Mrs Cooper	Mr Feng
Mrs Greenberg	Ms Hoang
Mrs Kerr	Ms Lau
Mrs Millar	Ms Peng
Mrs Squires	Mrs Ward

Name: ……

Teacher: ……

Pymble Ladies' College

YEAR 11

MATHEMATICS ADVANCED

ASSESSMENT TASK 1

Term 1
2023

General Instructions

- Working time 55 minutes
- Write using black or blue pen.
- Erasable pens are not to be used.
- Approved calculators may be used.
- Diagrams are not drawn to scale.
- All necessary working should be shown in every question.
- Marks may be deducted for careless or untidy work.
- Reference sheets are provided.

Mark	**/42**
Rank	**/**
Highest Mark	**/42**

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Mathematics Advanced

Name: ……

Teacher's Name: ……

Section I – Multiple Choice Answer Sheet

Select the alternative (A), (B), (C) or (D) that best answers the question. Fill in the response oval completely.

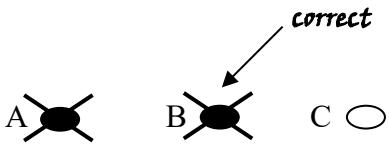
Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.



Assessment Task 1

1 A ○ B ○ C ○ D ○

2 A ○ B ○ C ○ D ○

3 A ○ B ○ C ○ D ○

4 A ○ B ○ C ○ D ○

5 A ○ B ○ C ○ D ○

6 A ○ B ○ C ○ D ○

7 A ○ B ○ C ○ D ○

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SECTION I
(7 Marks)

Answer using the Multiple Choice Answer Sheet provided.

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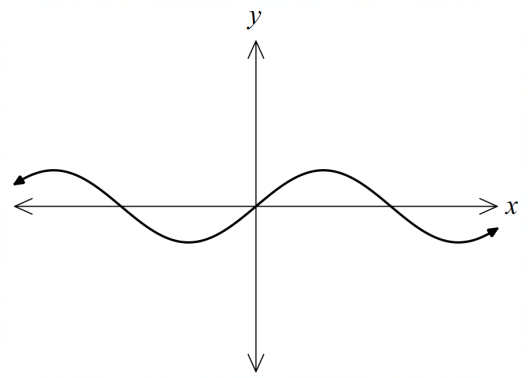
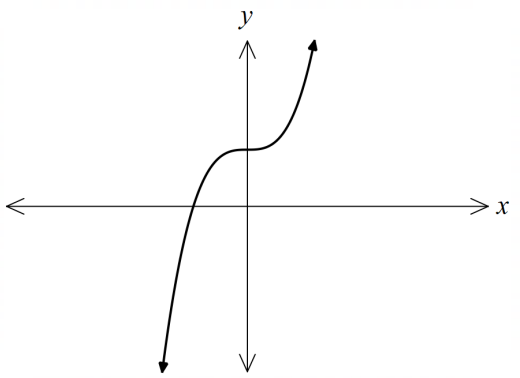
1 Which graph shows the graph of an odd function?

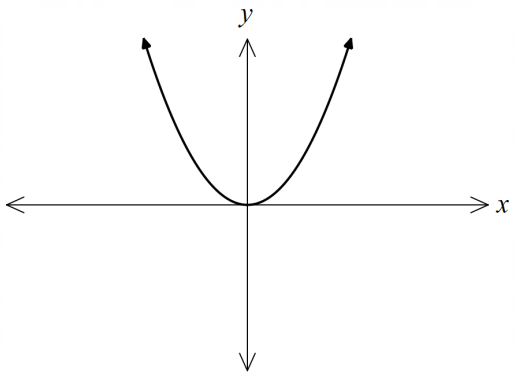
(A)



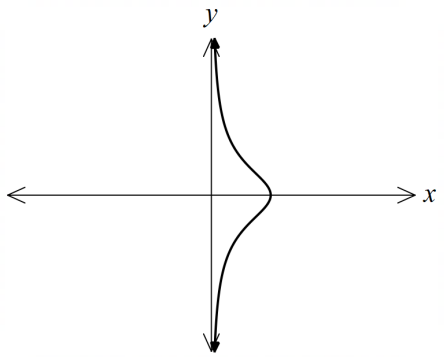
(B)



(C)



(D)



