Trinity Grammar School

Mathematics Department

2017

HALF-YEARLY EXAMINATION

HSC ASSESSMENT TASK 3

Year 12

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- BOSTES approved calculators for this course may be used
- A Reference Sheet for this course is supplied
- In Questions 11 – 16, show all relevant mathematical reasoning and/or calculations
- Write your NESA Student Number (Year 12 HSC) or Name (Year 11) and your Class teacher on the question paper and on any answer sheets or writing booklets used to write your responses to the questions submitted
- If you do not attempt a question you must submit an answer sheet or writing booklet for that question clearly indicating **N/A** and your Student Number or Name.

- **HSC Assessment Weighting:** 30%

NESA Student Number
(Year 12 only)

Class Teacher:

……

Name:
(Year 10 or 11 only)

……

Do NOT write solutions on this question paper. Any working on this question paper will NOT be marked.

Date of examination:

Friday, 28 April 2017

Total marks – 100

Section I

10 marks

- Attempt all Questions
- Allow about **15** minutes for this section

Section II

90 marks

- Attempt all Questions
- Allow about **2** hours **45** minutes for this section

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Section I 10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section
- Shade the correct response on the multiple-choice answer sheet for Questions 1 – 10
- Each question is worth 1 mark

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1 What is the value of $\dfrac{(1.49)^2 - 1.98}{\sqrt{11.62 + 8.34 \times 2.72}}$ correct to three significant figures?

(A) 0.040
(B) 0.041
(C) 0.0409
(D) 0.0410

2 Which of the following is equivalent to $\sqrt{3} + \sqrt{27} - \sqrt{18}$?

(A) $\sqrt{3}$
(B) $\sqrt{12}$
(C) $4\sqrt{3} - 3\sqrt{2}$
(D) $10\sqrt{3} - 9\sqrt{2}$

3 What is the domain and range of the function $f(x) = \sqrt{1 - x^2}$?

(A) Domain: $0 \le x \le 1$, Range: $-1 \le y \le 1$
(B) Domain: $-1 \le x \le 1$, Range: $-1 \le y \le 1$
(C) Domain: $-1 \le x \le 1$, Range: $0 \le y \le 1$
(D) Domain: $0 \le x \le 1$, Range: $0 \le y \le 1$

4 The solution to $3x^2 + 7x > 6$ is:

(A) $x < -3$, $x > \frac{2}{3}$

(B) $x < -\frac{1}{2}$, $x > -\frac{1}{3}$

(C) $-\frac{1}{2} < x < -\frac{1}{3}$

(D) $-3 < x < \frac{2}{3}$

5 What is the gradient of the line joining $(-4,1)$ and $(4,7)$?

(A) -1

(B) $-\frac{3}{4}$

(C) $\frac{3}{4}$

(D) 1

6 Find the sum of the first 15 terms of the geometric series, $2.5 + 5 + 10 + ...$

(A) 30.5

(B) 300

(C) 40 960

(D) 81 917.5

7 For what values of m is $-4x^2 + 3x + m = 0$ negative definite?

(A) $m > -\frac{9}{16}$

(B) $m < -\frac{16}{9}$

(C) $m < -\frac{9}{16}$

(D) $m > -\frac{16}{9}$

8 What is the derivative of $3x^5 - 4x^2 + 2$?

(A) $15x^4 - 8x + 2$

(B) $15x^5 - 8x^2 + 2$

(C) $15x^4 - 8x$

(D) $15x^5 - 8x^2$

9 Find the expression for $f'(x)$ if $f(x) = \dfrac{2x-3}{3x-2}$?

(A) $f'(x) = \dfrac{-5}{(3x-2)}$

(B) $f'(x) = \dfrac{5}{(3x-2)}$

(C) $f'(x) = \dfrac{-5}{(3x-2)^2}$

(D) $f'(x) = \dfrac{5}{(3x-2)^2}$

10 The semi-circle $y = \sqrt{9-x^2}$ is rotated about the x-axis. Which of the following expressions is correct for the volume of the solid of revolution?

(A) $V = \pi\int_0^3 (9-x^2)dx$

(B) $V = 2\pi\int_0^3 (9-x^2)dx$

(C) $V = \pi\int_0^3 (9-y^2)dy$

(D) $V = 2\pi\int_0^3 (9-y^2)dy$

End of Section I
Section II commences on the next page

Section II 90 marks

- Attempt Questions 11 – 16
- Allow about 2 hours and 45 minutes for this section
- Answer each question in a SEPARATE writing booklet or answer sheet. Extra writing booklets are available.
- In Questions 11 – 16, your responses should include all relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet

(a) Evaluate $\int_0^1 (3x+1)^3\,dx$. **2**

(b) Find the limiting sum of the geometric series $1-\frac{1}{3}+\frac{1}{9}-\frac{1}{27}+\ldots$. **2**

(c) Solve the equation $x^2-5x+6=0$. **2**

(d) Find the primitive function of $3x^2-1$. **2**

(e) Factorise $m^2+2mn+n^2-1$. **2**

(f) Find integers a and b such that $(5-\sqrt{3})^2 = a+b\sqrt{3}$. **2**

(g) Solve $|2m+3| \leq 3$. Graph your solution on a number line. **3**

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet

(a) Differentiate:

(i) $\dfrac{1}{(2x+5)^3}$ **2**

(ii) $x^3(2x+1)$ **2**

(b) Find:

