TRINITY GRAMMAR SCHOOL
MATHEMATICS DEPARTMENT

YEAR 11 2009 ASSESSMENT TASK 3

MATHEMATICS/MATHEMATICS EXTENSION1 HALF-YEARLY EXAMINATION

Time Allowed – ***Two and a half hours plus 5 minutes reading time***

WEIGHTING 25% towards final result

Outcomes referred to: P1, P2, P4 and P5

INSTRUCTIONS:

1. Attempt **ALL** questions.

2. Show all necessary working.

3. **Begin** each question on a **new page.**

4. Each question is of equal value, **except Q9 -11 marks and Q10 – 8 marks.** Mark values are shown beside each part. Total marks 99.

5. Non-programmable silent Board of Studies approved calculators are permitted.

6. If requested, additional writing paper may be obtained from the examinations supervisor upon request.

7. Place your name and your teacher's name on each writing booklet.

Question 1: (Start a new page) (10 marks) **Marks**

(a) Find the value of $\frac{1}{2 \cdot 5 \times 0 \cdot 3125}$. Give your answer to 2 decimal places. **2**

(b) Given the Ideal Gas Equation $P = \frac{nRT}{V}$, find P (correct to 3 decimal places) when $n = 0 \cdot 0425$, $R = 0 \cdot 0963$, $T = 265$ and $V = 10 \cdot 4$. **2**

(c) Write $(2 \cdot 10004 \times 10^{-5}) \times (0 \cdot 00047 \times 10^{-3})$ in scientific notation (round off to 4 significant figures). **2**

(d) Increase 500 g by 16%. **1**

(e) Factorise $10x + 5y$. **1**

(f) Factorise $4a^2 - 9$. **2**

Question 2: (Start a new page) (10 marks) **Marks**

(a) A boat costing $\$3500$ is later sold for $\$2800$. Express the loss as a percentage of the cost price. **2**

(b) Express $0 \cdot \dot{3}\dot{4}$ as a common fraction. **2**

(c) Simplify $4\sqrt{45}$. **1**

(d) Simplify $2m^2p^3 \times (-4m^{-2}p^{-3})$. **2**

(e) Write $(2 + 3\sqrt{5})^2$ in the form of $a + b\sqrt{x}$. **3**

Question 3: (Start a new page) (10 marks) **Marks**

(a) Expand then Simplify $3(x-y)+3y-x$. **2**

(b) Factorise $a-b+ax-bx$. **2**

(c) Factorise $8c^3-27$. **2**

(d) Factorise $7x^2+32x-15$. **2**

(e) Factorise fully abx^2-a^3b. **2**

Question 4: (Start a new page) (10 marks) **Marks**

(a) Simplify $\frac{x^2+5x+6}{x^2-4}$. **2**

(b) Simplify $\frac{ax^2-4a}{bx-2b}\times\frac{b^2}{x^2-x-6}$. **2**

(c) Simplify $\frac{2}{x^2-3x-4}\div\frac{4}{x^2-4x}$. **3**

(d) Solve $\frac{x+4}{2}+\frac{x-6}{3}=1$. **3**

Question 5: (Start a new page) (10 marks) **Marks**

(a) Solve simultaneously
$y = 2x + 1$
$y = x^2 - 4x + 1$. **3**

(b) Solve $(x-4)(x+3) = 3(x-5)$. **3**

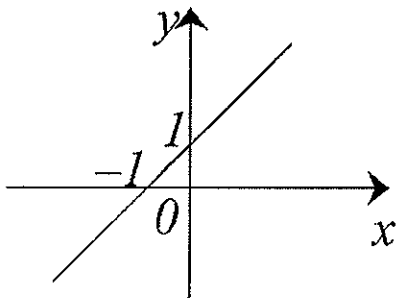
(c) Solve $|2x + 3| \geq 5$. **2**

(d) Solve $x^2 - 1 \leq 0$. **2**

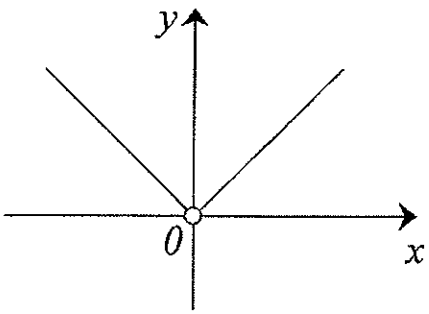
Question 6: (Start a new page) (10 marks) **Marks**

(a) Find the domain (D) and the range (R) of the function below.

