Trinity Grammar School

Mathematics Department

2015

HALF-YEARLY EXAMINATION

HSC ASSESSMENT TASK 3

Year 12

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Only approved calculators for this course are allowed in this task
- A table of Standard Integrals (if required) in this course is supplied
- Show all necessary working
- Write your Board of Studies Student Number (Year 12 HSC) or Name (Year 11) and your Class teacher on the question paper and on any answer sheets or writing booklets used to write your responses to the questions submitted
- If you do not attempt a question you must submit an answer sheet or writing booklet for that question clearly indicating **N/A** and your Student Number or Name.

- **HSC Assessment Weighting:** 30%

Board of Studies Student Number
(Year 12 only)

Class Teacher:

……

Name:
(Year 11 only)

……

Do NOT write solutions on this question paper. Any working on this question paper will NOT be marked.

Date of examination:

Wednesday, 22 April 2015

Total marks – 90

Section I

10 marks

- Attempt all Questions
- Allow about 15 minutes for this section

Section II

80 marks

- Attempt all Questions
- Allow about 2 hours 45 minutes for this section

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Section I 10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section
- Shade the correct response on the multiple-choice answer sheet for Questions 1 – 10
- Each question is worth 1 mark

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1 If $a = 2b^3 - 1$, what is the value of b when a is 13?

(A) $\sqrt[3]{6}$

(B) $\sqrt[3]{7}$

(C) $\sqrt[3]{14}$

(D) $\dfrac{\sqrt[3]{14}}{2}$

2 Consider the function defined by $f(x) = x^3 - 6x^2 + 9x + 2$. What are the coordinates of its stationary points and their nature?

(A) Maximum stationary point at $(1, 6)$ and minimum stationary point at $(3, 2)$

(B) Maximum stationary point at $(3, 2)$ and minimum stationary point at $(1, 6)$

(C) Maximum stationary point at $(6, 1)$ and minimum stationary point at $(2, 3)$

(D) Maximum stationary point at $(2, 3)$ and minimum stationary point at $(6, 1)$

3 Let α and β be roots of the equation $x^2 - 2x + 4 = 0$. What is the value of $\alpha\beta$?

(A) -4

(B) -2

(C) 2

(D) 4

4 What is the value of $\int_{-1}^{2} x^2 + 1 dx$?

(A) $\frac{10}{3}$

(B) 5

(C) 6

(D) 7

5 The fourth term of an arithmetic series is 27 and the seventh term is 12.

What is the common difference?

(A) -15

(B) -5

(C) 5

(D) 15

6 What is the solution to the equation $4^x - 9(2^x) + 8 = 0$?

(A) $x = 1$ and $x = 3$

(B) $x = 1$ and $x = 4$

(C) $x = 0$ and $x = 3$

(D) $x = 0$ and $x = 4$

7 What is the value of $f'(3)$ if $f(x) = 3x - x^3$?

(A) $f'(3) = -24$

(B) $f'(3) = -18$

(C) $f'(3) = -15$

(D) $f'(3) = 30$

8 Which of the following is true for the equation $7x^2 - 5x + 2 = 0$?

(A) No real roots

(B) One real root

(C) Two real distinct roots

(D) Three real roots

9 What is the equation of the normal to the curve $y = x^2 - 4x$ at $(1,-3)$?

(A) $x + 2y - 7 = 0$

(B) $x - 2y - 7 = 0$

(C) $2x - y - 5 = 0$

(D) $2x + y + 5 = 0$

10

The table below shows the values of a function $f(x) = \sqrt{25 - x^2}$ for six values of x.

x	0	1	2	3	4	5
$f(x)$	5.00	4.90	4.58	4.00	3.00	0.00

What value is an estimate for $\int_0^5 \sqrt{25 - x^2}\,dx$ using trapezoidal rule with these six values?

(A) 10.74

(B) 12.65

(C) 18.98

(D) 37.96

End of Section I
Section II commences on the next page

Section II 80 marks

- Attempt Questions 11 – 14
- Allow about 2 hours and 45 minutes for this section
- Answer each question in a SEPARATE writing booklet or answer sheet. Extra writing booklets are available.
- In Questions 11 – 14, your responses should include all relevant mathematical reasoning and/or calculations.

