer in exact form.

2

$$\begin{array}{l} 3^{2x}=16 \\ 2x=\log_3 16 \\ x=\frac{1}{2}\log_3 16 \text{ or } \log_3 4 \end{array}$$

Generally well done. One mark of rewriting into the logarithm form and another mark for rearranging to make x the subject. Half mark deducted if they made an error in simplification step.

Question 6 continues on Page 8

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(c) Consider $f(x) = \sqrt{2x+3}$.

(i) Find an expression for $f(3x+5)$. 1

$$f(3x+5) = \sqrt{2(3x+5)+3} = \sqrt{6x+13}$$

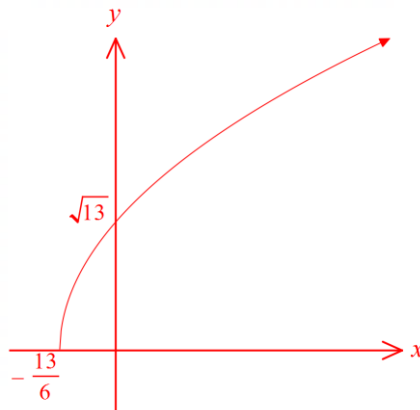
(ii) State the domain of $f(3x+5)$. 1

$$6x+13 \geq 0$$

$$x \geq -\frac{13}{6}$$

Generally well done. Half mark deducted for $x > -\frac{13}{6}$.

(iii) Sketch the graph of $y = f(3x+5)$. 2
Clearly indicate the intercepts if they exist.



One mark for correct shape of the graph, the other mark for indicating both intercepts correctly. No half marks awarded for this section.

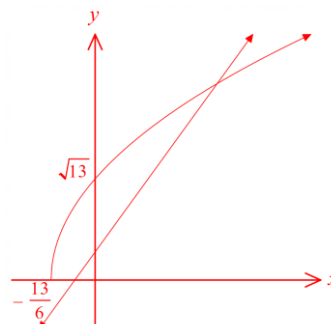
Question 6 continues on Page 9

(iv) Hence solve the equation $f(3x+5) = x+1$.

2

By sketching the graph of $y = x+1$ on the same axes, we can see that there should be only 1 positive solution.

$$\begin{aligned}\sqrt{6x+13} &= x+1 \\ 6x+13 &= (x+1)^2 \\ 6x+13 &= x^2 + 2x + 1 \\ x^2 - 4x - 12 &= 0 \\ (x-6)(x+2) &= 0 \\ x &= -2, 6\end{aligned}$$



Since $x > 0$ from the graph, then the solution must be $x = 6$

One mark for finding two possible solutions of x . Half mark for rejecting the right one, the other half mark for correctly verifying.

Many students verified algebraically (using substitution), even though the questions says “Hence,”. Only about 30% of students used their graphs from part iii) to reason their rejection of one solution.

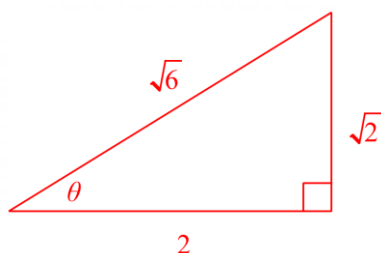
Some students wrote their final answer as a coordinates (6,7), and they are encouraged to be aware that they are to find the value of x in this question.

End of Question 6

Question 7 (11 marks)

- (a) Given that $\sec \alpha = \frac{\sqrt{6}}{2}$ and $\tan \alpha < 0$, find the exact value of $\sin \alpha$. 2

since $\sec \alpha = \frac{\sqrt{6}}{2} = \frac{H}{A}$, then $H = \sqrt{6}$, $A = 2$ and $O = \sqrt{(\sqrt{6})^2 - 2^2} = \sqrt{2}$



since $\sec \alpha > 0$ and $\tan \alpha < 0$, then α must be in the 4th quadrant and $\sin \alpha < 0$

$$\therefore \sin \alpha = -\frac{\sqrt{2}}{\sqrt{6}} = -\frac{1}{\sqrt{3}}$$

Mostly well done although a number of students did not recognise that this was a 4th quadrant angle and so left out the negative sign.

Please do not write $\sin \alpha = \frac{\sqrt{2}}{\sqrt{6}} = -\frac{\sqrt{2}}{\sqrt{6}}$ because one bit of working does not make sense

from the other. If you are referring to related first quadrant angle use a different pronumeral (e.g. β) or see solution for appropriate setting out.