2 Which inequality defines the domain of the function $f(x)=\dfrac{1}{\sqrt{x+3}}$?

(A) $x > -3$

(B) $x \geq -3$

(C) $x < -3$

(D) $x \leq -3$

3 Which of the following is true for the equation $3x^2 - x - 2 = 0$?

(A) No real roots

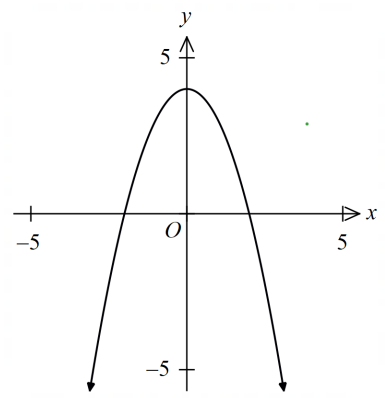
(B) One real root

(C) Two real distinct roots that are rational

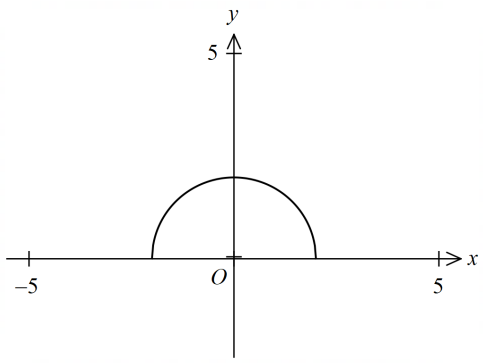
(D) Two real distinct roots that are irrational

4 Which of the following functions is one-to-one?

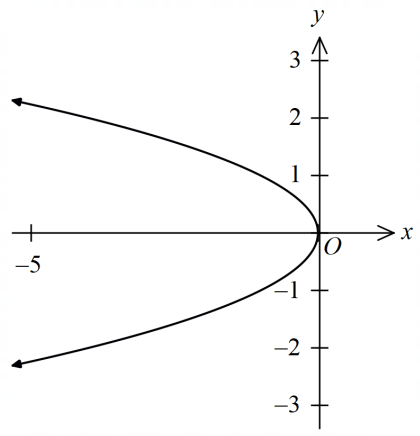
(A)



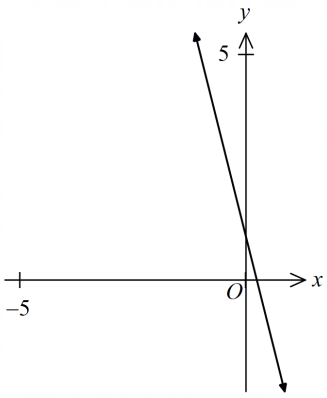
(B)



(C)



(D)



5 Which of the following is NOT equivalent to $\sqrt{24}$?

(A) $2\sqrt{6}$

(B) $\frac{1}{2}\sqrt{48}$

(C) $\sqrt{12}\times\sqrt{2}$

(D) $2\sqrt{3}\times\sqrt{2}$

6 The function $f(x)=\frac{x^2}{1-x^3}$ is which of the following:

(A) even

(B) odd

(C) neither

(D) cannot be determined

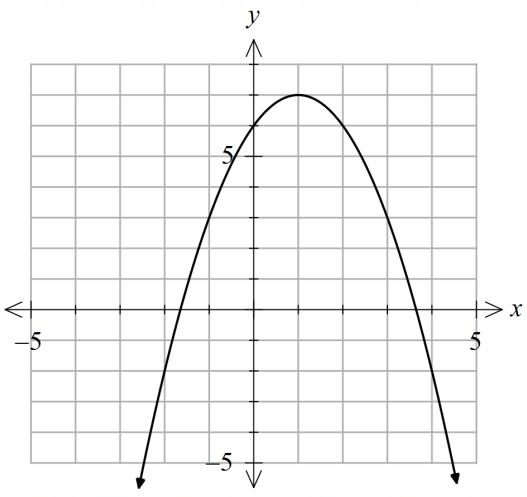
7 For the graph of the quadratic function below, what is the inequality that satisfies $f(x)>3$?

(A) $(-\infty,-1]\cup[3,\infty)$

(B) $(-\infty,-1)\cup(3,\infty)$

(C) $[-1,3]$

(D) $(-1,3)$



End of Section I

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SECTION II
(35 Marks)

Attempt all questions
Answer these questions in the spaces provided

Your responses should include relevant mathematical reasoning and/or calculations.

