



Trinity Grammar School

Mathematics Department

2017

HALF-YEARLY EXAMINATION

PRELIMINARY ASSESSMENT TASK 3

Year 11

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- BOSTES approved calculators for this course may be used
- A Reference Sheet for this course is supplied
- In Questions **11 – 16**, show all relevant mathematical reasoning and/or calculations
- Write your BOSTES Student Number (Year 12 HSC) or Name (Year 11) and your Class teacher on the question paper and on any answer sheets or writing booklets used to write your responses to the questions submitted
- **Preliminary Assessment Weighting:**
35%

Class Teacher:

.....

Name:

(Year 10 or 11 only)

.....

Do NOT write solutions on this question paper. Any working on this question paper will NOT be marked.

Date of examination:

Thursday, 27 April 2017

Total marks – 70

Section I

10 marks

- Attempt all Questions
- Allow about **15** minutes for this section

Section II

60 marks

- Attempt all Questions
- Allow about **1** hours **45** minutes for this section

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Section I 10 marks

- Attempt Questions 1 – 10.
 - Allow about 15 minutes for this section.
 - Shade the correct response on the multiple-choice answer sheet for Questions 1 – 10.
 - Each question is worth 1 mark.
-

1. What is 0.005 233 59 written in scientific notation, correct to 4 significant figures?

- (A) 5.2336×10^{-2}
(B) 5.234×10^{-2}
(C) 5.2336×10^{-3}
(D) 5.234×10^{-3}

2. Which of the following is equivalent to $\sqrt{3} + \sqrt{27} - \sqrt{18}$?

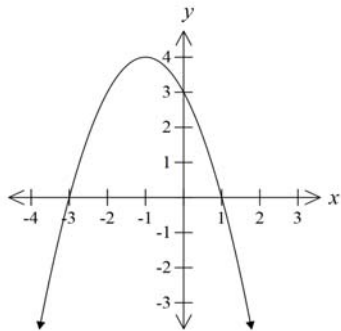
- (A) $\sqrt{3}$
(B) $\sqrt{12}$
(C) $4\sqrt{3} - 3\sqrt{2}$
(D) $10\sqrt{3} - 9\sqrt{2}$

3. Simplify $\frac{4a^2 - ab}{16a^2 - b^2}$.

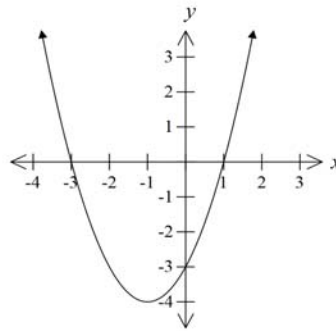
- (A) $\frac{a}{(4a - b)}$
(B) $\frac{4a - b}{(16a + b)}$
(C) $\frac{4a - b}{(16a - b)}$
(D) $\frac{a}{(4a + b)}$

4. Which graph best represents $y = x^2 + 2x - 3$?

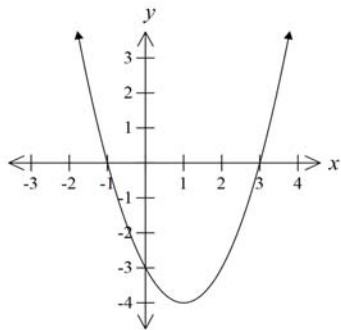
(A)



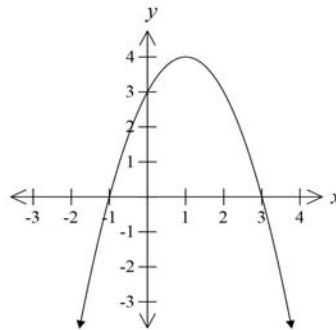
(B)



(C)



(D)