(i) $\int 2\,dx$ **1**

(ii) $\int \dfrac{x^2+3x+1}{x^4}\,dx$ **2**

(c) Evaluate $\int_1^4 \sqrt{x}\;dx$ **3**

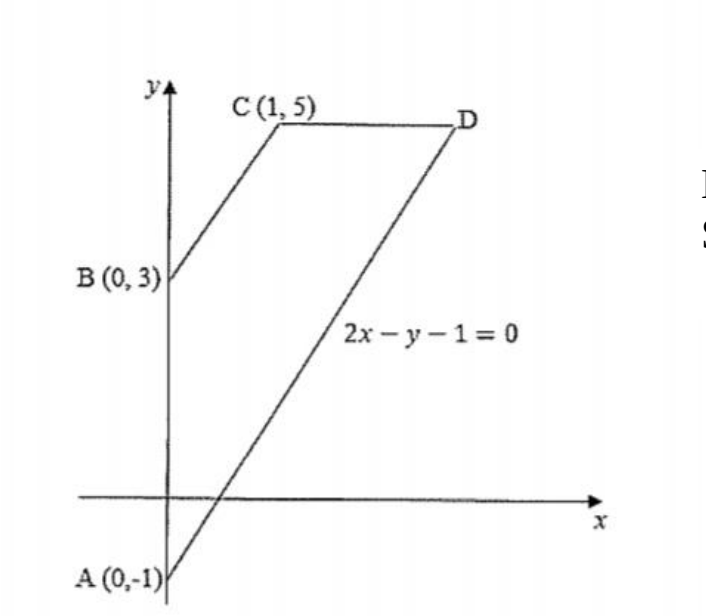
(d) Find the equation of the tangent to the curve $y = 3x^4 + 3x$ at the point $x = -1$. **2**

(e) Find the value of θ, if $2\sin\theta = -\sqrt{3}$, where $0 \le \theta \le 360°$ **3**

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet

(a) In the diagram, $ABCD$ is a quadrilateral. The equation of the line AD is $2x - y - 1 = 0$.



NOT TO SCALE

i. By comparing the gradients, show that $ABCD$ is a trapezium. **2**

ii. Find the exact length of BC. **1**

iii. Show that the perpendicular distance from B to AD is $\frac{4}{\sqrt{5}}$. **2**

iv. Hence, or otherwise, find the exact area of the trapezium $ABCD$. **2**

(b) Find the values of a, b and c such that **2**

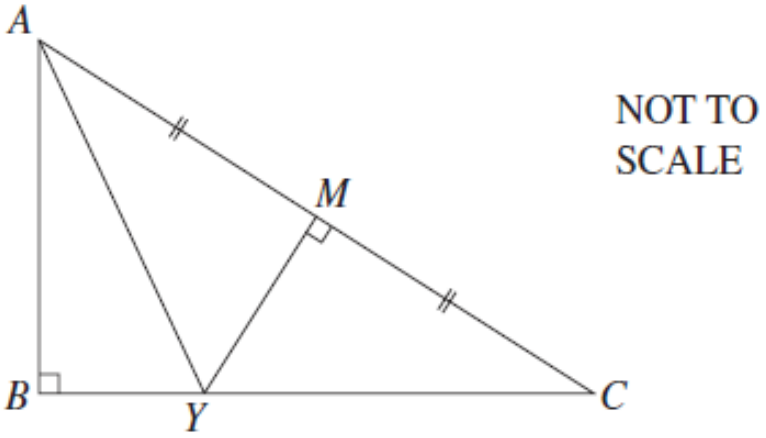
$3x^2 - 7x - 6 \equiv a(x-2)^2 + b(x-2) + c$ for all x.

Question 13 continues on page 9

Question 13 continued.

(c) Show that $\cot\theta + \tan\theta = \text{cosec}\,\theta\ \sec\theta$ **2**

(d)



The diagram shows a right-angled triangle ABC with $\angle ABC = 90°$. The point M is the midpoint of AC, and Y is the point where the perpendicular to AC at M meets BC.

Copy or trace the above diagram into your writing booklet.

i. Show that $\Delta AYM \equiv \Delta CYM$. **2**

ii. Suppose that it is also given that AY bisects $\angle BAC$. Find the size of $\angle YCM$. **2**

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet

(a) Use the Trapezoidal Rule with 5 function values to find an approximation to the value of $\int_0^2 2^x \, dx$ Give your answer correct to 3 decimal places. **3**

(b) Consider the function $f(x) = x^3 - 9x^2 + 24x + 2$.

i. Find the coordinates of the stationary points of the curve $y = f(x)$, and determine their nature. **2**

ii. Find any points of inflexion. **2**

iii. Sketch the curve for the domain $-1 \le x \le 6$. **2**

iv. For what values of x does $f'(x).f''(x) > 0$. **1**

(c) Josh is learning to drive. His first lesson is 40 minutes long. His second lesson is 45 minutes long. Each subsequent lesson is 5 minutes longer than the lesson before.

i. How long will Josh's tenth lesson be? **1**

ii. How many hours of lessons will Josh have completed after his tenth lesson? **2**

iii. During which lesson will Josh have completed his 120 hours of driving? **2**

End of Question 14

Question 15 (15 marks) Use a SEPARATE writing booklet

(a) If α and β are the roots of the equation $2x^2 + 5x - 1 = 0$ find the value of

i. $\alpha + \beta$ **1**

ii. $\alpha\beta$ **1**

iii. $\alpha^3 + \beta^3$ **2**

(b) Let $\log_a 2 = x$ and $\log_a 3 = y$.

