

Student Number: _____

Student Name: _____



Ascham School

2019

YEAR 11

Mathematics Advanced

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Diagrams are NOT drawn to scale

Total Marks:
100

Section I – 10 marks (pages 2–5)

- Attempt Questions 1–10
- Allow about 15 minutes for this section
- Give your answers on the multiple choice answer sheet

Section II – 90 marks (pages 7–21)

- Attempt Questions 11–16
- Allow about 2 hours and 45 minutes for this section
- Extra writing paper provided at the back

SECTION I (10 marks)

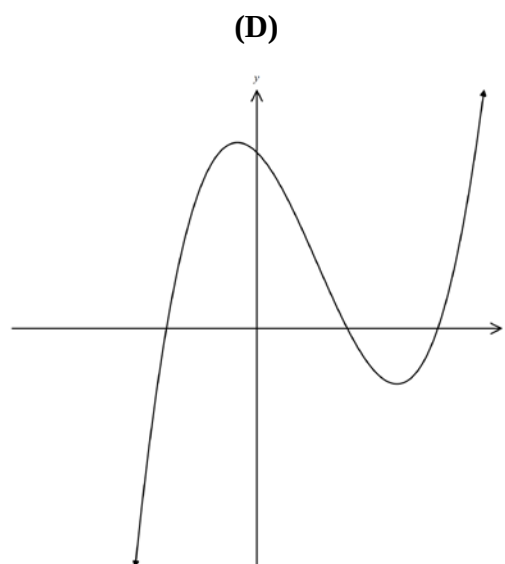
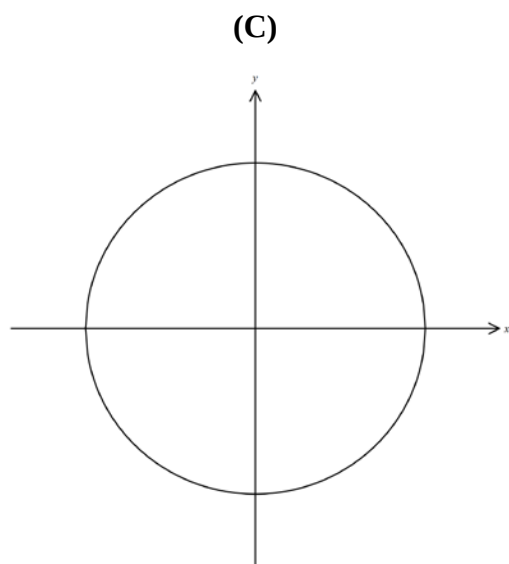
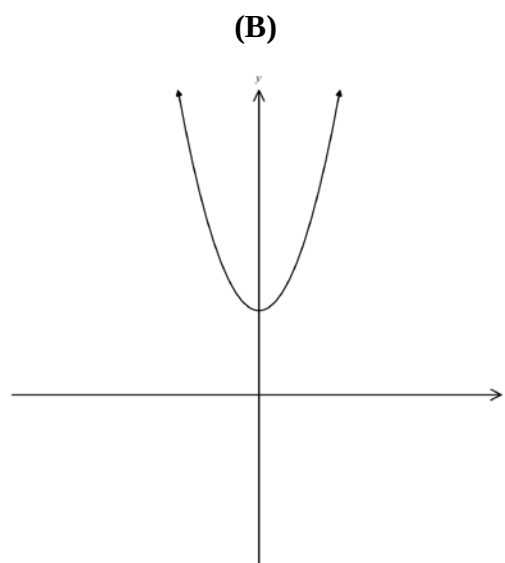
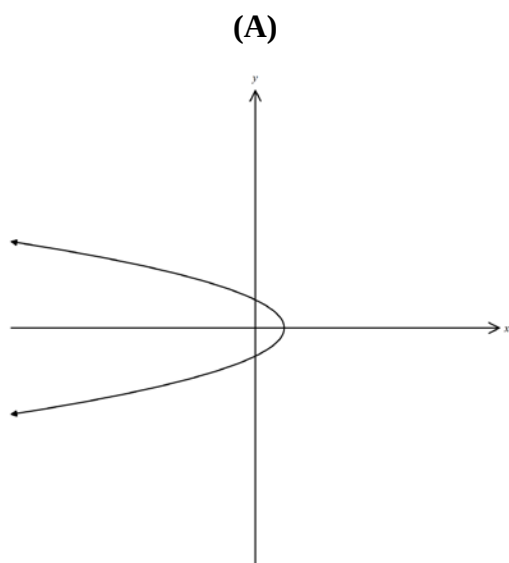
Attempt Question 1 – 10

Allow about 15 minutes for this section.

Use the multiple-choice answer sheet on the back page .