5. What is the solution to the inequality $|2 - 5x| < 7$?

(A) $x < -1$ or $x < \frac{9}{5}$

(B) $x > -1$ or $x > \frac{9}{5}$

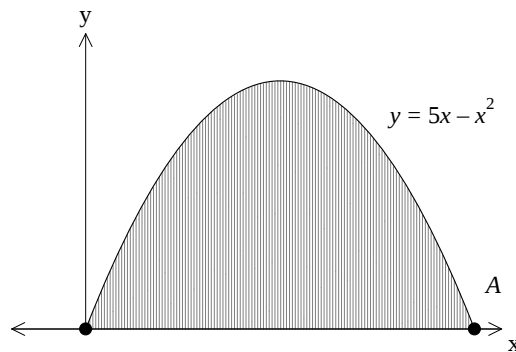
(C) $-1 < x < \frac{9}{5}$

(D) $-\frac{9}{5} < x < 1$

6. What is the centre and radius of the circle with the equation $(x+3)^2 + (y-4)^2 - 36 = 0$?

- (A) Centre $(-3, -4)$ and radius 36
- (B) Centre $(-3, 4)$ and radius 36
- (C) Centre $(-3, -4)$ and radius 6
- (D) Centre $(-3, 4)$ and radius 6

7. The diagram shows the graph of the function $y = 5x - x^2$.



What pair of inequalities specify the shaded region?

- (A) $y \leq 5x - x^2$ and $y \leq 0$.
- (B) $y \leq 5x - x^2$ and $y \geq 0$.
- (C) $y \geq 5x - x^2$ and $y \leq 0$.
- (D) $y \geq 5x - x^2$ and $y \geq 0$.

8. Which of the following is true for the function $f(x) = 3x^2 - x$?

- (A) Even function
- (B) Odd function
- (C) Neither odd or even
- (D) Zero function

9. If $a > b$, which of the following is always true?

(A) $a^2 > b^2$

(B) $\frac{1}{a} > \frac{1}{b}$

(C) $-a > -b$

(D) $2^a > 2^b$

10. The area of a rectangle is $80m^2$ and its length is x metres. Its perimeter, in metres, is:

(A) $2x + 160$

(B) $2x + \frac{80}{x}$

(C) $2x + \frac{40}{x}$

(D) $2x + \frac{160}{x}$

End of Section I
Section II commences on the next page

Section II 60 marks

- Attempt Questions 11 – 16.
- Allow about 1 hour and 45 minutes for this section.
- Answer each question in a SEPARATE writing booklet or answer sheet. Extra writing booklets are available.
- In Questions 11 – 16, your responses should include all relevant mathematical reasoning and/or calculations.

Question 11 (10 marks) Use a SEPARATE writing booklet.

(a) Factorise $2x^2 - 7x + 3$. 2

(b) Evaluate $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x^2 - 4}$. 2

(c) Simplify $\frac{a-b}{a^2-b^2} \div \frac{1}{a+b}$. 2

(d) Sketch $y = 1 - 2^x$, showing all essential features. 2

(e) If $\frac{5-2\sqrt{3}}{2-\sqrt{3}}$ can be expressed in the form $a + b\sqrt{3}$. 2

Find the value of a and b .

Question 12 (10 marks) Use a SEPARATE writing booklet.

(a) Sketch the region defined by $(x-2)^2 + (y-3)^2 \geq 4$. **3**

(b) Solve $8^{2-x} = 4^{2x}$. **2**

(c) Find the domain and range for the function $f(x) = \sqrt{1-x^2}$. **2**

(d) Solve simultaneously:

$$\begin{aligned} y &= \frac{2}{x} \\ 2x + 5y &= 12 \end{aligned} \quad \text{3}$$

Question 13 (10 marks) Use a SEPARATE writing booklet.

- (a) Consider the function $f(x) = \sqrt{x^2 - 9}$.
- (i) Find $f(3)$. 1
- (ii) For what value(s) of x is $f(x) = 4$? 2
- (iii) State whether the function, $f(x)$, is odd, even or neither.
You must justify your answer algebraically. 2
- (b) Express $0.3\dot{2}$ as a fraction in simplest form, using algebraic equations. 2
- (c) Given that $L = \frac{g}{4} \left(\frac{T}{\pi} \right)^2$, find the value of T when $L = 1.5$, $g = 9.8$ 3
and $T > 0$. Give answer correct to 2 decimal places.

Question 14 (10 marks) Use a SEPARATE writing booklet

- (a) After 10% GST is added, the new price is \$33 000 000. 2

What was the price before GST?

