Student Number: _____

The Scots College
Sydney Australia

2024 **Year 12**

Mathematics Advanced
Assessment Task 2

Wednesday 26th March, 2024

Weighting: 30%

General Instructions

- Reading time –5 minutes
- Working time – 2 hours
- Write using black pen.
- NESA Approved calculators may be used.
- A reference sheet is provided.
- For questions in Section II, show mathematical reasoning and/or calculations.
- Students may bring in one double sided A4 page with their own handwritten notes (NO PHOTOCOPIES).

Total marks: 81

Section I – 10 marks

- Attempt Questions 1 - 10
- Allow about 15 minutes for this section

Section II – 71 marks

- Attempt Questions 11 - 24
- Allow about 1 hours and 45 minutes for this section

Section I
10 marks
Attempt Questions 1– 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10

1. If $f(x) = x^5 + x^3 - 2x$, what is the value of $f'(1)$?

A. -2

B. 0

C. 1

D. 6

2. What is the effective annual interest rate for a loan advertised at 7% p.a compounded daily? (use 365 days in a year)

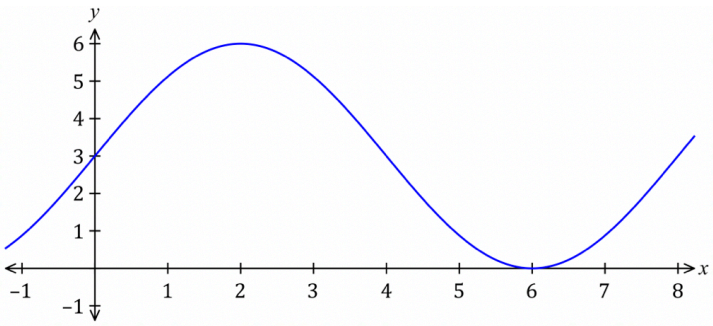
A. 7.25%

B. 7.23%

C. 7.00%

D. 0.70%

3. Which of the following equations is likely to be the rule for the graph of the trigonometric function shown above?



A. $y = 3 + 3\sin\left(\frac{\pi x}{4}\right)$

B. $y = 3 + 3\cos\left(\frac{\pi x}{4}\right)$

C. $y = 3 + 3\sin\left(\frac{x}{4}\right)$

D. $y = 3 + 3\cos\left(\frac{x}{4}\right)$

4. Lachlan deposits \$750 every 6 months for 4 years into an account earning 6% p.a compounded biannually.

The table below shows the future value interest factors for an annuity of \$1

Periods (n)	Future Value of an Annuity of \$1				
	Interest Rate				
	2%	3%	4%	5%	6%
2	2.020	2.030	2.040	2.050	2.060
4	4.122	4.184	4.247	4.310	4.375
6	6.308	6.468	6.633	6.802	6.975
8	8.583	8.892	9.214	9.549	9.898
10	10.950	11.464	12.006	12.578	13.181

What is the future value of Lachlan's investment?

A. \$3138

B. \$3281.25

C. \$6669

D. \$7423.50

5. What is the gradient of the tangent to the curve $y = (x-2)^3$ at its y-intercept?

A. -24

B. -8

C. 0

D. 12

6. If the numbers $3,\ x,\ 7$ are the first three terms of a geometric series, which of the following is a possible value of x ?

A. $\sqrt{21}$

B. 5

C. 6

D. 21

7. What is the period of the function $y = \tan 2x$

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. 2π

D. 4π

8. How many solutions does the equation $e^x(\sin x + 1) = 0$ have for $0 \leq x \leq 2\pi$?

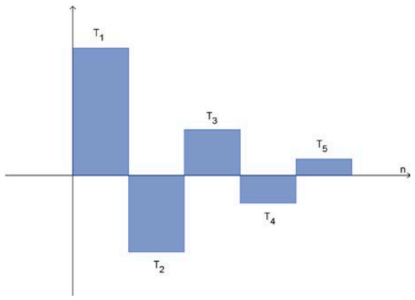
A. 0

B. 1

C. 2

D. 3

9. The values for a geometric sequence $T_n = ar^{n-1}$ are plotted on a bar graph:



Which statement must be true?

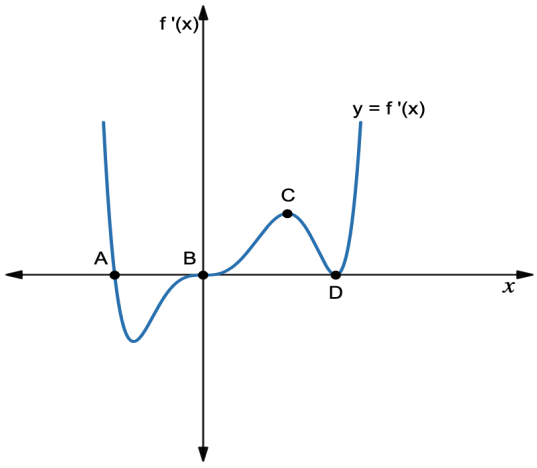
A. $a < 0,\ -1 < r < 0$

B. $a < 0,\ 0 < r < 1$

C. $a > 0,\ -1 < r < 0$

D. $a > 0,\ 0 < r < 1$

10. The diagram shows a sketch of the gradient function $y = f'(x)$, passing through the points *A, B, C* and *D*.



Which point represents the horizontal point of inflection of the curve $y = f(x)$?

A. Point *A*

B. Point *B*

C. Point *C*

D. Point *D*

Section II

71 marks
Attempt Questions 11 – 24
Allow about 1 hour and 45 minutes for this section.

Question 11 (8 marks)

Differentiate with respect to x.

