Trinity Grammar School

Mathematics Department

2016

HALF-YEARLY EXAMINATION

PRELIMINARY ASSESSMENT TASK 3

Year 11

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- BOSTES approved calculators for this course may be used
- A Reference Sheet for this course is supplied
- In Questions **11 – 16**, show all relevant mathematical reasoning and/or calculations
- Write your BOSTES Student Number (Year 12 HSC) or Name (Year 11) and your Class teacher on the question paper and on any answer sheets or writing booklets used to write your responses to the questions submitted
- **Preliminary** Assessment **Weighting:** 35%

BOSTES Student Number
(Year 12 only)

Class Teacher:

……

Name:
(Year 10 or 11 only)

……

Do NOT write solutions on this question paper. Any working on this question paper will NOT be marked.

Date of examination:

Wednesday, 27 April 2016

Total marks – 80

Section I

10 marks

- Attempt all Questions
- Allow about **15** minutes for this section

Section II

70 marks

- Attempt all Questions
- Allow about **1** hours **45** minutes for this section

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Section I 10 marks

- Attempt Questions 1 – 10.
- Allow about 15 minutes for this section.
- Shade the correct response on the multiple-choice answer sheet for Questions 1 – **10.**
- Each question is worth 1 mark.

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1. What is 3.098 94 correct to three significant figures?

(A) 3.09

(B) 3.10

(C) 3.098

(D) 3.099

2. Which of the following is an expression for $x^2 - 5x - 6$?

(A) $(x + 2)(x - 3)$

(B) $(x - 1)(x + 6)$

(C) $(x + 1)(x - 6)$

(D) $(x - 2)(x + 3)$

3. Which of the following does not represent a function?

(A) $x = 1$

(B) $y = 1$

(C) $x = \sqrt{y}$

(D) $y = \sqrt{x}$

4. What are the solutions of $2x^2 - 5x - 1 = 0$?

(A) $x = \frac{-5 \pm \sqrt{17}}{4}$

(B) $x = \frac{5 \pm \sqrt{17}}{4}$

(C) $x = \frac{-5 \pm \sqrt{33}}{4}$

(D) $x = \frac{5 \pm \sqrt{33}}{4}$

5. What is the solution of $64 - 32x = 55 - 35x$?

(A) $x = \frac{119}{3}$

(B) $x = -\frac{119}{3}$

(C) $x = -\frac{9}{3}$

(D) $x = \frac{9}{3}$

6. The rationalised version of $\frac{2}{3-2\sqrt{3}}$ is?

(A) $\frac{6-4\sqrt{3}}{3}$

(B) $\frac{-6-4\sqrt{3}}{3}$

(C) $\frac{6+4\sqrt{3}}{3}$

(D) $\frac{6-4\sqrt{3}}{-3}$

7. A circle of radius 7 and centre (1, 5) is represented by the following equation:

(A) $(x + 1)^2 + (y + 5)^2 = 7$

(B) $(x - 1)^2 + (y + 5)^2 = 7^2$

(C) $(x - 1)^2 + (y - 5)^2 = 7^2$

(D) $(x - 1)^2 + (y + 5)^2 = 7$

8. The range of the function $y = 4 - |2x|$ is given by:

(A) $y \geq 4$

(B) $y \leq 4$

(C) $0 \leq y \leq 4$

(D) $y \leq 0$, $y \geq 4$

9. The solutions to $|x+1|=4$ are:

(A) $x=4$

(B) $x=3$ and $x=-5$

(C) $x=5$ and $x=-4$

(D) $x=-5$

10. The $\lim_{x\to 2}\frac{x^3-8}{x-2}$ is

(A) 0

(B) 2

(C) 12

(D) Undefined

End of Section I

Section II commences on the next page

Section II 70 marks

- Attempt Questions 11 – 17.
- Allow about 1 hours and 45 minutes for this section.
- Answer each question in a SEPARATE writing booklet or answer sheet. Extra writing booklets are available.
- In Questions 11 – 17, your responses should include all relevant mathematical reasoning and/or calculations.

