TRINITY GRAMMAR SCHOOL

Mathematics Department

2012

TASK 3 - HALF YEARLY EXAMINATION

Year 12

Mathematics

General Instructions

- Reading time – 5 minutes.
- Working time – 3 hours.
- Weighting 30%
- Write using black or blue pen. Black pen is preferred.
- Board-approved calculators may be used.
- A table of standard integrals is provided at the back of this paper.
- Show all necessary working in Questions 11 – 16.
- Write your Board of Studies Student Number **and** Class Teacher on the writing booklet(s) **or** sheet(s) submitted.

Total marks – 100

Section I Pages 2 – 4

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section.

Section II Pages 5 – 11

90 marks

- Attempt Questions 11 – 16
- Allow about 2 hours 45 minutes for this section.

SECTION 1 Total marks (10)

- Circle the correct response on the answer sheet provided.
- Each question is worth 1 mark.

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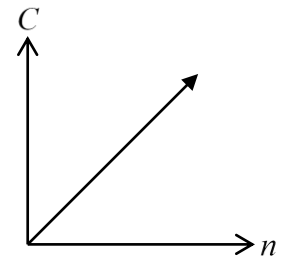
1 A ship sails 9 nautical miles north, then 12 nautical miles east. What is the ship's bearing from its starting point?

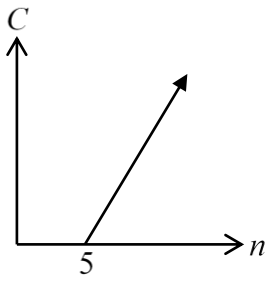
(A) 037° (B) 053° (C) 217° (D) 233°

2 The cost, C, of producing textbooks is given by the formula $C = 14n + 5$. Which of the graphs below best represents this equation?

(A)

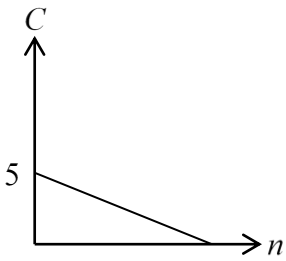


(B)

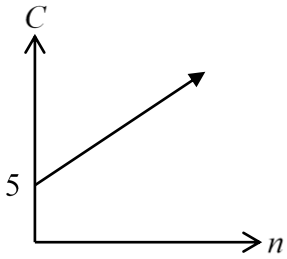


Not to scale

(C)



(D)

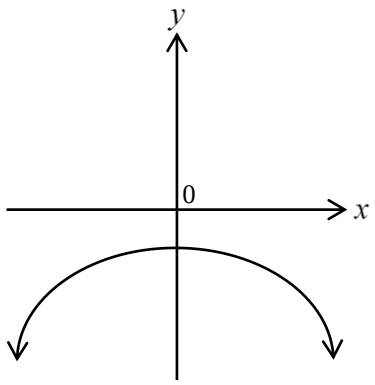


3 The force, F, on a particular moving object, moving with a velocity of V, is defined by the equation $F = \frac{3V^2}{4}$

Which of the following could be a value for V if the value of F is 12?

(A) 3 (B) 4 (C) 9 (D) 16

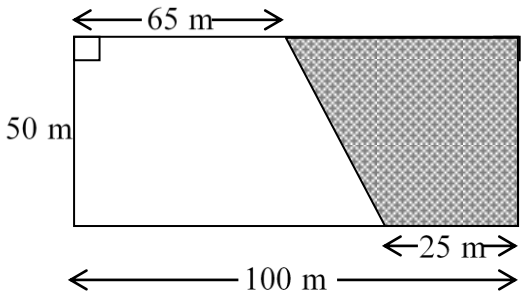
4



The parabola shown could have the equation.

(A) $y = x^2 - 3$ (B) $y = -x^2 + 3$ (C) $y = -x^2 - 3$ (D) $y = x^2 + 3$

5



Not to scale

The diagram shows a yard that consists of a rectangle split into two trapezia.

Find the shaded area to the nearest square metre.