Find an expression for $\log_a 12$ in terms of x and y . **2**

(c) Consider the function $x^2 + 2x + 28y - 111 = 0$.

i. Show that the function can be written in the form $(x + 1)^2 = -28(y - 4)$. **1**

ii. Find the coordinates of the vertex. **1**

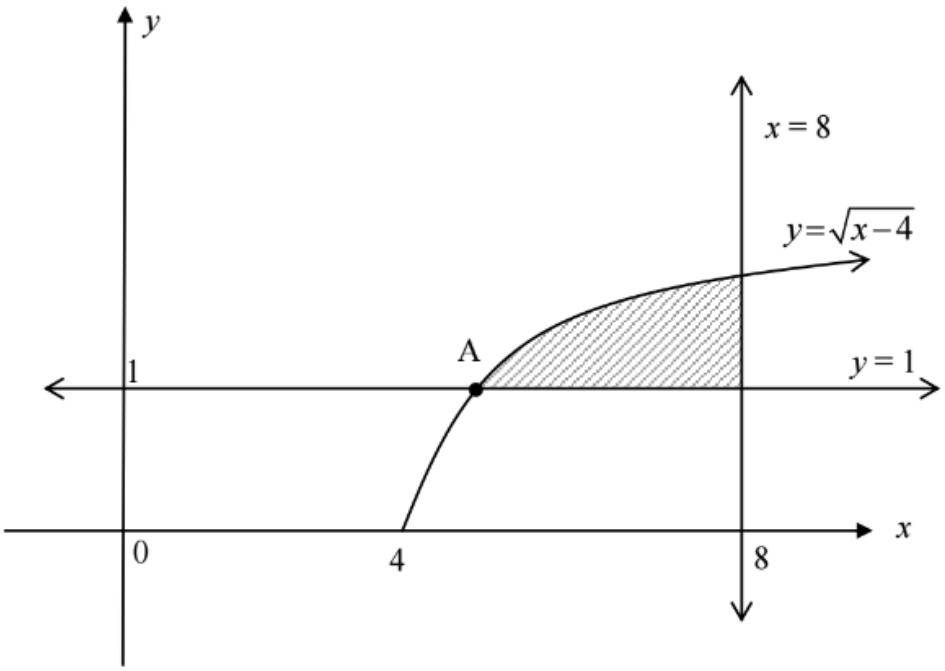
iii. Find the coordinates of the focus. **1**

iv. Sketch the parabola clearly showing the vertex, focus and directrix. **2**

Question 15 continues on page 12

Question 15 continued.

(d) The region between $y = \sqrt{x-4}$, $y = 1$ and $x = 8$ is rotated about the x - axis.



i. Show that the coordinates of A are (5,1). **1**

ii. Find the exact volume of the solid of revolution. **3**

End of Question 15

Question 16 (15 marks) Use a SEPARATE writing booklet

(a) Find the area of the shaded region bounded by the curve $y = \sqrt{x+1}$, the y-axis, and the lines $y = 0$ and $y = 3$. **3**

(b) Mr Pythagoras borrowed \$50 000 at the beginning of 2017 to renovate his house. His bank agreed to charge him 6% p.a. As a special deal, the bank decided not to charge any interest in the first 3 months. Interest is calculated at the beginning of each month. The debt is to be repaid in equal monthly repayments over 5 years.

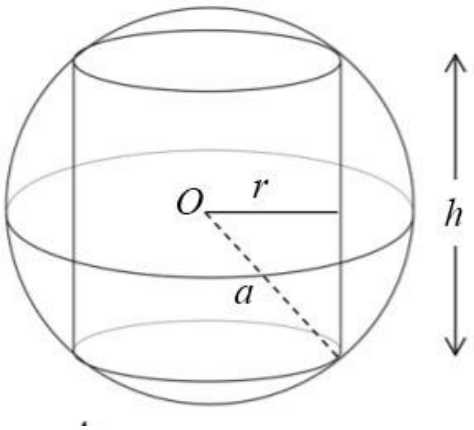
i. Find an expression for A_3. **1**

ii. Show that $A_5 = (50000 - 3M)(1.005)^2 - M(1 + 1.005)$. **1**

iii. Show that $A_{60} = (50000 - 3M)(1.005)^{57} - 200M(1.005^{57} - 1)$. **2**

iv. Find M, to the nearest dollar. **2**

(c) A right cylinder of radius r units and height h units is inscribed in a sphere of radius a units centred at O as shown below.



i. Write an expression for r^2 in terms of a and h. **1**

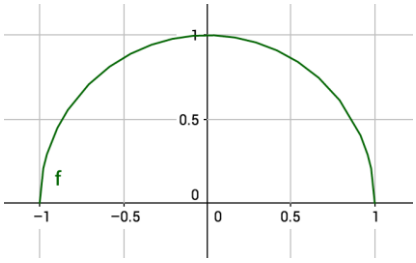
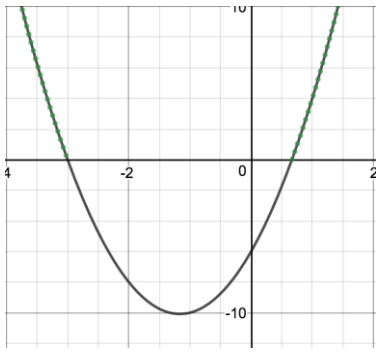
ii. Show that the volume of the cylinder is given by $V = \pi(a^2h - \frac{h^3}{4})$. **1**

iii. Hence or otherwise, show that if the size of the sphere remains constant, the maximum volume of the cylinder is $\frac{4\pi\sqrt{3}a^3}{9}$ $units^3$. **4**

