TRINITY GRAMMAR SCHOOL

Mathematics Department

2012

HALF YEARLY EXAMINATION

ASSESSMENT TASK 3

Year 11

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – $2\frac{1}{2}$ hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used.
- Show all necessary working in Questions 11 – 16
- Write your NAME **and** Class Teacher on the writing booklet(s) **or** sheet(s) submitted.
- Weighting 30%

Total marks – 100

Section I Pages 2 – 5

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II Pages 6 – 12

90 marks

- Attempt Questions 11 – 16
- Allow about 2 hours 15 minutes for this section

Section I 10 marks

- Shade the correct response on the answer sheet provided
- Each question is worth 1 mark

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1. Simplify: $5^{m+1} \times 5^{m+3}$

a) 25^{2m+4}

b) 5^{m^2+3}

c) 5^{2m+4}

d) 5^{m^2+4m+3}

2. Solve for *a*: $\frac{a}{10} + \frac{a}{5} = 3$

a) $a = 10$

b) $a = 1$

c) $a = 15$

d) $a = \frac{1}{2}$

3. Solve for *n*: $\frac{n-7}{4} - \frac{n-1}{9} = 1$

a) $n = 12$

b) $n = 19$

c) $n = 12\frac{3}{5}$

d) $n = 13\frac{3}{5}$

4. After a 20% discount on a fridge, Rudy paid $429 for his new fridge. Calculate the original price of the fridge before the discount correct to the nearest dollar.

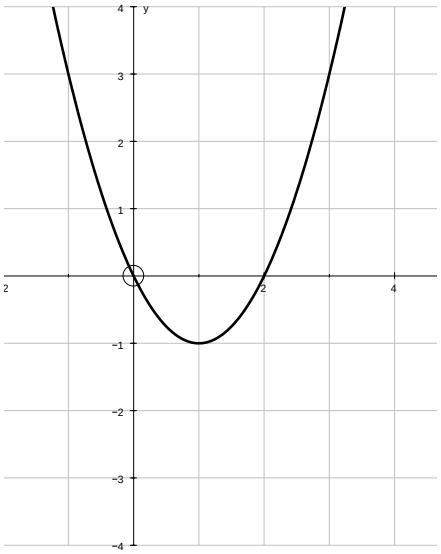
a) $2145

b) $536

c) $515

d) $343

5. The equation of the following function is:



a) $y = x^2 + x - 1$

b) $y = x^2 + 2x$

c) $y = x^2 + 2x - 2$

d) $y = x^2 - 2x$

6. Factorise: $y^3 + 8z^3$

a) $(y + 2z)(y^2 - 2yz + 4z^2)$

b) $y + 2z \quad y^2 + 2yz + 4z^2$

c) $y + 2z \quad y^2 + 4yz + 4z^2$

d) $(y - 2z)(y^2 + 2yz + 4z^2)$

7. What is the equation of a line passing through (2,-1) and parallel to the line represented by the equation $y = 2x + 1$

a) $y = -\frac{1}{2}x$

b) $y = -\frac{1}{2}x + 1$

c) $y = 2x - 5$

d) $y = 2x - 1$

8.

a) 35^o

b) 41^o

c) 44^o

d) 46^o

9.

The length of side AC is closest to:

a) 57 m

b) 67 m

c) 77 m

d) 81 m

10.

