



Trinity Grammar School

Mathematics Department

2013

HALF-YEARLY EXAMINATION

PRELIMINARY ASSESSMENT TASK 3

Year 11

Mathematics

General Instructions

- Reading time – 5 minutes
- Writing time – 2 hours
- Write using black or blue pen
- Only approved calculators for this course are allowed in this task
- A table of Standard Integrals (if required) in this course is supplied
- Show all necessary working
- Write your Board of Studies Student Number (Year 12 HSC) or Name (Year 11) and your Class teacher on the question paper and on any answer sheets or booklets used to write your responses to the questions submitted
- If you do not attempt a question you must submit an answer sheet or writing booklet for that question clearly indicating **N/A** and your Student Number or Name.
- **Preliminary Assessment Weighting: 30%**

Board of Studies Student Number

(Year 12 only)

--	--	--	--	--	--	--	--

Class Teacher:

.....
Name:

(Year 11 students)

.....

Do NOT write solutions on this question paper. Any working on this question paper will NOT be marked.

Total marks – 80

Section 1

10 marks

- Attempt all Questions
- Allow about **15** minutes for this section

Section II

70 marks

- Attempt all Questions
- Allow about **1** hour **45** minutes for this section

Blank page

Section I **10 marks**

- Shade the correct response on the answer sheet provided
 - Each question is worth 1 mark
-

1 Simplify $3(x - 2) - 2(x - 1)$.

(A) $x - 4$

(B) $x - 3$

(C) $x - 1$

(D) $x - 8$

2 Express $2\sqrt{45}$ in its simplest form.

(A) $3\sqrt{5}$

(B) $4\sqrt{5}$

(C) $6\sqrt{5}$

(D) $18\sqrt{5}$

3 The exact value of **sin 120°** is

(A) $\frac{\sqrt{3}}{2}$

(B) $\frac{2}{\sqrt{3}}$

(C) $\frac{1}{2}$

(D) $\sqrt{3}$

4 What constant must be added to complete the square for $a^2 - 4a$?

(A) $+2$

(B) -4

(C) $+4$

(D) $+8$

5 $0.\overline{127}$ as a fraction in its simplest form is

(A) $\frac{127}{1000}$

(B) $\frac{127}{999}$

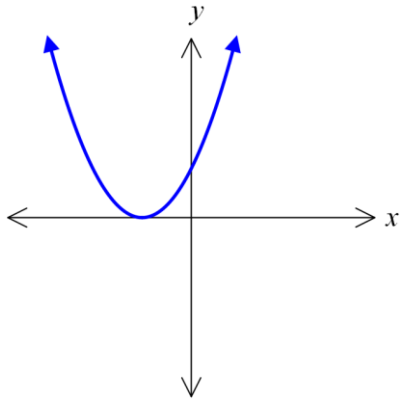
(C) $\frac{23}{180}$

(D) $\frac{127}{990}$

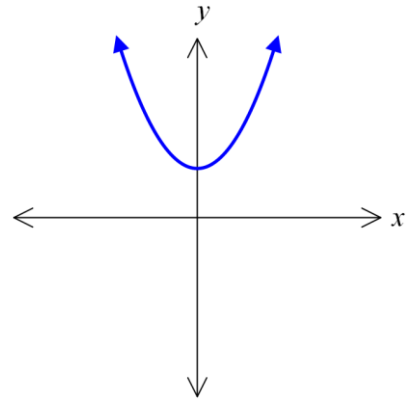
Question 6 commences on the next page.

6 Which of the graphs could represent $y = x^2 + 2x + 1$?

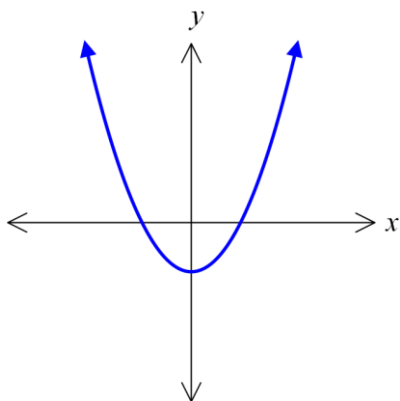
(A)