End of Question 16

End of Section II
End of Examination

Solutions for 2 Unit Mathematics Half-Yearly (2017)

1	=0.04099347305 = 0.0410 **D**
2	$=\sqrt{3}+3\sqrt{3}-3\sqrt{2}$ $=4\sqrt{3}-3\sqrt{2}$ **C**
3	Domain: $-1 \leq x \leq 1$ Range: $0 \leq y \leq 1$ **C** 
4	$3x^2+7x-6>0$ $(3x-2)(x+3)>0$ $x<-3$ and $x>\frac{2}{3}$ **A** 
5	$m=\frac{7-1}{4--4}$ $m=\frac{6}{8}=\frac{3}{4}$ **C**
6	$S_{15}=\frac{2.5(2^{15}-1)}{2-1}$ $S_{15}=81917.5$ **D (all students received this mark regardless)**

7	$a<0$ and $\Delta<0$ $\therefore\ b^2-4ac<0$ $3^2-4\times-4\times m<0$ $9+16m<0$ $16m<-9$ $m<-\frac{9}{16}$ **C**
8	$V=\pi\int_{-3}^{3}(9-x^2)\,dx$ **C**
9	$f'(x)=\frac{(3x-2)(2)-(2x-3)(3)}{(3x-2)^2}$ $f'(x)=\frac{6x-4-6x+9}{(3x-2)^2}$ $f'(x)=\frac{5}{(3x-2)^2}$ **D**
10	$y^2=9-x^2$ $V=\pi\int_{-3}^{3}(9-x^2)\,dx$ **B**

Section II – Question 11 (Pay)

Question 11 (a)

Criteria	Marks
* Provides correct solution	2
* Provides correct primitive or follow through from one mistake	1

Sample answer:

$$= \left[\frac{(3x+1)^4}{4(3)} \right]_0^1$$

$$= \left(\frac{(3(1)+1)^4}{4(3)} \right) - \left(\frac{(3(0)+1)^4}{4(3)} \right)$$

$$= \frac{64}{3} - \frac{1}{12}$$

$$= \frac{85}{4} \text{ or } 21\frac{1}{4} \text{ or } 21.25$$

Question 11 (b)

Criteria	Marks
* Provides correct solution	2
* Attempts to use limiting sum formula with one mistake or $-\frac{1}{3}$ is seen somewhere	1

Sample answer:

$$\frac{3x^3}{3} - x + c$$

$$S_\infty = \frac{1}{1 - \left(-\frac{1}{3} \right)}$$

$$S_\infty = \frac{3}{4}$$

Question 11 (c)

Criteria	Marks
* Provides correct solution	2
* Provides one correct answer, or equivalent merit	1

Sample answer:

$$(x-3)(x-2) = 0$$

$$x = 3 \text{ and } x = 2$$

Question 11 (d)

Criteria	Marks
* Provides correct solution	2
* Provides correct primitive with no $+c$	1

Sample answer:

$\frac{3x^3}{3} - x + c$

$x^3 - x + c$

Question 11 (e)

Criteria	Marks
* Provides correct solution	2
* Recognises the perfect square $(m+n)^2$	1

Sample answer:

$(m+n)^2 - 1$

$(m+n+1)(m+n-1)$

Question 11 (f)

Criteria	Marks
* Provides correct solution	2
* Provides correct expression but does not find the values of $\underline{a}$ and b, provides one correct solution or equivalent merit (follow through one mistake)	1

Sample answer:

$25 - 10\sqrt{3} + 3$

$28 - 10\sqrt{3}$

$a = 28$ and $b = -10$

Question 11 (e)

Criteria	Marks
* Provides correct solution and graph	3
* Provides two correct solutions (no graph) or one correct solution and correct graph (ft)	2
* Provides one correct solution	1

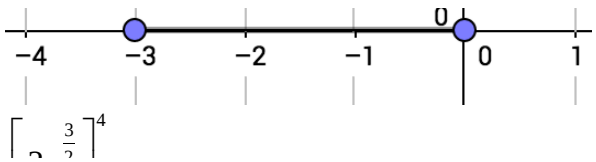
Sample answer:

$2m+3 \leq 3$ and $2m+3 > -3$

$2m \leq 0$ $2m > -6$

$m \leq 0$ and $m > -3$

OR $-3 < m \leq 0$



Question 12 (Geddes)

Question 12 (a) (i)

Criteria	Marks
* Provides correct solution	2
* Attempts to use the quotient rule with one mistake	1

Sample answer:

$(2x+5)^{-3}$

$\frac{d}{dx} = -3(2x+5)^{-4} \times 2$

$= -6(2x+5)^{-4}$ or $= -\frac{6}{(2x+5)^4}$

Question 12 (a) (ii)

Criteria	Marks
* Provides correct solution	2
* Attempts to use the product rule with one mistake or unsimplified	1

Sample answer:

$2x^4 + x^3$

$\frac{d}{dx} = 8x^3 + 3x^2$

OR

$\frac{d}{dx} = (2x+1) \times 3x^2 + x^3 \times 2$

$= 6x^3 + 3x^2 + 2x^3$

$= 8x^3 + 3x^2$

Question 12 (b) (i)

Criteria	Marks
* Provides correct solution	1

Sample answer:

$2x + c$

Question 12 (b) (ii)

Criteria	Marks
* Provides correct solution	2
* Attempts to split the fraction and write with negative indices	1

Sample answer:

$\int \left(\frac{x^2}{x^4} + \frac{3x}{x^4} + \frac{1}{x^4} \right) dx = \int \left(x^{-2} + 3x^{-3} + x^{-4} \right) dx$

$= \frac{x^{-1}}{-1} + \frac{3x^{-2}}{-2} + \frac{x^{-3}}{-3} + c$ OR $-\frac{1}{x} - \frac{3}{2x^2} - \frac{1}{3x^3} + c$

Question 12 (c)

Criteria	Marks
* Provides correct solution	**3**
* Provides correct integration and correct substitution **OR** provides correct integration and one mistake only with follow through **OR** one mistake with integration and correct follow through.	**2**
* Correct integration only **OR** attempt to substitute into incorrect integration	**1**

Sample answer:

$$\int_1^4 x^{\frac{1}{2}}\,dx = \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^4 = \left[\frac{2x^{\frac{3}{2}}}{3}\right]_1^4$$

$$= \left[\left(\frac{2(4)^{\frac{3}{2}}}{3}\right) - \left(\frac{2(1)^{\frac{3}{2}}}{3}\right)\right]$$

$$= \left(\frac{16}{3}\right) - \left(\frac{2}{3}\right) = \frac{14}{3} \text{ OR } 4\frac{2}{3}$$

Question 12 (d)

Criteria	Marks
* Provides correct solution	**2**
* Provides correct gradient	**1**

Sample answer:

$$\frac{dy}{dx} = 12x^3 + 3$$

when $x = -1$, $\frac{dy}{dx} = 12(-1)^3 + 3 = -9$

$\therefore\ y - 0 = -9(x+1)$

$y = -9x - 9$ **OR** $9x + y + 9 = 0$

Question 12 (e)

Criteria	Marks
* Provides correct solution	**3**
* Provides one correct solution	**2**
* Rearranges correctly to get $\sin\theta = -\frac{\sqrt{3}}{2}$	**1**

Sample answer:

$\sin q = \frac{-\sqrt{3}}{2}$

$\theta = 240^\circ \quad \text{and} \quad 300^\circ$

Question 13 (LAMMIMAN)

Question 13 (a) (i)

Criteria	Marks
* Provides correct solution	2
* Provides one correct gradient	1

Sample answer:

$m_{BC} = \frac{5 - 3}{1 - 0}$ $\qquad$ $y = 2x - 1$

$m_{BC} = 2$ $\qquad$ $m_{AD} = 2$

$m_{BC} = m_{AD}$ (Parallel Lines) $\quad \therefore ABCD$ is a trapezium

Question 13 (a) (ii)

Criteria	Marks
* Provides correct solution	1

Sample answer:

$d = \sqrt{(5 - 3)^2 + (1 - 0)^2}$

$d = \sqrt{5}\, units$

Question 13 (a) (iii)

Criteria	Marks
* Provides correct solution	2
* Provides one correct gradient	1

Sample answer:

$\therefore d = \frac{|2 \times 0 + (-1) \times 3 + (-1)|}{\sqrt{2^2 + (-1)^2}}$

$\therefore d = \frac{4}{\sqrt{5}}$

Question 13 (a) (iv)

Criteria	Marks
* Provides correct solution	2
* Provides distance of AD $\left(3\sqrt{5}\right)$ **OR** coordinates of D is seen anywhere	1

Sample answer:

$A = \frac{1}{2} \times \frac{4}{\sqrt{5}} \times \left(\sqrt{5} + 3\sqrt{5}\right) = 8 units^2$

Question 13 (b)

Criteria	Marks
* Provides correct solution	2
* Provides equivalent equation (3rd line below) **OR** two correct solutions	1

Sample answer:

- $a\left(x^2 - 4x + 4\right) + bx - 2b + c$
- $ax^2 - 4ax + 4a + bx - 2b + c$
- $ax^2 + \left(-4a + b\right)x + 4a - 2b + c$

$\therefore a = 3$ $\quad -4a + b = -7$ $\quad 4a - 2b + c = -6$

$-12 + b = -7$ $\quad 12 - 10 + c = -6$

$b = 5$ $\quad c = -8$

Question 13 (c)

Criteria	Marks
* Provides correct solution	2
* Provides an expression with a common denominator	1

Sample answer:

$LHS = \cot\theta + \tan\theta$

$= \dfrac{\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{\cos\theta}$ $\quad = \dfrac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta}$

$= \dfrac{1}{\sin\theta\cos\theta}$ $\quad = \dfrac{1}{\sin\theta} \times \dfrac{1}{\cos\theta}$

$= \operatorname{cosec}\theta\sec\theta \quad = RHS$

Question 13 (d) (i)

Criteria	Marks
* Provides correct solution with reasoning	**2**
* Provides any two statements with reasoning	**1**

Sample answer:

In AYM and CYM :
$AM = CM (given)$
YM is common
$\angle AMY = \angle CMY (MY \perp AC = 90^\circ)$
$\therefore \quad AYM \equiv CYM (SAS)$