(a) $y = 2x^2\cos x$ **2**

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(b) $y = \ln\sqrt{x}$ **2**

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(c) $y = \dfrac{e}{x}$ **2**

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(d) $y = \dfrac{x^2 + 1}{x - 1}$ **2**

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Question 12 (3 marks)

For what values of x is the graph of $y = x^3 + 3x^2 - 9x - 22$ concave up? **3**

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Question 13 (3 marks)

Show that $\frac{d}{dx}\text{cosec } x = -\text{cosec } x \cot x$ **3**

Question 14 (3 marks)

The table gives the present value interest factors for an annuity of $1 per period, for various rates, *r* and number of periods *N*.

Present value interest factors					
N	**Interest rate per period (as a decimal) (*r*)**				
	0.0025	**0.005**	**0.0075**	**0.008**	**0.009**
71	64.9814	59.6412	54.8929	54.0075	52.2966
72	65.8169	60.3395	55.4769	54.5710	52.8212
73	66.6502	61.0343	56.0564	55.1299	53.3411
74	67.4815	61.7257	56.6317	55.6845	53.8564
75	68.3108	62.4137	57.2027	56.2346	54.3671
76	69.1379	63.0982	57.7694	56.7803	54.8732

(a) What is the present value of an annuity of $2000 per month for 73 months if the interest rate is 0.75% per month? **1**

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(b) Hayden takes out a loan of $50 000 to buy a car. It is to be repaid in equal monthly instalments over 6 years with an interest rate of 10.8% per annum. **1**

What are the monthly repayments? (rounded to the nearest dollar)

i) What is Hayden's monthly repayment? (rounded to the nearest dollar)

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ii) What is the total interest paid on this loan? **1**

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Question 15 (4 marks)

The second term of an arithmetic progression is 6 and the ninth term is 62.

(a) Find the common difference **2**

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(b) What is the first term greater than 500? **2**

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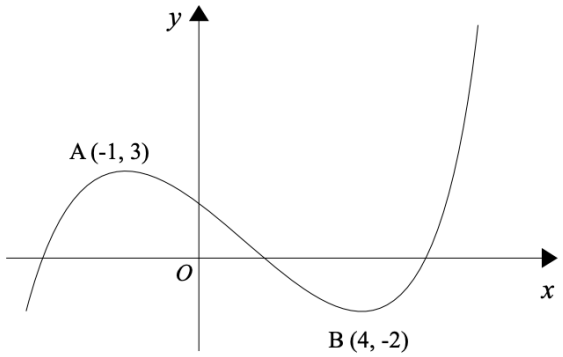
Question 16 (4 marks)

The diagram shows the graph of $y = f(x)$.



The curve has a maximum at point A(-1, 3) and a minimum point at B(4, -2).

Find the coordinates of B' mapped by the transformation $f(2x)$ **1**

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Find the coordinates of A' mapped by the transformation $3f(-x)$ **1**

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Write down the constants *a* and *b* such that the curve with equation $y = f(x + a) + b$ has a maximum point at the origin *O*. **2**

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Question 17 (3 marks)

Consider the function $f(x) = \ln x$.

(a) If $f(x) = \ln x$ is transformed to $g(x) = \ln\left(\frac{1}{x}\right)$, describe the transformation that took place. **1**

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(b) Show that $g(x) = \ln\left(\frac{1}{x}\right)$ is decreasing for all values of x in the natural domain. **2**

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Question 18 (6 marks)

The number of eggs being laid on a chicken farm is modelled by the function

$$N = 40 + 8\sin\frac{\pi}{12}t$$

Where *t* is the number of hours after midday.

Sketch the graph of the number of eggs vs. the time $0 \leq t \leq 24$ **3**

Find the first time *t,* after midday, when the number of eggs being laid is 36 **3**

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Question 19 (5 marks).

For the series $1\frac{1}{2} + 4\frac{1}{4} + 7\frac{1}{8} + 10\frac{1}{16} + \ldots$

(a) Show that the nth term is given by $T_n = (3n - 2) + \left(\frac{1}{2}\right)^n$ **3**

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(b) Find S_9 **2**

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Question 20 (3 marks)

A new boat depreciates by 11% each year. How long does it take for the boat's value to fall below 25% of its original value? Answer to the nearest year. **3**

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Question 21 (8 marks)

The temperature of an oven can be modelled by the equation

$$T = 23 + 177(1.3)^{-0.5t}$$

where T is the temperature of the oven, in °C, t minutes after it is turned off.

(a) What is the temperature of the oven when it is initially turned off ? **1**

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(b) At what rate is the temperature of the oven changing after 8 minutes? Give your answer to one decimal place. **2**

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(c) How long will it take for the oven to cool to 32℃ ? Answer to one decimal place. **3**

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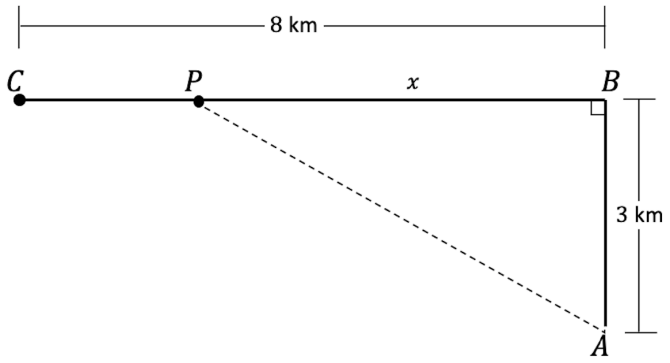
(d) Sketch a graph of the function $T = 23 + 177(1.3)^{-0.5t}$, showing important features. **2**

Question 22 (6 marks)

Finn and Miles run in a cross country race where the route is to go from *A* to *B* which is 3 kilometres, then from *B* to *C* which is 8 kilometres. Finn thinks it might be faster to take a shortcut and instead run from *A* to another point *P* which lies on *BC,* and then continue the rest of the route as normal.



When running the normal route both boys run at 10 kilometres per hour. However, when taking the shortcut Finn runs at a slower pace of 8 kilometres per hour from *A* to *P* because there are a number of hills.