(A) 1500 (B) 2500 (C) 3500 (D) 5000

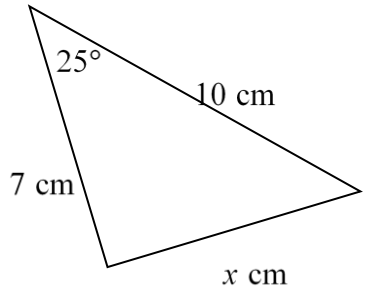
6 Find $\log_{10} 4^3$ correct to 3 significant figures.

(A) 0.218 (B) 1.806 (C) 0.22 (D) 1.81

7 $16 - 4(x - 5) =$

(A) $12x + 60$ (B) $12x - 60$ (C) $-4 - 4x$ (D) $36 - 4x$

8



Not to scale

Which of the following equations should be used to find the value of x in the triangle above?

(A) $\tan 25° = \frac{x}{10}$

(B) $x^2 + 7^2 = 10^2$

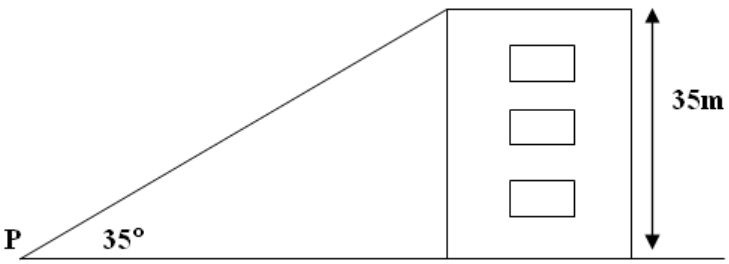
(C) $x^2 = 7^2 + 10^2 - 2 \times 7 \times 10 \times \cos 25°$

(D) $\frac{x}{\sin 25} = \frac{7}{\sin 10°}$

9 The water from a flat rectangular roof 12 metres by 15 metres is collected in a cylindrical tank of diameter 2.2 metres. What rise in water level in the tank is caused by rainfall of 1 cm ?

(A) 0.12 m (B) 0.47 m (C) 12 m (D) 47 m

10



The angle of elevation of the top of a building from a point P on the ground is 35°.
If the building is 35 metres tall, find the distance from the base of the building to the point P, correct to the nearest metre.

(A) 20 (B) 25 (C) 43 (D) 50

SECTION 2 Total marks (90)

- Begin each question in a new writing booklet or on a new answer sheet.
- Show all necessary working.
- Each question is worth 15 marks.

