

YEAR 11 MATHEMATICS

MID-SEMESTER EXAM

QUESTION 1. (19 marks)

(a) Simplify $\frac{10x+15y}{5x+5y}$ [1]

(b) Factorise $6a^2 - 3a - 3$ [2]

(c) Solve $\frac{x}{3} - \frac{x+1}{2} = 5$ [2]

(d) Show on a number line the solution to $|x+4| \leq 5$ [3]

(e) Solve $x^2 - 7x - 4 = 0$ Answer in exact form. [2]

(f) Factorise $3x^3 - 2x^2 - 6x + 4$ [2]

(g) Simplify $\frac{x^2-3x+2}{x-1} \times \frac{x+1}{x^2-x-2}$ [2]

(h) A number when added to twice its square equals 55.
Find the number(s) that satisfy this condition. [3]

(i) Solve $\frac{1}{x+1} - \frac{1}{2x-1} = 0$ [2]

(j) The volume of a square pyramid is given by $V = \frac{1}{3}x^2h$, where x is the base length
and h is the length of the perpendicular height. Find x , correct to two decimal places,
if $V = 53.77 \text{ m}^3$ and $h = 2.37 \text{ m}$ [2]

QUESTION 2 (26 marks) START A NEW PAGE

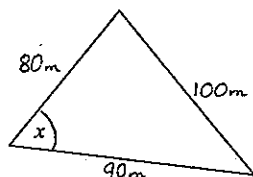
- (a) Write the exact values for:
- (i) $\tan 30^\circ$ [1]
 (ii) $\sec 135^\circ$ [1]

- (b) Find all the values for θ , for $0^\circ \leq \theta \leq 360^\circ$:
- (i) $\sin \theta = 0$ [2]
 (ii) $\cot \theta = \sqrt{3}$ [2]
 (iii) $2 \cos \theta + 1 = 0$ [3]

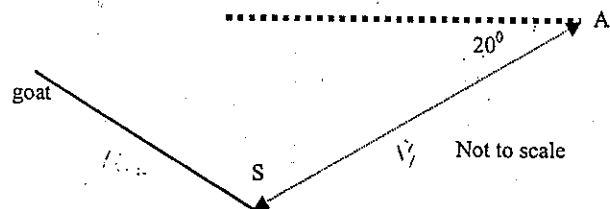
- (c) Simplify:
- (i) $\frac{\sin^2 \theta + \cos^2 \theta}{\tan^2 \theta}$ [2]
 (ii) $1 - \sin^2 (180^\circ + \theta)$ [2]
 (iii) $(\sec^2 \theta - 1) \tan (90^\circ - \theta)$ [3]

- (d) Given that $\tan Y = \frac{2}{3}$, and $\sin Y < 0$, find the exact value of $\sec Y$. [2]

- (e) The sides of a triangular field have lengths 80m, 90m, and 100m.



- (i) Calculate the size of the angle 'x' to the nearest degree. [2]
 (ii) Calculate the area of the triangle. (Answer to 2 significant figures) [2]
 (f) A wooden stake, S, is 13 m from a point, A, on straight fence. SA makes an angle of 20° with the fence. If a goat is tethered to S by a 10m rope, at what angle to the fence is the rope, if the goat reaches the closest possible point on the fence to A. Draw a neat diagram to illustrate your solution.



[4]

QUESTION 3 (19 marks) START A NEW PAGE

- (a) A function $F(x)$ is defined: $F(x) = \begin{cases} x^2 - 2 & \text{when } -3 \leq x \leq 0 \\ 2x + 3 & \text{when } 0 < x \leq 3 \end{cases}$

- (i) Find $(\alpha) F(1)$ [1]
 $(\beta) F(-2)$ [1]

- (ii) Find the range of the function $F(x)$ [1]

- (b) Consider the function $y = -\sqrt{16 - x^2}$

- (i) Write down the domain of the function [1]

- (ii) Write down the range of the function [1]

- (iii) Describe in geometrical terms the number plane graph of $y = -\sqrt{16 - x^2}$ [2]

- (c) Using a clear sketch, show the region determined by the intersection of the following inequalities. Make clear any x and y intercepts.

$$\begin{aligned} y &\geq x^2 - 4 \\ y &\leq 2 \end{aligned} \quad [3]$$

- (d) Sketch neatly the graphs of the following functions. Make clear any x and y intercepts.