(a) If x is the distance from *B* to *P.* Show that the time taken for Finn to go from *A* to *P,* then *P* to *C* is: **2**

$$T = \frac{\sqrt{9 + x^2}}{8} + \frac{8 - x}{10}$$

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(b) Find the value of x that minimises the time it will take Finn via the shortcut. **3**

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(c) Who's route was faster, Miles or Finn's ? Justify your answer by showing necessary working.. **1**

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Question 23 (8 marks)

Consider the function $f(x) = xe^x$

(a) Find the coordinates of the stationary point and determine its nature. **3**

(b) Find the coordinates of the point of inflection. **2**

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(c) Sketch the graph of the function showing all important features. **3**

Question 24 (7 marks)

Harry has just retired with $1 000 000 in his superannuation account. This account earns interest at 6% p.a. compounded annually. He intends to withdraw money in equal amounts every year immediately after interest is compounded.

Let An be the amount of money in his account after the n^{th}withdrawal of $\$W$

(a) Show that an expression for the amount remaining in the account after two withdrawals is **1**

$$A_2 = 1\,000\,000(1.06)^2 - W(1 + 1.06)$$

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(b) If Harry wants the money to last exactly 20 withdrawals, show that W = $87 185, to the nearest dollar. **3**

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(c) After his 10th withdrawal, the bank changes its interest rate to 4% p.a on the remaining balance. How many further withdrawals can Harry make if W remains at \$87 185? **3**

End of Examination ☺

Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

Student Number: Solutions

The Scots College
Sydney Australia

Q11-15 MOBR
16-19 MODO
20-22 MCLE
23-24 LGAZ
+multis

2024 **Year 12**

Mathematics Advanced
Assessment Task 2

Wednesday 26th March, 2024

Weighting: 30%

General Instructions

- Reading time –5 minutes
- Working time – 2 hours
- Write using black pen
- NESA Approved calculators may be used
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Total marks 80:

Section I – 10 marks

- Attempt Question 1 - 10
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Section II – 70 marks

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1. If $f(x) = x^5 + x^3 - 2x$, what is the value of $f'(1)$?

A. -2

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C. 1

D. 6

2. What is the effective annual interest rate for a loan advertised at 7% p.a compounded daily? (use 365 days in a year)

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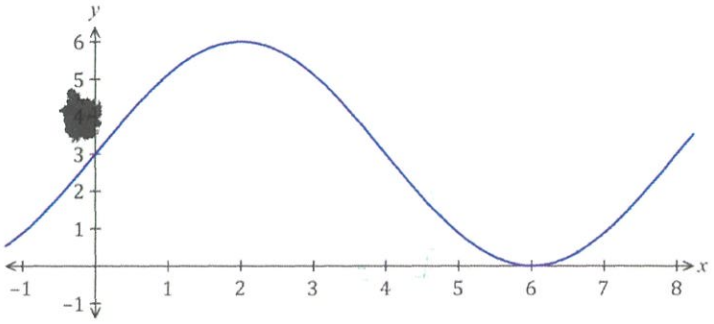
3. Which of the following equations is likely to be the rule for the graph of the trigonometric function shown above?



A. $y = 3 + 3\sin\left(\frac{\pi x}{4}\right)$

B. $y = 3 + 3\cos\left(\frac{\pi x}{4}\right)$

C. $y = 3 + 3\sin\left(\frac{x}{4}\right)$

D. $y = 3 + 3\cos\left(\frac{x}{4}\right)$

4. Lachlan deposits \$750 every 6 months for 4 years into an account earning 6% p.a compounded biannually.

The table below shows the future value interest factors for an annuity of \$1

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What is the future value of Lachlan's investment?

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B. 5

C. 6

D. 21

7. What is the period of the function $y = \tan 2x$

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B. $\frac{\pi}{2}$

C. 2π

D. 4π

8. How many solutions does the equation $e^x(\sin x + 1) = 0$ have for $0 \leq x \leq 2\pi$?

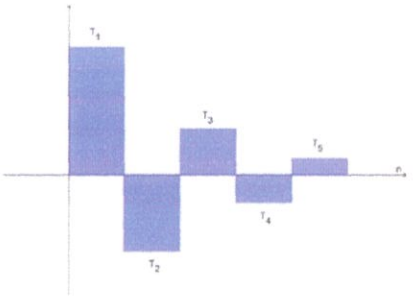
A. 0

B. 1

C. 2

D. 3

9. The values for a geometric sequence $T_n = ar^{n-1}$ are plotted on a bar graph:



Which statement must be true?

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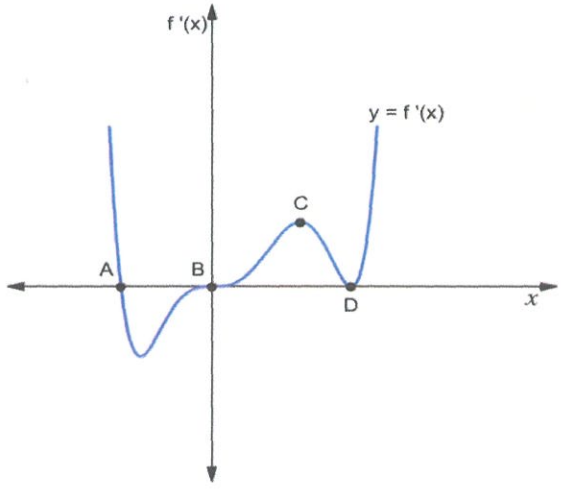
B. $a < 0,\ 0 < r < 1$

C. $a > 0,\ -1 < r < 0$

D. $a > 0,\ 0 < r < 1$

10. The diagram shows a sketch of the gradient function $y = f'(x)$, passing through the points *A*, *B*, *C* and *D*.



Which point represents the horizontal point of inflection of the curve $y = f(x)$?

A. Point *A*

B. Point *B*

C. Point *C*

D. Point *D*

Student Number: Solutions.

Year 12 Mathematics Advanced

Section I Multiple Choice Answer Sheet

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

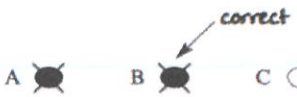
Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ✕ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word ~~correct~~ and drawing an arrow as follows.