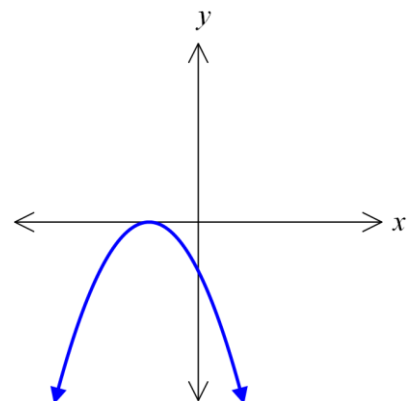
(C)



(B)



(D)



7 Write in index form $\frac{1}{\sqrt{2x-3}}$.

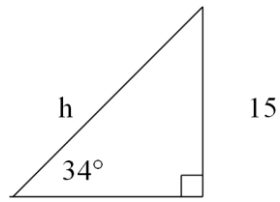
(A) $(2x - 3)^{1/2}$

(B) $\frac{1}{(2x - 3)^2}$

(C) $(2x - 3)^{-2}$

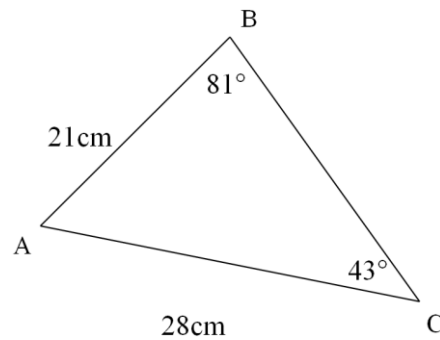
(D) $(2x - 3)^{-1/2}$

- 8 The correct equation used to find the value of the pronumeral in the diagram is



- (A) $h = \frac{15}{\sin 34^\circ}$
- (B) $h = \frac{15}{\cos 34^\circ}$
- (C) $h = \frac{15}{\tan 34^\circ}$
- (D) $h = \frac{\sin 34^\circ}{15}$

- 9 The area of triangle ABC is closet to



- (A) 200 cm^2
- (B) 201 cm^2
- (C) 290 cm^2
- (D) 244 cm^2

10 Simplify $2^{2m+3} \times 8^{m-1}$.

(A) 2^{3m+2}

(B) 2^{6m^2-9}

(C) $2^{3m+2} \times 8$

(D) 2^{5m}

End of Section I

Section II commences on the next page.

Section II 70 marks

- Begin each question in a new writing booklet or answer sheet
 - Show all necessary working
-

Question 11 (10 marks)

a) Evaluate $\sqrt[3]{6.91 \times 10^{-5}}$ correct to three significant figures. **2**

b) Factorise $2x^2 + x - 28$ **2**

c) Simplify $\frac{2x+3}{3} - \frac{x-2}{4}$ **2**

d) Solve the inequality $|3x - 5| \geq 2$ and graph the solution on a real number line. **4**

Question 12 commences on the next page.

Question 12 (10 marks)**Begin a NEW answer booklet**

a) Express $(2\sqrt{3} + 1)(2 - \sqrt{3})$ in the form $a\sqrt{3} + b$. 2

b) Solve the equation $\frac{2x - 1}{5} = \frac{3x + 2}{4}$ 2

c) Simplify fully $\frac{x^2 - 9}{x^2 + x - 12}$ 4

d) A shirt was discounted by 15% to a selling price of \$63.75.
Find the original marked price of the shirt before the discount was applied. 2

Question 13 commences on the next page.

Question 13 (10 marks)**Begin a NEW answer booklet**

a) Simplify $\sqrt{72} + 2\sqrt{50}$ 2

b) Solve simultaneously 3

$$5p - q = 13$$

$$2p - q = 7$$

c) Solve $2 - 9x \geq 11$ 2

d) Find the value of a and of b such that 3

$$\frac{15}{\sqrt{7} + \sqrt{2}} = a\sqrt{7} - \sqrt{b}$$

Question 14 commences on the next page.

Question 14 (10 marks)**Begin a NEW answer booklet**

a) Simplify:

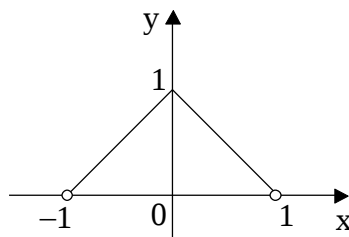
3

$$\frac{x^2 + 13x + 36}{x^2 - 16} \div \frac{x^2 + 18x + 81}{x^2 + 7x + 12}$$

Leave your answer in fully factored form.

b) If $f(x) = x^3 + 3$, find the value of x when $f(x) = -5$.**2**

c) State the range of the following function:

1d) i) Write down the domain of $y = \frac{1}{x+2}$.**1**ii) Sketch the graph $y = \frac{1}{x+2}$.