Question 11 (15 marks) Use a SEPARATE writing booklet **Marks**

(a) Evaluate $2e^4$, correct to three significant figures. **2**

(b) Solve $2x^2 = x$ **2**

(c) Factorise fully $8x^3 - 1000$ **2**

(d) Simplify $\dfrac{6x^2 + 5x - 6}{4x^2 + 4x - 3}$ **2**

(e) Evaluate $|-3 \times 4| - |2 \times -5|$ **2**

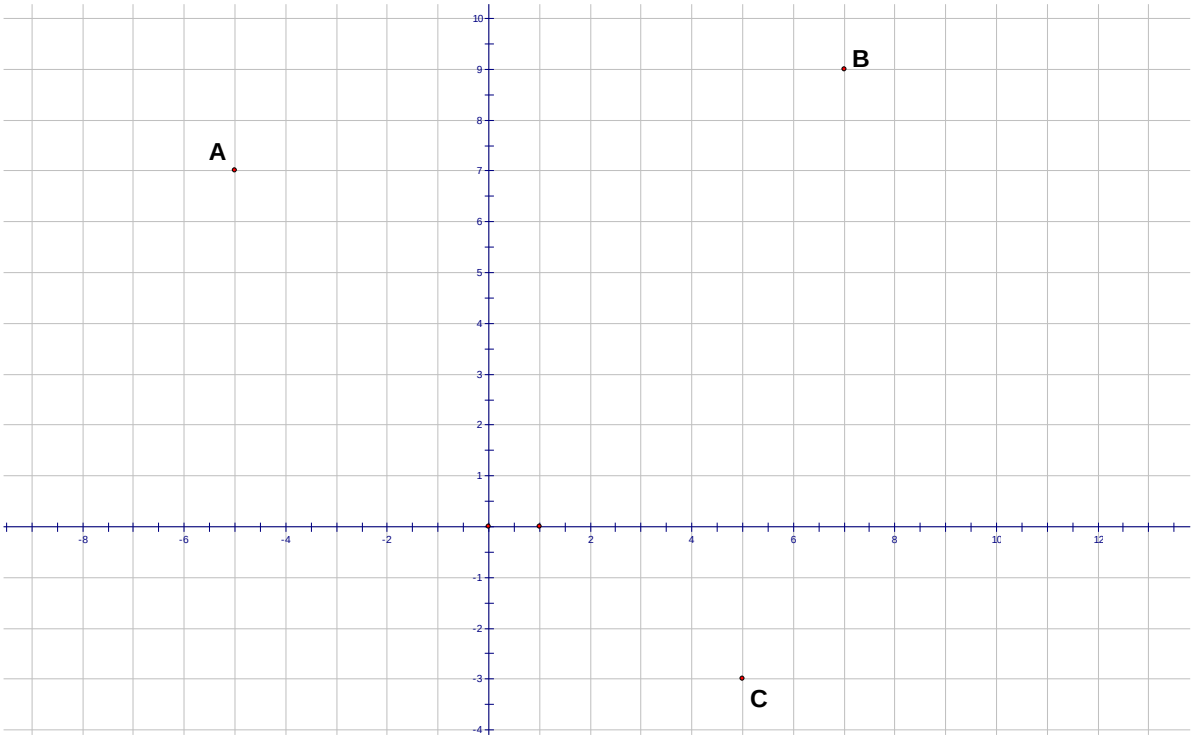
(f) Find the value of $log_2 7$ correct to 3 decimal places. **2**

(g) Find the points of intersection of the functions; **3**

$$x^2 - 4x + y = 0 \text{ and } y = -2x + 5$$

Question 12 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) The points A(-5, 7), B(7, 9) and C(5,-3) are shown on the diagram below.



(i) Show that AB has equation $x - 6y + 47 = 0$. **2**

(ii) Show that the length of AB = the length of BC. **2**

(iii) Show that the midpoint, M, of AC is (0, 2). **1**

(iv) Find *l*, the equation of the perpendicular bisector of AC. **2**

(v) Show that *l* passes through B. **1**

(vi) Find the perpendicular distance from M to AB. **2**

(vii) Find the area of ΔABC. **2**

(b) Find the equation of the normal to the curve $y = (2e^x + 1)^2$ at the point where $x = 0$ **3**

Question 13 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) Convert $\frac{3\pi}{2}$ radians into degrees. **1**

(b) Differentiate each of the following with respect to x.

(i) $y = 3e^{4x-1} + \frac{1}{e^x}$ **2**

(ii) $y = 2x \ln x$ **2**

(iii) $y = \ln(e^x + 1)$ **2**

(iv) $y = (e^x + \ln x)^3$ **2**

(c) Evaluate $\int_0^2 (3x - 1)^3 \, dx$ **3**

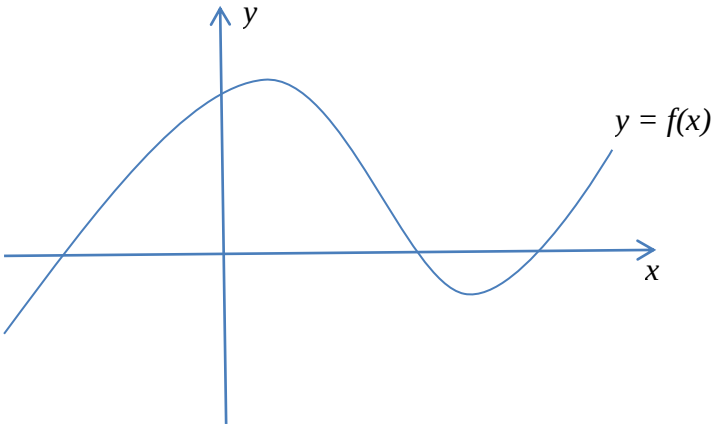
(d) (i) Solve $x^2 - 9x \geq 0$ **2**

(ii) Hence or otherwise find the domain of $y = \ln(x^2 - 9x)$ **1**

Question 14 (15 marks) Use a SEPARATE writing booklet.