- (b) Solve $\cos 2x = \frac{\sqrt{2}}{2}$ for $0 \leq x \leq 2\pi$. 2

since $0 \leq x \leq 2\pi$ then $0 \leq 2x \leq 4\pi$

$$\cos 2x = \frac{\sqrt{2}}{2}$$

$$2x = \frac{\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}, \frac{15\pi}{4}$$

$$x = \frac{\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{15\pi}{8}$$

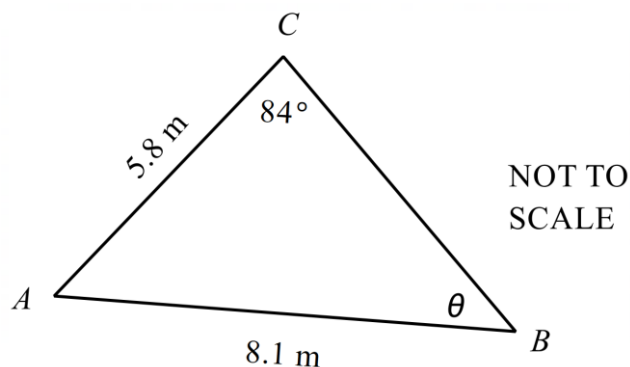
Some students worked in degrees and then converted their final answer to radians. This approach can be time-consuming and you are encouraged to switch to just working in radians when required.

Some students did not consider that $0 \leq 2x \leq 4\pi$ and so were missing some solutions.

Question 7 continues on Page 11

(c) $\triangle ABC$ is given below. Use the information to find the area of the triangle.

4



$$\frac{\sin \theta}{AC} = \frac{\sin 84^\circ}{AB}$$

$$\sin \theta = \frac{5.8 \sin 84^\circ}{8.1}$$

$$\theta = 45^\circ 24', 134^\circ 36'$$

since $84^\circ + 134^\circ 36' > 180$, then $\theta = 134^\circ 36'$ is rejected.

$$\therefore \theta = 45^\circ 24'$$

$$\text{Hence } \angle CAB = 180 - 84^\circ - 45^\circ 24' = 50^\circ 36'$$

$$A = \frac{1}{2}(5.8)(8.1)\sin 50^\circ 36' = 18.15 \text{ m}^2$$

Some students, when using the sine rule, did not show evidence that they had considered the ambiguous case. This was not penalised this time but could be in the future.

Some students just found an angle using the sine rule and then stopped. Please make sure you go back and check you have answered the question.

Some students rounded their results before the final answer and this led to problems with the accuracy of their final answer. This was penalised $\frac{1}{2}$ mark, Please do not round your answers until the end of a question.

- (d) Solve the equation $\log_2(x-3) + \log_2(x+4) = 3$. Leave your answer(s) in exact value. 3

$$\log_2(x-3) + \log_2(x+4) = 3$$

$$\log_2(x-3)(x+4) = 3$$

$$(x-3)(x+4) = 2^3$$

$$x^2 + x - 12 = 8$$

$$x^2 + x - 20 = 0$$

$$(x+5)(x-4) = 0$$

$$x = -5, 4$$

By considering the domains of the logs in the original equation $x > 3$ and $x > -4$, $\therefore x > 3$ hence the solution is $x = 4$.

Some students could not correctly apply log laws.

Most students did not consider whether their solutions to the quadratic equation satisfied the original log equation. Some students said $x > 0$ since you can't take the log of a negative number but in this example we needed $x > 3$.

End of Question 7



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Mathematics Advanced

Section II Answer Booklet 2

Booklet 2 – Attempt Questions 8-9 (19 marks)

Answer the questions in the space provided. These spaces provide guidance for the expected length of response.

In questions 6 to 9, your responses should include relevant mathematical reasoning and/or calculations.

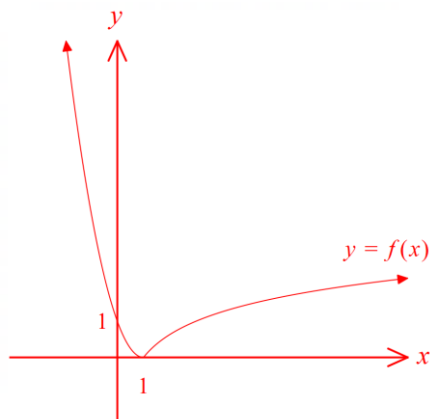
Extra writing space is provided at the end of each booklet. If you use this space, clearly indicate which question you are answering.

Question 8 (10 marks)

(a) Let $f(x) = \begin{cases} (x-1)^2, & x < 1 \\ \log_3 x, & x \geq 1 \end{cases}$.

(i) Sketch $y = f(x)$.

2



Generally well attempted question. Marks were deducted for not labelling the axis, points on the graphs or the intercept on the y axis and wrong sketch.

(ii) Determine the value of $f(-3)$.