You need to show all intercepts (if any) and asymptote(s).

3**Question 15 commences on the next page.**

Question 15 (10 marks)**Begin a NEW answer booklet**

- a) Determine whether the functions are odd, even or neither.
Justify your solutions.

i) $f(x) = \frac{1}{(x^2 - 4)}$ 2

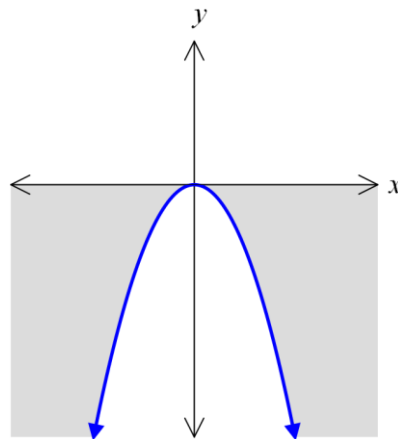
ii) $f(x) = x^3 + 1$ 2

- b) Consider

$$f(x) = \begin{cases} 3 & x \leq -1 \\ 2x - 3 & x > -1 \end{cases}$$

Evaluate $f(-1) + f(1)$. 2

- c) Identify and state the inequations to describe the following shaded region: 2



- d) Find the domain and range of the relation 2

$$x^2 + y^2 = 49$$

Question 16 (10 marks)**Begin a NEW answer booklet**

a) Find the exact value of

i) $\sin 210^\circ$. **1**

ii) $\cos(-300^\circ)$. **1**

iii) $\cos^2 x + \cos^2(90 - x)$. **1**

b) Prove that $(\sin \theta + \cos \theta)(\sin \theta - \cos \theta) = 2 \sin^2 \theta - 1$. **2**

c) An office building and a house are 52 metres apart. From the top of the office building the angle of depression to the base of the house is 15° .Find the height of the office building, correct to the nearest metre. **2**

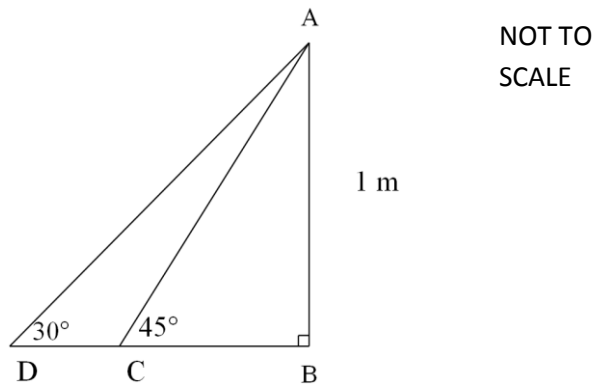
d) A ship sailed 12 nautical miles north from O and then 20 nautical miles west from a point A to a point B.

Find the bearing of point B from O (to the nearest degree). **3****Question 17 commences on the next page.**

Question 17 (10 marks)**Begin a NEW answer booklet**

- a) If $\cos \theta = -0.6$ and $\sin \theta < 0$ find the exact value of $\tan \theta$. 2

- b) In the following diagram $AB = 1$ m. $\angle ACB = 45^\circ$ and $\angle ADC = 30^\circ$.



Copy the diagram into your answer booklet.