Question 11 (20 marks) Use a SEPARATE writing booklet

a) Simplify giving the answer correct to 3 significant figures.

$$\frac{4.7 + \sqrt{3.2}}{1 + 4.15^2}$$

2

b) Solve: $-3 \leq 1 + 2x < 5$

3

c) If $\left(2 - \sqrt{3}\right)^2 = a - \sqrt{b}$ find the value of a and b.

2

d) Differentiate the following functions:

i) $(3x + 8)^6$

1

ii) $x^2(2x + 3)^3$

2

iii) $\frac{x^2}{3x + 1}$

2

e) The second term of a geometric series is 19 and the 5^{th} term is 152.

i) Find the first term and the common ratio.

2

ii) Find the sum of the first six terms.

2

f) Factorise: $12x^2 - 3y^2$

2

g) Express in its simplest form:

$$\left(8a^6\right)^{-\frac{1}{3}} \times a^2$$

2

End of Question 11.

Question 12 (20 marks) Use a SEPARATE writing booklet

---

a) The gradient function for a curve is given by $\frac{dy}{dx} = 6x^2 - 4x + 1$.

If the curve passes through the point (1, 3), find the equation of the curve.

3

b) Find the indefinite integrals:

i) $\int \left(x^2 - \frac{1}{x^2} \right) dx$

2

ii) $\int \sqrt{3-6x} \; dx$

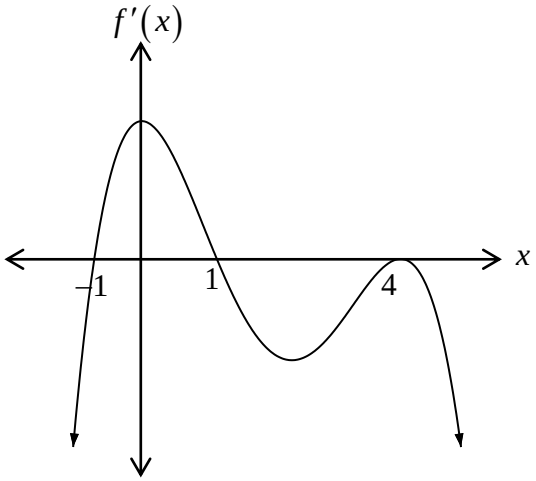
2

c) Use Simpson's rule with three function values to find an approximation to:

$$\int_1^3 \sqrt{x + \frac{3}{x}} \, dx$$

3

d)



The graph of $y = f'(x)$ is sketched above.

i) Write down the values of x where stationary points will occur in the graph of $y = f(x)$. **1**

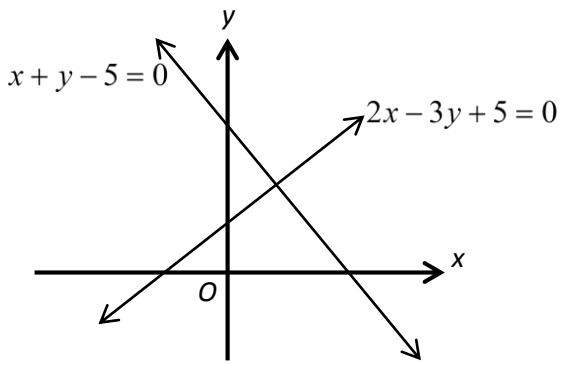
ii) Sketch $y = f(x)$ given that it passes through $(0,0)$ and $(4,-2)$. Clearly show any turning points or points of inflection. **2**

e) i) For what values of x does the geometric series $1+4x^2+16x^4+64x^6+...$ have a limiting sum?

2

(ii) Find the limiting sum when $x = \frac{1}{3}$.

f) The diagram shows the graphs of the lines $x + y - 5 = 0$ and $2x - 3y + 5 = 0$.



i) Find the coordinates of the point of intersection of the two lines.

1

ii) Copy the diagram into your writing booklet, and shade the region which satisfies both the inequalities $x + y - 5 \leq 0$ and $2x - 3y + 5 \leq 0$.

2

End of Question 12.