- (b) Express $a^{-3} \div 4a^{-4} \times 3a^5$ with positive indices. 2

- (c) Solve $2x^2 - 13x - 7 < 0$. 2

- (d) Simplify fully:

(i) $\frac{a^3 - b^3}{5a^2 + 5ab + 5b^2}$ 2

(ii) $\frac{a - b}{a^{-1} - b^{-1}}$ 2

Question 15 (10 marks) Use a SEPARATE writing booklet

(a) Solve the equation $\frac{3x-1}{5x+1} = \frac{3x-2}{5x+2}$. 2

(b) Simplify $5\sqrt{3} + \sqrt{4r} - 2\sqrt{12} + \sqrt{9r}$. 2

(c) Sketch the function $f(x) = \frac{2}{x-1} + 1$, and write down its domain and range. 4

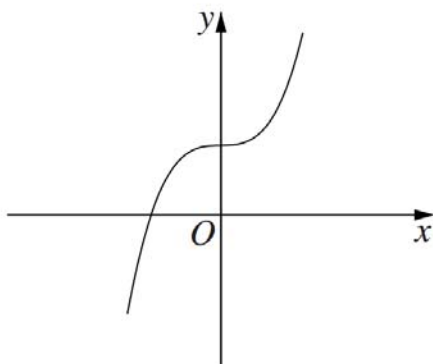
(d) Solve the equation $\frac{1-2x}{2x} + 2 = \frac{2}{x}, x \neq 0$. 2

Question 16 (10 marks) Use a SEPARATE writing booklet

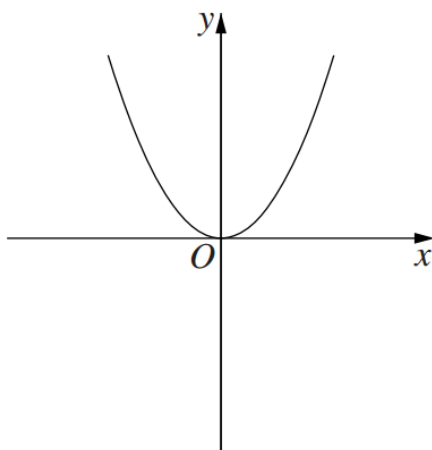
(a) State whether each of the following functions is odd, even or neither:

3

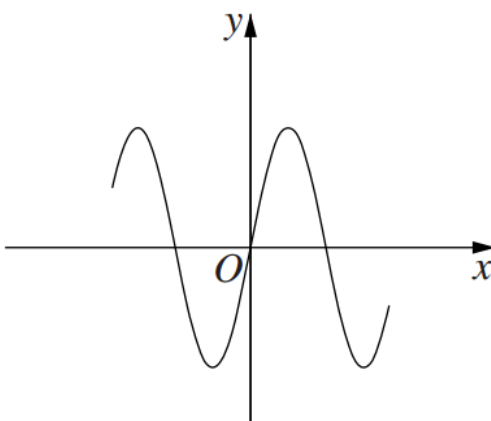
(i)



(ii)



(iii)



Question 16 continues on the next page

(b) Simplify $\frac{\frac{x}{a} + \frac{y}{b}}{\frac{b}{y} + \frac{a}{x}}$. 3

(c) Prove that $\frac{\sqrt[n]{a^{n+1}}}{\sqrt[n]{a}} = a$. 2

(d) Graph the solution of $4x \leq 15 \leq -9x$ on a number line. 2

End of Section II
End of Examination

IMA TASK 3 SOLUTIONS 2017

(1) (D) 5.234×10^{-3}

(2) (C) $\sqrt{3} + \sqrt{9 \times \sqrt{3}} - \sqrt[18]{6 \times \sqrt{3}} = \sqrt{3} + 3\sqrt{3} - \sqrt{9 \times \sqrt{2}}$
 $= 4\sqrt{3} - 3\sqrt{2}.$

(3) (D) $\frac{4a^2 - ab}{16a^2 - b^2} = \frac{a(4a - b)}{(4a - b)(4a + b)}$
 $= \frac{a}{4a + b}.$

(4) (B) $x^2 + 2x - 3 = 0$ $a = 1$
 $(x - 1)(x + 3) = 0$ $a > 0$
 $x = 1, -3$ concave up.