Question 8 (4 marks) **Marks**

(a) Solve for m $\dfrac{a^5\sqrt{a}}{a^{-2}} = a^m$ **2**

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(b) Express $y = 2x^2 - 12x + 11$ in the form $y = a(x-h)^2 + k.$ **2**

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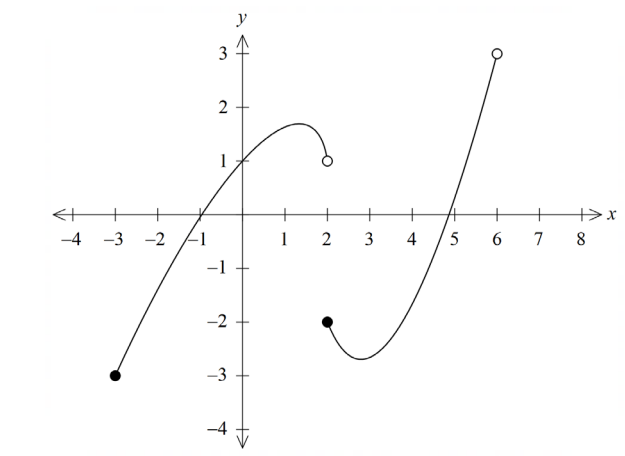
Question 9 (4 marks) **Marks**

(a) State the domain and range of the following functions. **2**



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(b) $y = 1 - \sqrt{1 - x}$ **2**

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Question 10 (6 marks) **Marks**

(a) A function is defined as $f(x) = x^3 - 2x$. **2**

Show this function is either even, odd or neither.

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(b) The function $y = f(x)$ is defined as follows: **2**

$$f(x) = \begin{cases} x^2 & \text{for } x \le 2 \\ 2x + 1, & \text{for } x > 2 \end{cases}$$

Evaluate $f(-1) + 2f(2) - f(3)$.

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(c) If $g(x) = x^2 + 3x + 5$, show that $g(a+1) = g(a) + 2(a+2)$. **2**

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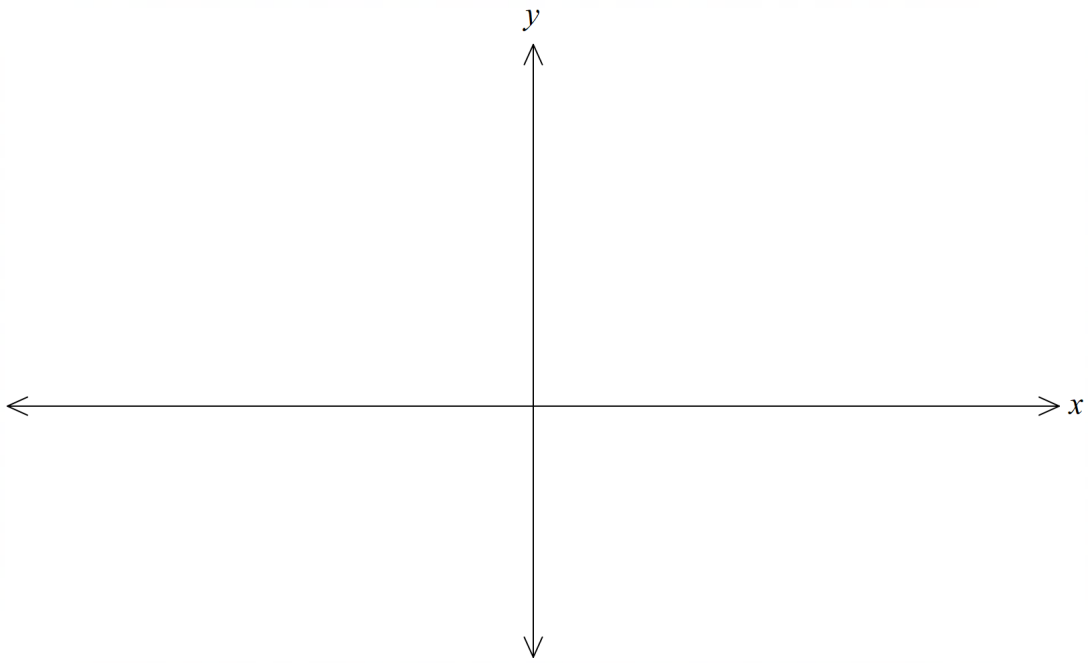
Question 11 (6 marks) **Marks**

(a) (i) On the axis provided, sketch the graph $y = x(x-4)^2$, showing all intercepts. **2**