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

1. Which graph can be described as being one-to-many?



2. The exact value of $\sin \frac{5\pi}{3}$ is:

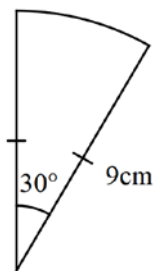
(A) $\frac{\sqrt{3}}{2}$

(B) $-\frac{\sqrt{3}}{2}$

(C) $\frac{1}{2}$

(D) $-\frac{1}{2}$

3. The length of the arc formed by an angle of 30° and a radius of 9cm is:



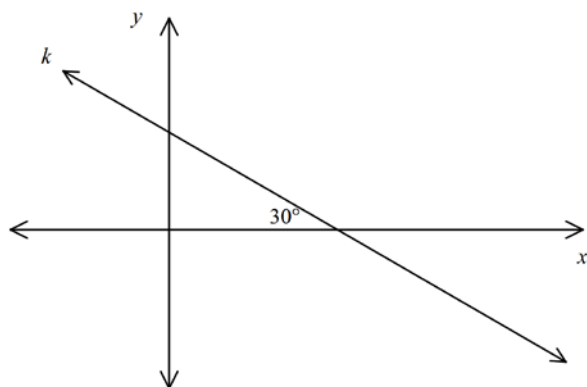
- (A) $\frac{3\pi}{2}$ cm (B) 270 cm
(C) $\frac{27\pi}{4}$ cm (D) 1215 cm

4. The value of n for which the table below is a discrete probability distribution is:

x	0	1	2	3	4
$p(x)$	$\frac{1}{20}$	$\frac{1}{15}$	$\frac{1}{10}$	$\frac{1}{5}$	n

- (A) $\frac{5}{12}$ (B) 0 (C) $\frac{7}{12}$ (D) $\frac{1}{5}$

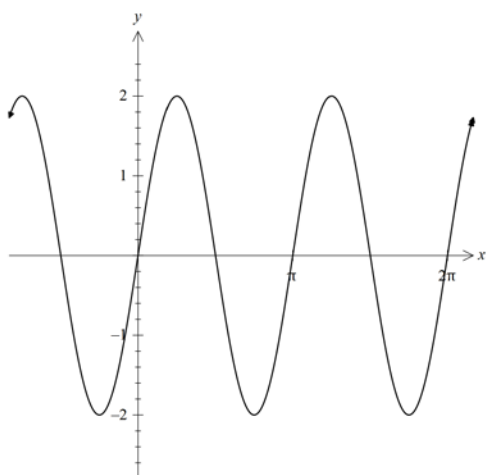
5. The slope of line k is:



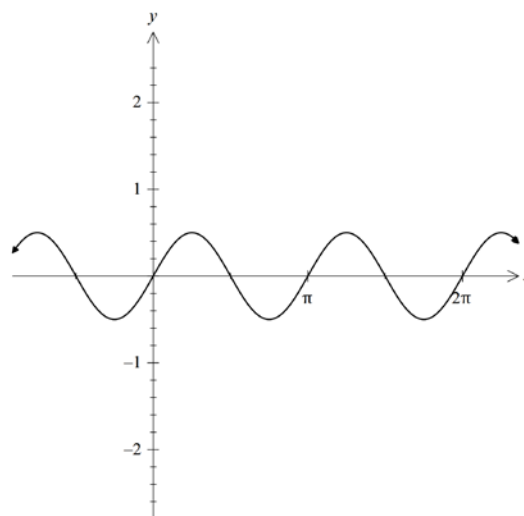
- (A) $\frac{1}{\sqrt{3}}$ (B) $-\frac{1}{\sqrt{3}}$ (C) $\sqrt{3}$ (D) $-\sqrt{3}$

6. Which diagram shows the graph $y = 2 \sin \frac{x}{2}$?

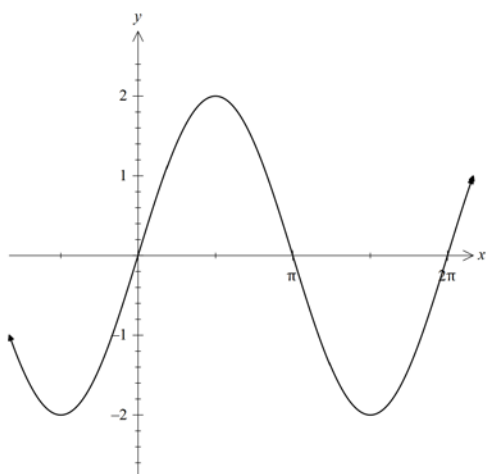
(A)



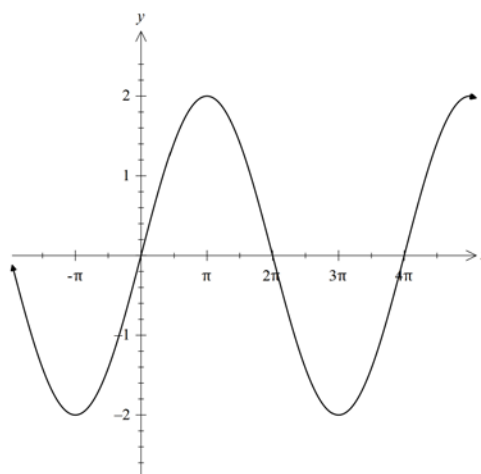
(B)



(C)



(D)