(5) (C) $-7 < 2 - 5x < 7$
 $-9 < -5x < 5$
 $-\frac{9}{-5} > x > \frac{5}{-5}$
 $\frac{9}{5} > x > -1$
 $\therefore -1 < x < \frac{9}{5}$

(6) (D) $x^2 + 6x + 9 + y^2 - 8y + 16 = 11 + 9 + 16$
 $(x + 3)^2 + (y - 4)^2 = 36$
 $(x - a)^2 + (y - b)^2 = r^2$
 Circle, centre $(-3, 4)$, radius = 6

7 (B) $y \leq 5x - x^2, \quad y \geq 0$

8 (C)
$$\begin{aligned} f(-x) &= 3(-x)^2 - (-x) \\ &= 3x^2 + x \\ &\neq f(x) \\ &\neq -f(x) \\ \therefore &\text{ neither} \end{aligned}$$

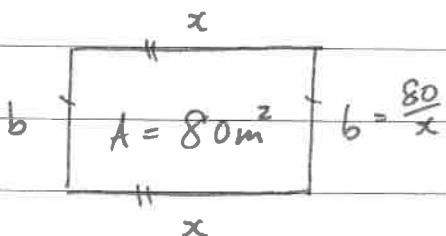
9 (D) If $a=4, b=3 \quad a > b \quad 4 > 3$

$$\begin{aligned} 4^2 &> 3^2 \quad \checkmark \\ \frac{1}{4} &> \frac{1}{3} \quad \times \\ -4 &> -3 \quad \times \\ 2^4 &> 2^3 \quad \checkmark \end{aligned}$$

If $a=-2, b=-3 \quad -2 > -3$

$$\begin{aligned} (-2)^2 &> (-3)^2 \quad \times \\ 2^{-2} &> 2^{-3} \\ \frac{1}{4} &> \frac{1}{8} \quad \checkmark \end{aligned}$$

10 (D)



$A = 80 \text{ m}^2$

$b = \frac{80}{x}$

$$\begin{aligned} A &= bx = 80 \\ \therefore b &= \frac{80}{x} \end{aligned}$$

$$\begin{aligned} P &= 2x + 2\left(\frac{80}{x}\right) \\ P &= 2x + \frac{160}{x} \end{aligned}$$

(11)

(a) $2x^2 - 7x + 3 = (2x - 1)(x - 3)$

(b) Step 1: $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x^2 - 4} = \frac{2^3 - 8}{2^2 - 4} = \frac{8 - 8}{4 - 4} = \frac{0}{0}$
undefined.

Step 2: $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x^2 - 4} = \lim_{x \rightarrow 2} \frac{(x-2)(x^2 + 2x + 4)}{(x-2)(x+2)}$

$= \lim_{x \rightarrow 2} \frac{(x^2 + 2x + 4)}{(x+2)}$

$= \frac{(2)^2 + 2(2) + 4}{2 + 2}$

Step 3:

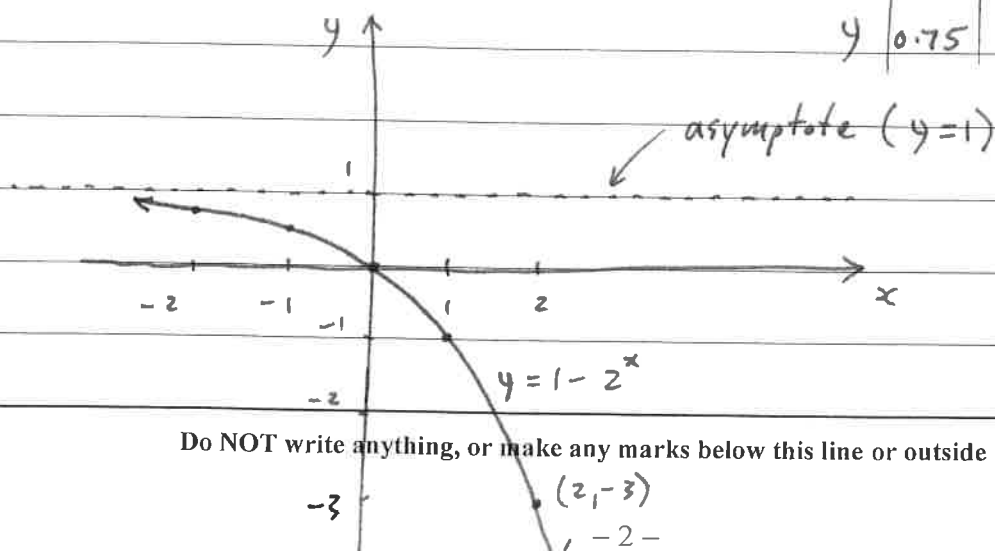
$= \frac{12}{4}$
 $= 3$

(c) $\frac{a-b}{(a-b)(a+b)} \times \frac{a+b}{1} = \frac{a-b}{a-b}$
 $= 1$

(d)

$y = -2^x + 1$

x	-2	-1	0	1	2
y	0.75	0.5	0	-1	-3



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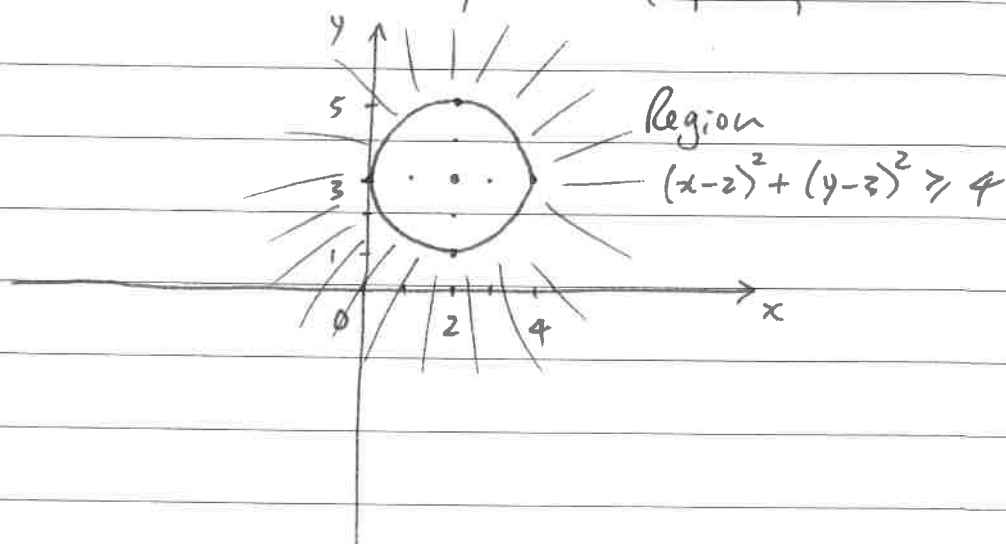
(11) (e)
$$\frac{5-2\sqrt{3}}{2-\sqrt{3}} = \frac{5-2\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$$

$$= \frac{10 + 5\sqrt{3} - 4\sqrt{3} - 6}{4 - 3}$$

$$= \frac{4 + \sqrt{3}}{1}$$

$$= 4 + \sqrt{3} \quad \left. \begin{array}{l} a = 4 \\ b = 1 \end{array} \right\}$$

(12) (a) Consider $(x-2)^2 + (y-3)^2 = 4$
 $(x-a)^2 + (y-b)^2 = r^2$
 Circle, centre $(2, 3)$, radius = 2