A ✕ B ✕ C ○ D ○

Question 1	A ○	B ○	C ○	D ●
Question 2	A ●	B ○	C ○	D ○
Question 3	A ●	B ○	C ○	D ○
Question 4	A ○	B ○	C ●	D ○
Question 5	A ○	B ○	C ○	D ●
Question 6	A ●	B ○	C ○	D ○
Question 7	A ○	B ●	C ○	D ○
Question 8	A ○	B ●	C ○	D ○
Question 9	A ○	B ○	C ●	D ○
Question 10	A ○	B ○	C ○	D ●

Section II

70 marks

Attempt Questions 11 – 24

Allow about 1 hour and 45 minutes for this section

Question 11 (8 marks)

Differentiate with respect to x.

(a) $y = 2x^2\cos x$ 2

$y' = 2x^2 \cdot -\sin x + \cos x \cdot 4x$ (1) $v = 2x^2$ $u = \cos x$

$v' = 4x$ $u' = -\sin x$

$= 2x(2\cos x - x\sin x)$ (½)

OR $-2x(x\sin x - 2\cos x)$

(b) $y = \ln\sqrt{x}$ 2

$y = \ln x^{\frac{1}{2}}$ (1) $f(x) = x^{\frac{1}{2}}$

$= \frac{1}{2}\ln x$ $f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$

$y' = \frac{1}{2} \times \frac{1}{x}$ (1). $\frac{\frac{1}{2}x^{-\frac{1}{2}}}{\sqrt{x}}$ (1)

$= \frac{1}{2x}$

(c) $y = \frac{e}{x}$ 2

$y = ex^{-1}$ (½).

$y' = -ex^{-2}$ (1) $\frac{x - e}{x^2}$ (1) if show u/u'/v/v'

(½) if not.

$= \frac{-e}{x^2}$ (½).

(d) $y = \dfrac{x^2+1}{x-1}$ **2**

$y' = \dfrac{(x-1).2x - (x^2+1).1}{(x-1)^2.}$ (1) $v = x-1$ $u = x^2+1$

$v' = 1$ $u' = 2x$.

$= \dfrac{2x^2 - 2x - x^2 - 1}{(x-1)^2.}$

$= \dfrac{x^2-2x-1}{(x-1)^2}$ (1) $\dfrac{x^2-2x+1}{(x-1)^2}$ $(-\frac{1}{2})$.

if factorised + simplified to $y' = 1$ $(-\frac{1}{2})$.

Question 12 (3 marks)

For what values of x is the graph of $y = x^3 + 3x^2 - 9x - 22$ concave up? **3**

$\dfrac{dy}{dx} = 3x^2 + 6x - 9$ (1)

$\dfrac{d^2y}{dx^2} = 6x + 6.$ (1)

$6x + 6 > 0$

$6x > -6$

$x > -1$ (1).

Question 13 (3 marks)

Show that $\frac{d}{dx}\operatorname{cosec} x = -\operatorname{cosec} x \cot x$ 3

LHS $= \frac{d}{dx} \operatorname{cosec} x.$

$= \frac{d}{dx}(\sin x)^{-1}$ (1).

$= -1(\sin x)^{-2} \cdot \cos x.$ (1)

$= \frac{-1 \cos x}{\sin^2 x}.$

$= \frac{-1}{\sin x} \times \frac{\cos x}{\sin x}$ (1)

$= -\operatorname{cosec} x \cot x.$

$= \text{RHS}.$

$\cot x \times \frac{-1}{\sin x}$

* if missing proof and have worked backwards

$\frac{-1}{\sin x} \times \frac{\cos x}{\sin x}$.

$= -\operatorname{cosec} x \cot x$

(1)/3.

Question 14 (3 marks)

The table gives the present value interest factors for an annuity of \$1 per period, for various rates, r and number of periods N.

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74	67.4815	61.7257	56.6317	55.6845	53.8564
75	68.3108	62.4137	57.2027	56.2346	54.3671
76	69.1379	63.0982	57.7694	56.7803	54.8732

(a) What is the present value of an annuity of \$2000 per month for 73 months if the interest rate is 0.75% per month? **1**

$PV = 2000 \times 56.0564$ $(\frac{1}{2})$ rate

$= \$112\,112.80$ $(\frac{1}{2})$.

(b) Hayden takes out a loan of \$50 000 to buy a car. It is to be repaid in equal monthly instalments over 6 years with an interest rate of 10.8% per annum. **1**

What are the monthly repayments? (rounded to the nearest dollar)

i) What is Hayden's monthly repayment? (rounded to the nearest dollar)

$n = 72 \quad r = 52.8212$ $(\frac{1}{2})$ rate

$50\,000 = a \times 52.8212$

$a = \frac{50\,000}{52.8212}$

monthly repayment $= \$946.5896269$

$= \$947$ $(\frac{1}{2})$.

ii) What is the total interest paid on this loan? **1**

Interest $= 72 \times 946.59 - 50\,000$ $(\frac{1}{2})$

$= \$18154.48$ $(\frac{1}{2})$.

OR $72 \times 947 - 50\,000 = \$18184$.

$-\frac{1}{2}$ if not subtracting 50 000

Question 15 (4 marks)

The second term of an arithmetic progression is 6 and the ninth term is 62.

(a) Find the common difference 2

$6 + 7d = 62$ (1)

$7d = 56$

$d = 8$ (1)

$a + d = 6$

$a + 8d = 62$ (1).

$7d = 56$

$d = 8$ (1)

(b) Find the value of the first term which is greater than 500 (or which term?) 2/3

$T_n > 500$

$-2 + (n-1) \times 8 > 500$ (1).