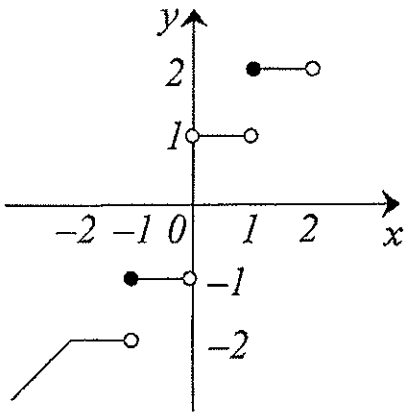
(i) **1**



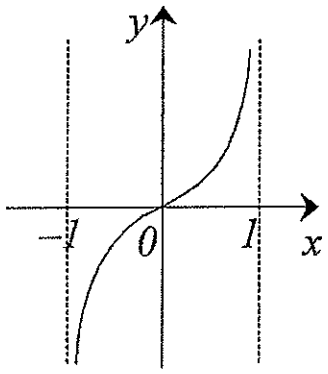
(ii) **2**



(iii) **2**



(iv) **2**



(b) Shade the region(s) in the XY-plane which represent the simultaneous solution of $y \leq x^2$ and $2x + 3y - 6 \leq 0$. **3**

Question 7: (Start a new Page) (10 marks) **Marks**

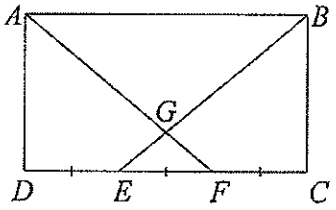
(a) Determine whether function $f(x) = x^2 - x$ is an even function, an odd function or neither. **2**

(b) Draw the graphs of $y = |x|$ and $y = x + 4$ on the same set of axes.
Find the coordinates of the point of intersection of these two graphs. **3**

(c) A function is defined as: $f(x) = \begin{cases} -x^2 \text{ when } x \leq 0 \\ 5x - 4 \text{ when } x > 0 \end{cases}$

Evaluate $f(-2) + f(3) - f(0)$. **2**

(d) $ABCD$ is a rectangle. E and F are two points on CD such that $CF = FE = ED$.



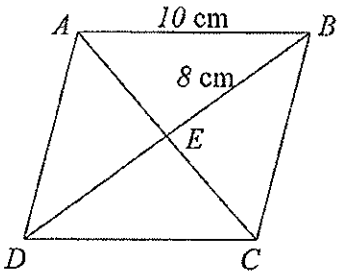
Prove that:

(i) $\Delta AFD \equiv \Delta BEC$. **2**

(ii) ΔAGB is an isosceles triangle. **1**

Question 8: (Start a new Page) (10 marks) **Marks**

(a) Find the area of the rhombus $ABCD$ given $AB = 10$ cm and $EB = 8$ cm. **2**



(b) A(2, –2), B(–2, –3) and C(0, 2) are the vertices of a triangle ABC.

(i) Draw a sketch diagram of the triangle.

(ii) Find the length of the line AC and the gradient of AC. **2**

(iii) Find the equation of the line AC in the general form. **2**

(iv) Calculate the perpendicular distance of B from the side AC and hence find the area of ΔABC. **3**

(v) Find the co-ordinates of D such that $ABCD$ is a parallelogram. **1**

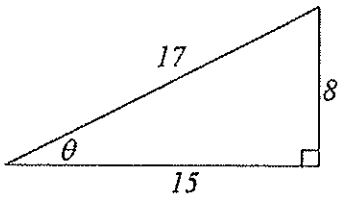
Question 9: (Start a new page) (11 marks) **Marks**

(a) In the triangle shown, find:

(i) $\operatorname{cosec}\theta$ **1**

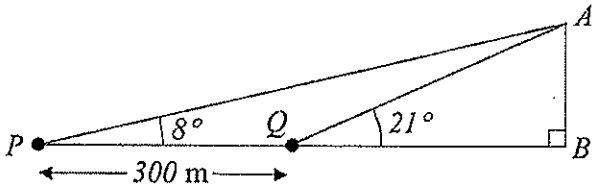
(ii) $\sec\theta$ **1**

(iii) $\cot\theta$ **1**



(b) Solve the equation $2\cos x = \sqrt{3}$, where $0 \leq x \leq 360$. **2**

(c)



From position P, Anne finds that the angle of elevation of the top A of a rock pillar AB is $8°$. After walking 300 m directly towards the pillar to the point Q she finds that the angle of elevation of A is $21°$.