Question 13 (d) (ii)

Criteria	Marks
* Provides correct solution with reasoning	**2**
* Provides correct solution *without* reasoning **OR** provides first line of working with reasoning	**1**

Sample answer:

$\angle MAY = \angle MCY = x$ (corresponding angles in congruent triangles are equal)
$\angle YAB = YAM = x$ (angles bisected are equal)

$\therefore 90 + x + x + x = 180^\circ$ (angle sum of a triangle equals 180°)
$3x + 90 = 180$
$x = 30^\circ$

$\therefore \angle YCM = 30^\circ$

Question 14 (PAY)

Question 14 (a)

Criteria	Marks
* Provides correct solution	**3**
* Provides correct substitution into Trapezoidal Rule **OR** substitution with one mistake only with correct follow through	**2**
* Provides correct height	**1**

Sample answer:

$$A \approx \frac{0.5}{2}\left[2^0 + 2\left(2^{0.5} + 2^1 + 2^{1.5}\right) + 2^2\right]$$

$$A \approx 4.371 units^2$$

Question 14 (b) (i)

Criteria	Marks
* Provides correct solution	**2**
* Provides coordinates of stationary points without determining their nature **OR** correct solution with one mistake in calculations	**1**

Sample answer:

$f'(x) = 3x^2 - 18x + 24$

Let $f'(x) = 0$ for stationary points

$\therefore\ 3x^2 - 18x + 24 = 0$

$3(x^2 - 6x + 8) = 0$

$(x - 4)(x - 2) = 0$

$x = 4 \qquad x = 2$

$y = 18 \qquad y = 22$

$f''(x) = 6x - 18$

when $x = 4$, $f''(x) = 6$ $\therefore\ f''(x) > 0$ (Concave Up)

when $x = 2$, $f''(x) = -6$ $\therefore\ f''(x) < 0$ (Concave down)

$\therefore\ (4,18)$ is a local minimum stationary point

$(2,22)$ is a local maximum stationary point

Question 14 (b) (ii)

Criteria	Marks
* Provides correct solution with test	**2**
* Provides correct solution without test	**1**

Sample answer:

Let $f''(x) = 0$ for points of inflexion

$6x - 18 = 0$

$x = 3$

$y = 20$

Test:

x	2.9	3	3.1
$f''(x)$	-0.6	0	0.6

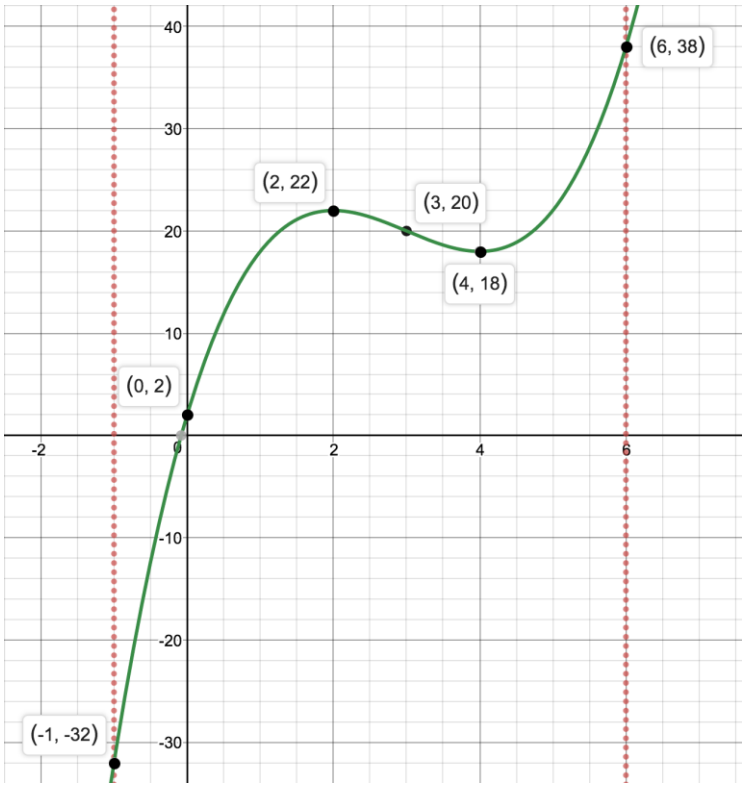
\ change in concavity

$(3, 20)$ is a point of inflexion

Question 14 (b) (iii)

Criteria	Marks
* Provides correct solution	**2**
* Provides at least 3 coordinates **OR** all coordinates with inaccurate graph (incorrect changes of concavity)	**1**

Sample answer:



Question 14 (b) (iv)

Criteria	Marks
* Provides correct solution with test	1

Sample answer:

$f'(x) \times f''(x) > 0$

$\therefore (-) \times (-)$ and $(+) \times (+)$

$f'(x) < 0$ and $f''(x) < 0$ and $f'(x) > 0$ and $f''(x) > 0$

$2 < x < 3$ and $x > 4$

Question 14 (c) (i)

Criteria	Marks
* Provides correct solution	1

Sample answer:

$40 + 45 + 50 + \ldots$

$a = 40$ and $d = 5$

$T_{10} = 40 + (10 - 1) \times 5$

$= 85$ minutes

Question 14 (c) (ii)

Criteria	Marks
* Provides correct solution	2
* Attempts to substitute into correct formulae	1

