THE SCOTS COLLEGE

HSC 2 UNIT MATHEMATICS

YEAR 12 ASSESSMENT 3

19TH JUNE 2018

GENERAL INSTRUCTIONS

- Working time – 45 minutes + 2 minutes reading time
- Write using black or blue pen
- Board-approved calculators may be used
- A Reference Sheet is provided
- Show all necessary working
- Attempt all questions 1- 4
- Clearly label your answer sheets with your candidate number .

WEIGHTING

25%

TOTAL MARKS

39

QUESTION 1 SHOW ALL WORKING (12 MARKS)
START A NEW ANSWER SHEET

a) **A front end loader removes Vm^3 of soil in t minutes,** [3]
where $V = 20t - \frac{t^2}{60}$

Find the rate at which the soil is being removed after 30 minutes.

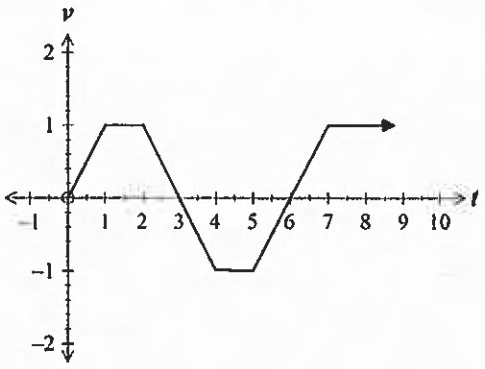
b) **Given the infinite series** [4]

$$\frac{x}{3} + \frac{2x^2}{9} + \frac{4x^3}{27} + \cdots$$

i) **Show that it is a geometric series**

ii) **Find the limiting sum in terms of x, in simplest form.**

c) **The diagram below describes the velocity, v, of an electrical pulse along a wire in metres/second. Answer the questions that follow.** [5]



i) **When is the pulse travelling in a negative direction?**
ii) **When is the pulse stopped?**
iii) **When is the acceleration zero for the first time?**
iv) **When is the pulse first back at the origin?**

QUESTION 4 SHOW ALL WORKING (11 MARKS)
START A NEW ANSWER SHEET

Rachel takes out a loan of $500 000. The loan is to be repaid in equal monthly repayments, $\$M$, at the end of each month, over 25 years. Reducible interest is charged at 4.8% per annum, calculated monthly. Let $\$A_n$, be the amount owing after the n^{th} repayment.

a) **Write down an expression for the amount owing after two months, $\$A_2$.** [2]

b) **Write an expression for $\$A_n$ and show that the monthly repayment is approximately $\$2864.98$.** [3]

c) **At the end of three years (36 months) Rachel pays off a lump sum of $100 000 in addition to her normal repayment. Show that her net balance is $\$366\,578.81$.** [3]

d) **How long will it take for her to pay off this balance using the same monthly repayments and reducible interest? Leave your answer to the nearest month.** [3]

****** End of Exam ******

QUESTION 2 SHOW ALL WORKING (7 MARKS)
START A NEW ANSWER SHEET

The amount A grams of a given carbon isotope in a dead tree trunk is given by

$$A = A_0 e^{-kt}$$

where A_0 and k are positive constants, and where the time t is measured in years from the death of the tree.

i) **Show that A satisfies the equation $\frac{dA}{dt} = -kA$** [1]

ii) **Find the value of k, if the amount of isotope is halved every 40,000 years. Give your answer to two significant figures.** [3]

iii) **For a particular dead tree trunk the amount of isotope is only 66 grams and the original amount in the living tree was 1.1 kg. How long ago did the tree die? Give your answer to the nearest 1000 years.** [3]

QUESTION 3 SHOW ALL WORKING (9 MARKS)
START A NEW ANSWER SHEET

a) **A particle is moving on the x-axis and is initially at the origin. Its velocity v metres per second, at time t seconds is given by**

$$v = \frac{2t}{8 + t^2}$$

i) **What is the initial velocity of the particle?** [1]

ii) **Find an expression for the acceleration of the particle.** [2]

iii) **Find the position of the particle when $t = 4$.** [3]

b) **A particle moves so that its distance in metres from the origin at time t seconds is given by $x = \frac{t^3}{3} - 5t^2 + 16t - 21$.** [3]
Find the times when that particle is stationary.

2U Task 3 2018.

HSC Mathematics Answer Booklet Task 3 Number:

Question Number : _____

Q1

(a) $V = 20t - \frac{t^2}{60}$

$\frac{dV}{dt} = 20 - \frac{2t}{60}$

$= 20 - \frac{t}{30}$ ✓

at $t = 30$, $\frac{dV}{dt} = 20 - \frac{30}{30}$ ✓

$= 20 - 1$

$= 19 \ m^3/min.$ ✓

(-½ no units)

(b) (i) Sufficient to show that

$\frac{T_2}{T_1} = \frac{T_3}{T_2} = r$

$\frac{T_2}{T_1} = \frac{2x^3/9}{x/3} = \frac{2x^3}{9} \times \frac{3}{x} = \frac{2x}{3}$

$\frac{T_3}{T_2} = \frac{4x^3/27}{2x^2/9} = \frac{4x^3}{27} \times \frac{9}{2x^2} = \frac{2x}{3}$

$\therefore r$ is consistent.