(12) (b)
$$(2^3)^{2-x} = (2^2)^{2x}$$

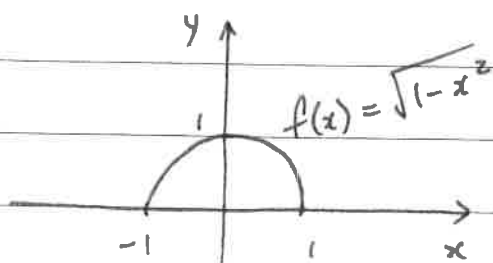
$$2^{6-3x} = 2^{4x}$$

$$6 - 3x = 4x$$

$$6 = 7x$$

$$x = \frac{6}{7}$$

12 (c)



semi-circle, radius = 1 unit

domain: $-1 \leq x \leq 1$

range: $0 \leq y \leq 1$

12 (d)

$$y = \frac{2}{x} \quad \text{--- (1)}$$

$$2x + 5y = 12 \quad \text{--- (2)}$$

$$\text{Subst. (1) into (2): } 2x + 5\left(\frac{2}{x}\right) = 12$$

$$2x + \frac{10}{x} = 12$$

$$2x^2 + 10 = 12x$$

$$2x^2 - 12x + 10 = 0$$

$$x^2 - 6x + 5 = 0$$

$$(x - 1)(x - 5) = 0$$

$$x = 1, 5$$

$$\text{Subst. } x = 1 \text{ into (1): } y = 2.$$

$$\text{Subst. } x = 5 \text{ into (1): } y = \frac{2}{5}$$

$\therefore$ solutions at $(1, 2)$ and $(5, \frac{2}{5})$

(13)

(a)

(i) $f(3) = \sqrt{(3)^2 - 9}$

$$f(3) = \sqrt{9 - 9}$$

$$f(3) = \sqrt{0}$$

$$f(3) = 0.$$

(ii) $f(x) = 4$

$$\therefore \sqrt{x^2 - 9} = 4$$

$$x^2 - 9 = 16$$

$$x^2 = 25$$

$$\therefore x = \pm 5.$$

(iii) $f(-x) = \sqrt{(-x)^2 - 9}$
 $= \sqrt{x^2 - 9}$
 $= f(x)$

$\therefore$ function is even since $f(-x) = f(x)$.

(13)

(b)

Let $x = 0.\dot{3}\dot{2}$

$$10x = 3.\dot{2}$$

$$100x = 32.\dot{2}$$

$$100x - 10x = 32.\dot{2} - 3.\dot{2}$$

$$90x = 29$$

$$x = \frac{29}{90}$$

$$\therefore 0.\dot{3}\dot{2} = \frac{29}{90}$$

13

(c)

$$T = ?$$

$$L = \frac{g}{4} \left(\frac{T}{\pi} \right)^2$$

$$L = 1.5$$

$$g = 9.8$$

$$1.5 = \frac{9.8}{4} \left(\frac{T}{\pi} \right)^2$$

$$1.5 = 2.45 \left(\frac{T}{\pi} \right)^2$$

$$\frac{1.5}{2.45} = \left(\frac{T}{\pi} \right)^2$$

$$\left(\frac{T}{\pi} \right)^2 = 0.612244898...$$

Since $T > 0$, $\frac{T}{\pi} = 0.7824607964...$

$$T = 2.45817309...$$

$$T = 2.46 \text{ (2d.p.)}$$

OR

$$L = \frac{g}{4} \left(\frac{T}{\pi} \right)^2$$

$$\frac{4L}{g} = \frac{T^2}{\pi^2}$$

$$\frac{4\pi^2 L}{g} = T^2$$

$$T = \sqrt{\frac{4\pi^2 L}{g}} \quad (T > 0)$$

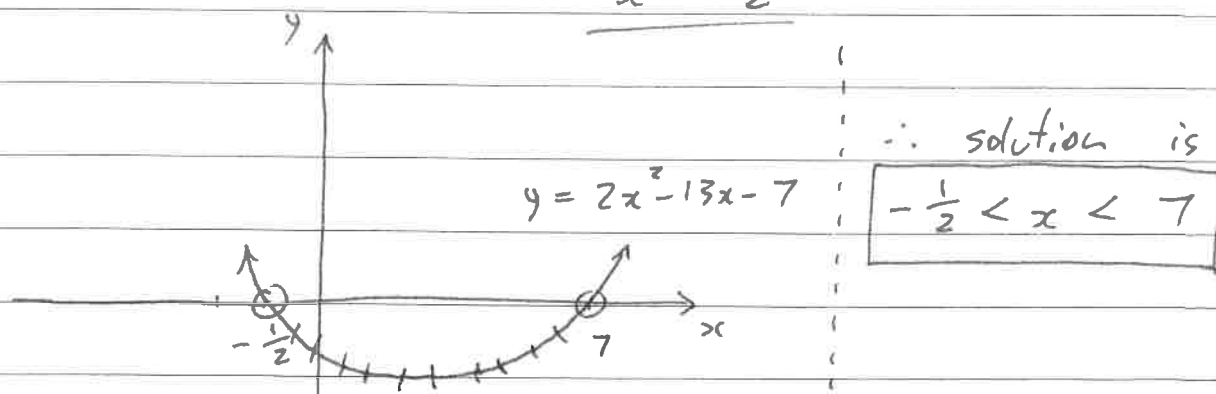
$$T = \sqrt{\frac{4 \times \pi^2 \times 1.5}{9.8}}$$