(ii) Hence, find the values of x for which $x(x-4)^2 > 0$. **2**

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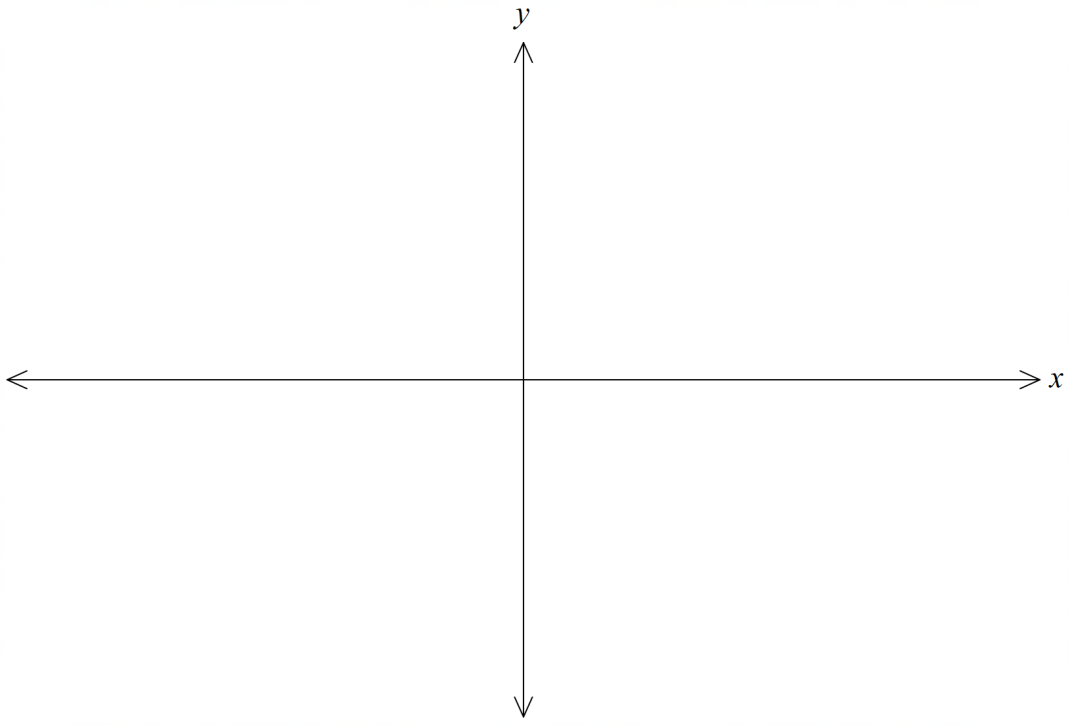
(iii) On the number plane below, sketch $y = f(-x)$ labelling all intercepts. **2**



Question 12 (7 marks) **Marks**

(a) Given that $9x^3 + 3x^2 - 12x - 4 = \left(3x^2 - 4\right)\left(3x + 1\right)$, hence, or otherwise, solve $18x^4 + 6x^3 - 24x^2 - 8x = 0$. **2**

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(b) For what values of m does the equation $6x^2 - 12mx - 7m + 3 = 0$ have equal roots. **3**

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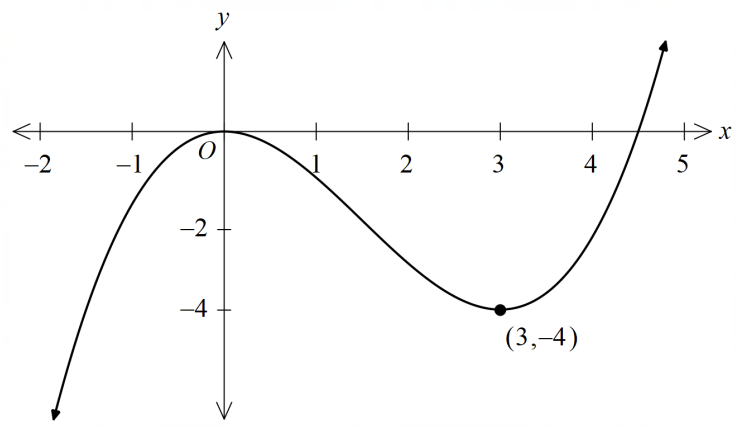
Question 12 continues on next page

Question 12 continued.

(c) The graph of $y = f(x)$ is sketched below.



For what values of k does the equation $f(x) = k$ have one solution? **2**

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Question 13 (4 marks) **Marks**

Let $f(x) = -x^2 + x + 4$ and $g(x) = x^2 - 2$.

(a) Find $g(f(3))$. **2**

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(b) Express $f(g(x))$ in the form $ax^4 + bx^2 + c$, where *a, b* and *c* are non-zero integers. **2**

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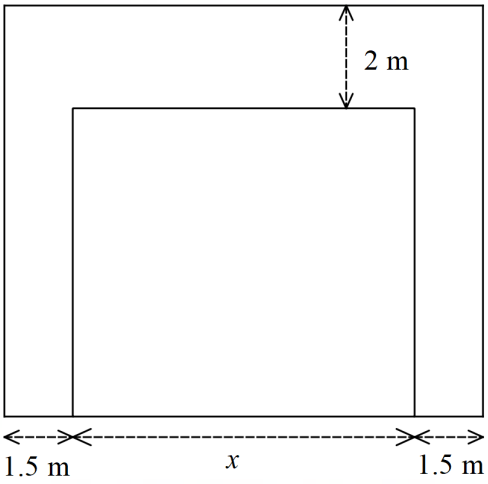
Question 14 (4 marks) **Marks**

A square garage door of length x metres is surrounded on three sides, by brickwork. The brick work is 1.5 metres wide on each side of the door and has a height of 2 metres above the door, as shown below.