7. What is the solution to the equation $5^x = 4$?

(A) $x = \frac{\ln 4}{5}$

(B) $x = \frac{4}{\ln 5}$

(C) $x = \frac{\ln 4}{\ln 5}$

(D) $x = \ln \left(\frac{4}{5} \right)$

8. The derivative of $y = xe^{2x}$ is:

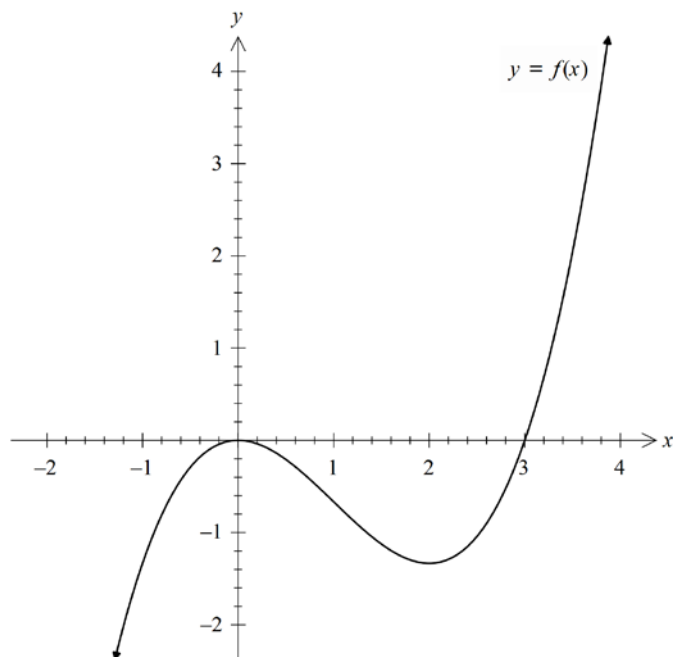
(A) $y' = x^2 e^x$

(B) $y' = e^{2x}(1 + 2x)$

(C) $y' = 2xe^{2x}$

(D) $y' = e^{2x} - 2xe^x$

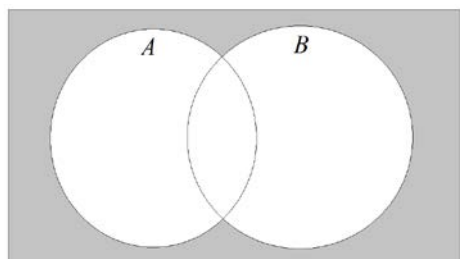
9. Which statement below is **TRUE** for the graph of $f(x)$:



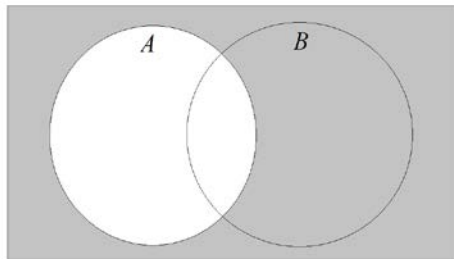
- (A) $f(x)$ is an even function (B) $f(x)$ is an odd function
 (C) $f'(x) = 0$ at $x = 0$ and $x = 1$ (D) $f'(x) > 0$ for $(-\infty, 0) \cup (2, \infty)$

10. Which Venn diagram represents the shaded region of $\bar{A} \cap B$?

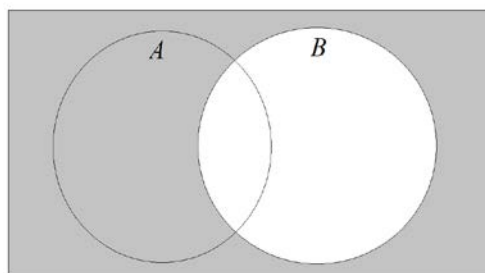
(A)



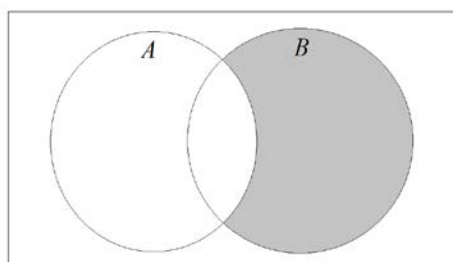
(B)



(C)



(D)



End of Multiple Choice

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Section II**90 marks****Attempt Questions 11–16****Allow about 2 hour and 45 minutes for this section**

Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses.

Your responses should include relevant mathematical reasoning and/or calculations.

Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Question 11 (15 marks)

- (a) Factorise $a^3 - a^2$. **2**

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- (b) Given that $f(x) = x^2 - 2x - 1$, evaluate $f(2)$. **2**

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- (c) Differentiate:

$$y = (x^3 - 2)^5 \quad \text{2}$$

.....

$$y = \frac{1}{4x^4} + 2 \quad \text{3}$$

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Differentiate:

$$y = \frac{x^2 - 1}{3 - x}$$

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(d) Evaluate a and b if $a + b\sqrt{3} = \frac{1 - \sqrt{3}}{2 + \sqrt{3}}$.

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Question 12 (15 marks)

- (a) Simplify
- $2\log x - \log xy$
- .

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- (b) Write down the discriminant of
- $kx^2 + 2x - 1 = 0$
- .