$$T = 2.46 \text{ (2d.p.)}$$

14 (a) 110% is \$33 000 000
 $\therefore 10\%$ is \$3 000 000 $\left\{ \begin{array}{l} \div 11 \\ \times 10 \end{array} \right.$
 $\therefore 100\%$ is \$30 000 000
 $\therefore$ the price before G.S.T. was \$30 000 000

14 (b) $a^{-3} \div 4a^{-4} \times 3a^5$
 $= \frac{1}{a^3} \div \frac{4}{a^4} \times \frac{3a^5}{1}$
 $= \frac{1}{a^3} \times \frac{a^4}{4} \times \frac{3a^5}{1}$
 $= \frac{3a^6}{4}$

14 (c) Consider $2x^2 - 13x - 7 = 0$
 $(2x+1)(x-7) = 0$
 $2x+1=0 \quad x-7=0$
 $2x=-1 \quad x=7$
 $x=-\frac{1}{2}$



$$\begin{aligned} \textcircled{14} \quad (d) \quad (i) \quad & \frac{a^3 - b^3}{5a^2 + 5ab + 5b^2} = \frac{(a-b)(\cancel{a^2 + ab + b^2})}{5(\cancel{a^2 + ab + b^2})} \\ & = \frac{a-b}{5} \end{aligned}$$

$$\begin{aligned} \textcircled{14} \quad (d) \quad (ii) \quad & \frac{a-b}{\frac{1}{a} - \frac{1}{b}} = (a-b) \div \left(\frac{1}{a} - \frac{1}{b} \right) \\ & = (a-b) \div \left(\frac{b-a}{ab} \right) \\ & = \frac{a-b}{1} \times \frac{ab}{b-a} \\ & = \frac{a-b}{b-a} \times ab \\ & = -1 \times ab \\ & = -ab \end{aligned}$$

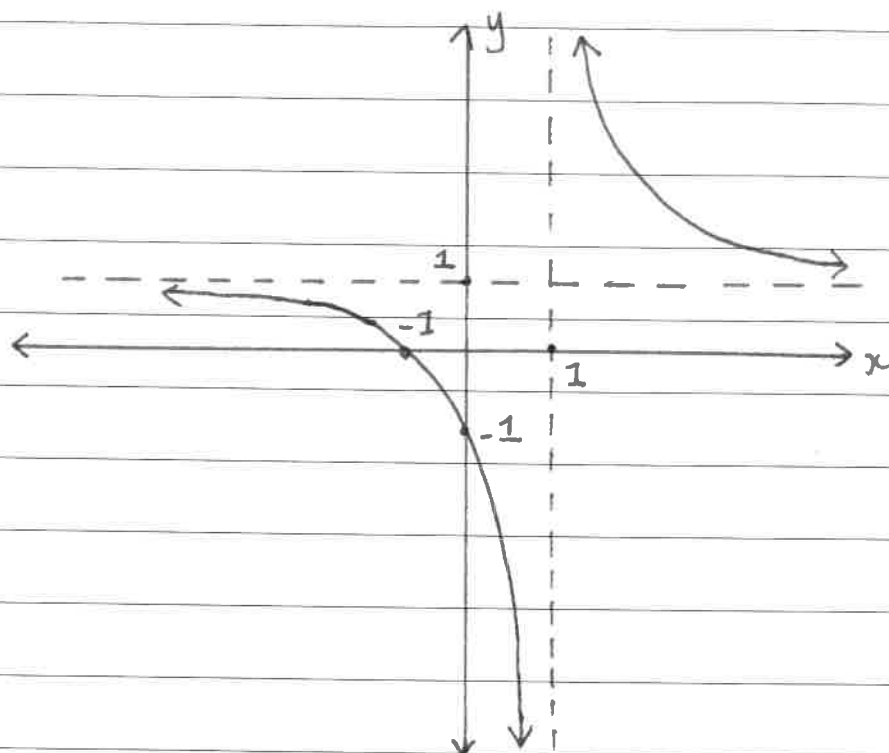
$$\begin{aligned} \textcircled{15} \quad (a) \quad & (3x-1)(5x+2) = (3x-2)(5x+1) \\ \cancel{15x^2} + 6x - 5x - 2 & = \cancel{15x^2} + 3x - 10x - 2 \\ x - 2 & = -7x - 2 \\ 8x & = 0 \end{aligned}$$