NOT TO SCALE

(a) Find an expression for the area of the brickwork. **2**

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(b) To meet building regulations, the area of the brick wall must be exactly twice the area of the garage door. **2**

Find the value(s) of x that satisfy the building regulations.

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End of Assessment Task

NSW Education Standards Authority

2022 **HIGHER SCHOOL CERTIFICATE EXAMINATION**

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}\left(a + b\right)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{and } \alpha\beta\gamma = -\frac{d}{a}$$

Relations

$$\left(x - h\right)^2 + \left(y - k\right)^2 = r^2$$

Financial Mathematics

$$A = P\left(1 + r\right)^n$$

Sequences and series

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a\left(1 - r^n\right)}{1 - r} = \frac{a\left(r^n - 1\right)}{r - 1},\ r \neq 1$$

$$S = \frac{a}{1 - r},\ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

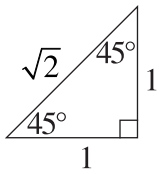
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

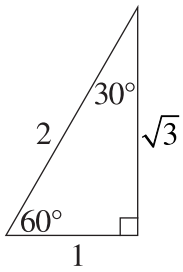
$$A = \frac{1}{2}ab\sin C$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A},\ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A},\ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A},\ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

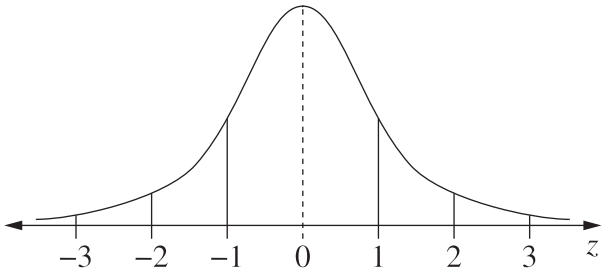
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)},\ P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1-p)^{n-r}$$

$$X \sim \text{Bin}(n, p)$$

$$\Rightarrow\ P(X = x) = \binom{n}{x} p^x (1-p)^{n-x},\ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1-p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)\left[f(x)\right]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+\left[f(x)\right]^2}$

Integral Calculus

$$\int f'(x)\left[f(x)\right]^n dx = \frac{1}{n+1}\left[f(x)\right]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}\,dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln\left|f(x)\right| + c$$

$$\int f'(x)a^{f(x)}\,dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - \left[f(x)\right]^2}}\,dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + \left[f(x)\right]^2}\,dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f\left(x_1\right) + \cdots + f\left(x_{n-1}\right)\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^{n} = x^{n} + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^{r} + \cdots + a^{n}$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^{2}+y^{2}}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_{1}x_{2} + y_{1}y_{2},$$

where $\underset{\sim}{u} = x_{1}\underset{\sim}{i} + y_{1}\underset{\sim}{j}$

and $\underset{\sim}{v} = x_{2}\underset{\sim}{i} + y_{2}\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$z = a + ib = r(\cos\theta + i\sin\theta)$$
$$= re^{i\theta}$$

$$\left[r(\cos\theta + i\sin\theta)\right]^{n} = r^{n}(\cos n\theta + i\sin n\theta)$$
$$= r^{n}e^{in\theta}$$

Mechanics

$$\frac{d^{2}x}{dt^{2}} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^{2}\right)$$

$$x = a\cos(nt+\alpha) + c$$

$$x = a\sin(nt+\alpha) + c$$

$$\ddot{x} = -n^{2}(x-c)$$

Mathematics Advanced

Name: .....

Teacher's Name: .....

Section I – Multiple Choice Answer Sheet

Select the alternative (A), (B), (C) or (D) that best answers the question. Fill in the response oval completely.

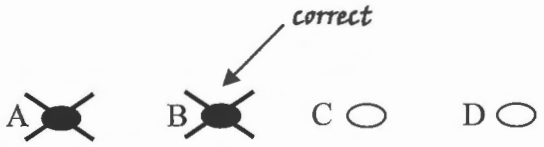
Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.



Assessment Task 1

1 A ● B ○ C ○ D ○

2 A ● B ○ C ○ D ○

3 A ○ B ○ C ● D ○

4 A ○ B ○ C ○ D ●

5 A ○ B ● C ○ D ○

6 A ○ B ○ C ● D ○

7 A ○ B ○ C ○ D ●

SECTION II
(35 Marks)

Attempt all questions
Answer these questions in the spaces provided

Your responses should include relevant mathematical reasoning and/or calculations.

