

Name: _____

Class: _____



TRINITY GRAMMAR SCHOOL
MATHEMATICS DEPARTMENT



YEAR 11 2010 ASSESSMENT TASK 3

MATHEMATICS / MATHEMATICS EXTENSION 1
HALF-YEARLY EXAMINATION

Monday, 19 April 2010

TIME ALLOWED: Two and a half hours (plus 5 minutes reading time).

WEIGHTING: 25% towards final result.

OUTCOMES REFERRED TO: P1, P2, P3, P4 and P5.

INSTRUCTIONS:

1. Attempt **ALL** questions.
2. Show all necessary working. Marks may be deducted for careless or poorly arranged work.
3. **Begin** each question on a **new page**.
4. There are 10 questions worth 10 marks each. **Total marks: 100.**
5. Marks for each part question are indicated on the paper.
6. Board of Studies approved calculators are permitted.
7. Write your name and your teacher's name on each sheet.

QUESTION 1 (10 marks)

Start each question on a new page.

Write your name and your teacher's name at the top of the page.

	Marks
(a) Evaluate $\frac{8.2 \times 10^{-2}}{\sqrt{5.6} - 4.2}$ correct to 3 decimal places.	2
(b) Given that $v = u + at$, find t if $v = 85$, $u = 25$ and $a = 10$.	2
(c) The speed of light in a vacuum is 299 792 458 m/s. Express in scientific notation, correct to 1 significant figure.	2
(d) Simplify $ -4 \times 5 $.	1
(e) Factorise $24a + 18b + 54c$.	1
(f) A boat lost 35% of its value after one year. If it was worth \$7800 after one year, what was the original value?	2

QUESTION 2 (10 marks)

Start each question on a new page.

Write your name and your teacher's name at the top of the page.

	Marks
(a) Express $0.\dot{6}\dot{7}$ as a fraction.	2
(b) Simplify $3\sqrt{54}$.	1
(c) Express $a^{-\frac{1}{3}}$ without negative or fractional indices.	1
(d) Simplify $2(6x^2 + 3x - 7) - 3(4x^2 - 2x + 1)$.	2
(e) Simplify $\frac{2x^2 + 5x - 3}{6x - 3}$.	2
(f) By rationalising the denominator express $\frac{11}{5 + \sqrt{3}}$ in simplest form.	2

QUESTION 3 (10 marks)

Start each question on a new page.

Write your name and your teacher's name at the top of the page.

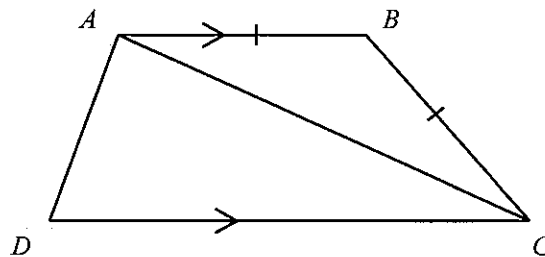
	Marks
(a) Solve $-\frac{2a}{5} > 4$.	1
(b) Expand and simplify $(3\sqrt{5} + \sqrt{2})^2$.	2
(c) Solve $\frac{x+2}{3} + \frac{x-2}{2} = 3$.	2
(d) Solve $x^2 - 4x - 3 = 0$, giving your answer in exact form.	2
(e) Solve $y = 3x + 2$ and $y = x^2 - 2$ simultaneously.	3

QUESTION 4 (10 marks)

Start each question on a new page.

Write your name and your teacher's name at the top of the page.

	Marks
(a) Solve $2^{2x-3} = \frac{1}{32}$.	2
(b) Solve $ 5+10k \leq 25$ and graph your solution on a number line.	2
(c) Fully simplify $\frac{x}{x+2} - \frac{6}{x^2+x-2}$.	3
(d) In the figure, $AB \parallel DC$ and $AB = BC$. Prove that AC bisects $\angle BCD$, giving reasons.	3



QUESTION 5 (10 marks)

Start each question on a new page.

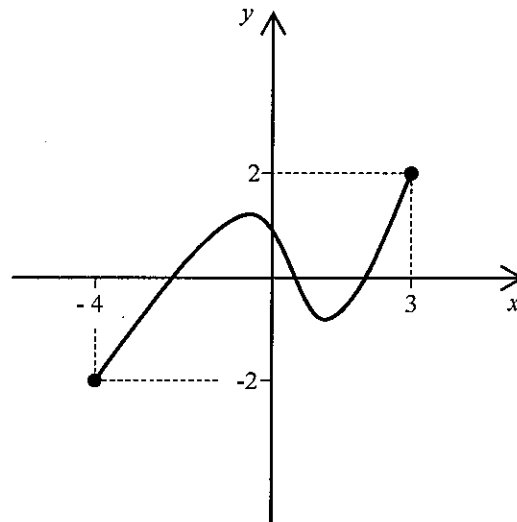
Write your name and your teacher's name at the top of the page.