Question 11 (10 marks) Use a SEPARATE writing booklet

a. Calculate $\frac{2}{\sqrt{7.4}+3.9^2}$, correct to one decimal place. 2

b. Simplify the following:

i. $\sqrt{80}$ 1

ii. $5\sqrt{8}-2\sqrt{2}$ 2

c. A surfboard was bought and then sold for \$350, at a loss of 60%.
What was the original price? 1

d. Write $1.3\dot{2}\dot{8}$ as a mixed numeral in simplest form. (Show full working) 3

e. Write 84 500 000 in scientific notation. 1

End of Question 11

Question 12 (10 marks) Use a SEPARATE writing booklet

a. Evaluate $6\cdot3\times10^{7}\times7\cdot2\times10^{6}$, writing your answer in scientific notation correct to three significant figures. 2

b. Simplify the following:

i. $50p^{10}q^{6}\div5p^{5}q^{2}$ 1

ii. $\left(6x^{4}\right)^{2}\times2x^{3}$ 2

c. Expand and simplify the following:

i. $\left(6x-4\right)\left(8x+3\right)$ 1

ii. $\left(y+7\right)\left(y-7\right)$ 1

d. Factorise the following:

i. $81-16a^{2}$ 1

ii. $x^{3}-64y^{3}$ 2

End of Question 12

Question 13 (10 marks) Use a SEPARATE writing booklet

a. Expand and simplify: $2\sqrt{3}\left(3\sqrt{3}-\sqrt{6}\right)$ 2

b. Express $\frac{2}{5-2\sqrt{5}}$ as a fraction with a rational denominator. 2

c. If $x = 2 - \sqrt{3}$, write $\frac{x}{x^2}$ with a rational denominator. 3

d. Show that $\frac{3-\sqrt{2}}{3+\sqrt{2}}+\frac{3+\sqrt{2}}{3-\sqrt{2}}$ is a rational number. 3

End of Question 13

Question 14 (10 marks) Use a SEPARATE writing booklet

a. Evaluate $3x^3 - 7x^2 - 6x$ when $x = -2$. 2

b. Expand and simplify: $5(2a+3) - 2(3a-2)$ 2

c. Simplify the following:

i. $a^4 \times a^6 \div a^9$ 1

ii. $\sqrt[3]{-125a^6b^{12}}$ 2

d. Solve:

$12(x-1) = 2(4x+14)$ 2

e. The distance to the visible horizon is found using the formula $D = \sqrt{\frac{h}{2}}$.

Find *h*, correct to two decimal places, when *D* = 26. 1

End of Question 14

Question 15 (10 marks) Use a SEPARATE writing booklet

a. Simplify the following:

i. $\frac{x^2-1}{x-3} \times \frac{x^2-3x}{2x-2}$ 2

ii. $\frac{2x+y}{x+3} - \frac{3x-2y}{x^2+x-6}$ 3

iii. $\frac{y^2-y}{3y^3+3y^2} \div \frac{y^2-1}{6y}$ 3

b. Solve the following inequality:

$\frac{-3x+2}{5} \geq 4$ 2

End of Question 15

Question 16 (10 marks) Use a SEPARATE writing booklet

a. Solve the following:

i. $2y^2 = 11y + 21$ 2

ii. $\frac{3x-2}{5} = \frac{x}{4}$ 2

b. Solve $x^2 - 2x - 9 = 0$, leaving your answer in simplest surd form. 2

c. Solve the following:

i. $|5x + 2| = 12$ 2

ii. $9^{x+1} = 27$ 2

End of Question 16

Question 17 (10 marks**)** Use a SEPARATE writing booklet

a. Solve the following equations simultaneously: 3

$$2x+3y=5$$
$$3x+4y=6$$

b. In your answer booklet, neatly draw a Cartesian plane (Number plane).

i. Draw a sketch of the function $y = |2x + 3|$. 2

ii. On the same plane, draw a sketch of the function $y = |5x + 4|$.