(i) Copy the diagram into your Writing Booklet and find $\angle PAQ$. **1**

(ii) Calculate the length AQ. **2**

(iii) Find the height of the rock pillar AB. **2**

Question 10: (Start a new page) (8 marks) **Marks**

(a) Prove that $\frac{\cot\theta \cos ec\theta}{1+\cot^2\theta} = \cos\theta$. **4**

(b) From a point O the point P bears $120°$ from the North and is $12{\cdot}3$ km away. The point Q is $15{\cdot}2$ km South West of O. Mark the relative positions of O, P, Q on a sketch. Calculate the distance PQ in kilometres. (Rounded off correct to one decimal place). **4**

YEAR 11 – HALF YEARLY 2009
MATHEMATICS

Question 1:

(a) $1 \cdot 28$ — (2) correct answer; (1) for $1\frac{7}{25}$

(b) $P = 0 \cdot 1042864\ldots$ (1)

$P = 0 \cdot 104$ (3 decimal places). (1)

(c) $9 \cdot 870188 \times 10^{-12}$ (2) if correct; (1) for not scientific not.

$= 9 \cdot 870 \times 10^{-12}$ (4 significant figures).

(d) $500g \times 116\%$

$= 580g$ (1)

(e) $5(2x + y)$ (1)

(f) $(2a - 3)(2a + 3)$ (2) correct; (1) for $(4a - 3)(4a + 3)$

Question 2:

(a) Loss $= \$700$

$= \frac{700}{3500} \times \frac{100}{1}$ —— (1)

$= 20\%$ —— (1)

(b) let $x = 0 \cdot 343434\ldots$

$100x = 34 \cdot 343434\ldots$

$99x = 34$ — (1)

$x = \frac{34}{99}$ —— (1)

(c) $4\sqrt{9 \times 5}$

$= 12\sqrt{5}$ —— (1)

(d) $-8 \times p^0 \times m^0$ —— (1)

$= -8$ —— (1)

(e) $4 + 12\sqrt{5} + 45$

$= 49 + 12\sqrt{5}$

(1) (1) (1)

Question 3 :

(a) $3x - 3y + 3y - x$ (1)

$= 2x$ (1)

(b) $(a-b) + x(a-b)$ (1)

$= (a-b)(1+x)$ (1)

(c) $(2c-3)(4c^2+6c+9)$ (1) (1)

(d) $(7x-3)(x+5)$ (1) (1)

(e) $ab(x^2-a^2)$ (1)

$= ab(x-a)(x+a)$ (1)

Question 4 :

(a) $\frac{(x+2)(x+3)}{(x-2)(x+2)}$ (M1)

$= \frac{x+3}{x-2}$ (1)

(b) $\frac{a(x-2)(x+2)}{b(x-2)} \times \frac{b}{(x-3)(x+2)}$ (M1)

$= \frac{ab}{x-3}$ (1)

(c) $\frac{1}{(x+1)(x-4)}$ (1) $\times \frac{x(x-4)}{4}$ (1)

$= \frac{x}{2(x+1)}$ (1)

(d) $3(x+4) + 2(x-6) = 6$ (1)

$3x + 12 + 2x - 12 = 6$

$5x = 6$ (1)

$x = \frac{6}{5}$

or $x = 1\frac{1}{5}$ (1)

Question 5:

(a) let $2x+1 = x^2 - 4x + 1$

$0 = x^2 - 6x$ — (1)

$0 = x(x-6)$

$x = 0$, $y = 1$ (1)

$x = 6$, $y = 13$ (1)

(b) $x^2 - 4x + 3x - 12 = 3x - 15$ (M1)

$x^2 - 4x + 3 = 0$

$(x-1)(x-3) = 0$

$x = 1$ or $x = 3$ (1) (1)