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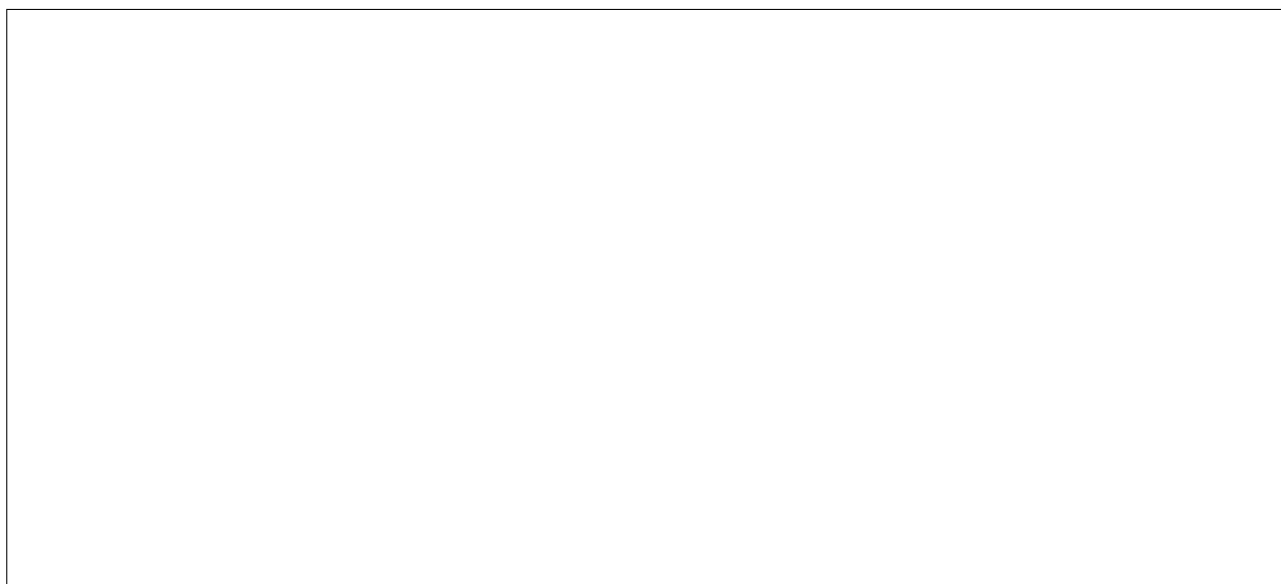
For what values of k will $kx^2 + 2x - 1 = 0$ have no roots?**2**

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- (c) Graph
- $f(x) = (x - 1)(x + 2)^2$
- , showing all intercepts.

2

State the domain and range for the function $f(x) = (x - 1)(x + 2)^2$.**2**

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(d) The mass in grams of an increasing particle is given the formula $M = t^3 - 2t + 15$, where t is time in minutes.

What is the initial mass of the particle? 1

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Find the average rate of change of M between $t = 0$ and $t = 2$. 2

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Find the rate at which the particle will be increasing at 4 minutes? 2

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(e) From a standard pack of 52 cards, two cards are drawn at random with replacement. Find the probability of drawing a red card followed by an ace. 1

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Question 13 (15 marks)

- (a) Is the function $f(x) = 3e^x$ odd, even or neither? Justify your answer algebraically. **2**

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- (b) If $\tan \theta = \frac{2}{3}$ and $\cos \theta < 0$, find the exact value of $\sin \theta$ **2**

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- (c) If $f(x) = 3x$ and $g(x) = x^2 - 3x$,
Find $f(x) - g(x)$, **1**

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- Find $f(g(x))$. **1**

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- (d) Solve $|3 - 2x| = 17$. 2

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- (e) Solve $2 \sin x = -1$ in the domain $[0^\circ, 360^\circ]$. 3

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- (f) There are 200 tickets sold in a raffle. The ticket for 2nd prize is drawn first, then the ticket for 1st prize is drawn. Sam bought 12 tickets and Bianca bought 15 tickets.

Find the probability that Sam won 1st prize and Bianca won 2nd prize. 2

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Find the probability that Sam wins both prizes. 2

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(a) Find the centre and radius of the circle with the equation $x^2 + y^2 - 6x - 4y + 4 = 0$. **3**

[illegible]

(b) Given that $f(x) = x^3 - 3x$, find the equation of the normal at $x = 2$. 4

[illegible]

- (c) Construct a probability distribution table for the number of heads when tossing two coins.

2

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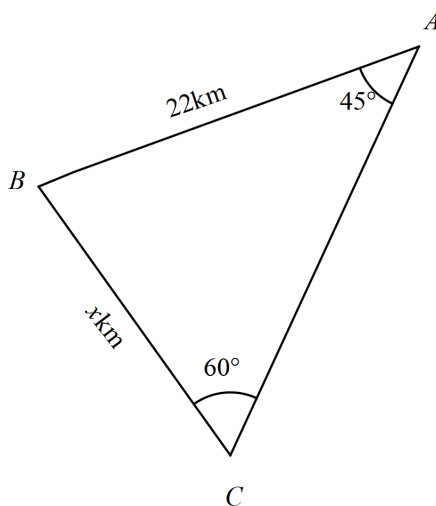
Is this probability distribution uniform? Briefly explain why or why not?