1

$$f(-3) = (-3-1)^2 = 16$$

Most students could do this part correctly.

(iii) Find the value(s) of x for which $f(x) = 2$. Leave your answer(s) in exact form.

3

From the graph, we know that there are two solutions to $f(x) = 2$

$$(x-1)^2 = 2$$

$$\log_3 x = 2$$

$$x-1 = \pm\sqrt{2}$$

$$x = 3^2$$

$$x = 1 - \sqrt{2} \text{ since } f(x) = (x-1)^2 \text{ when } x < 1$$

$$x = 9$$

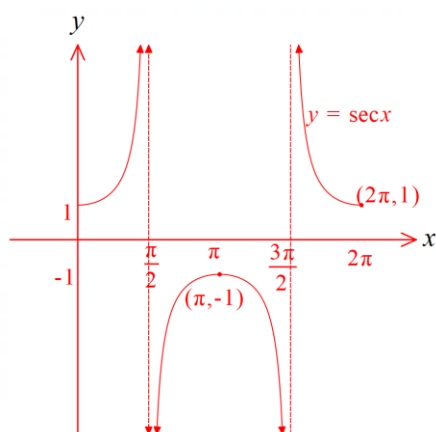
$$\therefore x = 1 - \sqrt{2}, 9$$

Students were able to find points where $f(x)=2$. Mark was deducted for not showing the reasoning in accepting or rejecting one solution and not calculating the other solution correctly.

Question 8 continues on Page 15

- (b) Sketch $y = \sec x$ for $0 \leq x \leq 2\pi$, showing all important features.

2



Most students were able to sketch the curve with significant features. Marks were deducted for not labelling points, axis, wrong sketch and extending the curve on the left side of the origin.

- (c) Prove that:

2

$$\frac{2\sec^2 \theta - 2}{\sin \theta} = 2 \tan \theta \sec \theta .$$

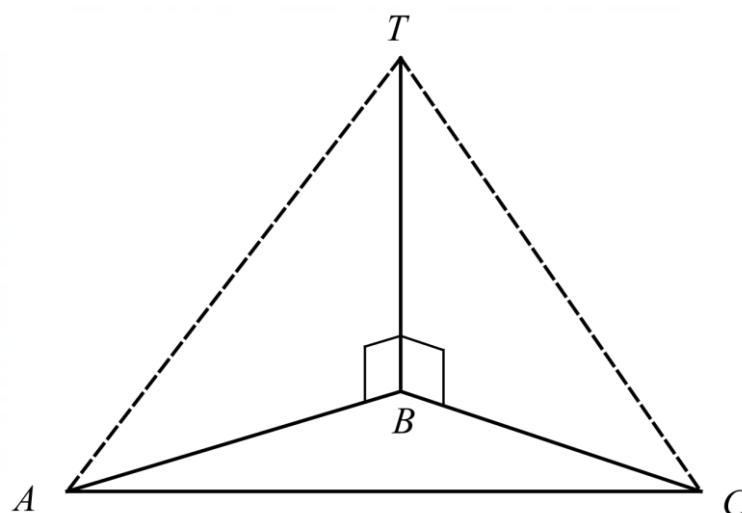
$$\begin{aligned} LHS &= \frac{2\sec^2 \theta - 2}{\sin \theta} \\ &= \frac{2(\sec^2 \theta - 1)}{\sin \theta} \\ &= \frac{2\tan^2 \theta}{\sin \theta} \\ &= \frac{2\sin^2 \theta}{\cos^2 \theta \sin \theta} \\ &= \frac{2\sin \theta}{\cos^2 \theta} \\ &= 2\left(\frac{\sin \theta}{\cos \theta}\right)\left(\frac{1}{\cos \theta}\right) \\ &= 2 \tan \theta \sec \theta \\ &= RHS \end{aligned}$$

Generally well attempted question. Common mistakes were writing wrong identities, incorrect steps in the working out and skipping the steps leading to the proof . Marks were deducted for these mistakes.

End of Question 8

Question 9 (9 marks)

- (a) Sarah observes the base of a high-rise building, B , at a bearing of 067° from point A . The angle of elevation of the top of the building, T , is 32° . She then walks 168 m east to point C and observes T at the same angle of elevation of 32° . Let the height of the building, BT , be h metres.



- (i) Show that $AB = h \cot 32^\circ$.

1

In $\triangle ABT$,

$$\tan 32^\circ = \frac{h}{AB}$$

$$AB = \frac{h}{\tan 32^\circ}$$

$$AB = h \cot 32^\circ$$

well done.