(c) (+) $2x + 3 \geq 5$ OR (−) $-2x - 3 \geq 5$

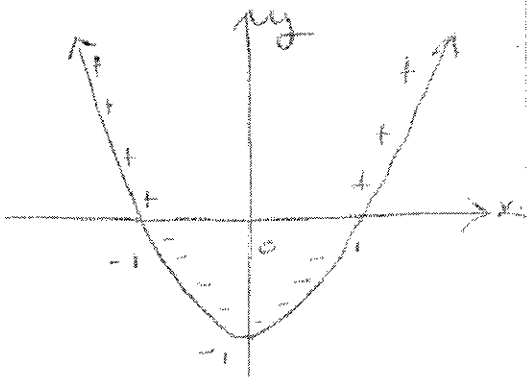
$2x \geq 2$

$x \geq 1$ (1)

$-2x \geq 8$

$x \leq -4$ (1)

(d) $(x-1)(x+1) \leq 0$



$-1 \leq x \leq 1$ (1) (1)

Question 6:

(a) (i) D: { all real values of x }

R: { all real values of y } (1)

(ii) D: { all real values except $x \neq 0$ } (1)

R: { $y > 0$ } (1)

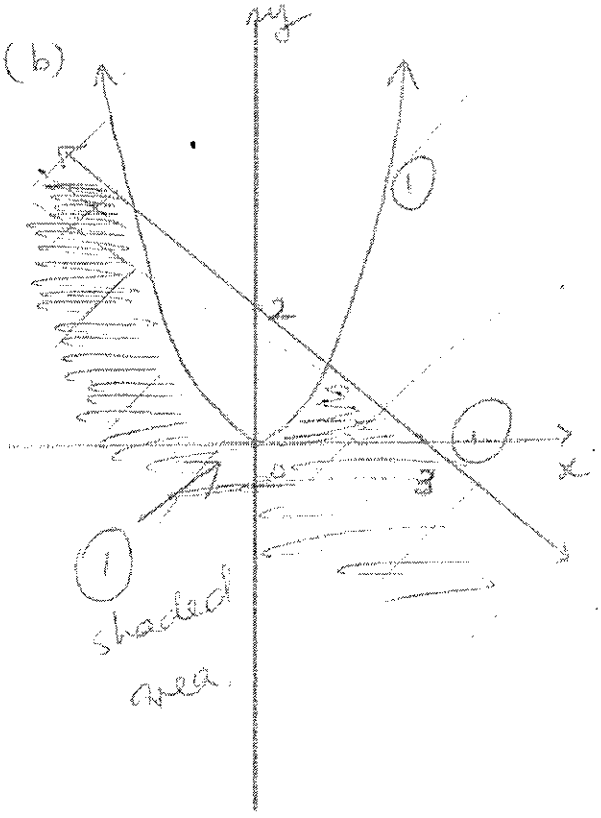
(iii) D: { $0 < x < 2$ and $x < 0$ } (1)

R: { $y \leq -2$ and $y = \pm 1, 2$ } (1)

(iv) D: { $-1 < x < 1$ } (1)

R: { all real values of y } (1)

(b)



Question 7:

(a) $f(x) = x^2 - x$

$f(-x) = x^2 + x$.

For even

$f(x) = f(-x)$

$x^2 - x \neq x^2 + x$

Not even.

For odd (M1)

$f(-x) = -f(x)$

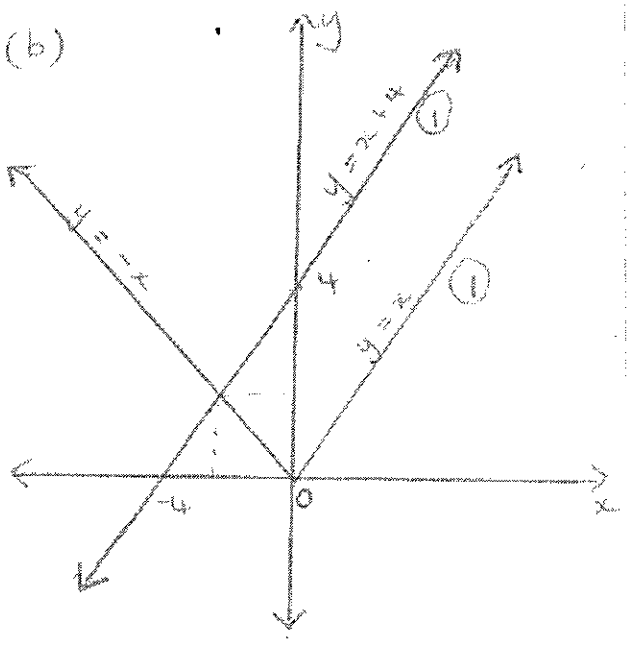
$x^2 + x = -(x^2 - x)$

$x^2 + x \neq -x^2 + x^2$

Not odd

∴ neither. (1)