1

iii. Using the information from your sketch, how <u>many</u> solutions are there for the equation $|2x + 3| = |5x + 4|$. 1

iv. Using the above information or otherwise find the solutions to the equation $|2x + 3| = |5x + 4|$. 2

c. A function $f(x)$ is defined as:

$$f(x) = \begin{cases} -5 & x \leq -3 \\ 2x & -3 < x < 1.5 \\ x^2 & x \geq 1.5 \end{cases}$$

Find the value of $f(-3)+ f(3)$. 1

End of Question 17

End of Section II
End of Examination

Solutions Yr 11 Preliminary 2U Half Yearly

1. 3.10 B

2. $(x+1)(x-6)$ C

3. $xy = 1$ A

4. $2x^2 - 5x - 1 = 0$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{5 \pm \sqrt{25+8}}{4}$$

$$x = \frac{5 \pm \sqrt{33}}{4}$$ D

5 $64 - 32x = 55 - 35x$

$64 + 3x = 55$

$3x = -9$

$x = -\frac{9}{3}$ C

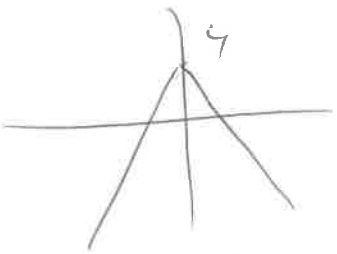
6 $\frac{2}{3-2\sqrt{3}} \cdot \frac{3+2\sqrt{3}}{3+2\sqrt{3}}$

$\frac{6+4\sqrt{3}}{9-12}$

$\frac{-6-4\sqrt{3}}{3}$ B

7 $(x-1)^2 + (y+5)^2 = 7^2$ B

8



$y \leq 4$ B

9 $|x+1| = 4$

for $x \geq -1$: $x+1 = 4$, $x = 3$

$x < -1$: $-(x+1) = 4$, $x+1 = -4$, $x = -5$

$3, -5$ C

10 $\lim_{x \to 2} \frac{(x-2)(x^2+2x+4)}{x-2}$

$\lim_{x \to 2} x^2 + 2x + 4$

C

12 B

B C A D C B B B C C

11 $\frac{2}{\sqrt{7.4} + 3.9^2}$

a. $0.11154...$ ✓

0.1 ✓

b. i $\sqrt{80}$

$= \sqrt{16 \times 5}$

$= 4\sqrt{5}$ ✓

ii $5\sqrt{8} - 2\sqrt{2}$

$10\sqrt{2} - 2\sqrt{2}$ ✓

$8\sqrt{2}$ ✓

c. $\frac{350}{0.4} = \$875$ ✓

d. $1.3\dot{2}\dot{8} =$

$1.3 + 0.028282828$

$\frac{13}{10} + 0.02828...$

$x = 0.02828...$

$100x = 2.828...$ ✓

$99x = 2.8$

$x = \frac{28}{990}$

$x = \frac{14}{495}$ ✓

$1.3\dot{2}\dot{8} = \frac{13}{10} + \frac{14}{495}$

$= 1\frac{65}{198}$ ✓

e. 84500000

$= 8.45 \times 10^7$ ✓

12

a $6.3 \times 10^7 \times 7.2 \times 10^6$

$= 4.536 \times 10^{14}$ ✓

$= 4.54 \times 10^{14}$ ✓

b i $50p^{10}q^6 \div 5p^5q^2$

$= 10p^5q^4$ ✓

ii $(6x^4)^2 \times 2x^3$

$72x^{11}$ 1 correct numerical
1 correct index.