- i) Show that $DC = (\sqrt{3} - 1)$ m. 2
- ii) Hence, show that $\sin 15^\circ = \frac{\sqrt{3} - 1}{2\sqrt{2}}$ 3
- c) Find the perimeter of a triangular paddock which has two sides 300 m and 800 m long and the angle between the two sides is 75° . Answer correct to 1 decimal place. 3

End of Section II**End of Examination**

Yr 11 Mathematics Half Yearly Exam

1. A

2. C

3. A

4. C

5. C

6. A

7. D

8. A

9. D

10. D

Q11

a) $0.041035 \dots$ ✓
 $= 0.0410$ ✓

(2)

b) $(2x-7)(x+4)$ ✓

(2)

c) $\frac{2x+3}{3} - \frac{x-2}{4} = \frac{8x+12-3x+6}{12}$ ✓

$= \frac{5x+18}{12}$ ✓

(2)

d) $|3x-5| > 2$

$3x-5 > 2$

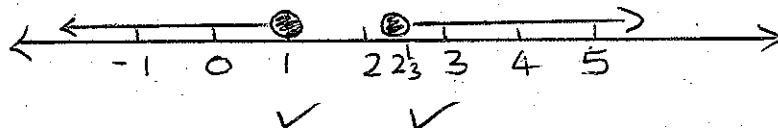
$3x > 7$

$x > \frac{7}{3}$ ✓

$3x-5 < -2$

$3x < 3$

$x < 1$ ✓



(4)

Q12

$$a) (2\sqrt{3}+1)(2-\sqrt{3}) = 4\sqrt{3} - 6 + 2 - \sqrt{3} \quad \checkmark$$

$$= 3\sqrt{3} - 4 \quad \checkmark$$

(2)

$$b) 4(2x-1) = 5(3x+2)$$

$$8x-4 = 15x+10 \quad \checkmark$$

$$-7x = 14$$

$$x = -2 \quad \checkmark$$

(2)

$$c) \frac{x^2-9}{x^2+x-12} = \frac{(x+3)(x-3)}{(x-3)(x+4)} \quad \checkmark$$

$$= \frac{x+3}{x+4} \quad \checkmark$$

(4)

$$d) 85\% = \$63.75 \quad \checkmark$$

$$100\% = \$75.00 \quad \checkmark$$

(2)

Q13

$$a) \sqrt{72} + 2\sqrt{50} = 6\sqrt{2} + 10\sqrt{2} \quad \checkmark$$

$$= 16\sqrt{2} \quad \checkmark$$

(2)

$$b) \begin{array}{l} 5p - q = 13 \quad (1) \\ 2p - q = 7 \quad (2) \end{array}$$

$$(1) - (2) \quad \checkmark$$

$$3p = 6$$

$$p = 2 \quad \checkmark$$

subs in (2)

$$4 - q = 7$$

$$-q = 3$$

$$q = -3 \quad \checkmark$$

(3)

$$\begin{aligned}
 c) \quad 2 - 9x &\geq 11 \\
 -9x &\geq 9 \quad \checkmark \\
 x &\leq -1 \quad \checkmark
 \end{aligned}$$

②

$$\begin{aligned}
 d) \quad \frac{15}{\sqrt{7} + \sqrt{2}} \times \frac{\sqrt{7} - \sqrt{2}}{\sqrt{7} - \sqrt{2}} &= \frac{15\sqrt{7} - 15\sqrt{2}}{7 - 2} \\
 &= 3\sqrt{7} - 3\sqrt{2} \\
 &= 3\sqrt{7} - \sqrt{18} \quad \checkmark
 \end{aligned}$$

$$\begin{aligned}
 a &= 3 \quad \checkmark \\
 b &= 18 \quad \checkmark
 \end{aligned}$$

③

Q14

$$\begin{aligned}
 a) \quad \frac{x^2 + 13x + 36}{x^2 - 16} &= \frac{x^2 + 18x + 81}{x^2 + 7x + 12} \\
 &= \frac{(x+4)(x+9) \checkmark}{(x+4)(x-4)} \times \frac{(x+4)(x+3) \checkmark}{(x+9)(x+9)} \\
 &= \frac{(x+4)(x+3) \checkmark}{(x-4)(x+9)}
 \end{aligned}$$

③

$$\begin{aligned}
 b) \quad f(x) &= x^3 + 3 \\
 f(x) &= -5 \\
 x^3 + 3 &= -5 \quad \checkmark \\
 x^3 &= -8 \\
 x &= -2 \quad \checkmark
 \end{aligned}$$