$n - 1 > \frac{502}{8}$

$n > \frac{502}{8} + 1$

$n > 63.75$ ($\frac{1}{2}$)

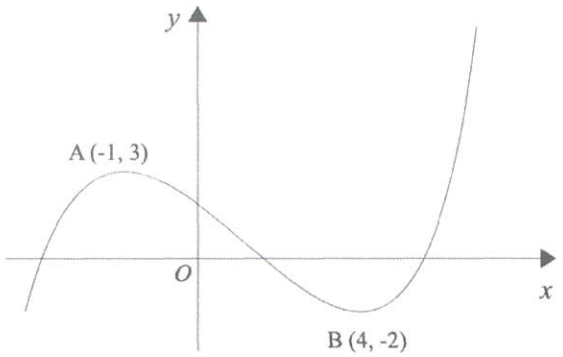
$n = 64$ ($\frac{1}{2}$).

$a + 8 = 6$

$a = -2$ ($\frac{1}{2}$).

Question 16 (4 marks)

The diagram shows the graph of $y = f(x)$.



The curve has a maximum at point A(-1, 3) and a minimum point at B(4, -2).

Find the coordinates of B' mapped by the transformation $f(2x)$ **1**

B' (2, -2)
(1).

Find the coordinates of A' mapped by the transformation $3f(-x)$ **1**

A' (1, 9).
$(\frac{1}{2})$ $(\frac{1}{2})$.

Write down the constants a and b such that the curve with equation $y = f(x + a) + b$ has a maximum point at the origin O. **2**

$y = f(x-1) - 3$

$a = -1$ (1)

$b = -3$ (1)

Question 17 (2 marks)

Consider the function $f(x) = \ln x$.

(a) If $f(x) = \ln x$ is transformed to $g(x) = \ln\left(\frac{1}{x}\right)$, describe the transformation that took place. 1

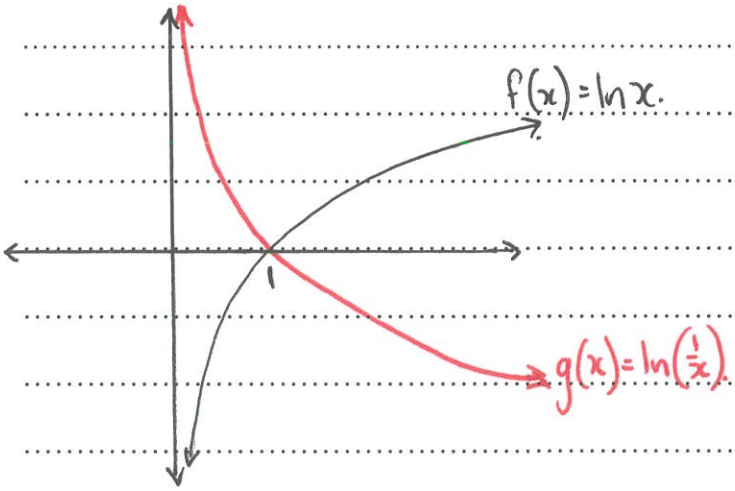
$g(x) = \ln\left(\frac{1}{x}\right)$

$= \ln x^{-1}$

$= -\ln x$

∴ vertical dilation s.f -1 OR reflection across the x-axis (1).

(b) Hence or otherwise, show that $g(x) = \ln\left(\frac{1}{x}\right)$ is decreasing for all values of x.



$g(x) = \ln\left(\frac{1}{x}\right)$

$= \ln x^{-1}$

$= -\ln x.$

$g'(x) = \frac{-1}{x}$ (1).

$\frac{-1}{x} < 0$

for $x > 0$ since $\ln\left(\frac{1}{x}\right)$ $(0 < x < \infty)$. (1).

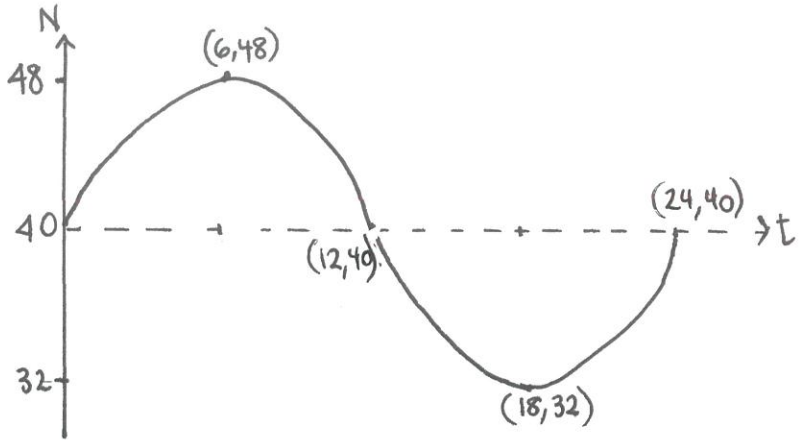
Question 18 (6 marks)

The number of eggs being laid on a chicken farm is modelled by the function

$$N = 40 + 8\sin\frac{\pi}{12}t$$

Where t is the number of hours after midday.

Sketch the graph of the number of eggs vs. the time $0 \leq t \leq 24$ **3**



(1) range + centre
(1) shape.
(1) max, mins ?

Find the first time t, after midday, when the number of eggs being laid is 36 **3**

$36 = 40 + 8\sin\frac{\pi}{12}t$ (½).

$\sin\frac{\pi}{12}t = -\frac{4}{8}$

basic $\angle = \frac{\pi}{6}$ (½)

$\frac{\pi}{12}t = \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$

$= \frac{7\pi}{6}, \frac{11\pi}{6}$ (1).

$t = \frac{7\pi}{6} \div \frac{\pi}{12}, \frac{11\pi}{6} \div \frac{\pi}{12}.$

$= 14, 22$ hours (½)

∴ 36 eggs first laid at 2am (½).

Question 19 (5 marks). ?????