Marks

(a) **2**



Copy or trace the function above onto your page. On the same axes, sketch a possible graph of the gradient function $f'(x)$

(b)

(i) Copy and complete the table below correct to 2 decimal places **2**

x	1	1.5	2	2.5	3
$y = 1 + 2\ln x$					

(ii) Use the five function values from part i) and 2 applications of Simpson's Rule to estimate the value of **2**

$$\int_1^3 1 + 2\ln x \ dx$$

(c) Find

(i) $\int \left(\frac{1}{x} + x \right) dx$ **2**

(ii) $\int \sqrt{2x+5} \ dx$ **1**

(iii) $\int \frac{3x^2 + 2x + 1}{x^2} \ dx$ **2**

(d) Given that $\log_a 5 = x$ and $\log_a 3 = y$

find an expression in terms of x and y for

(i) $\log_a 75$ **1**

(ii) $\log_a 0.6$ **1**

(e) Find the exact arc length of a circle of radius is 4cm and angle subtended is 120^0. **2**

Question 15 (15 marks) Use a SEPARATE writing booklet**.** **Marks**

(a) The equation $3x^2 - 8x + 9 = 0$ has roots $\propto$ and β

Evaluate:

(i) $\propto + \beta$ **1**

(ii) $\propto \beta$ **1**

(iii) $\propto^2 + \beta^2$ **1**

(b) Consider the function $y = 4x^3 + 6x^2 - 144x + 1$

(i) Find the stationary points and determine their nature **3**

(ii) Find the point of inflexion **1**

(iii) Find the maximum value of the function in the domain $-5 \le x \le 7$ **1**

(iv) Sketch the graph of $y = 4x^3 + 6x^2 - 144x + 1$ in this domain **2**

(c) A piece of wire 24 cm long is cut into 2 pieces. One of the pieces is bent into a square with sides of x cm. The other piece is bent to form a right angled isosceles triangle. The equal sides of the triangle are of length y cm.

(i) Show that $y = \dfrac{24 - 4x}{2 + \sqrt{2}}$ **2**

(ii) Find an exact expression for the total area covered by the square and the triangle. **1**

(iii) Find the exact value for x so that the total area is a minimum. **2**

Question 16 (15 marks) Use a SEPARATE writing booklet. **Marks**

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(a) For the parabola with equation $x^2 - 6x - 8y + 17 = 0$ find,

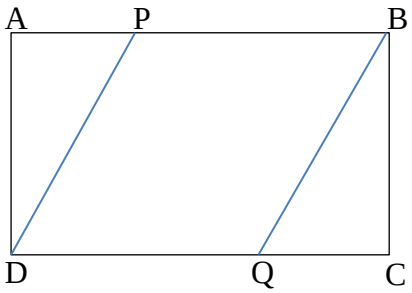
(i) the co-ordinates of the vertex. **2**

(ii) the co-ordinates of the focus. **1**

(iii) the equation of the axis of symmetry. **1**

(b) The arc of the curve $y = e^x - 1$ between $x = 1$ and $x = 3$ is rotated about the x axis. Calculate the volume of the solid generated giving your answer to 3 significant figures. **3**

(c)



In the diagram, ABCD is a rectangle. The points P and Q are on the intervals AB and CD respectively, and AP = CQ.