Question 13 (20 marks) Use a SEPARATE writing booklet

---

a) Find the area enclosed between the curve $y = x^2 - 4$ and the x-axis, bounded by the ordinates $x = 1$ and $x = 4$.

3

b) Consider the curve given by $y = 7 + 4x^3 - 3x^4$.

i) Find the coordinates of the stationary points and determine their nature.

4

ii) Sketch the curve for the domain $-1 \le x \le 2$.

2

c) i) Write down the discriminant of $x^2 - 2kx + 6k$.

2

ii) For what values of k is $x^2 - 2kx + 6k$ positive definite?

2

d) Express $2x^2 + 3x - 6$ in the form $a(x+1)^2 + b(x+1) + c$.

3

e) A parabola has equation $8y = x^2 + 2x + 9$.

i) Write down the coordinates of the vertex of the parabola. **2**

ii) Write down the coordinates of the focus of the parabola. **1**

iii) What is the equation of the directrix of the parabola? **1**

End of Question 13.

Question 14 (20 marks) Use a SEPARATE writing booklet

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a) If $\cos\theta = -\frac{2}{3}$ and $\tan\theta > 0$, find the exact value of $\sin\theta$ (without finding the value of θ).

2

b) Solve $x^2 + 2x - 15 > 0$

2

c) On the day of his daughter's birth, Stephen deposits \$2 000 in a special account for his daughter's university education. Each year on her birthday, he deposits a further \$1 000 into the account. He continues to do this up to and including her 17th birthday. The account earns 7% per annum interest, compounded annually.

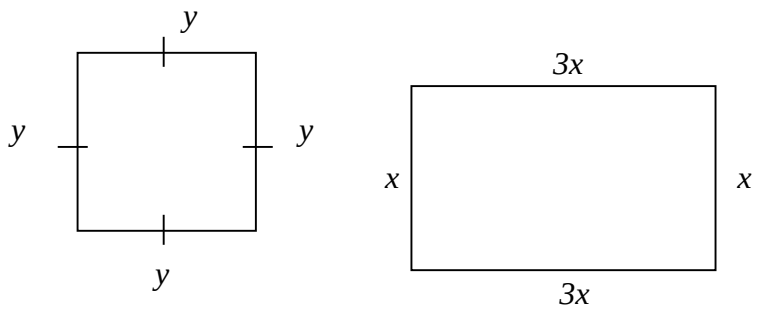
Stephen decides to close the account and withdraw all the money on his daughter's 18th birthday. How much money will he receive ?

4

d) Find the volume of the solid of revolution formed by rotating the curve $y = x + \frac{1}{x}$ about the x axis between $x = 1$ and $x = 3$.

3

d) A piece of wire 12 cm long is cut into two parts. The first piece is bent to form a square, the second is bent to form a rectangle whose length is 3 times its breadth.



i) Show that $2x + y = 3$.

1

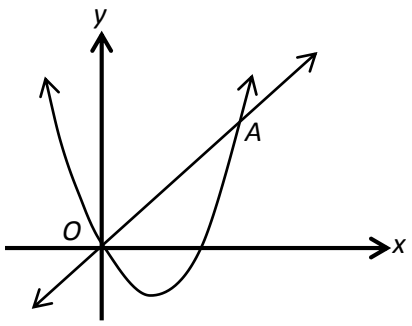
ii) Write an expression for the sum of their areas in terms of x.

2

iii) Find the dimensions of each figure if the sum of their areas is a minimum.

2

e)



The line $y = 2x$ and the parabola $y = x(x-3)$ meet at $O(0, 0)$ and A.

i) Find the x coordinate of A.

1

ii) Hence find the area bounded by the line and the parabola.

3

End of Question 14.

End of Section II
End of Examination

TRINITY GRAMMAR SCHOOL
MATHEMATICS DEPARTMENT

HSC MATHEMATICS

2015 ASSESSMENT TASK 3
ANSWER SHEET

Name: SOLUTIONS

Teacher: _____

Select the alternative A, B, C or D that best answers the question. Circle the letter completely.