$$x = 0$$

check: $\frac{-1}{1} = \frac{-2}{2}$
true.

$$\begin{aligned}
 (15) \quad (b) \quad & 5\sqrt{3} + \sqrt{4 \times \sqrt{r}} - 2\sqrt{4 \times \sqrt{3}} + \sqrt{9 \times \sqrt{r}} \\
 &= 5\sqrt{3} + 2\sqrt{r} - 4\sqrt{3} + 3\sqrt{r} \\
 &= \sqrt{3} + 5\sqrt{r}.
 \end{aligned}$$

$$15(c) \quad f(x) = \frac{2}{x-1} + 1$$



Domain: All real x , $x \neq 1$

Range: All real y , $y \neq 1$

$$\begin{aligned}
 (15) \quad (d) \quad & \frac{\cancel{2x}}{1} \times \frac{1-2x}{\cancel{2x}} + 2 \times 2x = \frac{2}{\cancel{x}} \times \frac{\cancel{2x}}{1} \\
 & 1-2x + 4x = 4 \\
 & 1 + 2x = 4 \\
 & 2x = 3 \\
 & \underline{x = \frac{3}{2}}
 \end{aligned}$$

Do NOT write anything, or make any marks below this line or outside borders

- 16 (a) (i) neither
(ii) even
(iii) odd

$$16 (b) \left(\frac{x}{b} + \frac{y}{a} \right) \div \left(\frac{a}{y} + \frac{b}{x} \right)$$

$$= \left(\frac{ax + by}{ab} \right) \div \left(\frac{ax + by}{yx} \right)$$

$$= \frac{\cancel{(ax + by)}}{ab} \times \frac{xy}{\cancel{(ax + by)}}$$

$$= \frac{xy}{ab}$$

$$16 (c) \text{ LHS} = \frac{{}^n\sqrt{a^{n+1}}}{{}^n\sqrt{a}}$$

$$= \sqrt[n]{\frac{a^{n+1}}{a^1}}$$

$$= \sqrt[n]{a^{n+1-1}}$$

$$= \sqrt[n]{a^n}$$

$$= a$$

$$= \text{RHS.}$$

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16 (d)

$$4x \leq 15$$

$$x \leq \frac{15}{4}$$

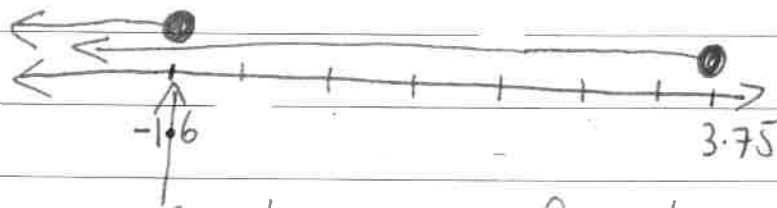
$$x \leq 3.75$$

$$15 \leq -9x$$

$$-9x \geq 15$$

$$x \leq -\frac{15}{9}$$

$$x \leq -1.6$$



Overlap occurs from this point onwards

$$\therefore x \leq -\frac{15}{9}$$