c i $(6x-4)(8x+3)$

$48x^2 + 18x - 32x - 12$

$48x^2 - 14x - 12$ ✓

ii $(y+7)(y-7)$

$y^2 - 49$ ✓

d. i $81 - 16a^2$

$(9+4a)(9-4a)$ ✓

ii $x^3 - 64y^3$ //

$(x-4y)(x^2+4xy+16y^2)$

13 $2\sqrt{3}(3\sqrt{3} - \sqrt{6})$

$6 \times 3 - 2\sqrt{18}$ ✓

$18 - 6\sqrt{2}$ ✓

b. $\frac{2}{5-2\sqrt{5}} \cdot \frac{5+2\sqrt{5}}{5+2\sqrt{5}}$ ✓

$\frac{10+4\sqrt{5}}{25-20}$

$\frac{10+4\sqrt{5}}{5}$ ✓

c. $x = 2-\sqrt{3}$

$x^2 = (2-\sqrt{3})(2-\sqrt{3})$

$= 7 - 4\sqrt{3}$ ✓

$\frac{x}{x^2} = \frac{2-\sqrt{3}}{7-4\sqrt{3}}$

$= \frac{2-\sqrt{3}}{7-4\sqrt{3}} \cdot \frac{7+4\sqrt{3}}{7+4\sqrt{3}}$ ✓

$= \frac{14+8\sqrt{3}-7\sqrt{3}-12}{49-48}$

$= 2 + \sqrt{3}$ ✓

Alt sol^{n}

$\frac{x}{x^2} = \frac{1}{x}$ ✓

$= \frac{1}{2-\sqrt{3}}$

$= \frac{1}{2-\sqrt{3}} \cdot \frac{2+\sqrt{3}}{2+\sqrt{3}}$ ✓

$= \frac{2+\sqrt{3}}{4-3}$

$= 2+\sqrt{3}$ ✓

d. $\frac{3-\sqrt{2}}{3+\sqrt{2}} + \frac{3+\sqrt{2}}{3-\sqrt{2}}$

$\frac{3-\sqrt{2}}{3+\sqrt{2}} \cdot \frac{3-\sqrt{2}}{3-\sqrt{2}} + \frac{3+\sqrt{2}}{3-\sqrt{2}} \cdot \frac{3+\sqrt{2}}{3+\sqrt{2}}$ ✓

$= \frac{9-6\sqrt{2}+2}{9-2} + \frac{9+6\sqrt{2}+2}{9-2}$

$= \frac{9-6\sqrt{2}+2+9+6\sqrt{2}+2}{7}$ ✓

$= \frac{22}{7}$ ✓ which is rational

14.