1.	A	**B**	C	D
2.	**A**	B	C	D
3.	A	B	C	**D**
4.	A	B	**C**	D
5.	A	**B**	C	D
6.	A	B	**C**	D
7.	**A**	B	C	D
8.	**A**	B	C	D
9.	A	**B**	C	D
10.	A	B	**C**	D

Y12 MATHEMATICS HALF YEARLY

QUESTION 11

a) $0.356090...$ ✓

$= 0.356$ ✓

b) $-3 \leq 1 + 2x < 5$

$-4 \leq 2x < 4$. ✓

$-2 \leq x < 2$.

✓ ✓

c) $(2 - \sqrt{3})^2 = a - \sqrt{b}$

$4 - 4\sqrt{3} + 3 = 7 - 4\sqrt{3}$

$= 7 - \sqrt{48}$

$a = 7$ ✓

$b = 48$ ✓

d) i) $y = (3x+8)^6$

$\frac{dy}{dx} = 6(3x+8)^5 \times 3$

$= 18(3x+8)^5$ ✓

ii) $y = x^2(2x+3)^3$

$= uv$.

$u = x^2$ $\quad v = (2x+3)^3$

$\frac{du}{dx} = 2x$ $\quad \frac{dv}{dx} = 6(2x+3)^2$.

$\frac{dy}{dx} = 6x^2(2x+3)^2 + 2x(2x+3)^3$ ✓

$= 2x(2x+3)^2(3x + 2x + 3)$

$= 2x(2x+3)^2(5x+3)$ ✓

iii) $y = \frac{x^2}{3x+1}$

$= \frac{u}{v}$

$u = x^2$ $\quad v = 3x+1$

$\frac{du}{dx} = 2x$ $\quad \frac{dv}{dx} = 3$.

$$\frac{dy}{dx} = \frac{(3x+1) \times 2x - x^2 \times 3}{(3x+1)^2} \checkmark$$

$$= \frac{6x^2 + 2x - 3x^2}{(3x+1)^2}$$

$$= \frac{3x^2 + 2x}{(3x+1)^2} \checkmark$$

e) i) $T_2 = 19$ $\qquad T_5 = 152$.

$ar = 19$ $\qquad ar^4 = 152$.

$a = \frac{19}{r}$

$$\frac{19}{r} \times r^4 = 152.$$

$$19r^3 = 152.$$

$$r^3 = 8$$

$$r = 2 \checkmark$$

$$a = \frac{19}{2} \checkmark$$

ii) $S_6 = \frac{a(r^n - 1)}{r-1}$

$$= \frac{19/2 (2^6 - 1)}{2-1} \checkmark$$

$$= 598\ 1/2 \checkmark$$

f) $12x^2 - 3y^2 = 3(2x-y)(2x+y)$ ✓ ✓

g) $(8a^6)^{-1/3} \times a^2 = \frac{1}{2a^2} \checkmark \times a^2$

$$= \frac{1}{2} \checkmark$$

QUESTION 12.

a) $\frac{dy}{dx} = 6x^2 - 4x + 1$

$y = 2x^3 - 2x^2 + x + c.$

$(1, 3)$ ✓

$3 = 2(1)^3 - 2(1)^2 + (1) + c.$

$3 = 1 + c$

$2 = c$ ✓

$y = 2x^3 - 2x^2 + x + 2$ ✓

b) i) $\int x^2 - \frac{1}{x^2}\, dx.$

$= \frac{x^3}{3} - \frac{x^{-1}}{-1} + c.$ ✓

$= \frac{x^3}{3} + \frac{1}{x} + c.$ ✓

ii) $\int \sqrt{3 - 6x}\, dx.$

$= \int (3 - 6x)^{1/2}\, dx.$

$= \frac{(3-6x)^{3/2}}{3/2 \times -6} + c.$ ✓

$= -\frac{(3-6x)^{3/2}}{9} + c.$ ✓

c)