The length of the longer diagonal of this parallelogram is closest to:

a) 26.5 cm

b) 30.1 cm

c) 30.8 cm

d) 34.5 cm

End of Section I

Section II 90 marks

- Begin each question in a **new** writing booklet or on a new answer sheet
- Show all necessary working
- Each question is worth 15 marks

Question 11 (15 marks)

(a) Evaluate $\dfrac{3\cdot 4^2 + 2\cdot 5^4}{5\cdot 56 \times 10^{-3}}$ correct to 3 significant figures. **1**

(b) Simplify $36x^4y^{-2} \div (6xy)^3$ **2**

(c) Expand and simplify $8\ a-2b\ -6\ 2a-3b$ **2**

(d) Express $0.7\dot{3}$ as a fraction in its simplest form. **2**

(e) Simplify $\dfrac{a^2-2a-3}{a^3+1}$ **2**

(f) Simplify $4\sqrt{18}+3\sqrt{12}-2\sqrt{50}$ **2**

(g) Given the equation $v^2 = u^2 + 2as$

(i) make u the subject of the equation. **1**

(ii) Hence or otherwise find the value(s) of u when,

$v=12,\ a=-14,\ \ s=4$ **1**

(h) Factorise fully $x^3 - y^3 + y^2 - x^2$ **2**

Question 12 (15 marks) **Begin a NEW answer booklet or answer sheet**

(a) Solve $1 \leq \frac{2x-3}{5} < 5$, and plot the solution on a number line. **3**

(b) If $x - \sqrt{y} = (3 - \sqrt{2})^2$, find the value of x and y. **3**

(c) Fully factorise $4x^3 - 32$ **2**

(d) Simplify the following

i) $\frac{x^2 - 2x}{y^2 - 4y + 4} \times \frac{y-2}{x}$ **2**

ii) $\frac{3}{x-y} + \frac{x}{x^2 - y^2}$ **3**

(e) Simplify $\frac{\sqrt{6}}{\sqrt{5}+\sqrt{2}}$ by rationalising the denominator. **2**

Question 13 (15 marks) **Begin a NEW answer booklet or answer sheet**

(a) Solve simultaneously

$$x - y = 2$$

$$y = x^2 - 4$$

3

(b) Solve the following equations

(i) $6x^2 + 13x - 5 = 0$ **2**

(ii) $\frac{x}{6} - \frac{x-1}{3} = 5$ **3**

(iii) $|1 - 3x| = 8$ **2**

(c) Solve the following inequation

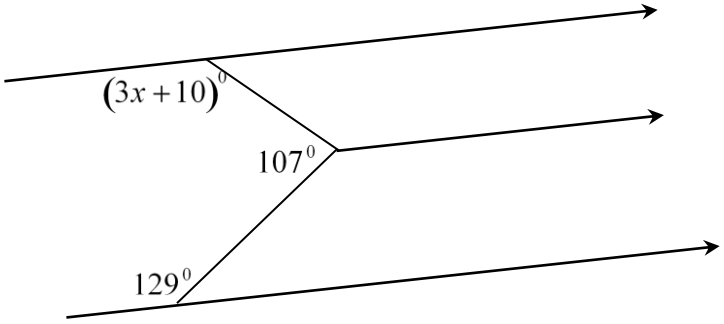
$$\left|\frac{x}{2} - 1\right| < 5$$ **3**

(d) By using the quadratic formula, or otherwise solve the following equation, leaving answer in simplified surd form.

$-9x + 3x^2 = 2$ **2**

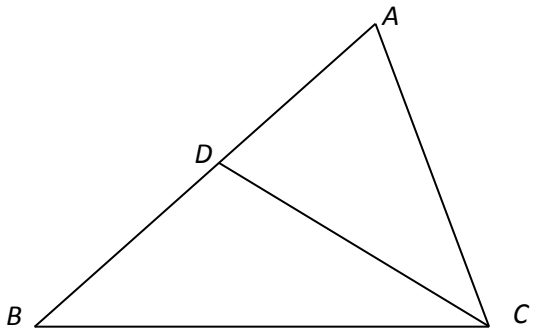
Question 14 (15 marks) **Begin a NEW answer booklet or answer sheet**

(a) Redraw the diagram below and then find the value of x, giving all reasons.