Question 8 (4 marks) **Marks**

(a) Solve for m $\dfrac{a^5\sqrt{a}}{a^{-2}} = a^m$ 2

$\dfrac{a^{11/2}}{a^{-2}} = a^{15/2}$ | 2 | correct

$= a^{11/2-(-2)}$ | 1 | error

$\therefore m = 15/2$

(b) Express $y = 2x^2 - 12x + 11$ in the form $y = a(x-h)^2 + k$. 2

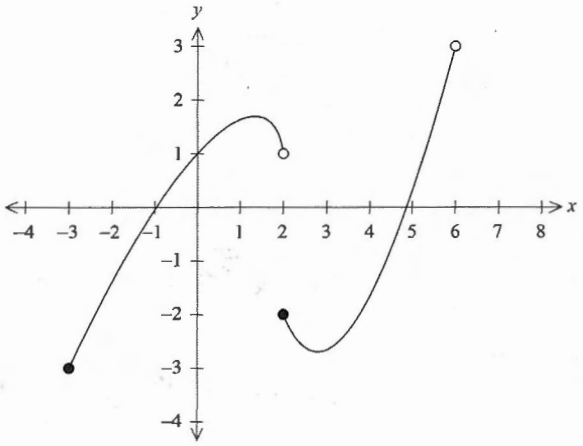
$y = 2(x^2 - 6x + 9) + 11 - 18$

$= 2(x-3)^2 - 7$ | 2 | correct

1	correctly completing the Square having $(x-3)^2$

Question 9 (4 marks) **Marks**

(a) State the domain and range of the following functions. **2**



(1) max if no $x \in$ $y \in$ and both D and R correct

Domain : $x \in [-3, 6)$

Range : $y \in [-3, 3)$

2	Both correct
1	one correct

(b) $y = 1 - \sqrt{1-x}$ **2**

Domain : $1 - x \geq 0 \quad x \leq 1$

Range : $y \leq 1$

2	Both correct
1	one correct

(1) max if no $x \in$ $y \in$ and both D and R correct.

Question 10 (6 marks) **Marks**

(a) A function is defined as $f(x) = x^3 - 2x$. **2**

Show this function is either even, odd or neither.

$f(a) = a^3 - 2a$

$f(-a) = (-a)^3 - 2(-a)$

$= -a^3 + 2a$

$= -(a^3 - 2a)$

$= -f(-a)$

$\therefore$ function is odd

2	Correct
1.	Subs $x=-a$ or equivalent

(b) The function $y = f(x)$ is defined as follows: **2**

$$f(x) = \begin{cases} x^2 & \text{for } x \le 2 \\ 2x+1, & \text{for } x > 2 \end{cases}$$

Evaluate $f(-1) + 2f(2) - f(3)$.