(i) Prove that the triangles APD and BCQ are congruent. **2**

(ii) Prove that PBQD is a parallelogram. **2**

(d) The fifth term of an arithmetic series is 14 and the sum of the first 10 terms is 165. Find the first term of the series. **3**

(e) Find the limiting sum of $6 + 3 + 1\frac{1}{2} + \cdots$ **1**

-- End of Paper -

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x \, , \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln \left(x + \sqrt{x^2 - a^2} \right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln \left(x + \sqrt{x^2 + a^2} \right)$$

Note $\ln x = \log_e x, \quad x > 0$

TRINITY GRAMMAR SCHOOL
2012, Year 12 Mathematics
Half Yearly Examination
SECTION I
ANSWER SHEET

Name: …… **Class Teacher: ……..**

Be sure to write your answers for Section I on this answer sheet. After you have selected an answer, **CIRCLE** the correct answer. To change an answer, erase your previous mark completely, and then record your new answer. Mark only one answer for each question.

Q1.	A	B	C	D
Q2.	A	B	C	D
Q3.	A	B	C	D
Q4.	A	B	C	D
Q5.	A	B	C	D
Q6.	A	B	C	D
Q7.	A	B	C	D
Q8.	A	B	C	D
Q9.	A	B	C	D
Q10.	A	B	C	D

TRINITY GRAMMAR SCHOOL
2012, Year 12 Mathematics
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SECTION I
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Name: ..... **Class Teacher:** .....

Be sure to write your answers for Section I on this answer sheet. After you have selected an answer, **CIRCLE** the correct answer. To change an answer, erase your previous mark completely, and then record your new answer. Mark only one answer for each question.

Q1.	A	(B)	C	D
Q2.	A	B	C	(D)
Q3.	A	(B)	C	D
Q4.	A	B	(C)	D
Q5.	(A)	B	C	D
Q6.	A	B	C	(D)
Q7.	A	B	C	(D)
Q8.	A	B	(C)	D
Q9.	A	(B)	C	D
Q10.	A	B	C	(D)

Question 11

a) 109. ✓ ✓(3 s.f.) 2

b) $2x^2 - x = 0$

$x(2x - 1) = 0$ ✓

$x = 0 / x = \frac{1}{2}$ ✓ 2

c) $8x^3 - 1000 = 8(x^3 - 125)$ ✓ OR $(2x-10)(4x^2+20x+100)$

$2(x-5)4(x^2+5x+25)$

$= 8(x^3 - 5^3)$

$= 8(x-5)(x^2 + 5x + 25)$ ✓ 2

d) $\frac{6x^2 + 5x - 6}{4x^2 + 4x - 3} = \frac{6x^2 + 9x - 4x - 6}{4x^2 + 6x - 2x - 3}$

$= \frac{3x(2x+3) - 2(2x+3)}{2x(2x+3) - 1(2x+3)}$

$= \frac{(3x-2)(2x+3)}{(2x-1)(2x+3)}$ ✓

$= \frac{3x-2}{2x-1}$ ✓ 2

e) $|-12| - |-10| = 12 - 10 = 2$ ✓ ✓ 2

f) $\log_2 7 = \frac{\log_{10} 7}{\log_{10} 2} = 2,807$ ✓ ✓ 0.845 ✓ 0.301 2

g) $x^2 - 4x + y = 0$

$x^2 - 4x - 2x + 5 = 0$

$x^2 - 4x - 2x + 5 = 0$

$x^2 - 6x + 5 = 0$

$(x-5)(x-1) = 0$ ✓

$x = 5 : x = 1$ ✓

When $x = 1$, $y = -2 + 5 = 3$

When $x = 5$, $y = -10 + 5 = -5$ ✓

∴ Pts of intersection ✓ are $(1, 3)$ and $(5, -5)$ 2

Question 12

a) (i) $m_{AB} = \frac{9-7}{7--5} = \frac{2}{12} = \frac{1}{6}$ (1)

$y - 7 = \frac{1}{6}(x + 5)$

$6y - 42 = x + 5$ (1)

$x - 6y + 47 = 0$ 2

(ii) $d_{AB} = \sqrt{(7--5)^2 + (9-7)^2}$

$= \sqrt{12^2 + 2^2}$

$= \sqrt{148}$ (1)