Sample answer:

$S_{10} = \frac{10}{2}\left[2(40) + (10 - 1) \times 5\right]$ **OR** $S_{10} = \frac{10}{2}\left[40 + 85\right]$

$S_{10} = 625$ minutes **OR** 10 hours and 25 minutes

Question 14 (c) (iii)

Criteria	Marks
* Provides correct solution	2
* Provides quadratic equation (Line 1)	1

Sample answer:

$7200 = \frac{n}{2}\left[2(40) + (n - 1) \times 5\right]$

$14400 = 80n + 5n^2 - 5n$

$5n^2 + 75n - 14400 = 0$

$n = \frac{-75 \pm \sqrt{(75)^2 - 4(5 \times -14400)}}{2 \times 5}$

$n = 46.687$ $\therefore$ 47th lesson

Question 15 (DUNBAR)

Question 15 (a) (i)

Criteria	Marks
* Provides correct solution	1

Sample answer:

$a+b=\dfrac{-b}{a}$ $=\dfrac{-5}{2}$ **OR** -2.5

Question 15 (a) (ii)

Criteria	Marks
* Provides correct solution	1

Sample answer:

$ab=\dfrac{c}{a}$ $=\dfrac{-1}{2}$ **OR** -0.5

Question 15 (a) (iii)

Criteria	Marks
* Provides correct solution	2
* Provides factorisation of the sum of a cube (1st Line)	1

Sample answer:

$$\begin{aligned}a^3+b^3&=(a+b)(a^2-ab+b^2)\\&=(a+b)(a^2+b^2-ab)\\&=(a+b)\left[(a+b)^2-2ab-ab\right]\\&=(a+b)\left[(a+b)^2-3ab\right]\\&=\frac{-5}{2}\times\left[\left(\frac{-5}{2}\right)^2-3\left(\frac{-1}{2}\right)\right]\end{aligned}$$

$=\dfrac{-155}{8}$ **OR** $-19\dfrac{3}{8}$ **OR** -19.375

Question 15 (b)

Criteria	Marks
* Provides correct solution	2
* Provides first line of working	1

Sample answer:

$$\begin{aligned}\log_a 12 &= \log_a 3 + \log_a 4 \\ &= \log_a 3 + \log_a 2^2 \\ &= \log_a 3 + 2\log_a 2 \\ &= y + 2x\end{aligned}$$

Question 15 (c) (i)

Criteria	Marks
* Provides correct solution	1

Sample answer:

$$\begin{aligned}x^2 + 2x &= -28y + 111 \\ (x+1)^2 - 1 &= -28y + 111 \\ (x+1)^2 &= -28y + 112 \\ (x+1)^2 &= -28(y-4) \text{ as required}\end{aligned}$$

Question 15 (c) (ii)

Criteria	Marks
* Provides correct solution	1

Sample answer:

Vertex $= (-1, 4)$

Question 15 (c) (iii)

Criteria	Marks
* Provides correct solution	1

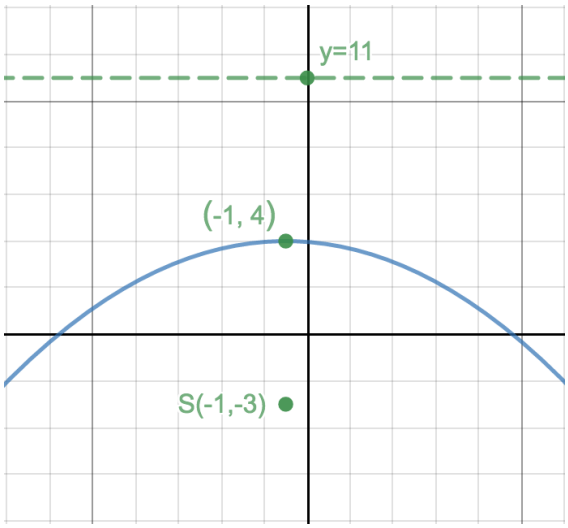
Sample answer:

Focus $= (-1, -3)$

Question 15 (c) (iv)

Criteria	Marks
* Provides correct solution	**2**
* Provides directrix **OR** correct graph with vertex and focus	**1**

Sample answer:



Question 15 (d) (i)

Criteria	Marks
* Provides correct solution	**2**
* Provides first line of working	**1**

Sample answer:

$\sqrt{x-4}=1$

$x-4=1$

$x=5 \quad y=1$

$\therefore \ A(5,1)$ *as required*

Question 15 (d) (ii)

Criteria	Marks
* Provides correct solution	**3**
* Provides correct integration and substitution (1 mistake)	**2**
* Provides correct integration	**1**

Sample answer:

$y^2 = x - 4$

$$V = \pi\left[\int_5^8 (x - 4)\,dx - \int_5^8 1\,dx\right]$$

$$= \pi\left(\left[\frac{x^2}{2} - 4x\right]_5^8 - \left[x\right]_5^8\right)$$

$$= \pi\left[\left(\frac{8^2}{2} - 4(8)\right) - \left(\frac{5^2}{2} - 4(5)\right) - (8 - 5)\right]$$

$$= \frac{9\pi}{2}\ units^3 \quad \textbf{OR} \quad 4.5\pi\ units^3$$

Question 16 (GIULIANI)

Question 16 (a)