x	1	2	3	
y	2	1.87	2	
k	1	4	1	
ky	2	7.48	2	11.48

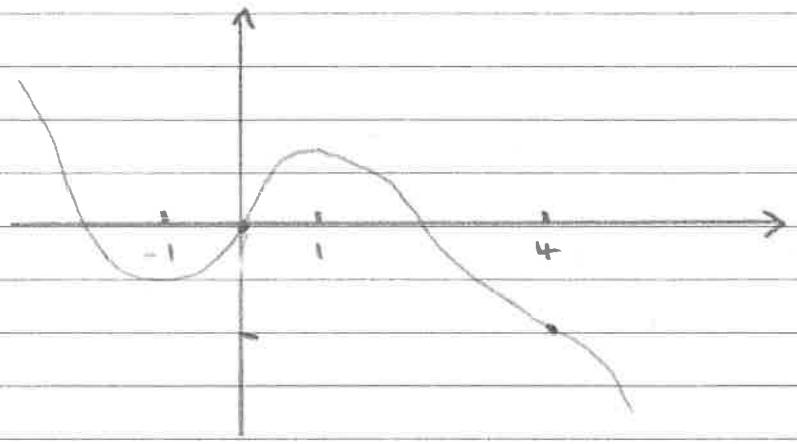
✓ ✓

$A \doteq \frac{h}{3} \times 11.48.$

$\doteq \frac{1}{3} \times 11.48$

$\doteq 3.83 \text{ units}^2$ ✓

d) i) $x = -1, 1, 4$ ✓



Shape ✓
Intercepts ✓

e) i) Limiting sum $-1 < r < 1$

$r = 4x^2$

$-1 < 4x^2 < 1$ $\quad 4x^2 > 0$

$\therefore x^2 < \frac{1}{4}$.

$x^2 - \frac{1}{4} < 0$

$(x + \frac{1}{2})(x - \frac{1}{2}) < 0$ ✓

$-\frac{1}{2} < x < \frac{1}{2}$ ✓

ii) When $x = 1/3$

$r = 4x^2$

$= \frac{4}{9}$ ✓

$a = 1$

$S_\infty = \frac{a}{1-r}$

$= \frac{1}{1-4/9}$

$= \frac{9}{5}$ ✓

f) i) $x + y - 5 = 0$

$2x - 3y + 5 = 0$

$2x + 2y = 10$

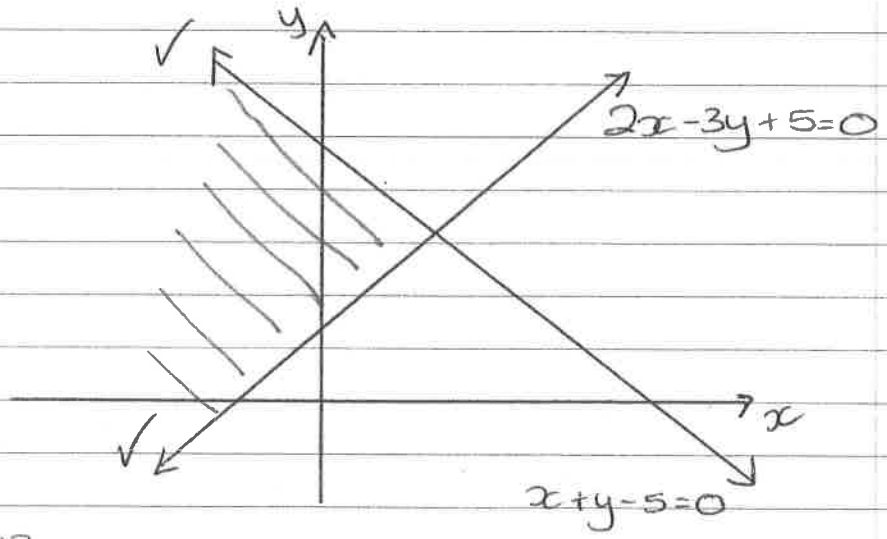
$2x - 3y = -5$

$5y = 15$

$y = 3$ ✓

$x = 2$.