$d_{BC} = \sqrt{(5-7)^2 + (-3-9)^2}$

$= \sqrt{(-2)^2 + (-12)^2}$

$= \sqrt{148}$ (1)

$\therefore d_{AB} = d_{BC}$ 2

(iii) $M_{AC} = \left(\frac{-5+5}{2}, \frac{7+-3}{2}\right)$ (1)

$= (0, 2)$ 1

(iv) $m_{AC} = \frac{-3-7}{5--5} = \frac{-10}{10} = -1$

$\therefore$ Gradient of normal $= 1$ (1)

$y - 2 = 1(x - 0)$

$y = x + 2$ (1)

$x - y + 2 = 0$ 2

(v) If $y = x + 2$ and $x = 7$, $y = 7 + 2 = 9$

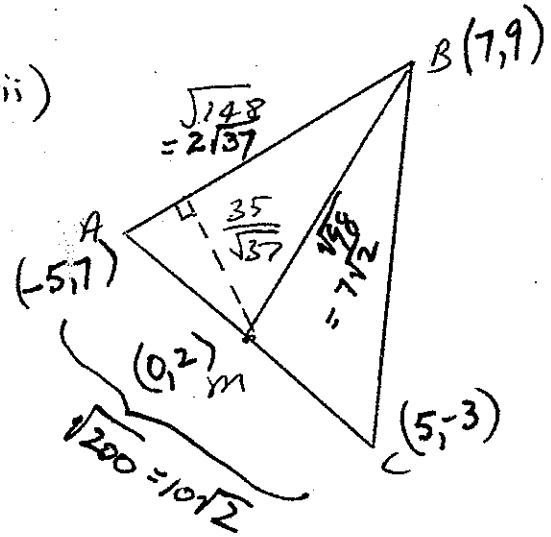
$\therefore B(7, 9)$ is on l. (1) 1

(vi) $d = \dfrac{|ax + by + c|}{\sqrt{a^2 + b^2}}$

$= \dfrac{|1 \times 0 + 6 \times 2 + 47|}{\sqrt{1^2 + 6^2}}$ (M1)

$= \dfrac{35}{\sqrt{37}}$ (1) 2

(vii)



$A = \frac{1}{2} \times \frac{35}{\sqrt{37}} \times \sqrt{148} \times 2$ (M1)

$= 70$ sq units (1) 2

b) $y = (2e^x + 1)^2$

$y' = 2(2e^x + 1) \times 2e^x$

$= 4e^x(2e^x + 1)$ (1)

$= 12$ when $x = 0$

$\therefore$ Gradient of normal $= -\frac{1}{12}$

$y - 9 = -\frac{1}{12}(x - 0)$ (1)