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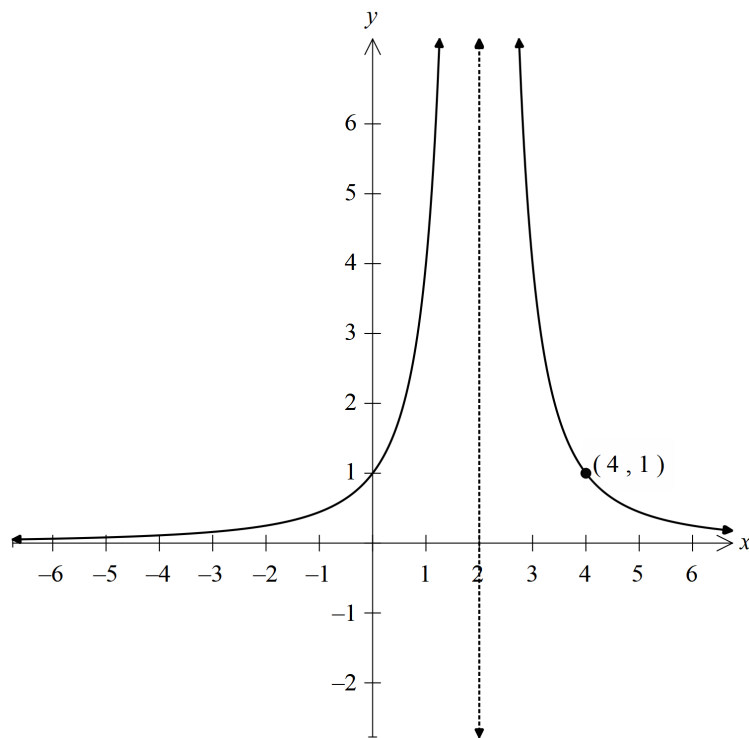
- (d) In the diagram, Beth's house B is 22km away from Abby's house at A , and $\angle BCA$ and $\angle BAC$ are 60° and 45° respectively. By letting the distance BC by x kilometres, find the exact distance from Beth's house to the cafe at C .

3



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- (e) Shown below is the graph of $y = f(x)$. Sketch the graph of $y = f(-x)$. Show all important features

2

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Question 15 (15 marks)

- (a) Find the points of the curve $y = \frac{x^3}{3} + x^2 - 3x$ where the tangent is horizontal. **3**

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- (b) Sketch the graph $y = 3 \cos(x) + 2$ in the domain $[-180^\circ, 180^\circ]$ in the space below. **3**

(c) Solve $\log_2 x + \log_2 (x + 2) = 3$. 3

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(d) Jenna tosses three coins. Two of the coins are unbiased however one coin has heads on both sides. Given that she obtains at least one tail, what is the probability that she obtains at least two heads? 3

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(e) Solve $\tan^2 \frac{x}{2} = 3$ in the domain $[-2\pi, 2\pi]$. 3

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(a) Construct a probability distribution table given the probability distribution function

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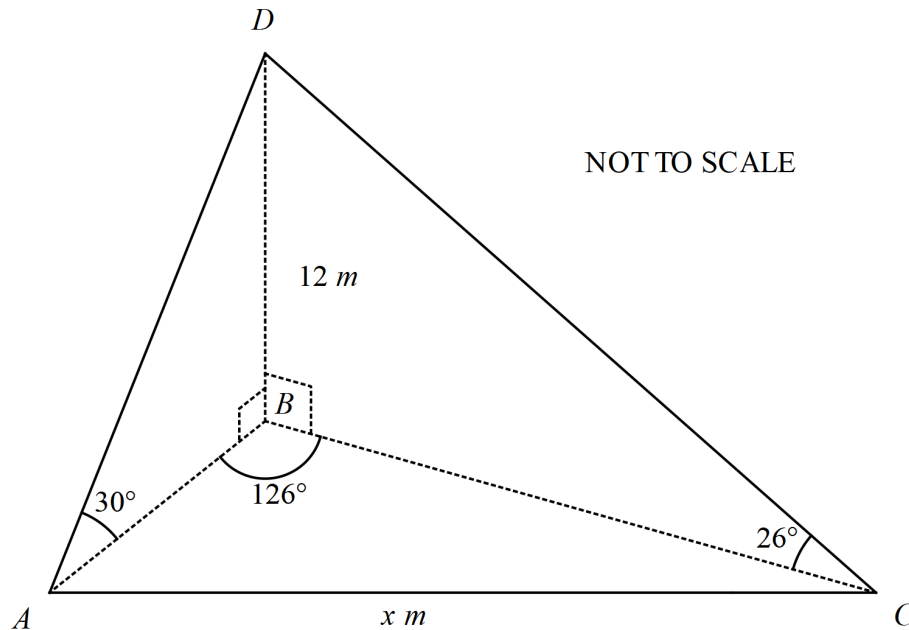
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[illegible]

- (b) In the diagram BD represents a vertical tower of height 12m standing on level ground. From A and C at ground level, the angles of elevation to the top of the tower are 30° and 26° respectively. Let x represent the distance between A and C . Solve for x , correct to 2 decimal places.

3

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

(c) Simplify $\frac{1}{\sin x - 1} - \frac{1}{\sin x + 1}$ 3

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(d) Solve the following pair of simultaneous equations:

$$\begin{cases} e^{x+y} = \frac{1}{e} \\ e^{3x+2y} = 1 \end{cases}$$

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End of Section II - End of Paper

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Extra Writing Paper for Question 11

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Extra Writing Paper for Question 12

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Extra Writing Paper for Question 13

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Extra Writing Paper for Question 14

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Extra Writing Paper for Question 15

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Extra Writing Paper for Question 16

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Student Name

Student Number

Teacher: JH AF MN LK GAS SW

ASCHAM SCHOOL

YEAR 11 PRELIMINARY MATHEMATICS ADVANCED EXAMINATION - 2019

SECTION I ANSWER SHEET

- | | | | | | | | | |
|-----|---|-----------------------|---|-----------------------|---|-----------------------|---|-----------------------|
| 1. | A | ☐ | B | ☐ | C | ☐ | D | ☐ |
| 2. | A | ☐ | B | ☐ | C | ☐ | D | ☐ |
| 3. | A | ☐ | B | ☐ | C | ☐ | D | ☐ |
| 4. | A | ☐ | B | ☐ | C | ☐ | D | ☐ |
| 5. | A | ☐ | B | ☐ | C | ☐ | D | ☐ |
| 6. | A | ☐ | B | ☐ | C | ☐ | D | ☐ |
| 7. | A | ☐ | B | ☐ | C | ☐ | D | ☐ |
| 8. | A | ☐ | B | ☐ | C | ☐ | D | ☐ |
| 9. | A | ☐ | B | ☐ | C | ☐ | D | ☐ |
| 10. | A | ☐ | B | ☐ | C | ☐ | D | ☐ |