For the series $1\frac{1}{2} + 4\frac{1}{4} + 7\frac{1}{8} + 10\frac{1}{16} + \ldots$

(a) Show that the nth term is given by $T_n = (3n - 2) + \left(\frac{1}{2}\right)^n$ **3**

$1 + \frac{1}{2} + 4 + \frac{1}{4} + 7 + \frac{1}{8} + 10 + \frac{1}{16}$

$1 + 4 + 7 + 10 + \ldots$ + $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots$ (1)

arithmetic — geometric

$T_{n_1} = 1 + (n-1) \times 3$

$= 1 + 3n - 3$ (1).

$= 3n - 2$

$T_{n_2} = \frac{1}{2} \times \frac{1}{2}^{n-1}$

$= \left(\frac{1}{2}\right)^n$ (1)

$\therefore T_n = (3n - 2) + \left(\frac{1}{2}\right)^n$

Find S_9 **2**

$S_9 = \frac{9}{2}\left[2 \times 1 + (9-1) \times 3\right] + \frac{1}{2} \times \left(\frac{1 - \left(\frac{1}{2}\right)^9}{1 - \frac{1}{2}}\right)$ (1)

$= 117 + \frac{511}{512}$

$= 117\frac{511}{512}$ (1)

Question 20 (3 marks)

A new boat depreciates by 11% each year. How long does it take for the boat's value to fall below 25% of its original value? **3**

$T_0 = 1$

$T_1 = 0.89$

$T_2 = 0.89 \times 0.89$ (1).

$0.89^n < 0.25$ (1).

$\log_e 0.89^n < \log_e 0.25$

$n \ln 0.89 < \ln 0.25$.

$n > \dfrac{\ln 0.25}{\ln 0.89}$

$n > 11.89606936$ ($\frac{1}{2}$)

∴ It will take 12 years. ($\frac{1}{2}$).

Question 21 (8 marks)

The temperature of an oven can be modelled by the equation

$$T = 23 + 177(1.3)^{-0.5t}$$

where T is the temperature of the oven, in °C, t minutes after it is turned off.

(a) What is the temperature of the oven when it is initially turned off? **1**

when $t=0$ $\quad -0.5\times 0$

$T = 23 + 177(1.3)$ $\quad (\frac{1}{2})$

$= 200°C$ $(\frac{1}{2})$.

(b) At what rate is the temperature of the oven changing after 8 minutes? Give your answer to one decimal place. **2**

$\frac{dT}{dt} = -0.5\times 177\times \ln(1.3)\times (1.3)^{-0.5t}$

$= -\ln(1.3)\times 88.5\times (1.3)^{-0.5t}$ (1).

at $t=8$ $\quad \frac{dT}{dt} = -\ln(1.3)\times 88.5\times (1.3)^{-0.5\times(8)}$

$= -8.129700433$

$= -8.1\,°C$/minute (1).

(c) How long will it take for the oven to cool to 32°C ? Answer to one decimal place. **3**

$32 = 23 + 177(1.3)^{-0.5t}$

$\frac{9}{177} = (1.3)^{-0.5t}$ (1).

$\ln\left(\frac{9}{177}\right) = \ln(1.3)^{-0.5t}$ (1).

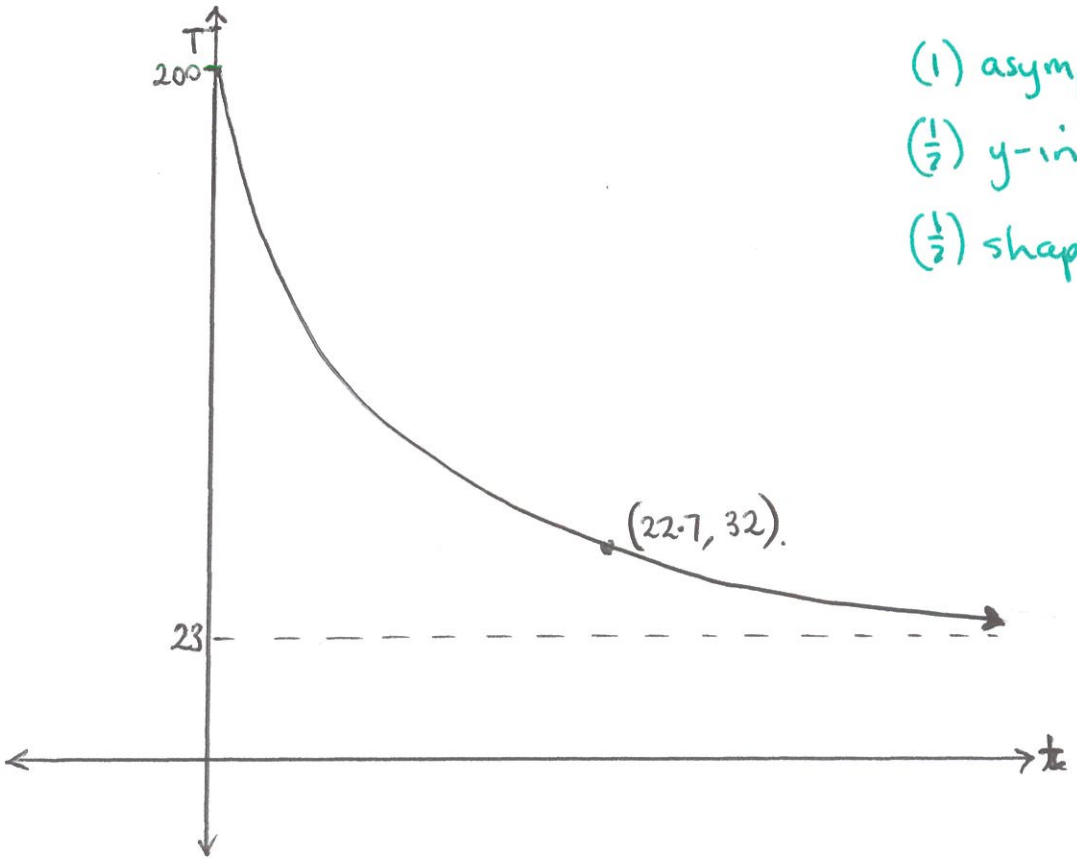
$\ln\left(\frac{9}{177}\right) = -0.5t \ln(1.3)$.