$12y - 108 = -x$

$x + 12y - 108 = 0$ (1) 3

OR $y = -\frac{1}{12}x + 9$

Question 13

a) $\frac{3\pi}{2} \times \frac{180}{\pi} = 270^\circ$ 1

b) (i) $y = 3e^{4x-1} + \frac{1}{e^x}$

$y' = 12e^{4x-1} - \frac{1}{e^x}$ 2

(ii) $y = 2x \ln x$

$y' = \ln x \times 2 + 2x \times \frac{1}{x}$

$= 2\ln x + 2$ 2

(iii) $y = \ln(e^x + 1)$

$y' = \frac{e^x}{e^x + 1}$ 2

(iv) $y = (e^x + \ln x)^3$

$y' = 3(e^x + \ln x)^2 \times \left(e^x + \frac{1}{x}\right)$

$= 3\left(e^x + \frac{1}{x}\right)(e^x + \ln x)^2$ 2

c) $\int_0^2 (3x-1)^3 \, dx = \frac{1}{12}(3x-1)^4 \Big]_0^2$

$= \frac{1}{12} \times 5^4 - \frac{1}{12}(-1)^4$

$= 52$ 3

d) (i) $x^2 - 9x \geq 0$

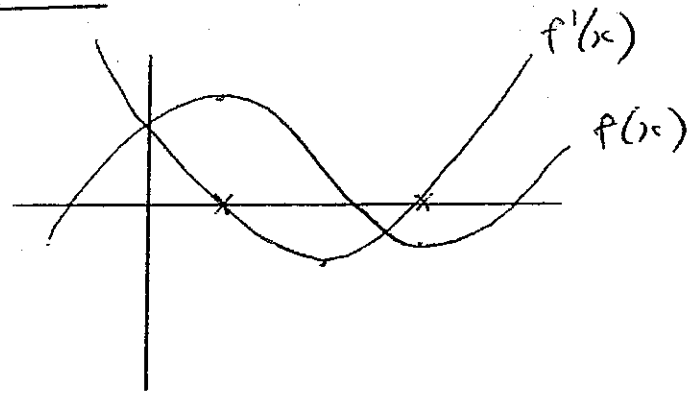
$x(x-9) \geq 0$

$x \leq 0, \; x \geq 9$ 2

(ii) The domain is $x < 0, \; x > 9$ 1

Question 14

a)



2

b)

x	1	1.5	2	2.5	3
(i) $y = 1 + 2\ln x$	1	1.81	2.39	2.83	3.20

2

(ii) $\int_1^3 1 + 2\ln x \, dx \doteq \frac{0.5}{3}\left[(1+3.2) + 4(1.81+2.83) + 2(2.39)\right]$

$= 4.59$ (1 decimal place) 2

c) (i) $\int \frac{1}{x} + x \, dx = \ln x + \frac{1}{2}x^2 + c$ 2

(ii) $\int (2x+5)^{\frac{1}{2}} dx = \frac{2}{3}\frac{(2x+5)^{3/2}}{2} + c$

$= \frac{\sqrt{(2x+5)^3}}{3} + c$ 1

(iii) $\int 3 + 2x^{-1} + x^{-2} \, dx = 3x + 2\ln x - \frac{1}{x} + c$ 2

① progress ①

d) (i) $\log_a 75 = \log_a 5^2 + \log_a 3$

$= 2\log_a 5 + \log_a 3$

$= 2x + y$ 1

(ii) $\log_a 0.6 = \log_a \frac{3}{5}$

$= \log_a 3 - \log_a 5$

$= y - x$ 1

e) $\ell = r\theta$

$= 4 \times 120 \times \frac{\pi}{180}$

$= 4 \times \frac{2\pi}{3}$

$= \frac{8\pi}{3}$ cm

2

if not exact form?

Question 15

a) (i) $\alpha+\beta = \frac{-b}{a} = -\frac{(-8)}{3} = \frac{8}{3}$ ✓

(ii) $\alpha\beta = \frac{c}{a} = \frac{9}{3} = 3$ ✓

(iii) $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= \left(\frac{8}{3}\right)^2 - 2\times 3$