Student Name

Student Number

Teacher: JH AF MN LK GAS SW

ASCHAM SCHOOL

YEAR 11 PRELIMINARY MATHEMATICS ADVANCED EXAMINATION - 2019

SECTION I ANSWER SHEET

- | | | | | | | | | |
|-----|---|----------------------------------|---|----------------------------------|---|----------------------------------|---|----------------------------------|
| 1. | A | ☒ | B | ☐ | C | ☐ | D | ☐ |
| 2. | A | ☐ | B | ☒ | C | ☐ | D | ☐ |
| 3. | A | ☒ | B | ☐ | C | ☐ | D | ☐ |
| 4. | A | ☐ | B | ☐ | C | ☒ | D | ☐ |
| 5. | A | ☐ | B | ☒ | C | ☐ | D | ☐ |
| 6. | A | ☐ | B | ☐ | C | ☐ | D | ☒ |
| 7. | A | ☐ | B | ☐ | C | ☒ | D | ☐ |
| 8. | A | ☐ | B | ☒ | C | ☐ | D | ☐ |
| 9. | A | ☐ | B | ☐ | C | ☐ | D | ☒ |
| 10. | A | ☐ | B | ☐ | C | ☐ | D | ☒ |

Section II**90 marks****Attempt Questions 11–16.****Allow about 2 hour and 45 minutes for this section**

Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses.

Your responses should include relevant mathematical reasoning and/or calculations.

Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Question 11 (15 marks)

- (a) Factorise
- $a^3 - a^2$
- .

2

$$a^3 - a^2 = a^2(a - 1)$$

- (b) Given that
- $f(x) = x^2 - 2x - 1$
- , evaluate
- $f(2)$
- .

2

$$f(2) = 2^2 - 2(2) - 1$$

$$= -1$$

- (c) Differentiate:

$$y = (x^3 - 2)^5$$

2

$$y' = 5(x^3 - 2)^4 \times 3x^2$$

$$= 15x^2(x^3 - 2)^4$$

$$y = \frac{1}{4x^4} + 2$$

3

$$y = \frac{1}{4}x^{-4} + 2$$

$$y' = \frac{-4}{4}x^{-5}$$

$$= \frac{-1}{x^5}$$

Differentiate:

$$y = \frac{x^2 - 1}{3 - x}$$

3

$$u = x^2 - 1$$

$$v = 3 - x$$

$$u' = 2x$$

$$v' = -1$$

$$y' = \frac{u'v - v'u}{v^2}$$

$$= \frac{2x(3-x) - (-1)(x^2-1)}{(3-x)^2}$$

$$= \frac{6x - 2x^2 + x^2 - 1}{(3-x)^2}$$

$$= \frac{6x - x^2 - 1}{(3-x)^2}$$

(d) Evaluate a and b if $a + b\sqrt{3} = \frac{1 - \sqrt{3}}{2 + \sqrt{3}}$.

3

$$a + b\sqrt{3} = \frac{(1 - \sqrt{3}) \times (2 - \sqrt{3})}{2 + \sqrt{3} \times (2 - \sqrt{3})}$$

$$= \frac{2 - \sqrt{3} - 2\sqrt{3} + 3}{4 - 3}$$

$$= 5 - 3\sqrt{3}$$

$$\therefore a = 5, b = -3$$

Question 12 (15 marks)

- (a) Simplify
- $2\log x - \log xy$
- .

2

$$\begin{aligned}
 2\log x - \log xy &= \log x^2 - \log xy \\
 &= \log\left(\frac{x^2}{xy}\right) \\
 &= \log\left(\frac{x}{y}\right)
 \end{aligned}$$

- (b) Write down the discriminant of
- $kx^2 + 2x - 1 = 0$
- .