2

(b)



(i) Given $\angle ABC = \angle ACD$, show that $\Delta ABC \,|||\, \Delta ACD$. **2**

(ii) Hence show that $AC^2 = AB \times AD$. **1**

(iii) Find the length of AB if AC=6cm, AD=4cm. **1**

(c) Find the largest possible domain for the function $y = \sqrt{2x-3}$. **2**

(d) For the function $f(x) = \begin{cases} -1 & x \leq -2 \\ 2x & -2 < x < 0 \\ 3 - x^2 & x \geq 0 \end{cases}$,

evaluate $f(-2) + f\left(\frac{1}{2}\right) + 2f(4)$

3

(e) Sketch the following graphs showing <u>all</u> the essential features.

(i) $y = x^2 - 4x$ **2**

(ii) $y = |x - 1|$ **2**

Question 15 (15 marks) **Begin a NEW answer booklet or answer sheet**

(a) Find the **exact** value of

(i) $\cos 135°$ **1**

ii) $\tan(-60°)$ **1**

(b) If $\tan\theta = -\frac{3}{4}$ and $\sin\theta < 0$, find $\cos\theta$ **2**

(c) Solve $2\sin\theta + 1 = 0$ for $0 \leq \theta \leq 360°$ **2**

(d) Simplify: $\frac{\sin(90°-\theta)}{\sin(180°-\theta)}$ **2**

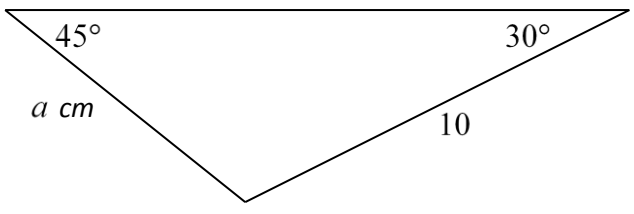
(e) A ship sets sail from Sydney for 200 km on a bearing of 040º.
It then turns on to a bearing of 157 º and sails for another 345km.

i) Draw a neat sketch of the information. **1**

ii) How far is the ship from Sydney, to the nearest km? **2**

iii)What is the bearing of the ship from Sydney, to the nearest degree? **2**

(f) Find the **exact** value of *a* in the diagram below. **2**



Question 16 (15 marks) **Begin a NEW answer booklet or answer sheet**

(a) (i) Show whether the function $f\ x\ = \frac{1}{x-1} + 2$ is odd, even or neither. **3**

(ii) Neatly and clearly sketch the graph of $y = \frac{1}{x-1} + 2$,

showing all intercepts and asymptotes. **3**

(iii) State the domain and range of the function, $f\ x\ = \frac{1}{x-1} + 2$. **2**

(iv) Write down the equation of the vertical asymptote of $f\ x\ = \frac{1}{x-1} + 2$.

1

(b) Show that $\tan\theta - \frac{\sin^3\theta}{\cos\theta} = \sin\theta\cos\theta$ **2**

(c) (i) Show that the equation for the area of an equilateral triangle,

with side length *a*, can be written as $Area = \frac{a^2\sqrt{3}}{4}$ **2**

(ii) If the area of the same equilateral triangle is $16\sqrt{3}$ $\boldsymbol{cm}^2$, find the length of its sides. **2**

End of Section II

End of Examination

TRINITY GRAMMAR SCHOOL
2012, Year 11 Mathematics
Assessment Task 3
SECTION I
ANSWER SHEET

Name: …… **Class Teacher: ……..**

Be sure to write your answers for Section I on this answer sheet. After you have selected an answer, **CIRCLE** the correct answer. To change an answer, erase your previous mark completely, and then record your new answer. Mark only one answer for each question.

Q1.	A	B	C	D
Q2.	A	B	C	D
Q3.	A	B	C	D
Q4.	A	B	C	D
Q5.	A	B	C	D
Q6.	A	B	C	D
Q7.	A	B	C	D
Q8.	A	B	C	D
Q9.	A	B	C	D
Q10.	A	B	C	D

TRINITY GRAMMAR SCHOOL
2012, Year 11 Mathematics
Assessment Task 3
SECTION I
ANSWER SHEET

Name: ....SOLUTIONS..... **Class Teacher:** .....