$= 1\frac{1}{9}$ ✓

b) $y = 4x^3 + 6x^2 - 144x + 1$

(i) $\frac{dy}{dx} = 12x^2 + 12x - 144 = 0$ for stat. pts

$12(x^2 + x - 12) = 0$

$12(x+4)(x-3) = 0$

$x = -4, 3$ ✓

If $x = -4$, $y = 417$

If $x = 3$, $y = -269$

$\frac{d^2y}{dx^2} = 24x + 12 < 0$ when $x = -4$

$\therefore (-4, 417)$ is a max ✓

$\frac{d^2y}{dx^2} = 24x + 12 > 0$ when $x = 3$

$\therefore (3, -269)$ is a min ✓

(ii) Pt of inflexion exists when $\frac{d^2y}{dx^2} = 0$

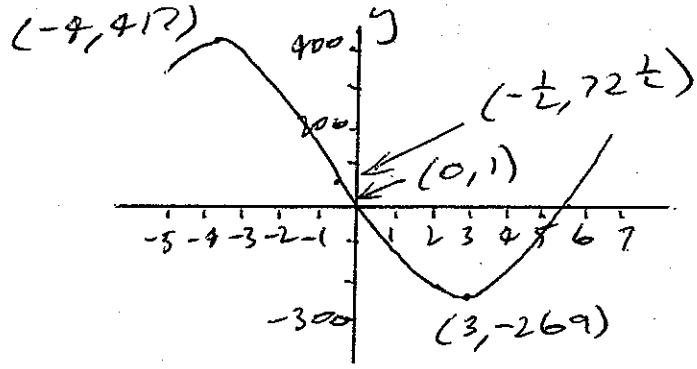
$\therefore 24x + 12 = 0$

$x = -\frac{1}{2}$ ✓

pt of inflexion at $\left(-\frac{1}{2}, 74\right)$ ✓

(iii) Max when $x = 7$, $y = 4(7)^3 + 6(7)^2 - 144(7) + 1 = 659$

(iv)



✓✓

c) (i) $4x + 2y + \sqrt{2}y = 24$ ✓✓

$4x + (2+\sqrt{2})y = 24$

$y = \dfrac{24 - 4x}{2+\sqrt{2}}$

(ii) $A = x^2 + \frac{1}{2}y^2$ ✓

$= x^2 + \frac{1}{2}\left(\dfrac{24-4x}{2+\sqrt{2}}\right)^2$

(iii) $\dfrac{dA}{dx} = 2x + 2x\,\frac{1}{2} \times \left(\dfrac{-4}{2+2\sqrt{2}}\right)\left(\dfrac{24-4x}{2+2\sqrt{2}}\right) = 0$ ✓

$2x - \dfrac{4(24-4x)}{(2+2\sqrt{2})^2} = 0$

$2x(2+2\sqrt{2})^2 = 4(24-4x)$

$2x(2+2\sqrt{2})^2 = 96 - 16x$

$16x + 2x(2+2\sqrt{2})^2 = 96$

$x = \dfrac{96}{16 + 2(2+2\sqrt{2})^2}$ ✓

$= 3.74$ (corrected to 2 dp)

Question 16

a) $x^2 - 6x - 8y + 17 = 0$

$(x-3)^2 - 9 = 8y - 17$

$(x-3)^2 = 8y - 8$

$(x-3)^2 = 8(y-1)$

(i) Vertex at $(3, 1)$ 1

(ii) Since $4a = 8$

$\therefore a = 2$

$\therefore$ Focus at $(3, 3)$ 1

(iii) Axis of symmetry at $x = 3$ 1

b) $V = \pi \int_1^3 (e^x - 1)^2 \, dx$ ✓

$= \pi \int_1^3 (e^{2x} - 2e^x + 1) \, dx$

$= \pi \left[\frac{1}{2} e^{2x} - 2e^x + x \right]_1^3$ ✓

$= \pi \left[\left(\frac{1}{2} e^6 - 2e^3 + 3 \right) - \left(\frac{1}{2} e^2 - 2e + 1 \right) \right]$

$= 519$ (correct to 3 sig. fig) ✓ 3

c) (i) $\triangle APD \equiv \triangle BCQ$ (SAS test) 2

(ii) $DP = BQ$ (corresponding sides of congruent triangles)

$PB = DQ$ Since $AB = DC$ (opp sides of rect.) and $AP = QC$ (Given)

$\therefore$ $PBQD$ is a parallelogram since opposite sides are equal. 2

d) $T_5 = a + (5-1)d = 14$

$\therefore a + 4d = 14$ ✓

Also, $S_{10} = \frac{10}{2}[2a + (10-1)d] = 165$ ✓

$5(2a + 9d) = 165$

$2a + 9d = 33$

Now solve simultaneously

$a + 4d = 14$ ①

$2a + 9d = 33$ ②

① × 2 $\quad 2a + 8d = 28$

$2a + 9d = 33$

$\therefore d = 5$

Sub $d = 5$, in $a + 4d = 14$

$a + 20 = 14$

$\therefore a = -6$ ✓ 3

e) $S_\infty = \frac{a}{1-r}$

$= \frac{6}{1-\frac{1}{2}}$

$= 12$