(ii) $S_\infty = \frac{a}{1-r}$

$= \frac{x/3}{1 - 2x/3}$ ✓

$= \frac{x/3}{\frac{3-2x}{3}}$

$= \frac{x}{3-2x}$ ✓

(c) (i) For $3 < t < 6$ ✓

(ii) 0, 3, 6. ✓✓

(iii) for $1 < t < 2$ or $t = 1$ ✓

(iv) $t = 6$. ✓

Q2

i) $A = A_0 e^{-kt}$

$\frac{dA}{dt} = -k A_0 e^{-kt}$

$\frac{dA}{dt} = -kA$ ✓

ii) $A_0 \rightarrow \frac{1}{2} A_0$

$\Rightarrow \frac{1}{2} A_0 = A_0 e^{-kt}$ ✓

$e^{-kt} = \frac{1}{2}$

$\ln e^{-kt} = \ln(\frac{1}{2})$ } ✓

$-kt = -\ln 2$

$k = \frac{\ln 2}{t}$

$k = \frac{\ln 2}{40000}$

$= 1.7 \times 10^{-5}$. ✓

(iii) $66 = 1100 \, e^{-(\ln 2/4000)t}$

$\frac{6}{100} = e^{-(\ln 2/4000)t}$

$\ln(0.06) = -(\ln 2/4000)t$

$t = \frac{\ln(0.06)}{-(\ln 2/4000)}$

$= 16235.57$

$= 16000$

$= 16$ thousand years. ✓✓✓

Q3

a)

i) $v = \frac{2t}{8+t^2}$

$t=0$, $v = \frac{2\times 0}{8+0^2} = 0$ ✓

ii) $\frac{dv}{dt} = \frac{(8+t^2)(2) - (2t)(2t)}{(8+t^2)^2}$ ✓

$a = \frac{16 + 2t^2 - 4t^2}{(8+t^2)^2}$ ✗

$a = \frac{16 - 2t^2}{(8+t^2)^2}$ ✓

iii) $x = \int v\,dt$

$= \int \frac{2t}{8+t^2}\,dt$

$x = \ln(8+t^2) + c$ ✓

$x=0, t=0 \Rightarrow 0 = \ln(8+0) + c$

$c = -\ln 8$

$\therefore x = \ln(8+t^2) - \ln 8$ ✓

at $t=4$, $x = \ln(8+16) - \ln 8$

$= \ln \frac{24}{8}$

$= \ln 3$ } either ✓

or ≈ 1.0986

(b)

$x = \frac{t^3}{3} - 5t^2 + 16t - 21$

$v = \frac{dx}{dt} = \frac{3t^2}{3} - 10t + 16$ ✓

$v=0$ (stationary)

$\Rightarrow t^2 - 10t + 16 = 0$

$(t-8)(t-2) = 0$

particle is stationary at

$t=8$ and $t=2$ ✓✓

Q4 Loan 500 000

$n = 25\times 12 = 300$

$r = 4.8\%$ $r = \frac{4.8}{1200} = 0.004$

$A_2 = 500000 \times 1.004^2 - M\left\{\frac{(1+r)^2-1}{1+r-1}\right\}$

$= 500000 \times 1.004^2 - M\left\{\frac{1.004^2-1}{0.004}\right\}$

or $= 500000 \times 1.004^2 - M\{1 + 1.004\}$ //

(b) $A_n = 500000 \times 1.004^n - M\left\{\frac{(1+r)^n-1}{r}\right\}$

$= 500000 \times 1.004^n - M\left\{\frac{(1.004)^n-1}{0.004}\right\}$

$A_n = 0 \Rightarrow M = \frac{500000 \times 1.004^{300}}{\left\{\frac{1.004^{300}-1}{0.004}\right\}}$ ///

$= 2864.98$

(c) $A_{36} = 500000 \times 1.004^{36} - 2864.98\left\{\frac{1.004^{36}-1}{0.004}\right\}$

$A_{36} = 466578.81$

Balance $= 466578.81 - 100000$ ///

$= 366578.81$

(d) $0 = 366578.81 \times 1.004^n - 2864.98\left\{\frac{1.004^n-1}{0.004}\right\}$

$\Rightarrow 0 = 366578.81 \times 1.004^n - \frac{2864.98}{0.004}\times 1.004^n + \frac{2864.98}{0.004}$

$\Rightarrow 1.004^n\left\{\frac{2864.98}{0.004} - 366578.81\right\} = \frac{2864.98}{0.004}$

$\Rightarrow 1.004^n = \frac{2864.98/0.004}{\left\{\frac{2864.98}{0.004} - 366578.81\right\}}$ ✓

$\Rightarrow n = \frac{\ln\left\{\frac{716245}{716245 - 366578.81}\right\}}{\ln 1.004}$ ✓

$n = 179.62$ ✓

≈ 180 months