$-0.5t = \frac{\ln \frac{9}{177}}{\ln(1.3)}$ $(\frac{1}{2})$

$t = 22.7083148$ $(\frac{1}{2})$.

$t = 22.7$ minutes (1dp).

(d) Sketch a graph of the function $T = 23 + 177(1.3)^{-0.5t}$, showing important features. **2**



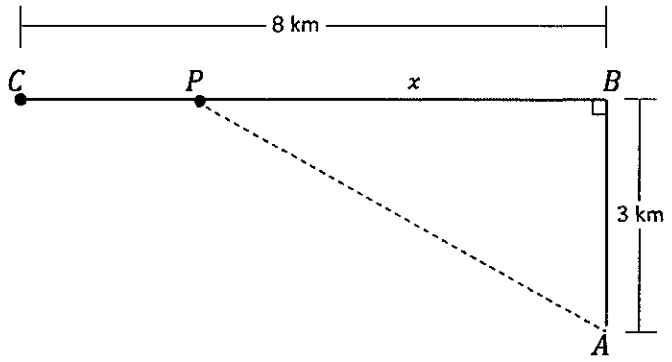
(1) asymptote.

$(\frac{1}{2})$ y-int

$(\frac{1}{2})$ shape.

Question 22 (6 marks)

Finn and Miles run in a cross country race where the route is to go from A to B which is 3 kilometres, then from B to C which is 8 kilometres. Finn thinks it might be faster to take a shortcut and instead run from A to another point P which lies on BC, and then continue the rest of the route as normal.



When running the normal route both boys run at 10 kilometres per hour. However, when taking the shortcut Finn runs at a slower pace of 8 kilometres per hour from A to P because there are a number of hills.

a) If x is the distance from B to P. Show that the time taken for Finn to go from A to P, then P to C is: **2**

$$T = \frac{\sqrt{9+x^2}}{8} + \frac{8-x}{10}$$

distance

$AP^2 = 3^2 + x^2$ $\quad CP = CB - PB$

$AP = \sqrt{9+x^2}$ $\quad = 8 - x.$

$S = \frac{D}{T}$ $\quad T = \frac{D}{S}$

time

$T_{AP} = \frac{\sqrt{9+x^2}}{8}$ $\quad T_{CP} = \frac{8-x}{10}$

$T = \frac{\sqrt{9+x^2}}{8} + \frac{8-x}{10}$

b) Find the value of x that minimises the time it will take Finn via the shortcut. 3

$\frac{dT}{dx} = \frac{1}{2} \times \frac{1}{8}(9+x^2)^{-\frac{1}{2}} \times 2x + \left(-\frac{1}{10}\right)$.

$= \frac{x}{8\sqrt{9+x^2}} - \frac{1}{10}$ (1)

$\frac{dT}{dx} = 0$

$\frac{x}{8\sqrt{9+x^2}} = \frac{1}{10}$

$\frac{10x}{8} = \sqrt{9+x^2}$

$\frac{25x^2}{16} = 9+x^2$

$25x^2 = 144 + 16x^2$.

$25x^2 - 16x^2 = 144$.

$9x^2 = 144$.

$x^2 = \pm\sqrt{\frac{144}{9}}$

$x = \pm 4$ (1) $x > 0$

x	3	4	5
$f'(x)$	-0.012	0	0.007
	\	−	/

(1).

$\therefore x = 4$ is a minimum

c) Who's route was faster, Miles or Finn's ? Justify. 1

Miles: $T = \frac{11}{10}$

$= 1.1\text{hr}$

$= 1\text{hr } 6\text{min}$ $(\frac{1}{2})$

Finn: $T = \frac{\sqrt{9+4^2}}{8} + \frac{8-4}{10}$

$= 1.025$

$= 1^\circ\ 1\text{min } 30\text{seconds}$ $(\frac{1}{2})$.

$\therefore$ Finn's route was faster.

Question 23 (8 marks)

Consider the function $f(x) = xe^x$

(a) Find the coordinates of the stationary point and determine its nature. **3**

$f'(x) = x.e^x + e^x.1$ $\quad v = x \quad u = e^x$

$= e^x(x+1)$. (1) $\quad v' = 1 \quad u' = e^x$

$f'(x) = 0$

$e^x(x+1) = 0$

$e^x = 0$ OR $x+1 = 0$

no soln $\quad x = -1$.

$f(-1) = (-1).e^{-1}$

$= \frac{-1}{e}$ $\quad \left(-1, -\frac{1}{e}\right)$ (1).

$f''(x) = e^x.1 + (x+1).e^x$ $\quad v = e^x \quad u = x+1$

$= e^x(1+x+1)$ $\quad v' = e^x \quad u' = 1$

$= e^x(2+x)$.

$f''(-1) = e^{-1}(2+-1)$. (1) test.

$= \frac{1}{e} > 0$ concave ↑ ∴ minimum @ $\left(-1, -\frac{1}{e}\right)$.