Marks

(a) State the domain and range for each of the following three functions.

i)

2



ii) $y = 3 - x$.

2

iii) $y = |x| + 2$.

2

(b) Sketch the following functions.

i) $y = 3^x$.

2

ii) $y = \frac{4}{x-1}$.

2

QUESTION 6 (10 marks)

Start each question on a new page.

Write your name and your teacher's name at the top of the page.

Marks

- (a) Determine whether the function $f(x) = \frac{x}{x^2 - 1}$ is an even function, an odd function or neither. 2
- (b) The function $f(x)$ is defined by the rule
- $$f(x) = \begin{cases} x & \text{for } -4 \leq x < 0 \\ \sqrt{16 - x^2} & \text{for } 0 \leq x \leq 4 \end{cases}$$
- i) Evaluate $f(0)$. 1
- ii) Sketch $y = f(x)$. 3
- (c) Find the coordinates of the vertex of the parabola $y = x^2 - 2x - 15$. 2
- (d) Sketch the function $y = |2x - 4|$, showing all x and y intercepts. 2

QUESTION 7 (10 marks)

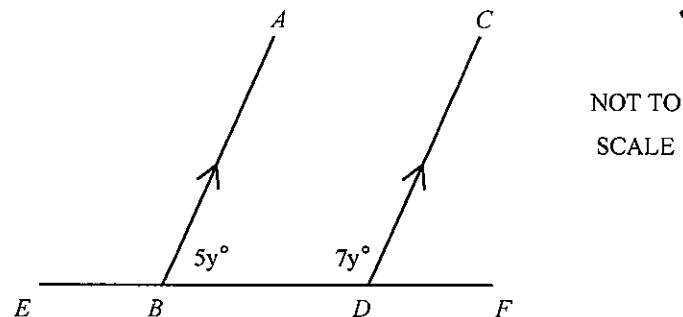
Start each question on a new page.

Write your name and your teacher's name at the top of the page.

Marks

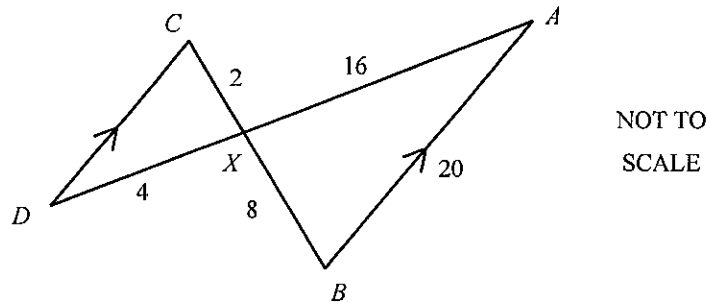
- (a) In the diagram $AB \parallel CD$. Find the value of y , giving reasons.

2



- (b) Given $\triangle XCD \parallel \triangle XBA$, find the length of CD .

2



- (c) Simplify $\frac{x^2 - x}{3x^3 + 3x^2} \div \frac{x^2 - 1}{3x}$.

3

- (d) If $g(x) = \frac{x^2}{x+4}$, find the values of x for which $g(x) = 2$.

3

QUESTION 8 (10 marks)

Start each question on a new page.

Write your name and your teacher's name at the top of the page.

	Marks
(a) The exterior angles of a regular polygon are each 24° . How many sides does the polygon have?	2
(b) Calculate the side length of a rhombus whose diagonals are 10m and 24m long.	2
(c) Sketch the region $y > x^2 - 9$ on the number plane.	3
(d) The width, w , of a rectangle is one third the size of its length. If the width is increased by two units while the length remains constant, find the increase in area in terms of w .	3

QUESTION 9 (10 marks)

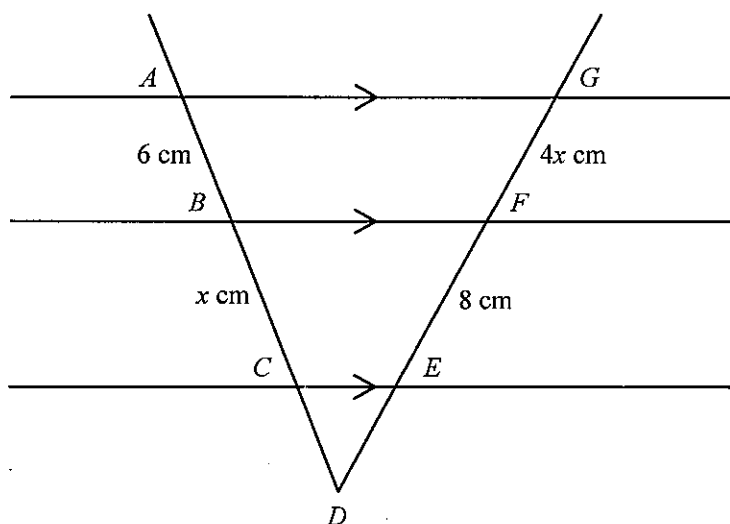
Start each question on a new page.