Question 9 continues on Page 17

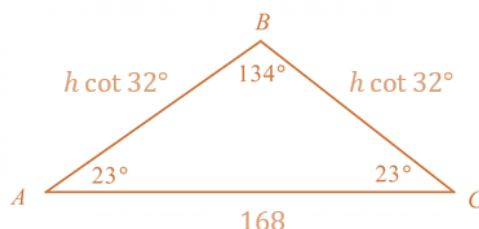
(ii) Hence find h , correct to 3 significant figures.

3

From (i) $AB = h \cot 32^\circ$, similarly $BC = h \cot 32^\circ$.

Since the bearing of B from A is 067° , then $\angle BAC = 23^\circ$.

Since $\triangle ABC$ is isosceles ($AB = BC$), then $\angle ABC = 180^\circ - 23^\circ - 23^\circ = 134^\circ$



Using sine rule,

$$\begin{aligned}\frac{AB}{\sin 23^\circ} &= \frac{AC}{\sin 134^\circ} \\ AB &= \frac{168 \sin 23^\circ}{\sin 134^\circ} \\ h \cot 32^\circ &= \frac{168 \sin 23^\circ}{\sin 134^\circ} \\ h &= \frac{168 \sin 23^\circ \tan 32^\circ}{\sin 134^\circ} \\ h &= 57.0 \text{ m (3 s.f.)}\end{aligned}$$

Generally poorly done. There were many mistakes going between the lines of algebra. This will continue to cost you marks in every question.

Draw diagrams! Drawing multiple lines on a 3D diagram leads to confusion, it is quick and easy to draw the top-down view and the side-view.

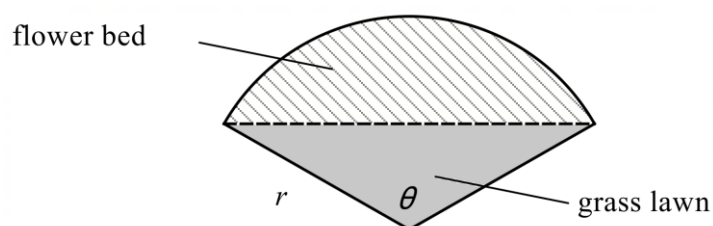
To work on the triangle on the ground, you could use sine rule, cosine rule or split the triangle as it was isosceles and use cosine.

If you used the fact that the triangle was isosceles, you had to justify this.

A common mistake was using 67° instead of 23° for the interior angle of the triangle. This can be easily solved by drawing the triangle you are working on.

Question 9 continues on Page 18

- (b) A garden in the shape of a minor sector is designed such that the flower bed and grass lawn are separated, as shown in the diagram below, where θ is in radians.



- (i) Find an expression for the area of the flower bed, A , in terms of r and θ . 1

$$A = A_{\text{sector}} - A_{\text{triangle}} = \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta = \frac{1}{2}r^2(\theta - \sin\theta)$$

well done.

- (ii) Given that $A = 15\pi - 9 \text{ m}^2$, $r = 6 \text{ m}$ and the ratio of the areas of the flower bed to the grass lawn is $\frac{5\pi - 3}{3}$, find the exact value(s) of θ . 4

$$\begin{aligned}
 A &= 15\pi - 9 \text{ and } \frac{A}{A_{\text{grass}}} = \frac{5\pi - 3}{3} \\
 \therefore A_{\text{grass}} &= \frac{3A}{5\pi - 3} = \frac{3(15\pi - 9)}{5\pi - 3} = 9 \\
 A_{\text{grass}} &= \frac{1}{2}r^2 \sin \theta \text{ and } r = 6 \\
 \therefore 9 &= \frac{1}{2}(6)^2 \sin \theta \\
 \sin \theta &= \frac{1}{2} \\
 \theta &= \frac{\pi}{6}, \frac{5\pi}{6} \\
 \text{when } \theta &= \frac{\pi}{6}, A = \frac{1}{2}(6)^2 \left(\frac{\pi}{6} - \sin \frac{\pi}{6} \right) = 3\pi - 9 \neq 15\pi - 9 \\
 \text{when } \theta &= \frac{5\pi}{6}, A = \frac{1}{2}(6)^2 \left(\frac{5\pi}{6} - \sin \frac{5\pi}{6} \right) = 15\pi - 9 \\
 \therefore \theta &= \frac{5\pi}{6}
 \end{aligned}$$

Several students found the answers for the angle, only a few then got the last mark by checking each possible solution.

Many students tried to work with the ratios in the form A:B, which lead to many mistakes. When equating ratios, writing them as a fraction allows for easier algebra.

There were multiple ways to approach the problem, however starting with the information from part (i) in this case didn't immediately lead to the answer. If you get stuck, go back and look at the rest of the information in the question.

End of Paper