- (i) $y = |2x - 1|$ [2]

- (ii) $F(x) = \begin{cases} x & \text{when } x > 1 \\ 1 & \text{when } -1 \leq x \leq 1 \\ -x & \text{when } x < -1 \end{cases}$ [3]

- (e) The equation of a circle is $x^2 + y^2 + 4x + 2y - 5 = 0$

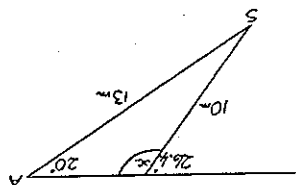
- Find (i) The coordinates of the centre of the circle [3]
 (ii) The length of the radius [1]

QUESTION 4 (19 marks) **START A NEW PAGE**

- (a) For the line $2x + 3y - 12 = 0$, find
- (i) the points of intersection with the x and y axes [2]
 - (ii) the gradient of the line [1]
- (b) If $(-1, 2)$ lies on the line $ax - 4y + 11 = 0$, find 'a'. [2]
- (c)
- (i) Plot the points $A(2, -1)$, $B(5, 3)$, and $C(0, 3)$ on a suitably scaled number plane. [2]
 - (ii) Find the midpoint M of AC. [2]
 - (iii) Show that MB is perpendicular to AC. [3]
 - (iv) Find the equation of AC in general form. [3]
 - (v) Find the perpendicular distance from B to AC [2]
 - (vi) Find, using the midpoint formula, the coordinates of D so that ABCD is a parallelogram. [2]

QUESTION TWO (cont)		(e) (i) $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$		(ii) Area = $\frac{1}{2}ab \sin C$		(f)	
P3	5.5	2	5.5	2	5.5	P1	5.5
P3	5.5	2	5.5	2	5.5	P2	5.5
P3	5.5	2	5.5	2	5.5	P3	5.5

(f) (Must clearly indicate which angle you answer refers to)
 Exterior angle = 26.4°
 OR
 $x = 153.6^\circ$ (obtuse angle)
 $x = 26.4^\circ$ (acute angle)



$$\frac{\sin x}{10} = \frac{\sin 20}{13}$$

$$\sin x = \frac{10}{13} \sin 20$$

$$x = 13.536^\circ$$

$$\text{Area} = \frac{1}{2}ab \sin C$$

$$= \frac{1}{2} \times 10 \times 13 \times \sin 20^\circ$$

$$= 22.8090 \sin 72^\circ$$

$$= 34.238 \dots \text{ (calc)}$$

$$\approx 34.00 \text{ m}^2 \text{ (2 s.f. fig)}$$

$$A = 71.79 \dots \text{ (calc)}$$

$$x \approx 72^\circ \text{ (nearest degree)}$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

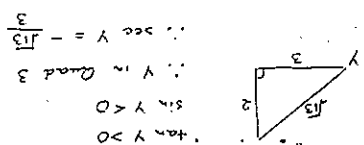
$$= \frac{80^2 + 90^2 - 100^2}{2 \times 80 \times 90}$$

$$= 0.3125$$

$$A = 71.79 \dots \text{ (calc)}$$

$$x \approx 72^\circ \text{ (nearest degree)}$$

QUESTION TWO		(i) $\tan 30^\circ = \frac{1}{\sqrt{3}}$		(ii) $\sec 135 = -\sqrt{2}$		(iii) $\sin \theta = 0$		(iv) $2 \cos \theta + 1 = 0$		(v) $\cos \theta = -1$		(vi) $\tan \theta = 0$	
P1	5.1	2	5.1	2	5.1	P1	5.1	2	5.1	2	5.1	2	5.1
P2	5.2	2	5.2	2	5.2	P2	5.2	2	5.2	2	5.2	2	5.2
P3	5.2	2	5.2	2	5.2	P3	5.2	2	5.2	2	5.2	2	5.2
P4	5.2	2	5.2	2	5.2	P4	5.2	2	5.2	2	5.2	2	5.2