Write your name and your teacher's name at the top of the page.

Marks

- (a) Find the value of x in the diagram below, leaving your answer as a simplified surd.

3



NOT TO
SCALE

- (b) Fully factorise $x^5 - 4x^3 - 8x^2 + 32$.

4

- (c) Sketch the region for which $y \leq x + 3$ and $x > -3$ hold simultaneously.

3

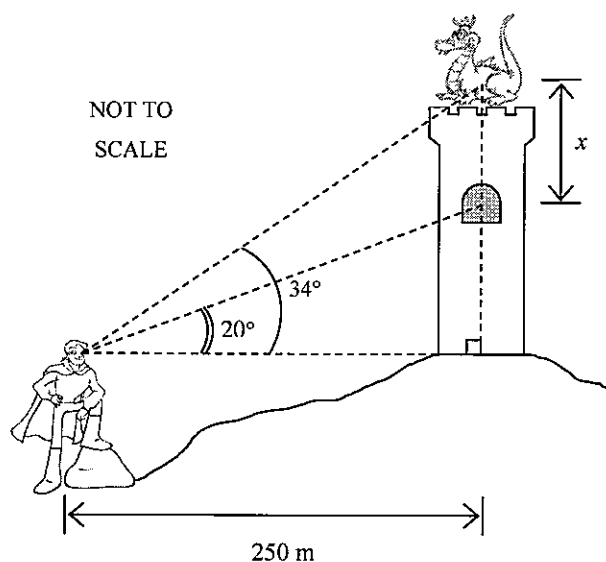
QUESTION 10 (10 marks)

Start each question on a new page.

Write your name and your teacher's name at the top of the page.

Marks

- (a) Write down the exact values of:
- i) $\sin 60^\circ$. 1
 - ii) $\cot 30^\circ$. 1
 - iii) $\frac{\cot 30^\circ}{\sin 60^\circ}$. 1
- (b) A plane flies 420m due east then turns and flies 670m due south. What is the bearing of the plane from its starting position to the nearest degree? 2
- (c) Use the complementary angle results to find a value of θ given $\sin \theta^\circ = \cos(\theta + 40)^\circ$. 2
- (d) Prince Charming observes the angle of elevation of a dragon sitting on top of a tower to be 34° at a location 250m from the tower's base. The angle of elevation of Princess Fiona's window in the tower is 20° from the same point. How high is the dragon above the window to the nearest metre? (i.e. find the value of x). 3



* Solutions *

①

YEAR 11 MATHEMATICS

ASSESSMENT TASK 3 - 2010

* Question 1:

$$\begin{aligned} \text{a) } & -0.044721546\dots \\ & = -0.045 \text{ (to 3 d.p.)} \end{aligned}$$

✓
✓

$$\begin{aligned} \text{b) } & v = u + at \\ & 85 = 25 + 10t \\ & 60 = 10t \\ & \therefore t = 6 \end{aligned}$$

✓
✓

$$\text{c) } 3 \times 10^8 \text{ m/s}$$

✓ ✓

$$\text{d) } 4 \times 5 = 20 \quad \checkmark$$

$$\begin{aligned} \text{e) } & 24a + 18b + 54c \\ & = 6(4a + 3b + 9c) \end{aligned}$$

✓

$$\text{f) } 65\% \text{ of value is } \$7800 \quad \checkmark$$

$$\therefore \text{original value} = \frac{7800}{65} \times 100$$

$$= \$12000 \quad \checkmark$$

• Question 2:

a) Let $x = 0.67777\dots$

$$\therefore 10x = 6.7777\dots \quad (1)$$

$$\therefore 100x = 67.7777\dots \quad (2)$$

✓ (process)

$$(2) - (1) \quad 90x = 61$$

$$\therefore x = \frac{61}{90}$$

✓

b) $3\sqrt{54} = 3 \times \sqrt{9} \times \sqrt{6}$
 $= 9\sqrt{6}$

✓

c) $\frac{1}{\sqrt[3]{a}}$

✓

d) $2(6x^2 + 3x - 7) - 3(4x^2 - 2x + 1)$

$$= 12x^2 + 6x - 14 - 12x^2 + 6x - 3$$

$$= 12x - 17$$

✓

✓

e) $\frac{2x^2 + 5x - 3}{6x - 3} = \frac{(2x-1)(x+3)}{3(2x-1)}$

✓

$$\frac{2x}{x} \times \frac{-1}{3}$$

$$= \frac{x+3}{3}$$

✓

f) $\frac{11}{5+\sqrt{3}} \times \frac{5-\sqrt{3}}{5-\sqrt{3}} = \frac{11(5-\sqrt{3})}{22}$