Criteria	Marks
* Provides correct solution	3
* Rearranges to make x the subject and correctly integrates **OR** fails to make x the subject but integrates and substitutes correctly	2
* Rearranges to make x the subject **OR** integrates $\sqrt{x+1}$ correctly	1

Sample answer:

$y^2 = x+1$

$x = y^2 - 1$

$A = \int_0^1 (y^2-1)\,dy \; + \; \int_1^3 (y^2-1)\,dy$

$A = \left[\frac{y^3}{3} - y\right]_0^1 + \left[\frac{y^3}{3} - y\right]_1^3$

$A = \left[\left(\frac{1}{3}-1\right)-(0)\right] + \left[(9-3)-\left(\frac{1}{3}-1\right)\right]$

$= \frac{22}{3}$ *units*^{2} **OR** $7\frac{1}{3}$ *units*^{2}

Question 16 (b) (i)

Criteria	Marks
* Provides correct solution	1

Sample answer:

$A_1 = 50000 - M$

$A_2 = (50000 - M) - M$

$A_3 = (50000 - 2M) - M$

$\therefore \; A_3 = 50000 - 3M$

Question 16 (b) (ii)

Criteria	Marks
* Provides correct solution	1

Sample answer:

$A_4 = (50000 - 3M)(1.005) - M$

$A_5 = [(50000 - 3M)(1.005) - M] \times 1.005 - M$

$A_5 = (50000 - 3M)(1.005)^2 - M(1.005) - M$

$A_5 = (50000 - 3M)(1.005)^2 - M(1+1.005)$ *as required*

Question 16 (b) (iii)

Criteria	Marks
* Provides correct solution	2
* Recognises Geometric Progression and attempt to simplify	1

Sample answer:

$$A_{60} = (50000 - 3M)(1.005)^{57} - M(1 + 1.005 + \ldots + 1.005^{56})$$

$$= (50000 - 3M)(1.005)^{57} - M \times \frac{1(1.005^{57} - 1)}{0.005}$$

$$= (50000 - 3M)(1.005)^{57} - 200M(1.005^{57} - 1) \text{ as required}$$

Question 16 (b) (iv)

Criteria	Marks
* Provides correct solution	2
* Attempt of expanding and factorising expression to equal zero	1

Sample answer:

$$0 = (50000 - 3M)(1.005)^{57} - 200M(1.005^{57} - 1)$$

$$200M(1.005^{57}) - 200M = 50000(1.005)^{57} - 3M(1.005)^{57}$$

$$203M(1.005)^{57} - 200M = 50000(1.005)^{57}$$

$$M\left[203(1.005)^{57} - 200\right] = 50000(1.005)^{57}$$

$$M = \frac{50000(1.005)^{57}}{\left[203(1.005)^{57} - 200\right]}$$

$M = \$952.56$ OR $\$953$ (nearest dollar)

Question 16 (c) (i)

Criteria	Marks
* Provides correct solution	2
* Recognises Geometric Progression and attempt to simplify	1

Sample answer:

$$a^2 = r^2 + \left(\frac{h}{2}\right)^2$$

$r^2 = a^2 - \left(\frac{h}{2}\right)^2$ **OR** $r^2 = a^2 - \frac{h^2}{4}$

Question 16 (c) (ii)

Criteria	Marks
* Provides correct solution	1

Sample answer:

$$V = \pi r^2 h$$
$$= \pi \cdot \left(a^2 - \frac{h^2}{4}\right) \cdot h$$

$V = \pi\left(a^2 h - \frac{h^3}{4}\right)$ *as required*

Question 16 (c) (iii)

Criteria	Marks
* Provides correct solution	**4**
* Provides correct derivative, expression for *h* and shows it is a maximum	**3**
* Provides correct derivative and expression for *h*	**2**
* Provides correct derivative	**1**

Sample answer:

$$\frac{dv}{dh} = p\left(a^2 - \frac{3h^2}{4}\right)$$

let $\frac{dv}{dh} = 0$ for min/max

$$\therefore \quad p\left(a^2 - \frac{3h^2}{4}\right) = 0$$

$$a^2 - \frac{3h^2}{4} = 0$$

$$\frac{3h^2}{4} = a^2$$

$$h = \pm\sqrt{\frac{4a^2}{3}} \quad ; \; h > 0 \qquad \therefore \quad h = \frac{2a}{\sqrt{3}}$$

$$\frac{d^2v}{dh^2} = p\left(-\frac{6h}{4}\right) = -\frac{3ph}{2}$$

when $h = \frac{2a}{\sqrt{3}}$; $\frac{d^2v}{dh^2} = -\frac{3\pi}{2}\cdot\frac{2a}{\sqrt{3}} = -\frac{6a\pi}{2\sqrt{3}}$ OR $-\frac{3a\pi}{\sqrt{3}}$

$$\therefore \quad \frac{d^2v}{dh^2} < 0 \text{ (Maximum)}$$

$$V = p\left(a^2\cdot\frac{2a}{\sqrt{3}} - \frac{\left(\frac{2a}{\sqrt{3}}\right)^3}{4}\right) = p\left(\frac{2a^3}{\sqrt{3}} - \frac{8a^3}{12\sqrt{3}}\right) = p\left(\frac{24a^3}{12\sqrt{3}} - \frac{8a^3}{12\sqrt{3}}\right) = p\left(\frac{16a^3}{12\sqrt{3}}\right)$$

$$= \frac{4\sqrt{3}pa^3}{9} \text{ units}^3 \quad \text{as required}$$