$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{1}{3}$$

$$\theta = \tan^{-1} \left(\frac{1}{3} \right) = 18.43^\circ$$

$$\therefore \theta = 18.43^\circ$$

$$\sin \theta = 0$$

$$\theta = 0^\circ, 180^\circ, 360^\circ$$

$$\cos \theta = -1$$

$$\theta = 180^\circ$$

$$\tan \theta = 0$$

$$\theta = 0^\circ, 180^\circ, 360^\circ$$

Yr 11 Half-Yearly EXAM - Task 2 21 May 2001 SOLUTIONS

QUESTION ONE

$$(a) \frac{10x + 15y}{5x + 5y} = \frac{5(2x + 3y)}{5(x + y)}$$

$$= \frac{2x + 3y}{x + y}$$

$$(b) 6a^2 - 3a - 3 = 3(2a^2 - a - 1)$$

$$= 3(2a + 1)(a - 1)$$

$$(c) \frac{x}{3} - \frac{x+1}{2} = 5$$

$$\frac{2x - 3(x+1)}{6} = 30$$

$$2x - 3x - 3 = 30$$

$$-x = 33$$

$$x = -33$$

$$(d) |x+4| \leq 5$$

$$x+4 \leq 5 \quad -(x+4) \leq 5$$

$$x \leq 1 \quad -x-4 \leq 5$$

$$-x \leq 9$$

$$x \geq -9$$



$$(e) x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(4)}}{2}$$

$$= 7 \pm \sqrt{5}$$

QUESTION ONE (cont)

$$(f) 3x^2 - 2x^2 - 6x + 4$$

$$= x^2(3x-2) - 2(3x-2)$$

$$= (x^2-2)(3x-2)$$

$$(g) \frac{x^2-3x+2}{x-1} \times \frac{x+1}{x^2-x-2}$$

$$= \frac{(x-2)(x-1)}{(x-1)(x-2)} \times \frac{(x+1)}{(x-2)(x+1)}$$

$$= 1$$

$$(h) x + 2x^2 = 55$$

$$2x^2 + x - 55 = 0$$

$$(2x+11)(x-5) = 0$$

$$x = -5\frac{1}{2} \text{ and } 5$$

$$(i) \frac{1}{x+1} - \frac{1}{2x-1} = 0$$

$$\frac{1}{x+1} = \frac{1}{2x-1}$$

$$2x-1 = x+1$$

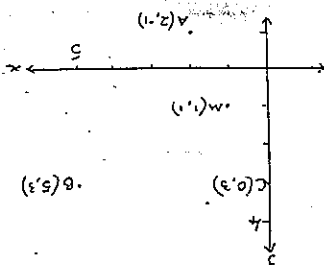
$$x = 2$$

$$(j) V = \frac{1}{3}x^2h$$

$$\sqrt{\frac{3V}{h}} = x$$

$$x = \sqrt{\frac{3 \times 53.77}{2.37}}$$

Marks	Content	Out
2	1.3	P1 P2 P4
2	1.3	P1 P3
3	1.4	P1 P2 P3 P4
2	1.4	P1 P3
2	1.4	P1 P2 P3

Out.	Marks	Ans.	Ques.
P1 P2 P3	2 6.1	(1) x intercepts @ $y = 0$ $2x - 12 = 0$ $2x = 12$ $x = 6 \rightarrow$ accepted $\text{Ans} = (6, 0) \rightarrow$ best answer y intercept @ $x = 0$ $3y - 12 = 0$ $3y = 12$ $y = 4 \rightarrow$ accepted $\text{Ans} = (0, 4) \rightarrow$ best answer	
P1 P3	1 6.2	(ii) $2x + 3y - 12 = 0$ $3y = -2x - 12$ $y = -\frac{2}{3}x - 4$ $m = -\frac{2}{3}$	
P3	2 6.2	$(-4) - 4(2) + 11 = 0$ $-a = -3$ $a = 3$	
P1 P4	2 6.8		