✓

$$= \frac{5-\sqrt{3}}{2}$$

✓

• Question 3:

$$a) -\frac{2a}{5} > 4$$

$$-2a > 20$$

$$\therefore a < -10 \quad \checkmark$$

$$\begin{aligned} b) (3\sqrt{5} + \sqrt{2})^2 &= 3\sqrt{5} \times 3\sqrt{5} + 2 \times 3\sqrt{5} \times \sqrt{2} + \sqrt{2} \times \sqrt{2} \quad \checkmark \\ &= 45 + 6\sqrt{10} + 2 \\ &= 47 + 6\sqrt{10} \quad \checkmark \end{aligned}$$

$$c) 6 \times \left(\frac{x+2}{3} + \frac{x-2}{2} \right) = 3 \times 6$$

$$2(x+2) + 3(x-2) = 18 \quad \checkmark$$

$$2x + 4 + 3x - 6 = 18$$

$$5x = 20$$

$$\therefore x = 4 \quad \checkmark$$

$$d) x^2 - 4x - 3 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = 1$$

$$b = -4$$

$$c = -3$$

$$= \frac{4 \pm \sqrt{16 - 4 \times 1 \times -3}}{2} \quad \checkmark$$

$$= \frac{4 \pm \sqrt{28}}{2} \quad \checkmark$$

$$= \frac{4 \pm 2\sqrt{7}}{2}$$

$$= 2 \pm \sqrt{7}$$

Alternatively:

$$x^2 - 4x + 4 = 3 + 4 \quad \checkmark$$

$$(x-2)^2 = 7$$

$$x-2 = \pm\sqrt{7}$$

$$\therefore x = 2 \pm \sqrt{7} \quad \checkmark$$

$$e) y = 3x + 2 \quad (1)$$

$$y = x^2 - 2 \quad (2)$$

$$\text{Set } (1) = (2) \quad 3x + 2 = x^2 - 2 \quad \checkmark$$

$$x^2 - 3x - 4 = 0$$

$$(x-4)(x+1) = 0$$

$$\therefore x = 4 \quad \text{or} \quad x = -1 \quad \checkmark$$

$$y = 4^2 - 2 \\ = 14$$

$$y = (-1)^2 - 2 \\ = -1 \quad \checkmark$$

$$\therefore x = 4, y = 14 \quad \text{or} \quad x = -1, y = -1$$

• Question 4:

$$a) 2^{2x-3} = \frac{1}{32}$$

$$2^{2x-3} = 2^{-5} \quad \checkmark$$

$$2x - 3 = -5$$

$$\therefore x = -1 \quad \checkmark$$

$$b) -25 \leq 5 + 10k \leq 25 \quad \checkmark$$

$$-30 \leq 10k \leq 20$$

$$-3 \leq k \leq 2$$



$$c) \frac{x}{x+2} - \frac{6}{x^2+x-2}$$

$$= \frac{x}{x+2} \times \frac{(x-1)}{(x-1)} - \frac{6}{(x+2)(x-1)} \quad \checkmark$$

$$= \frac{x^2 - x - 6}{(x+2)(x-1)} \quad \checkmark$$

$$= \frac{(x-3)\cancel{(x+2)}}{\cancel{(x+2)}(x-1)}$$

$$= \frac{x-3}{x-1} \quad \checkmark$$

d) $\triangle ABC$ is isosceles ($AB = BC$, given)

$\angle BAC = \angle BCA$ (base angles of isosceles $\triangle ABC$) $\checkmark$

$\angle BAC = \angle ACD$ (alternate $\angle$'s, $AB \parallel DC$) $\checkmark$

$\therefore \angle BCA = \angle ACD$ (both equal $\angle BAC$) $\checkmark$

$\therefore AC$ bisects $\angle BCD$

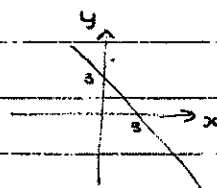
• Question 5:

a) i) D: $-4 \leq x \leq 3$ $\checkmark$

R: $-2 \leq y \leq 2$ $\checkmark$

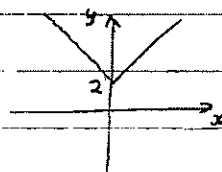
ii) D: all real x $\checkmark$

R: all real y $\checkmark$

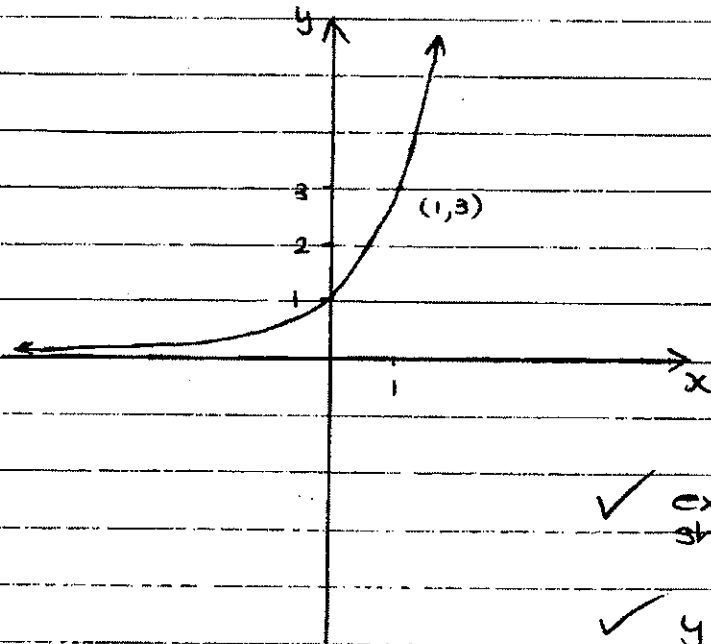


iii) D: all real x $\checkmark$

R: $y \geq 2$ $\checkmark$



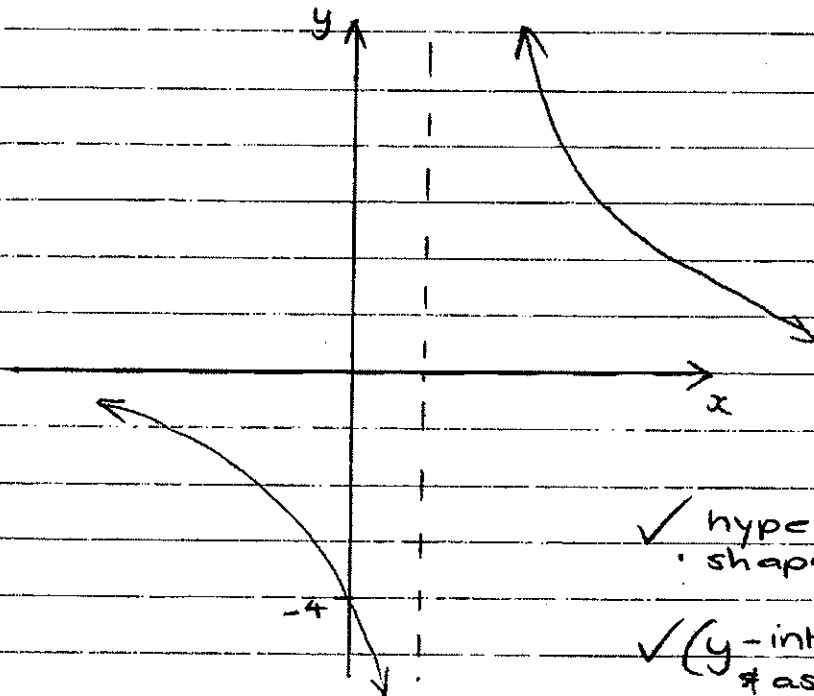
b) i) $y = 3^x$



✓ exponential shape

✓ y-intercept

ii) $y = \frac{4}{x-1}$



✓ hyperbolic shape

✓ (y-intercept) & asymptotes.

• Question 6:

$$a) f(-x) = \frac{(-x)}{(-x)^2 - 1}$$

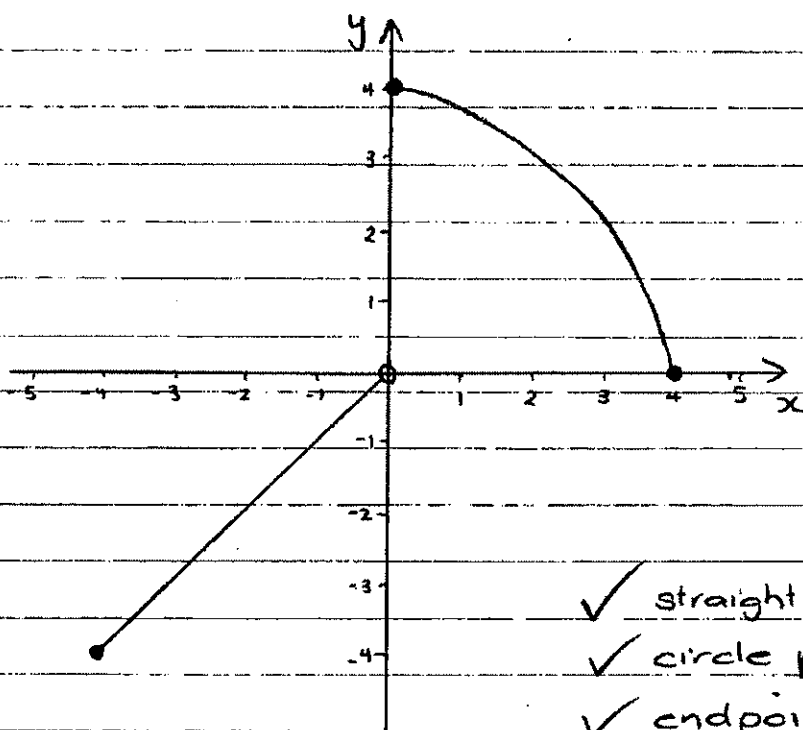
$$= \frac{-x}{x^2 - 1}$$

$$= -f(x) \quad \therefore \text{ODD} \quad \checkmark$$

$$b) i) f(0) = \sqrt{16 - 0^2}$$

$$= 4 \quad \checkmark$$

ii)