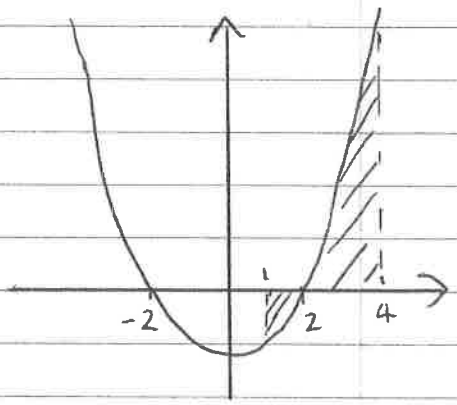
ii)



QUESTION 13

a) $y = x^2 - 4$

$y = (x+2)(x-2)$



$A_1 = \int_1^2 x^2 - 4 \, dx$

$= \left[\frac{x^3}{3} - 4x \right]_1^2$

$= \left(\frac{2^3}{3} - 4(2) \right) - \left(\frac{1^3}{3} - 4(1) \right)$

$= \frac{5}{3}$ ✓

$$A_2 = \int_2^4 x^2 - 4 \, dx$$

$$= \left[\frac{x^3}{3} - 4x \right]_2^4$$

$$= \left(\frac{4^3}{3} - 4(4) \right) - \left(\frac{2^3}{3} - 4(2) \right)$$

$$= \frac{32}{3} \checkmark$$

3

Total Area $= \frac{5}{3} + \frac{32}{3}$

$= \frac{37}{3}$ units^2 ✓

b) i) $y = 7 + 4x^3 - 3x^4$

$$\frac{dy}{dx} = 12x^2 - 12x^3$$

$12x^2 - 12x^3 = 0$

$12x^2(1 - x) = 0$

$x = 0$ ✓ or $x = 1$ ✓

$y = 7$ ✓ $y = 8$ ✓

4

$$\frac{d^2y}{dx^2} = 24x - 36x^2$$

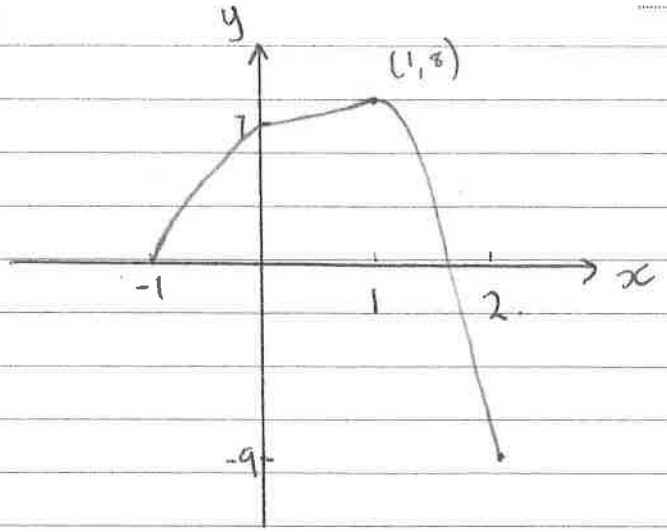
When $x = 0$

$\frac{d^2y}{dx^2} = 0$ point of inflexion ✓

When $x = 1$

$\frac{d^2y}{dx^2} = -12 < 0$ concave down. ✓ max point

ii)



Shape ✓
Intercepts ✓

c) i) $x^2 - 2kx + 6k$

$\Delta = b^2 - 4ac$ ✓
$= 4k^2 - 24k$ ✓

ii) $4k^2 - 24k < 0$
$4k(k-6) < 0$

$k > 0 \quad k - 6 < 0$
$0 < k < 6$ ✓

d) $2x^2 + 3x - 6 \equiv a(x+1)^2 + b(x+1) + c$

$RHS = a(x^2 + 2x + 1) + bx + b + c$
$= ax^2 + 2ax + a + bx + b + c$
$= ax^2 + (2a+b)x + (a+b+c)$

$\therefore a = 2$.

$2a + b = 3$
$4 + b = 3$
$b = -1$

$a + b + c = -6$
$2 - 1 + c = -6$
$c = -7$

$\therefore 2(x+1)^2 - (x+1) - 7$
✓ ✓ ✓

e) $8y = x^2 + 2x + 9$

$8y = x^2 + 2x + 1 + 8$

$8y - 8 = (x+1)^2$.