Be sure to write your answers for Section I on this answer sheet. After you have selected an answer, **CIRCLE** the correct answer. To change an answer, erase your previous mark completely, and then record your new answer. Mark only one answer for each question.

Q1.	A	B	(C)	D
Q2.	(A)	B	C	D
Q3.	A	(B)	C	D
Q4.	A	(B)	C	D
Q5.	A	B	C	(D)
Q6.	(A)	B	C	D
Q7.	A	B	(C)	D
Q8.	(A)	B	C	D
Q9.	A	B	(C)	D
Q10.	A	B	C	(D)

Question 11:

(a) $\frac{3.4^2 + 2.5^4}{5.56 \times 10^{-3}}$

$= 9104.766$

$= 9100$ (3sf) ✓

(b) $\frac{36x^4y^{-2}}{(6xy)^3}$

$= \frac{6^2 x^4 y^{-2}}{6^3 x^3 y^3}$

$= \frac{xy^{-5}}{6}$ ✓

$= \frac{x}{6y^5}$ ✓

(c) $8(a-2b) - 6(2a-3b)$

$= 8a - 16b - 12a + 18b$ ✓

$= -6a + 2b$ ✓

(d) $0.7\dot{3}$

$x = 0.7333$

$10x = 7.33$

$100x = 73.33$ ✓

$\therefore 90x = 66$

$x = \frac{66}{90}$

$= \frac{11}{15}$ ✓

(e) $\frac{a^2 - 2a - 3}{a^3 + 1}$

$= \frac{(a-3)(a+1)}{(a+1)(a^2-a+1)}$ ✓

$= \frac{a-3}{a^2-a+1}$ ✓

(f) $4\sqrt{18} + 3\sqrt{12} - 2\sqrt{50}$

$= 12\sqrt{2} + 6\sqrt{3} - 10\sqrt{2}$ ✓

$= 2\sqrt{2} + 6\sqrt{3}$ ✓

(g) i) $v^2 = u^2 + 2as$

$u^2 = v^2 - 2as$

$u = \pm\sqrt{v^2 - 2as}$ ✓

ii) $u = \pm\sqrt{(12)^2 - 14 \times 2 \times 4}$

$= \pm\sqrt{256}$

$= \pm 16$ ✓

(h) $x^3 - y^3 - x^2 + y^2$

$= (x-y)(x^2 + xy + y^2) - (x^2 - y^2)$ ✓

$= (x-y)(x^2 + xy + y^2) - (x-y)(x+y)$

$= (x-y)(x^2 + xy + y^2 - x - y)$ ✓

TOTAL = 15

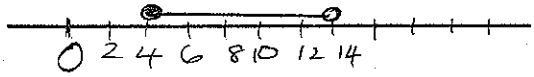
Question 12:

a) $1 \leq \frac{2x-3}{5} < 5$

$5 \leq 2x-3 < 25$ ✓

$8 \leq 2x < 28$

$4 \leq x < 14$ ✓



✓

b) $x - \sqrt{y} = (3-\sqrt{2})^2$

$= 9 - 6\sqrt{2} + 2$ ✓

$= 11 - 6\sqrt{2}$

$= 11 - \sqrt{72}$ ✓

$x = 11 \quad y = 72$ ✓

c) $4x^3 - 32$

$= 4(x^3 - 8)$ ✓

$= 4(x-2)(x^2 + 2x + 4)$ ✓

d) i) $\frac{x^2-2x}{y^2-4y+4} \times \frac{y-2}{x}$

$= \frac{x(x-2)}{(y-2)(y-2)} \times \frac{(y-2)}{x}$ ✓

$= \frac{x-2}{y-2}$ ✓

ii) $\frac{3}{x-y} + \frac{x}{x^2-y^2}$

$= \frac{3}{x-y} + \frac{x}{(x+y)(x-y)}$ ✓

$= \frac{3(x+y)+x}{(x+y)(x-y)}$ ✓

$= \frac{3x+3y+x}{(x+y)(x-y)}$

$= \frac{4x+3y}{(x+y)(x-y)}$ ✓

e) $\frac{\sqrt{6}}{\sqrt{5}+\sqrt{2}}$

$= \frac{\sqrt{6}}{\sqrt{5}+2} \times \frac{(\sqrt{5}-2)}{(\sqrt{5}-2)}$ ✓

$= \frac{\sqrt{30} - 2\sqrt{6}}{5-2}$

$= \frac{\sqrt{30} - 2\sqrt{3}}{3}$ ✓

TOTAL : 15

Question 13:

a) $x - y = 2$ ①

$y = x^2 - 4$ ②

from ① $y = x - 2$

∴ subst into ②

$x - 2 = x^2 - 4$ ✓

$0 = x^2 - x - 2$ ✓

$0 = (x-2)(x+1)$ ✓

$x = 2$ OR $x = -1$

$y = 0$ $y = -3$ ✓ (3)

b) i) $6x^2 + 13x - 5 = 0$

6 | 2 | −5

1 | 3 | −2

$(2x+5)(3x-1) = 0$ ✓

$x = -\frac{5}{2}$ OR $x = \frac{1}{3}$ ✓ (2)

ii) $\frac{x}{6} - \frac{x-1}{3} = 5$ L.C.D = 6

$x - 2(x-1) = 30$ ✓

$x - 2x + 2 = 30$ ✓

$-x + 2 = 30$

$-x = 28$

$x = -28$ ✓ (3)

iii) $|1 - 3x| = 8$

$1 - 3x = 8$ OR $-(1-3x) = 8$

$-3x = 7$ $-1 + 3x = 8$

$x = -\frac{7}{3}$ ✓ $3x = 9$

$x = 3$ ✓

(2)

c) $\left|\frac{x}{2} - 1\right| < 5$

$-5 < \frac{x}{2} - 1 < 5$ ✓

$-4 < \frac{x}{2} < 6$ ✓

$-8 < x < 12$ ✓ (3)

d) $-9x + 3x^2 = 2$

$3x^2 - 9x - 2 = 0$

$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(3)(-2)}}{2 \times 3}$ ✓

$= \frac{9 \pm \sqrt{81 + 24}}{6}$

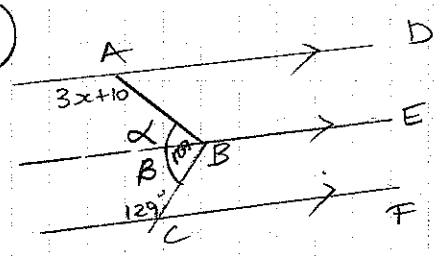
$= \frac{9 \pm \sqrt{105}}{6}$ ✓

(2)

TOTAL = 15

Question 14:

a)



$\alpha + \beta = 107°$

$\beta + 129° = 180°$ (co-int ∠s in ∥ lines) ✓

$\beta = 51°$

$107° - 51° = 56°$ (adj ∠s)

$\therefore 3x + 10° + 56° = 180°$ (co-int ∠s in ∥ lines)

$3x = 114°$

$x = 38°$ ✓

OR (2)

$\angle ABE = 3x + 10°$ (alt ∠s in ∥ lines)

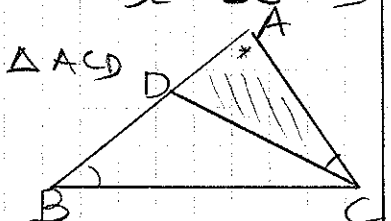
$\angle CBE = 129°$ (alt ∠s in ∥ lines) ✓