$\checkmark$ straight line portion
 $\checkmark$ circle portion
 $\checkmark$ endpoints

$$c) (x - 5)(x + 3) = 0$$

$$\therefore x\text{-intercepts } x = 5, x = -3 \quad \checkmark$$

$$\therefore \text{vertex @ } x = \frac{5 - 3}{2}$$

$$= 1$$

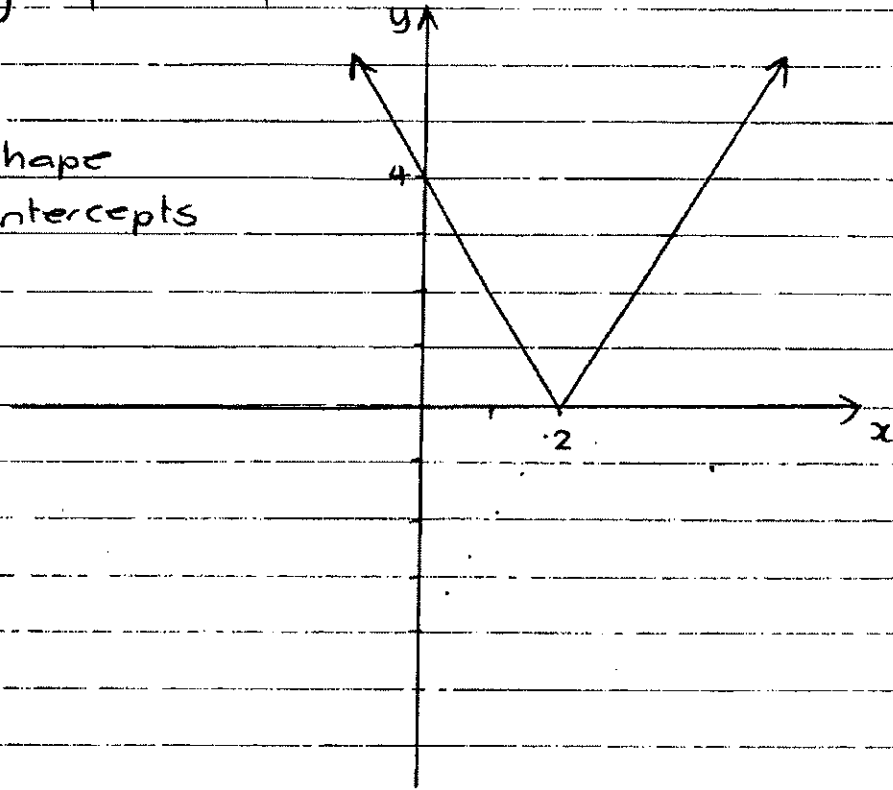
$$y = 1^2 - 2 - 15$$

$$= -16$$

$$\therefore \text{vertex is at } (1, -16) \quad \checkmark$$

d) $y = |2x - 4|$

✓ shape
✓ intercepts



• Question 7:

a) $5y^\circ + 7y^\circ = 180^\circ$ (co-interior $\angle$'s, $AB \parallel CD$) ✓
 $12y = 180$
 $y = 15$ ✓

b) $\frac{CD}{BA} = \frac{XC}{XB}$

$\frac{CD}{20} = \frac{2}{8}$ ✓

$\therefore CD = 5$ ✓

$$c) \frac{x^2 - x}{3x^3 + 3x^2} \div \frac{x^2 - 1}{3x}$$

$$= \frac{\cancel{x}(x-1)}{3\cancel{x}^2(x+1)} \times \frac{3\cancel{x}}{(x+1)(x-1)} \quad \checkmark \checkmark$$

$$= \frac{1}{(x+1)^2} \quad \checkmark$$

$$d) \quad 2 = \frac{x^2}{x+4} \quad \checkmark \quad (x \neq -4)$$

$$2x + 8 = x^2$$

$$x^2 - 2x - 8 = 0 \quad \checkmark$$

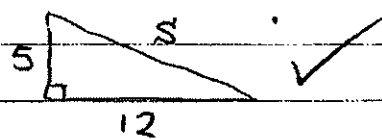
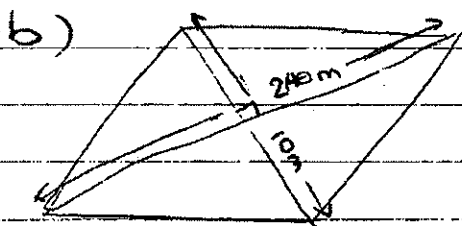
$$(x - 4)(x + 2) = 0$$

$$\therefore x = -2 \text{ or } 4 \quad \checkmark$$

• Question 8:

$$a) \frac{360^\circ}{n} = 24^\circ \quad \checkmark$$

$$\therefore n = 360 \div 24 \\ = 15 \quad \checkmark$$

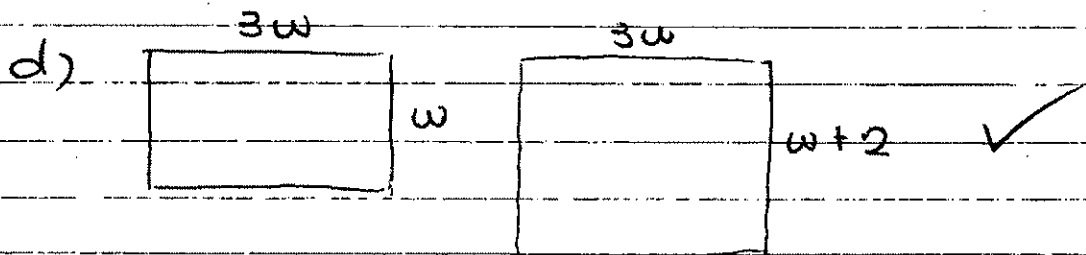
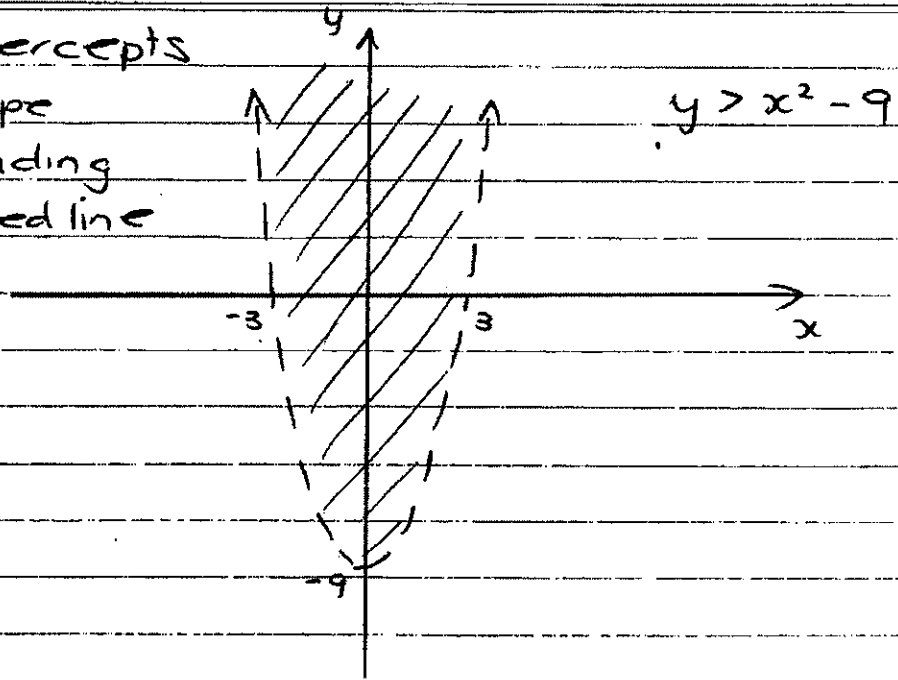


$$S^2 = 5^2 + 12^2 \\ = 169$$

$$\therefore S = 13$$

$\therefore$ side length is 13 m $\checkmark$

- c) ✓ intercepts
 ✓ shape
 ✓ { shading
 { dotted line



$$A_1 = 3w^2$$

$$A_2 = 3w(w+2) \\ = 3w^2 + 6w \quad \checkmark$$

$$\therefore \text{Increase in area} = 3w^2 + 6w - 3w^2 \\ = 6w \quad \checkmark$$

• Question 9:

a) $\frac{x}{6} = \frac{8}{4x} \quad \checkmark$

$$4x^2 = 48 \quad \checkmark$$

$$x^2 = 12$$

$$\therefore x = \sqrt{12} \quad (\text{since } x > 0) \quad \checkmark \\ = 2\sqrt{3}$$