a. $3x^3 - 7x^2 - 6x$

when $x = -2$

$3(-2)^3 - 7(-2)^2 - 6(-2)$

$-24 - 28 + 12$ ✓

-40 ✓

b $5(2a + 3) - 2(3a - 2)$

$10a + 15 - 6a + 4$ ✓

$4a + 19$ ✓

c i. $a^4 \times a^6 \div a^9$

a ✓

ii $\sqrt[3]{-125a^6b^{12}}$

$(-5^3 a^6 b^{12})^{1/3}$ ✓

$-5a^2b^4$ ✓

d $12(x - 1) = 2(4x + 14)$

$12x - 12 = 8x + 28$ ✓

$4x - 12 = 28$

$4x = 40$

$x = 10$ ✓

e. $D = \sqrt{\frac{h}{2}}$

$26 = \sqrt{\frac{h}{2}}$

$676 = \frac{h}{2}$

$h = 1352$ ✓

15

a. i. $\frac{x^2-1}{x-3} \times \frac{x^2-3x}{2x-2}$

$= \frac{(x+1)(x-1)}{x-3} \times \frac{x(x-3)}{2(x-1)}$ ✓

$= \frac{x(x+1)}{2}$ ✓

ii $\frac{2x+y}{x+3} - \frac{3x-2y}{x^2+x-6}$

$= \frac{2x+y}{x+3} - \frac{3x-2y}{(x+3)(x-2)}$ ✓

$= \frac{(2x+y)(x-2)}{(x+3)(x-2)} - \frac{3x-2y}{(x+3)(x-2)}$ ✓

$= \frac{2x^2-4x+xy-2y-(3x-2y)}{(x+3)(x-2)}$

$= \frac{2x^2-7x+xy}{(x+3)(x-2)}$

$= \frac{x(2x-5+y)}{(x+3)(x-2)}$ ✓

iii $\frac{y^2-y}{3y^3+3y^2} \div \frac{y^2-1}{6y}$

$\frac{y(y-1)}{3y^2(y+1)} \times \frac{2\,6y}{(y+1)(y-1)}$ ✓

$\frac{2}{(y+1)^2}$ ✓

b $\frac{-3x+2}{5} \geq 4$

$-3x+2 \geq 20$

$-3x \geq 18$ ✓

$x \leq -6$ ✓

16 a i $2y^2 = 11y + 21$

$2y^2 - 11y - 21 = 0$

$(2y+3)(y-7) = 0$ ✓

$2y + 3 = 0$ $\quad y - 7 = 0$

$2y = -3$

$y = -3/2$ $\quad y = 7$ ✓

ii $\frac{3x-2}{5} = \frac{x}{4}$

$4(3x-2) = 5x$

$12x - 8 = 5x$ ✓

$7x - 8 = 0$

$7x = 8$

$x = 8/7, 1\frac{1}{7}$ ✓

b $x^2 - 2x - 9 = 0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{2 \pm \sqrt{4+36}}{2}$ ✓

$x = \frac{2 \pm \sqrt{40}}{2}$

$x = 1 \pm \sqrt{10}$ ✓

c. i $|5x+2| = 12$

$x \geq -2/5$ $\quad x < -2/5$

$5x + 2 = 12$ $\quad 5x + 2 = -12$

$5x = 10$ $\quad 5x = -14$

$x = 2$ ✓ $\quad x = -2\frac{4}{5}$ ✓

ii $9^{x+1} = 27$

$3^{2(x+1)} = 3^3$ ✓

$2x + 2 = 3$

$2x = 1$

$x = 1/2$ ✓

17

a. $2x + 3y = 5$ ①

$3x + 4y = 6$ ②

② × 2 $6x + 8y = 12$ ③

① × 3 $6x + 9y = 15$ ④

④ − ③ $y = 3$ ✓

sub $y = 3$ into ①

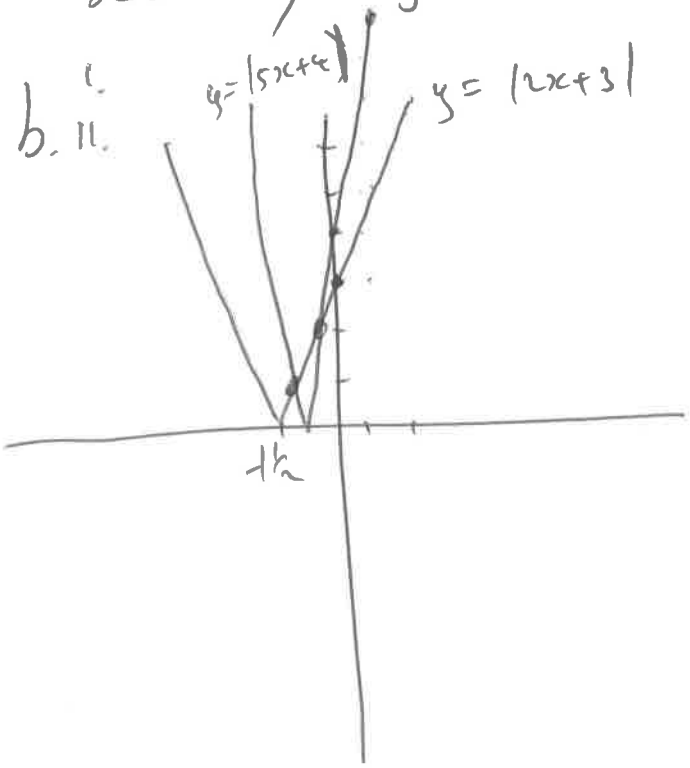
$2x + 9 = 5$

$2x = -4$ ✓

$x = -2$

$x = -2$, $y = 3$ ✓

b. i.



ii. 2 solutions

iii. $2x + 3 = 5x + 4$

$3 = 3x + 4$

$-1 = 3x$

$x = -\frac{1}{3}$ ✓

$2x + 3 = -(5x + 4)$

$2x + 3 = -5x - 4$

$7x + 3 = -4$

$7x = -7$

$x = -1$ ✓

$x = -1, -\frac{1}{3}$

c. $f(-3) = -5$

$f(3) = 9$

$f(-3) + f(3) = 9 - 5$

$= 4$ ✓