$= (-1)^2 + 2(2)^2 - (2(3)+1)$

$= 1 + 8 - 7$

$= 2$

2	Correct
1	1 error

(1) if correct equation
eg; $x^2 + 2x^2 - (2x+1)$

(c) If $g(x) = x^2 + 3x + 5$, show that $g(a+1) = g(a) + 2(a+2)$. **2**

$g(a+1) = (a+1)^2 + 3(a+1) + 5$

$= a^2 + 2a + 1 + 3a + 3 + 5$

$= a^2 + 5a + 9$

$g(a) = a^2 + 3a + 5$

$g(a) + 2(a+2) = a^2 + 3a + 5 + 2a + 4$

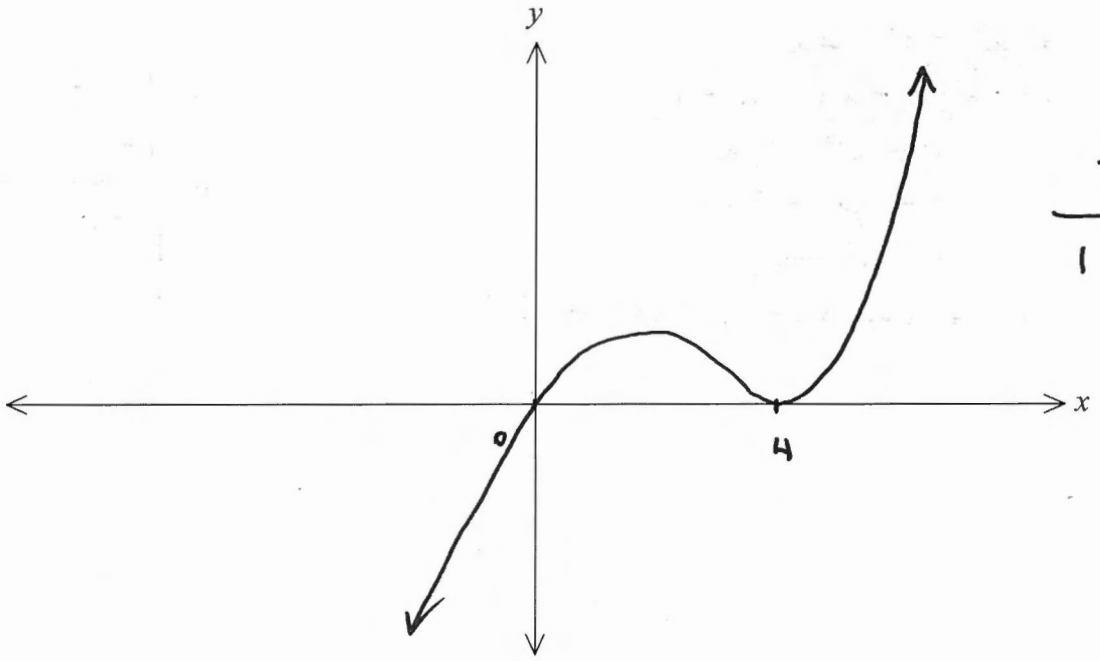
$= a^2 + 5a + 9$

$= g(a+1)$ As req'd

2	Correct
1	Substituting $x = a+1$ correctly

Question 11 (6 marks) **Marks**

(a) (i) On the axis provided, sketch the graph $y = x(x-4)^2$, showing all intercepts. **2**



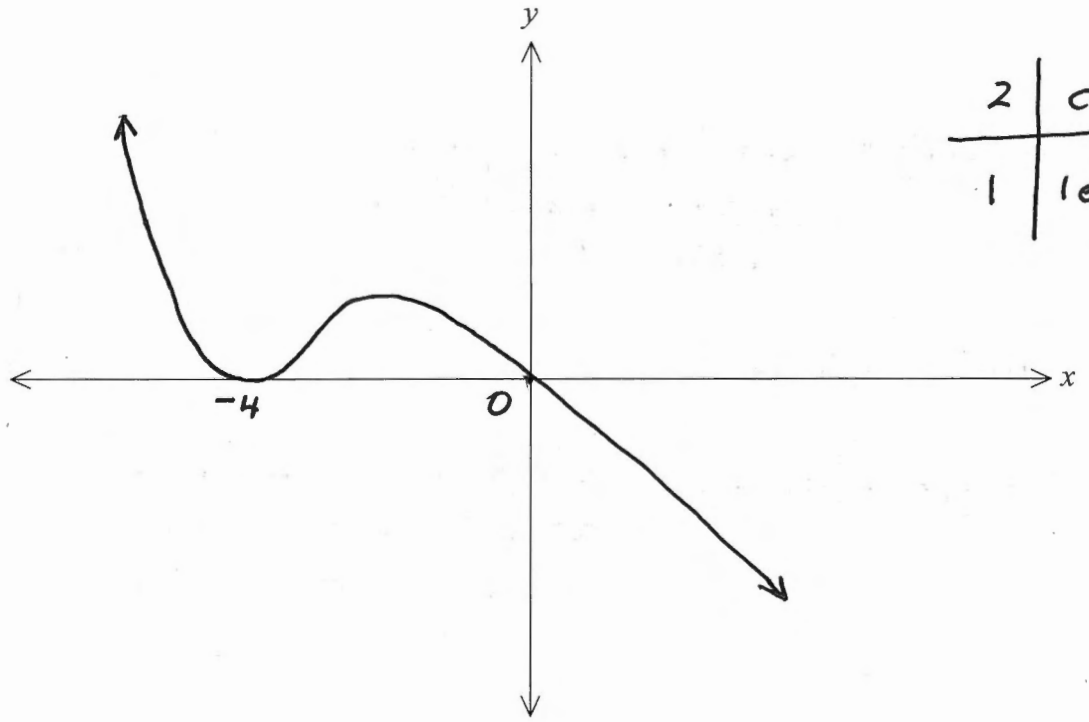
2	correct
1	error

(ii) Hence, find the values of x for which $x(x-4)^2 > 0$. **2**

$x \in (0,4) \cup (4,\infty)$

2	correct
1	error

(iii) On the number plane below, sketch $y = f(-x)$ labelling all intercepts. **2**



2	correct
1	error

Question 12 (7 marks) **Marks**

(a) Given that $9x^3+3x^2-12x-4=(3x^2-4)(3x+1)$, hence, or otherwise, solve $18x^4+6x^3-24x^2-8x=0$. **2**

$2x(9x^3+3x^2-12x-4)=0$

$2x(3x^2-4)(3x+1)=0$ *

$\therefore x=0$ or $x^2=4/3$ or $x=-1/3$

$x=\pm 2/\sqrt{3}$

2	Correct
1	Correct * factorisation

(b) For what values of m does the equation $6x^2-12mx-7m+3=0$ have equal roots. **3**