②

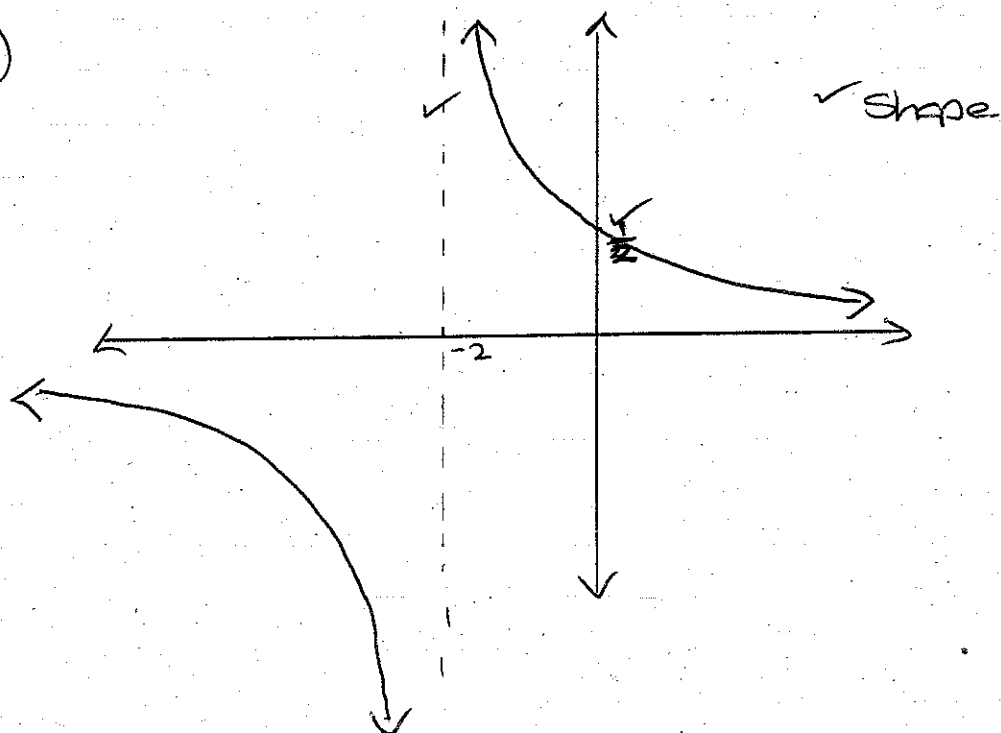
$$c) \quad \text{Range: } 0 < y \leq 1$$

①

d) i) Domain $x \neq -2$

①

ii)



③

Q15.

a) $f(x) = f(-x)$ Even function
 $f(-x) = -f(x)$ Odd function

i) $f(x) = \frac{1}{x^2 - 4}$

$$f(-x) = \frac{1}{(-x)^2 - 4}$$

$$= \frac{1}{x^2 - 4}$$

$$= f(x)$$

∴ Function is even.

②

ii) $f(x) = x^3 + 1$

$$\begin{aligned} f(-x) &= (-x)^3 + 1 \\ &= -x^3 + 1 \\ &\neq f(x) \quad \checkmark \\ \therefore \text{Not even.} \end{aligned}$$

$$\begin{aligned} -f(x) &= -(x^3 + 1) \\ &= -x^3 - 1 \\ &\neq f(-x) \\ \therefore \text{Not odd.} \end{aligned}$$

Function is neither
odd nor even $\checkmark$

(2)

b) $f(-1) = 3$ $\checkmark$
 $f(1) = 2 - 3 = -1$

$$\begin{aligned} f(-1) + f(1) &= 3 - 1 \\ &= 2 \quad \checkmark \end{aligned}$$

(2)

c) $y \geq -ax^2$ $\checkmark$

$y \leq 0$ $\checkmark$

(2)

d) Domain : $-7 \leq x \leq 7$ $\checkmark$
 Range : $-7 \leq y \leq 7$ $\checkmark$

(2)

Q16

a) i) $\sin 210^\circ = -\frac{1}{2}$ $\checkmark$

(1)

ii) $\cos(-300^\circ) = \frac{1}{2}$ $\checkmark$

(1)

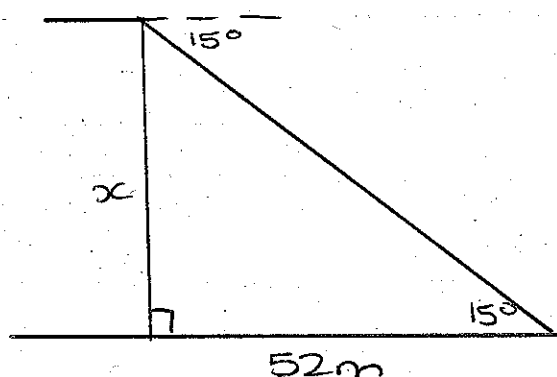
iii) $\cos^2 x + \cos^2(90^\circ - x) = \cos^2 x + \sin^2 x$
 $= 1$ $\checkmark$