$8(y-1) = (x+1)^2$. ✓

4

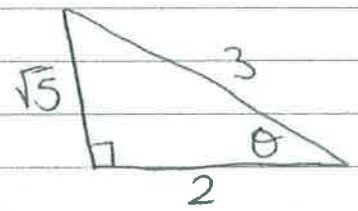
i) Vertex = $(-1, 1)$ ✓

ii) Focus = $(-1, 3)$ ✓

iii) Directrix = $y = -1$ ✓

QUESTION 14

a) $\cos\theta = -\frac{2}{3}$ $\quad \tan\theta > 0$



✓

S	A
T	C

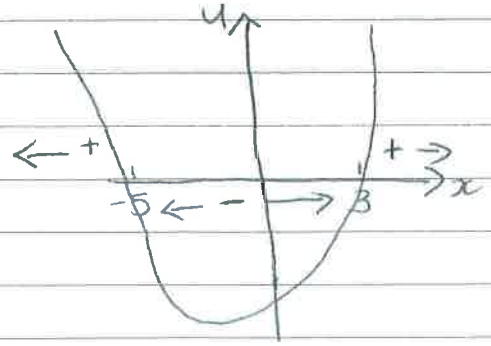
3rd Quad.

2

$\therefore \sin\theta = -\frac{\sqrt{5}}{3}$ ✓

b) $x^2 + 2x - 15 > 0$

$(x+5)(x-3) > 0$



$\therefore x < -5$ ✓ or $x > 3$ ✓

2

c) $A_1 = 2000(1+0.07)^{18}$

$A_2 = 1000(1+0.07)^{17}$

$A_3 = 1000(1+0.07)^{16}$

$\vdots$

$A_{17} = 1000(1+0.07)^{1}$

✓

$\text{Sum} = 1000(1.07)^1 + 1000(1.07)^2 \ldots + 1000(1.07)^{17} + 2000(1.07)^{18}$

$a = 1070$

$r = 1.07$

$n = 17$

✓ ✓

$$\text{Total} = \frac{1070(1.07^{17}-1)}{1.07-1} + 2000(1.07)^{18}$$

$= 32999.032\ldots + 6759.8645\ldots$

$= \$39758.90$ ✓

a) $V = \pi \int_1^3 y^2\,dx$

$= \pi \int_1^3 \left(x + \frac{1}{x}\right)^2 dx$

$= \pi \int_1^3 x^2 + 2 + \frac{1}{x^2}\,dx$ ✓

$= \pi \left[\frac{x^3}{3} + 2x - \frac{1}{x}\right]_1^3$ ✓

$= \pi\left[\left(\frac{3^3}{3} + 2(3) - \frac{1}{3}\right) - \left(\frac{1}{3} + 2 - 1\right)\right]$

$= \frac{40\pi}{3}$ units^3 ✓

e) i) $4y + 8x = 12$.

$4(2x + y) = 12$.

$2x + y = 3$. ✓

ii) $A = y^2 + 3x^2$. $\quad y = 3 - 2x$.

$A = (3-2x)^2 + 3x^2$. ✓

$= 9 - 12x + 4x^2 + 3x^2$.

$= 9 - 12x + 7x^2$ ✓

iii) $\frac{dA}{dx} = 14x - 12$.

$14x - 12 = 0$

$14x = 12$.

$x = \frac{6}{7}$

$y = 3 - 2\left(\frac{6}{7}\right)$

$= 9/7$

$\frac{d^2A}{dx^2} = 14 > 0$ concave up min point.

∴ Square $= \frac{9}{7} \times \frac{9}{7}$ ✓

Rectangle $= \frac{6}{7} \times \frac{18}{7}$ ✓

f) i) $y = 2x \qquad y = x^2 - 3x$.

$2x = x^2 - 3x$

$x^2 - 5x = 0$

$x(x-5) = 0$

$x = 0$ or $x = 5$

$y = 0 \qquad y = 10$

∴ $A = (5, 10)$ ✓

5

ii) $A = \int \text{Top} - \text{Bottom}\ dx$.

$= \int_0^5 2x - (x^2 - 3x)\ dx.$

$= \int_0^5 2x - x^2 + 3x\ dx.$ ✓

$= \left[\frac{5x^2}{2} - \frac{x^3}{3} \right]_0^5$ ✓

$= \left(\frac{5(5)^2}{2} - \frac{5^3}{3} \right) - 0$

$= \frac{125}{6}$ or $20\ 5/6\ \text{units}^2$ ✓