(b)



$y = -x$ —— (1)

$y = x + 4$ —— (2)

$-x = x + 4$

$-4 = 2x$

$-2 = x$

$y = 2$

point of intersection $(-2, 2)$ (1)

(c) $f(-2) = -4$

$f(3) = 11$ (M1)

$f(0) = 0$

∴ $-4 + 11 - 0$

$= 7$ (1)

(d)(i) Aim: To prove $\triangle AFD \equiv \triangle BEC$

Proof: $AD = BC$ —— opp sides

Height of rectangle are equal

(1) $\angle ADF = \angle BCE = 90°$

(angles in rectangle)

$DF = CE$ —— given.

(SAS) (1)

∴ $\triangle AFD \equiv \triangle BEC$

(ii) Since $\triangle AFD \equiv \triangle BEC$

$\angle AFD = \angle BEC$ —— corresponding $\angle$'s in congruent $\triangle$'s

$\angle ABE = \angle BEF$ —— alt $\angle$'s in $\parallel$ lines

$\angle BAF = \angle AFD$ —— " " "

(1)

∴ $\triangle ACB$ is isosceles since base angles are equal.

Question 8:

(a) using pythagoras

$AE^2 = 100 - 64$

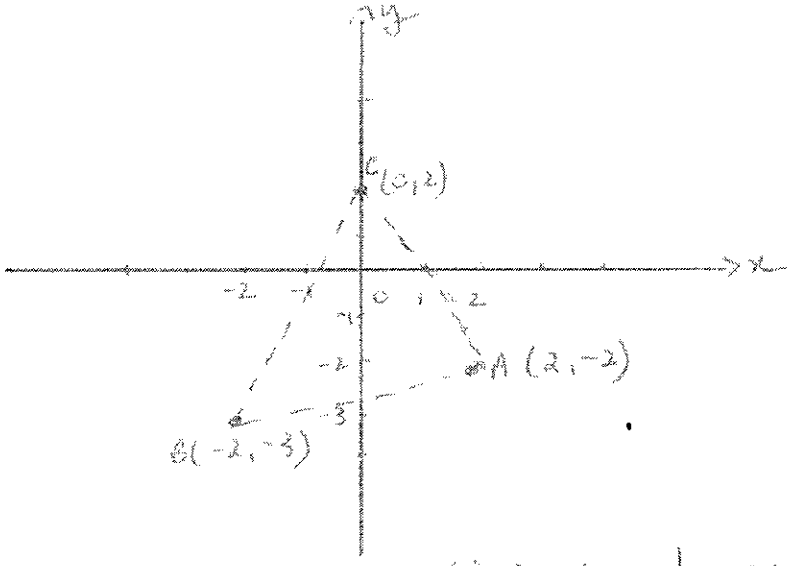
$AE^2 = 36$

$AE = 6\text{ cm}$ —— (1)

$\therefore$ Area $= \frac{1}{2} \times 16 \times 12$

Area $= 96\text{ cm}^2$ —— (1)

(b)



(ii) length $AC = \sqrt{(2-0)^2 + (-2-2)^2}$

$AC = \sqrt{4 + 16}$

$AC = \sqrt{20}$

$AC = 2\sqrt{5}$ (1)

$m_{AC} = \frac{-2-2}{2-0}$

$m_{AC} = \frac{-4}{2}$

$m = -2$ (1)

(iii) $y - 2 = -2(x - 0)$ (M1)

$y - 2 = -2x$

$2x + y - 2 = 0$ (1)

(iv) $d = \frac{|(-2)(2) + (1)(-3) - 2|}{\sqrt{2^2 + 1^2}}$ (M1)

$d = \frac{|-4 - 3 - 2|}{\sqrt{5}}$

$d = \frac{9}{\sqrt{5}}$ (1)

Area of $\Delta ABC = \frac{1}{2} \times 2\sqrt{5} \times \frac{9}{\sqrt{5}}$

Area $= 9$ units^2 (1)

(v) $D(4, 3)$ (1)

Question 9:

a) (i) $\frac{1}{\sin\theta} = \frac{1}{\frac{8}{17}} = \frac{17}{8}$ — (1)