Ques	Ans	Ques	Ans	Ques	Ans	Ques	Ans
Ques 1	Ans 1	Ques 2	Ans 2	Ques 3	Ans 3	Ques 4	Ans 4
Ques 5	Ans 5	Ques 6	Ans 6	Ques 7	Ans 7	Ques 8	Ans 8
Ques 9	Ans 9	Ques 10	Ans 10	Ques 11	Ans 11	Ques 12	Ans 12
Ques 13	Ans 13	Ques 14	Ans 14	Ques 15	Ans 15	Ques 16	Ans 16
Ques 17	Ans 17	Ques 18	Ans 18	Ques 19	Ans 19	Ques 20	Ans 20
Ques 21	Ans 21	Ques 22	Ans 22	Ques 23	Ans 23	Ques 24	Ans 24
Ques 25	Ans 25	Ques 26	Ans 26	Ques 27	Ans 27	Ques 28	Ans 28
Ques 29	Ans 29	Ques 30	Ans 30	Ques 31	Ans 31	Ques 32	Ans 32
Ques 33	Ans 33	Ques 34	Ans 34	Ques 35	Ans 35	Ques 36	Ans 36
Ques 37	Ans 37	Ques 38	Ans 38	Ques 39	Ans 39	Ques 40	Ans 40
Ques 41	Ans 41	Ques 42	Ans 42	Ques 43	Ans 43	Ques 44	Ans 44
Ques 45	Ans 45	Ques 46	Ans 46	Ques 47	Ans 47	Ques 48	Ans 48
Ques 49	Ans 49	Ques 50	Ans 50	Ques 51	Ans 51	Ques 52	Ans 52
Ques 53	Ans 53	Ques 54	Ans 54	Ques 55	Ans 55	Ques 56	Ans 56
Ques 57	Ans 57	Ques 58	Ans 58	Ques 59	Ans 59	Ques 60	Ans 60
Ques 61	Ans 61	Ques 62	Ans 62	Ques 63	Ans 63	Ques 64	Ans 64
Ques 65	Ans 65	Ques 66	Ans 66	Ques 67	Ans 67	Ques 68	Ans 68
Ques 69	Ans 69	Ques 70	Ans 70	Ques 71	Ans 71	Ques 72	Ans 72
Ques 73	Ans 73	Ques 74	Ans 74	Ques 75	Ans 75	Ques 76	Ans 76
Ques 77	Ans 77	Ques 78	Ans 78	Ques 79	Ans 79	Ques 80	Ans 80
Ques 81	Ans 81	Ques 82	Ans 82	Ques 83	Ans 83	Ques 84	Ans 84
Ques 85	Ans 85	Ques 86	Ans 86	Ques 87	Ans 87	Ques 88	Ans 88
Ques 89	Ans 89	Ques 90	Ans 90	Ques 91	Ans 91	Ques 92	Ans 92
Ques 93	Ans 93	Ques 94	Ans 94	Ques 95	Ans 95	Ques 96	Ans 96
Ques 97	Ans 97	Ques 98	Ans 98	Ques 99	Ans 99	Ques 100	Ans 100

QUESTION FOUR

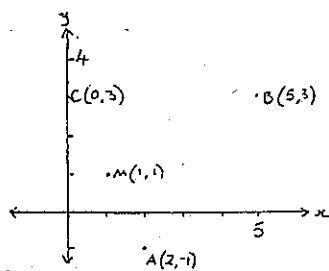
(a) (i) x intercepts @ $y = 0$
 $2x - 12 = 0$
 $2x = 12$
 $x = 6 \leftarrow \text{accepted}$
Ans = $(6, 0) \leftarrow \text{best answer}$

y intercept @ $x = 0$
 $3y - 12 = 0$
 $3y = 12$
 $y = 4 \leftarrow \text{accepted}$
Ans = $(0, 4) \leftarrow \text{best answer}$

(*) $2x + 3y - 12 = 0$
 $3y = -2x - 12$
 $y = -\frac{2}{3}x - 4$
 $m = -\frac{2}{3}$

b) $(-1) \cdot -4(2) + 11 = 0$
 $-a = -3$
 $a = 3$

c) (i)



QUESTION FOUR (cont)

(ii) $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$
 $= \left(\frac{2+0}{2}, \frac{-1+3}{2} \right)$
 $= (1, 1)$

(iii) Grad. MB = $\frac{3-1}{5-1}$
 $= \frac{1}{2}$

Grad AC = $\frac{-1-3}{2-0}$
 $= -2$

$m_1, m_2 = -1$
 $\frac{1}{2}x - 2 = -1$

(iv) $y - y_1 = m(x - x_1)$
 $y - 3 = -2(x - 0)$
 $y = -2x + 3$
 $2x + y - 3 = 0$

(v) $d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$
 $= \frac{|2(5) + 1(3) - 3|}{\sqrt{4 + 1}}$
 $= \frac{10}{\sqrt{5}} = 2\sqrt{5}$

In future,
Must mention that
BM $\perp$ AC for
full marks

(vi) $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$
 $(1, 1) = \left(\frac{x+5}{2}, \frac{y+3}{2} \right)$
 $x = -3$
 $y = -1$

Best answer: D(-3, -1)
Also accepted:
D(7, -1)
D(3, 7)