(1)

b) $(\sin \theta + \cos \theta)(\sin \theta - \cos \theta) = 2 \sin^2 \theta - 1$
 LHS = $\sin^2 \theta - \cos^2 \theta$
 $= \sin^2 \theta - (1 - \sin^2 \theta) \checkmark$
 $= \sin^2 \theta - 1 + \sin^2 \theta$
 $= 2 \sin^2 \theta - 1 \checkmark$
 $= \text{RHS.}$

(2)

c)



$$\tan 15^\circ = \frac{x}{52} \checkmark$$

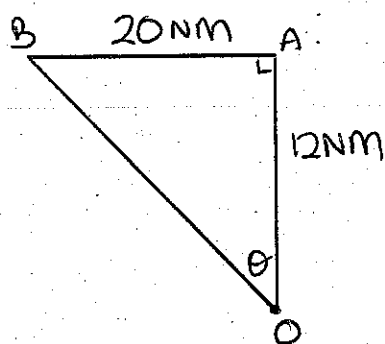
$$x = 52 \tan 15^\circ$$

$$= 13.933 \dots$$

$$= 14 \text{ m} \checkmark$$

(2)

d)



$$\tan \theta = \frac{20}{12} \checkmark$$

$$\theta = 59^\circ \checkmark$$

$$\therefore \text{Bearing is } 301^\circ \checkmark$$

(3)

Q17

a) $\cos \theta = -0.6$

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\sin^2 \theta = 1 - (-0.6)^2$$

$$= 0.64$$

$$\sin \theta = -0.8 \quad \checkmark$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$= \frac{-0.8}{-0.6}$$

$$= \frac{4}{3} \quad \checkmark$$

(2)

b) i) $\triangle ACB$ is isosceles triangle

$$\therefore CB = 1 \text{ m}$$

$$\angle DAB = 60^\circ$$

$$\tan 60^\circ = \frac{DB}{AB} \quad \checkmark$$

$$DB = 1 \times \tan 60^\circ$$

$$= \sqrt{3}$$

$$\therefore DC = DB - CB$$

$$= (\sqrt{3} - 1) \text{ m} \quad \checkmark$$

(2)

ii) $\sin 45^\circ = \frac{1}{\sqrt{2}}$

$$AC = \sqrt{2} \text{ m}$$

$$\angle DAC = 60^\circ - 45^\circ$$

$$= 15^\circ$$

In $\triangle ADC$ $\frac{\sin A}{a} = \frac{\sin D}{d}$

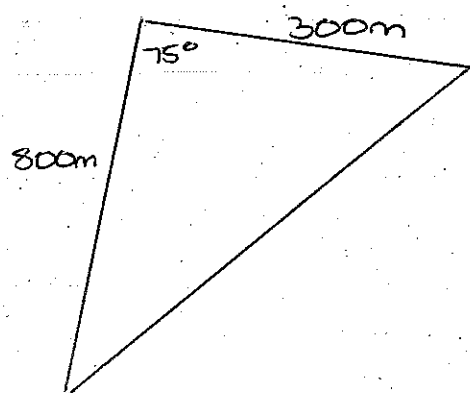
$$\frac{\sin 15^\circ}{\sqrt{3}-1} = \frac{\sin 30^\circ}{\sqrt{2}} \quad \checkmark$$

$$\frac{\sin 15^\circ}{\sqrt{3}-1} = \frac{\frac{1}{2}}{\sqrt{2}} \quad \checkmark$$

$$\sin 15^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}} \quad \checkmark$$

(3)

e)



$$c^2 = 800^2 + 300^2 - 2 \times 800 \times 300 \times \cos 75^\circ \quad \checkmark$$

$$c^2 = 605766.85 \dots$$

$$c = 778 \text{ m} \quad \checkmark$$

③

$$\therefore \text{Perimeter} = 1878 \text{ m} \quad \checkmark$$