$\Delta=(-12m)^2-4(6)(3-7m)$

$=144m^2-72+168m$

$=24(6m^2+7m-3)$

3	Correct
2	Correct Δ / error in factorising
1	$\Delta=0$

Equal roots $\Delta=0$ $\therefore 24(6m^2+7m-3)=0$

$3m\ -2$ $\quad 6m^2+7m-3=0$

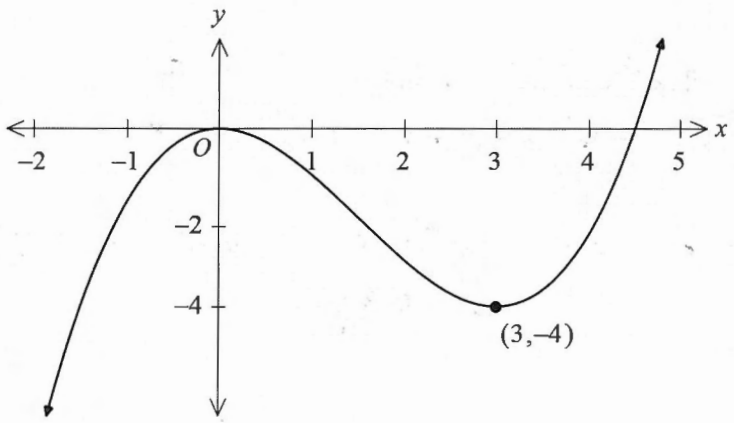
$2m\ \times 3$ $\quad (3m-2)(2m+3)=0$

$\therefore m=2/3$ or $m=-3/2$

Question 12 continues on next page

Question 12 continued.

(c) The graph of $y = f(x)$ is sketched below.



For what values of k does the equation $f(x) = k$ have one solution? **2**

$k > 0$ or $k < -4$ ..... 2 | correct

..... 1 | one correct

.....

.....

.....

Question 13 (4 marks) **Marks**

Let $f(x) = -x^2 + x + 4$ and $g(x) = x^2 - 2$.

(a) Find $g(f(3))$. **2**

$g(-3^2 + 3 + 4) = g(-2)$

$= (-2)^2 - 2$

$= 2$

2	correct
1	1 error

(b) Express $f(g(x))$ in the form $ax^4 + bx^2 + c$, where a, b and c are non-zero integers. **2**

$f(x^2 - 2) = -(x^2 - 2)^2 + (x^2 - 2) + 4$ *

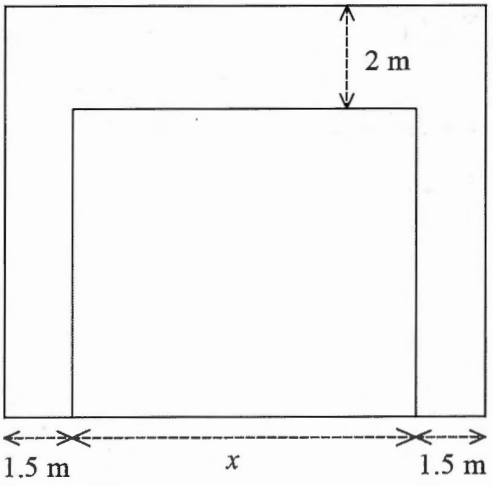
$= -(x^4 - 4x^2 + 4) + x^2 + 2$

$= -x^4 + 5x^2 - 2$

2	correct
1	correct + to *

Question 14 (4 marks) **Marks**

A square garage door of length x metres is surrounded on three sides, by brickwork. The brick work is 1.5 metres wide on each side of the door and has a height of 2 metres above the door, as shown below.



NOT TO SCALE

(a) Find an expression for the area of the brickwork. 2

$A_{Brickwork} = (x+3)(x+2) - x^2$ *

$= x^2 + 5x + 6 - x^2$

$= 5x + 6$

2 | correct

1 | correct to * | correct

(b) To meet building regulations, the area of the brick wall must be exactly twice the area of the garage door. 2

Find the value(s) of x that satisfy the building regulations.

$5x + 6 = 2x^2$

$2x^2 - 5x - 6 = 0$

$x = \dfrac{5 \pm \sqrt{25 + 48}}{4}$

$x = \dfrac{5 \pm \sqrt{73}}{4}$

But x is a length $\therefore x > 0$ $\therefore x = \dfrac{5 + \sqrt{73}}{4}$ m.

2 | correct

1 | correct quadratic and use of quadratic formula, not eliminating negative result

End of Assessment Task