$107° + 129° + 3x + 10° = 360°$ (∠s at a pt)

$\therefore 3x = 114$

$x = 38°$ ✓

b) Prove $\angle ABC \equiv \triangle ACD$



In $\triangle AC$ and $\triangle ACD$

$\angle ABC = \angle ACD$ (given)

$\angle BAC = \angle CAD$ (common)

$\therefore \angle ACB = \angle ADC$ (sum of ∠s in $\triangle$)

$\therefore \triangle ABC \equiv \triangle ACD$ ✓✓

ii) $\triangle ABC \equiv \triangle ACD$

$\frac{AB}{AC} = \frac{AC}{AD} = \frac{BC}{CD}$. ✓

$AB.AD = AC^2$ as required (1)

iii) $AC^2 = AB \times AD$

$6^2 = AB \times 4$

$AB = 9cm$ ✓ (1)

c) $2x - 3 \geq 0$ ✓

$2x \geq 3$

$x \geq 3/2$ ✓ (2)

d) $f(x) \begin{cases} -1 & x \leq -2 \\ 2x & -2 < x < 0 \\ 3 - x^2 & x \geq 0 \end{cases}$

$f(-2) + f(\frac{1}{2}) + 2f(4)$

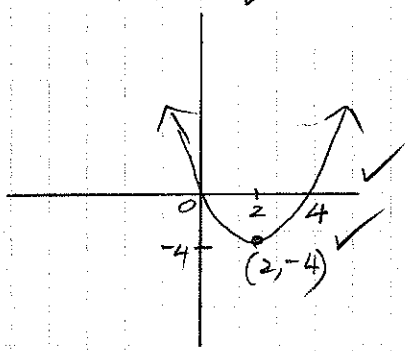
$= (-1) + \left[3 - (\frac{1}{2})^2\right] + 2\left[3 - (4)^2\right]$ ✓✓

$= -1 + \frac{11}{4} + 2(-13)$

$= -1 + \frac{11}{4} - 26$

$= -24\frac{1}{4}$ ✓ (3)

ei)



$y = x^2 - 4x$

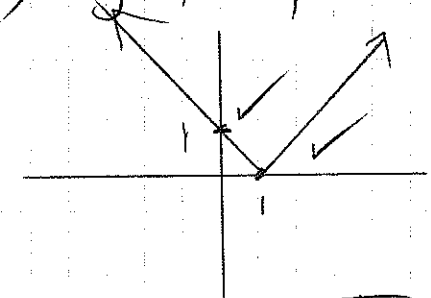
$y = x(x-4)$

subst $x = 2$

$y = (2)^2 - 4(2)$

$= -4$ (2)

ii) $y = |x - 1|$



(2)

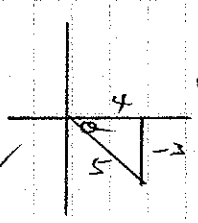
TOTAL = 15

Question 15:

a) i) $\cos 135^\circ$
$= -\cos 45^\circ$
$= -\frac{1}{\sqrt{2}}$ ✓ (1)

ii) $\tan(-60^\circ)$
$= -\tan 60^\circ$
$= -\sqrt{3}$ ✓ (1)

b) $\tan = -\frac{3}{4}$ ✓



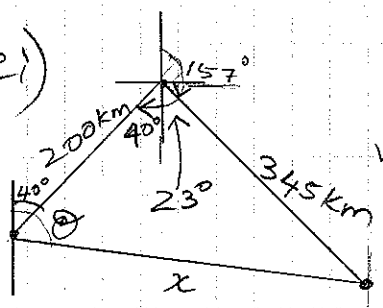
$\cos\theta = \frac{4}{5}$ ✓ (2)

c) $2\sin\theta + 1 = 0$
$\sin\theta = -\frac{1}{2}$ ✓

$\theta = 180^\circ + 30^\circ$ $\theta = 360^\circ - 30^\circ$
$= 210^\circ$ $= 330^\circ$ ✓ (2)

d) $\frac{\sin(90^\circ - \theta)}{\sin(180^\circ - \theta)}$
$= \frac{\cos\theta}{\sin\theta}$ ✓
$= \cot\theta$ ✓ (2)

e) i)