(b) Find the coordinates of the point of inflection. **2**

$f''(x) = 0 \qquad e^x(2+x) = 0$

$e^x = 0$ no soln OR $2+x = 0$

$x = -2$ (½)

$f(-2) = (-2).e^{-2}$

$= \frac{-2}{e^2}$ (½) $\left(-2, \frac{-2}{e^2}\right)$

x	-3	-2	-1
$f''(x)$	$-\frac{1}{e}$	0	$\frac{1}{e}$
	$\cap$		$\cup$

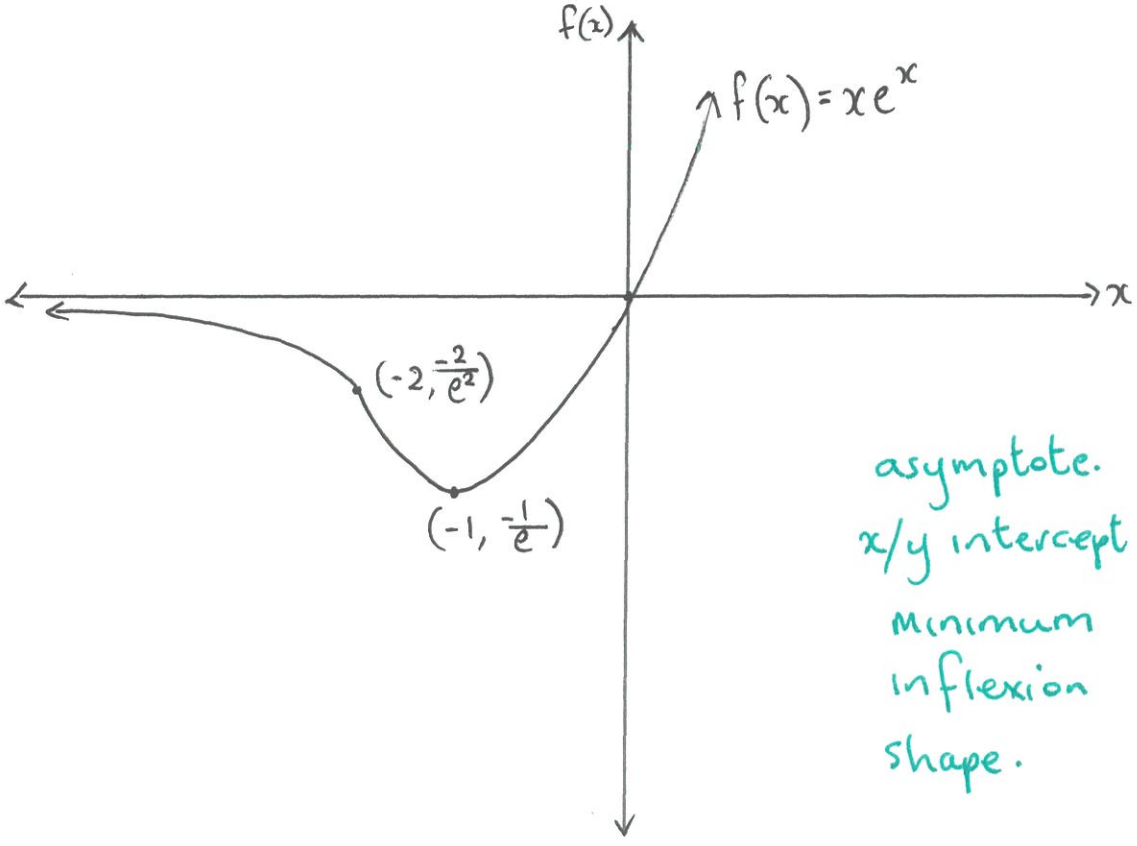
(1)

change in concavity ∴ pt of inflexion @ $\left(-2, \frac{-2}{e^2}\right)$

(c) Sketch the graph of the function showing all important features. **3**

as $x \to \infty$ $f(x) \to \infty$

as $x \to -\infty$ $f(x) \to 0^-$



asymptote.
x/y intercept
Minimum
inflexion
Shape.

Question 24 (7 marks)

Harry has just retired with \$1 000 000 in his superannuation account. This account earns interest at 6% p.a. compounded annually. He intends to withdraw money in equal amounts every year immediately after interest is compounded.

Let An be the amount of money in his account after the n^{th} withdrawal of $\$W$

(a) Show that an expression for the amount remaining in the account after two withdrawals is **1**

$$A_2 = 1\,000\,000(1.06)^2 - W(1.06 + 1)$$

$A_1 = 1000000(1.06) - W$ $(\frac{1}{2})$.

$A_2 = A_1(1.06) - W$

$= (1000000(1.06) - W)(1.06) - W$ $(\frac{1}{2})$.

$= 1000000(1.06)^2 - W(1.06) - W$

$= 1000000(1.06)^2 - W(1 + 1.06)$

(b) If Harry wants the money to last exactly 20 withdrawals, show that W = \$87 185, to the nearest dollar. **3**

$A_{20} = 1000000(1.06)^{20} - W(1 + 1.06 + 1.06^2 + \cdots + 1.06^{19})$ (1).

$= 1000000(1.06)^{20} - W\left(\frac{1(1.06^{20} - 1)}{1.06 - 1}\right)$ (1).

$A_{20} = 0$.

$W\left(\frac{1.06^{20} - 1}{0.06}\right) = 1000000(1.06)^{20}$

$W = 87184.55698$ (1).

$= \$87185$ (nearest \$).

(c) After his 10th withdrawal, the bank changes its interest rate to 4% p.a on the remaining balance. How many further withdrawals can Harry make if W stays the same? **3**

$A_{10} = 1000\,000(1{\cdot}06^{10}) - \dfrac{87185(1{\cdot}06^{10}-1)}{1{\cdot}06-1}$

$= \$641\,680{\cdot}0895$ (1).

$0 = 641680{\cdot}0895(1{\cdot}04^{n}) - \dfrac{87185(1{\cdot}04^{n}-1)}{1{\cdot}04-1}$ (1).

$= 641\,680{\cdot}0895(1{\cdot}04^{n}) - 2179625\times(1{\cdot}04^{n}) + 2179625$

$(1{\cdot}04^{n})(2179625 - 641680{\cdot}0895) = 2179625$

$1{\cdot}04^{n} = 1{\cdot}417232168$

$\ln(1{\cdot}04^{n}) = \ln 1{\cdot}417232168$

$n \ln(1{\cdot}04) = \ln 1{\cdot}417232168$

$n = \dfrac{\ln 1{\cdot}417232168}{\ln 1{\cdot}04}$

$n = 8{\cdot}890858021$ ($\frac{1}{2}$)

∴ Harry can make a further 8 withdrawals of \$87185 ($\frac{1}{2}$).

End of Examination ☺