(ii) $\frac{1}{\cos\theta} = \frac{1}{\frac{15}{17}} = \frac{17}{15}$ — (1)

(iii) $\frac{1}{\tan\theta} = \frac{1}{\frac{8}{15}} = \frac{15}{8}$ — (1)

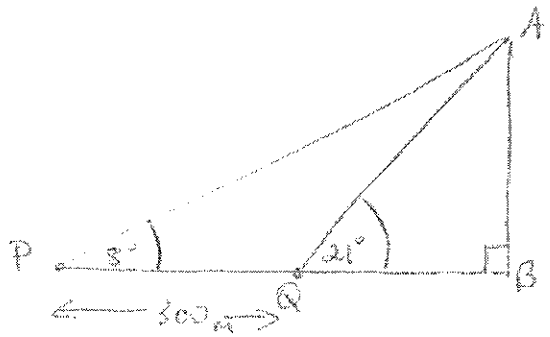
(b) $\cos x = \frac{\sqrt{3}}{2}$ — (1)

S	A ✓
T	C ✓

$x = 30°, 330°$

(1) (1)

c)



(i) $13° = \angle PAQ$ (1)

(ii) $\frac{AQ}{\sin 8°} = \frac{300}{\sin 13°}$ (M1)

$AQ = \frac{300 \times \sin 8°}{\sin 13°}$

$AQ = 185.6\text{ m}$ (1 decimal place)

(1)

(iii) $\frac{AB}{185.6} = \sin 21°$

(M1)

$AB = 185.6 \times \sin 21°$

$AB = 66.5\text{ m}$ (1 decimal pl).

(1)

Question 10 :

(a) LHS $\dfrac{\cot\theta \operatorname{cosec}\theta}{\operatorname{cosec}^2\theta}$ (1)

$= \dfrac{\cot\theta}{\operatorname{cosec}\theta}$

$= \dfrac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}}$ (1)

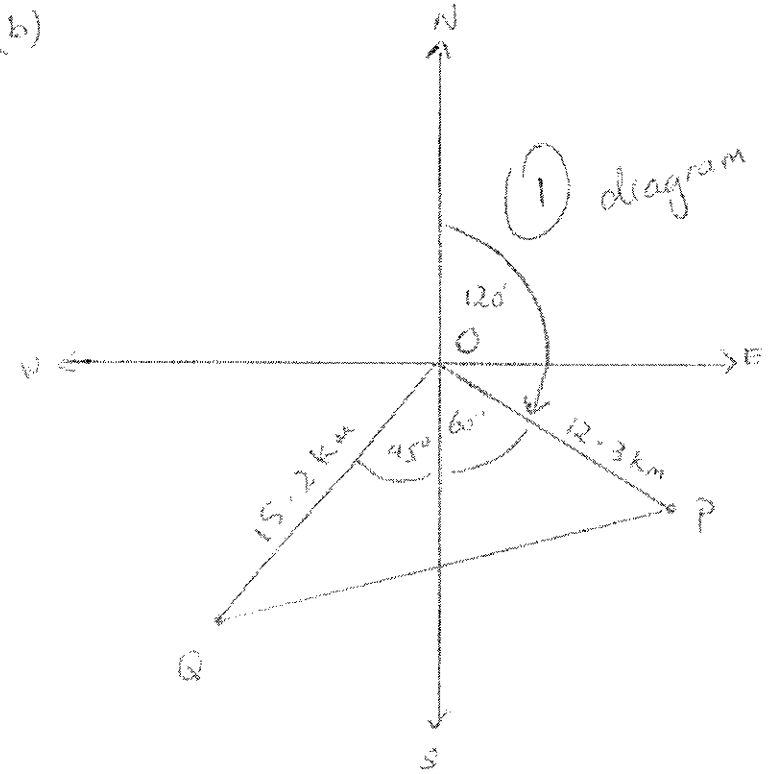
$= \dfrac{\cos\theta \times \sin\theta}{\sin\theta}$ (1)

$= \cos\theta$ (RHS) (1)

(1) mark if numerator was rearranged.

(2) for numerator and denominator was rearranged.

(b)



(1) diagram

(M1)

$PQ^2 = 12.3^2 + 15.2^2 - 2 \times 12.3 \times 15.2 \times \cos 101°$

$PQ^2 = 479.1076$ ... (1)

$PQ = 21.9$ km (1 dec. pl)

(1)