(1)

ii) $x^2 = 200^2 + 345^2 - 2 \times 200 \times 345 \times \cos 63^\circ$ ✓
$x^2 = 96374.3$
$x = 310$ km ✓ (2)

iii) $\frac{\sin\theta}{345} = \frac{\sin 63^\circ}{310}$ ✓
$\sin\theta = \frac{345 \times \sin 63^\circ}{310}$
$\theta = 83^\circ$ ✓ (2)

f) $\frac{a}{\sin 30^\circ} = \frac{10}{\sin 45^\circ}$
$a = \frac{10 \times \sin 30^\circ}{\sin 45^\circ}$ ✓
$= \frac{10 \times \frac{1}{2}}{\frac{1}{\sqrt{2}}}$
$a = 5\sqrt{2}$ ✓ (2)

TOTAL = 15

Question 16:

a) i) $f(-x) = \frac{1}{-x-1} + 2$

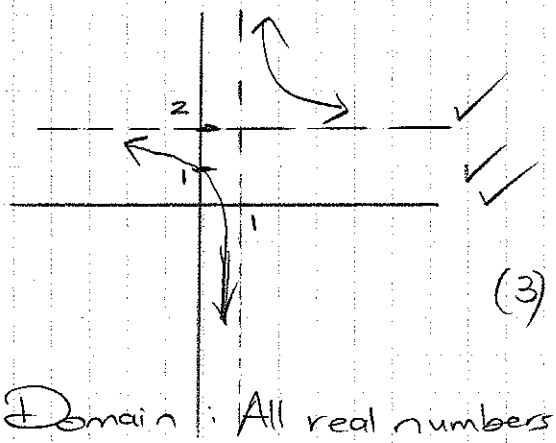
$= \frac{1}{-(x+1)} + 2$

$\neq f(x)$ ✓

$\neq -f(x)$ ✓

∴ neither ✓ (3)

ii)



✓ ✓ ✓ (3)

iii) Domain: All real numbers $x \neq 1$ ✓

Range: All real numbers $y \neq 2$ ✓ (2)

iv) Vertical Asymptote: $x = 1$ ✓ (1)

b) LHS: $\tan\theta - \frac{\sin^3\theta}{\cos\theta}$

$= \frac{\sin\theta}{\cos\theta} - \frac{\sin^3\theta}{\cos\theta}$ ✓

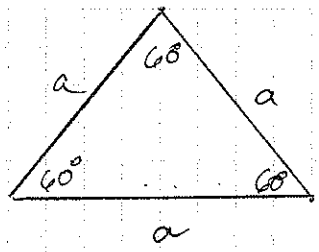
$= \frac{\sin\theta(1-\sin^2\theta)}{\cos\theta}$ ✓

$= \frac{\sin\theta(\cos^2\theta)\cos\theta}{\cos\theta}$

$= \sin\theta \cdot \cos\theta$

$=$ RHS. (2)

c) i)



$\text{Area} = \frac{1}{2}ab\sin C$

$= \frac{1}{2}a.a.\sin 60°$ ✓

$= \frac{1}{2}a^2\frac{\sqrt{3}}{2}$ ✓

$= \frac{a^2\sqrt{3}}{4}$ (2)

ii) Area $= 16\sqrt{3}$

$\therefore \frac{a^2\sqrt{3}}{4} = 16\sqrt{3}$ ✓

$\therefore \frac{a^2}{4} = 16$

$a^2 = 64$

$a = 8$ ✓

∴ length of its side $= 8$.

(2)

TOTAL = 15