1

$$\begin{aligned}
 \Delta &= b^2 - 4ac \\
 &= 4 + 4k
 \end{aligned}$$

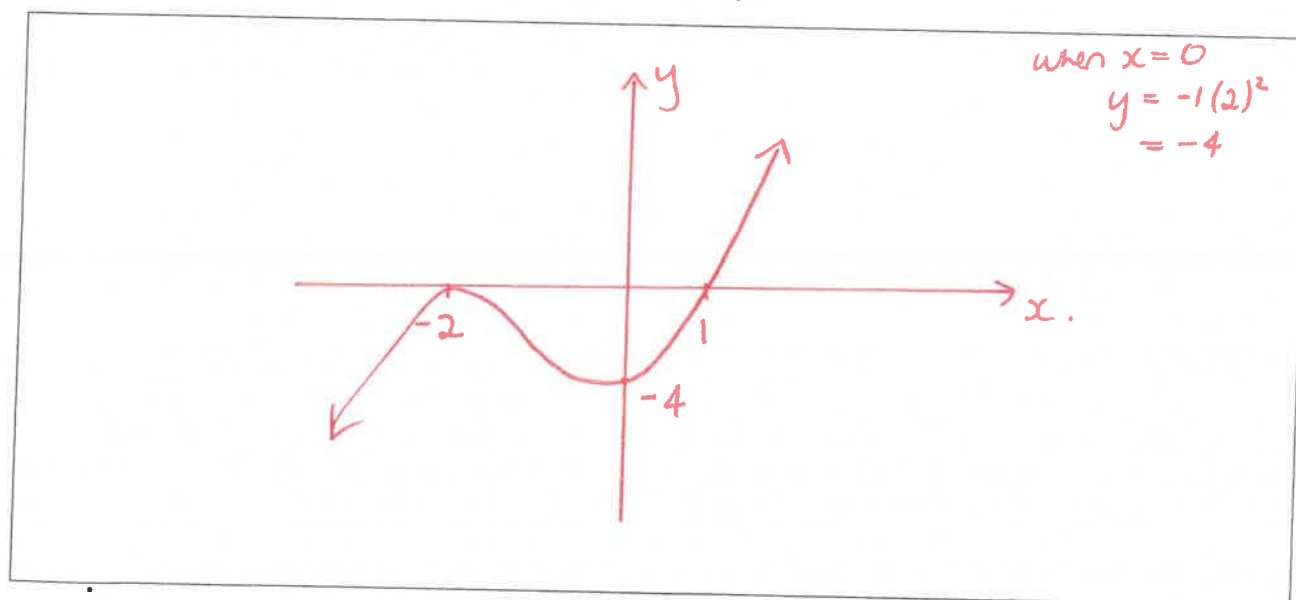
For what values of k will $kx^2 + 2x - 1 = 0$ have no roots?

2

$$\begin{aligned}
 \text{no roots } \Delta &< 0 & 4 + 4k &< 0 \\
 & & 4k &< -4 \\
 & & k &< -1
 \end{aligned}$$

- (c) Graph
- $f(x) = (x-1)(x+2)^2$
- , showing all intercepts.

2

State the domain and range for the function $f(x) = (x-1)(x+2)^2$.

2

$$D: (-\infty, \infty) \quad R: (-\infty, \infty)$$

- (d) The mass in grams of a melting ice block is given the formula $M = t^3 - 2t + 15$, where t is time in minutes.

What is the initial mass of the ice block?

1

$$t = 0 \quad M = 15 \text{ grams}$$

Find the average rate of change of M between $t = 0$ and $t = 2$.

2

$$t = 2 \quad M = 2^3 - 2(2) + 15 = 19$$

$$\text{av. rate} = \frac{19 - 15}{2 - 0} = 2 \text{ grams/min.}$$

Find the rate at what the ice block will be melting at 4 minutes?

2

$$\frac{dM}{dt} = 3t^2 - 2$$

$$\text{at } t = 4 \quad \frac{dM}{dt} = 3(4)^2 - 2 = 46 \text{ grams/min.}$$

- (e) From a standard pack of 52 cards, two cards are drawn at random with replacement. Find the probability of drawing a red card followed by an ace.

1

$$P(\text{red then ace}) = \frac{1}{2} \times \frac{4}{52} = \frac{1}{26}$$

Question 13 (15 marks)

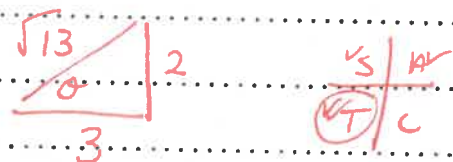
- (a) Is the function
- $f(x) = 3e^x$
- odd, even or neither? Justify your answer algebraically.

2

$$\begin{aligned}
 f(x) &= 3e^x & -f(x) &= -3e^x \\
 f(-x) &= 3e^{-x} & f(-x) &\neq -f(x) \\
 f(x) &\neq f(-x) & \therefore &\text{not odd} \\
 \therefore &\text{not even} \\
 \therefore &\text{neither}
 \end{aligned}$$

- (b) If
- $\tan \theta = \frac{2}{3}$
- and
- $\cos \theta < 0$
- , find the exact value of
- $\sin \theta$

2



$$\therefore \sin \theta = \frac{-2}{\sqrt{13}}$$

- (c) If
- $f(x) = 3x$
- and
- $g(x) = x^2 - 3x$
- ,

Find $f(x) - g(x)$,

2

$$\begin{aligned}
 f(x) - g(x) &= 3x - (x^2 - 3x) \\
 &= 3x - x^2 + 3x \\
 &= 6x - x^2 \\
 &= x(6 - x)
 \end{aligned}$$

Find $f(g(x))$.

1

$$\begin{aligned}
 f(g(x)) &= f(x^2 - 3x) \\
 &= 3(x^2 - 3x) \\
 &= 3x^2 - 9x \\
 &= 3x(x - 3)
 \end{aligned}$$

- (d) Solve
- $|3 - 2x| = 17$
- .

2

$$\begin{aligned}
 3 - 2x &= 17 & , & & 3 - 2x &= -17 \\
 -2x &= 14 & & & -2x &= -20 \\
 x &= -7 & & & x &= 10 \\
 \therefore x &= -7, 10
 \end{aligned}$$

- (e) Solve
- $2 \sin x = -1$
- in the domain
- $[0^\circ, 360^\circ]$
- .