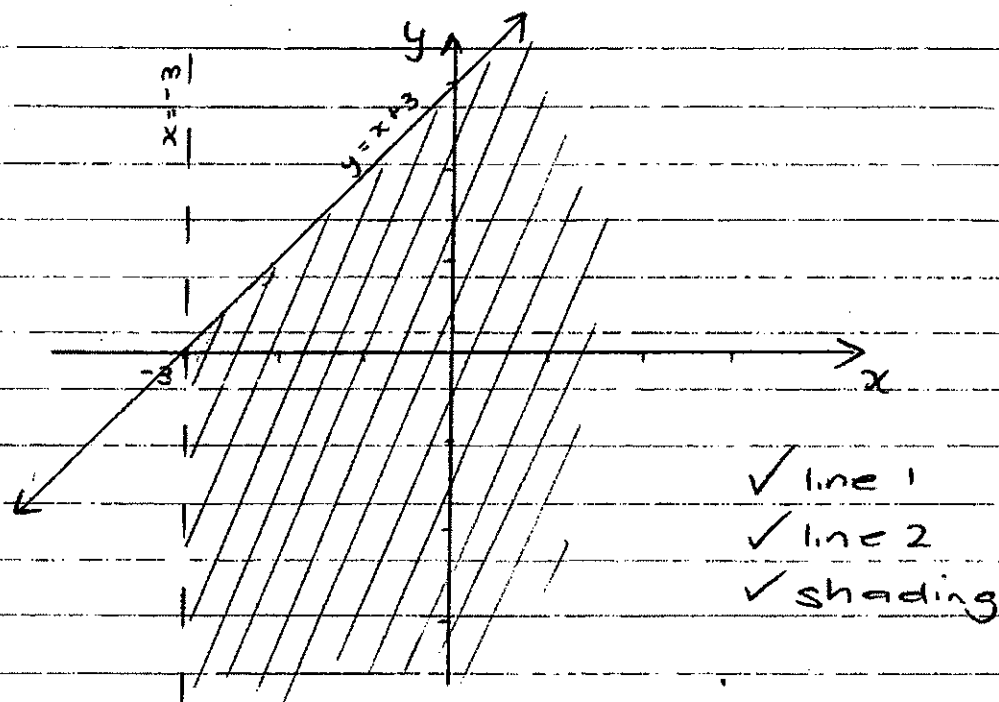
b) $x^5 - 4x^3 - 8x^2 + 32$

$= x^3(x^2 - 4) - 8(x^2 - 4)$ ✓

$= (x^3 - 8)(x^2 - 4)$ ✓

$= (x - 2)(x^2 + 2x + 4)(x + 2)(x - 2)$ ✓✓

c)



• Question 10:

a) i) $\sin 60^\circ = \frac{\sqrt{3}}{2}$ ✓

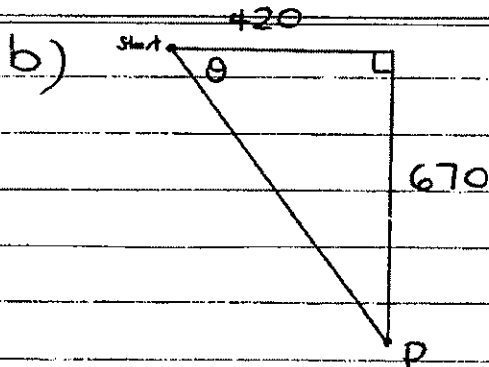
ii) $\cot 30^\circ = \frac{1}{\tan 30^\circ}$

$= \sqrt{3}$ ✓

iii) $\frac{\cot 30^\circ}{\sin 60^\circ} = \sqrt{3} \div \frac{\sqrt{3}}{2}$

$= \sqrt{3} \times \frac{2}{\sqrt{3}}$

$= 2$ ✓



$$\tan \theta = \frac{670}{420}$$

$$\therefore \theta = \tan^{-1}\left(\frac{670}{420}\right)$$

$$= 57.9178\dots^\circ$$

$$= 58^\circ \text{ (to nearest deg.)} \checkmark$$

$$\therefore \text{Bearing is } 90 + \theta$$

$$= 148^\circ \checkmark$$

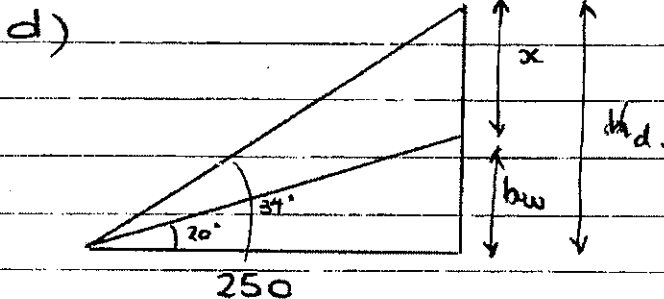
c) $\sin \theta^\circ = \cos (\theta + 40)^\circ$

$$\cos (90 - \theta)^\circ = \cos (\theta + 40)^\circ \checkmark$$

$$\angle 90 - \theta = \theta + 40$$

$$2\theta = 50$$

$$\therefore \theta = 25 \checkmark$$



$$\tan 20^\circ = \frac{h_{\text{window}}}{250}$$

$$\therefore h_{\text{window}} = 250 \tan 20^\circ (= 90.992558\dots) \checkmark$$

Similarly:

$$h_{\text{dragon}} = 250 \tan 34^\circ (= 168.627129\dots) \checkmark$$

$$x = h_{\text{dragon}} - h_{\text{window}}$$

$$= 250 \tan 34^\circ - 250 \tan 20^\circ$$

$$= 77.63457\dots$$

$$= 78 \text{ m (to nearest m)} \checkmark$$