3

$$\begin{aligned}
 2 \sin x &= -1 & [0^\circ, 360^\circ] \\
 \sin x &= \frac{-1}{2} & \text{ref} \angle = 30^\circ \\
 x &= 180^\circ + 30^\circ, 360^\circ - 30^\circ \\
 &= 210^\circ, 330^\circ
 \end{aligned}$$

$\frac{5}{11} \checkmark$

- (f) There are 200 tickets sold in a raffle. The ticket for 2nd prize is drawn first, then the ticket for 1st prize is drawn. Sam bought 12 tickets and Bianca bought 15 tickets.

Find the probability that Sam won 1st prize and Bianca won 2nd prize.

2

$$\begin{aligned}
 P(\text{Sam } 1^{\text{st}} \cap \text{Bianca } 2^{\text{nd}}) &= \frac{15}{200} \times \frac{12}{199} \\
 &= \frac{9}{1990}
 \end{aligned}$$

Find the probability that Sam wins both prizes.

2

$$\begin{aligned}
 P(\text{Sam both}) &= \frac{12}{200} \times \frac{11}{199} \\
 &= \frac{33}{9950}
 \end{aligned}$$

Question 14 (15 marks)

- (a) Find the centre and radius of the circle with the equation $x^2 + y^2 - 6x - 4y + 4 = 0$.

3

$$x^2 + y^2 - 6x - 4y + 4 = 0$$

$$x^2 - 6x + 3^2 + y^2 - 4y + 2^2 = -4 + 3^2 + 2^2$$

$$(x-3)^2 + (y-2)^2 = 9$$

$$\therefore \text{centre: } (3, 2)$$

$$\text{radius: } 3$$

- (b) Given that $f(x) = x^3 - 3x$, find the equation of the normal at $x = 2$.

3

$$f(x) = x^3 - 3x$$

$$f'(x) = 3x^2 - 3$$

$$f'(2) = 3(2)^2 - 3$$

$$= 9$$

$$\therefore m_T = 9, m_N = -\frac{1}{9}$$

$$f(2) = 2^3 - 3(2)$$

$$= 2$$

$$m_N = -\frac{1}{9} \quad (2, 2)$$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{1}{9}(x - 2)$$

$$9y - 18 = -x + 2$$

$$\therefore x + 9y - 20 = 0 \text{ is the equation of the normal}$$

- (c) Construct a probability distribution table for the number of heads when tossing two coins.

2

x	0	1	2
$p(x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

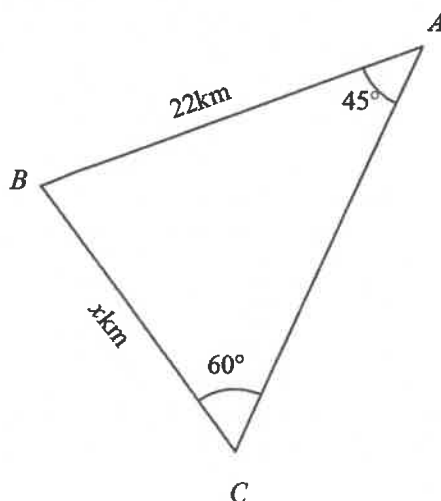
Is this probability distribution uniform? Briefly explain why or why not?

1

No. $p(0) \neq p(1)$
 $\therefore$ not uniform distribution as all $p(x)$ values must be the same.

- (d) In the diagram, Beth's house B is 22km away from Abby's house at A , and $\angle BCA$ and $\angle BAC$ are 60° and 45° respectively. By letting the distance BC by x kilometres, find the exact distance from Beth's house to the cafe.

3



$$\frac{x}{\sin 45} = \frac{22}{\sin 60}$$

$$\frac{x}{\frac{1}{\sqrt{2}}} = \frac{22}{\frac{\sqrt{3}}{2}}$$

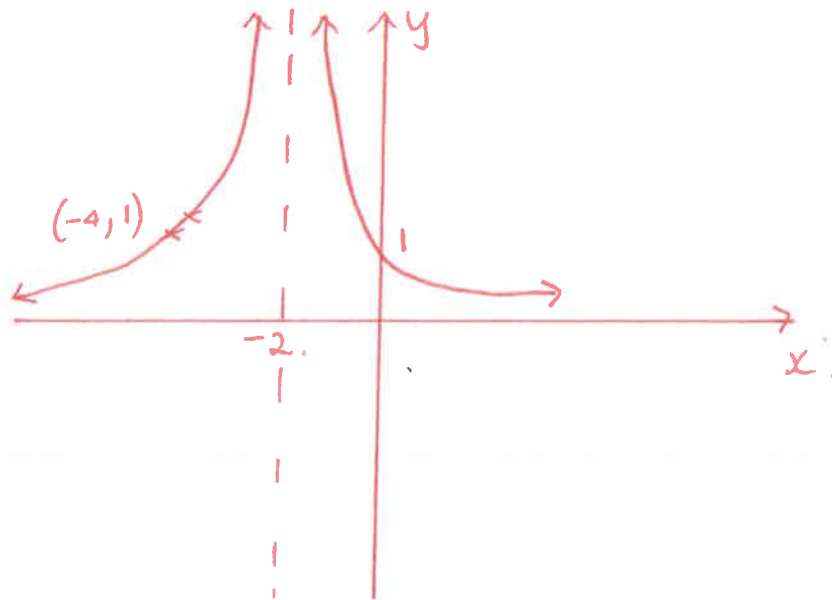
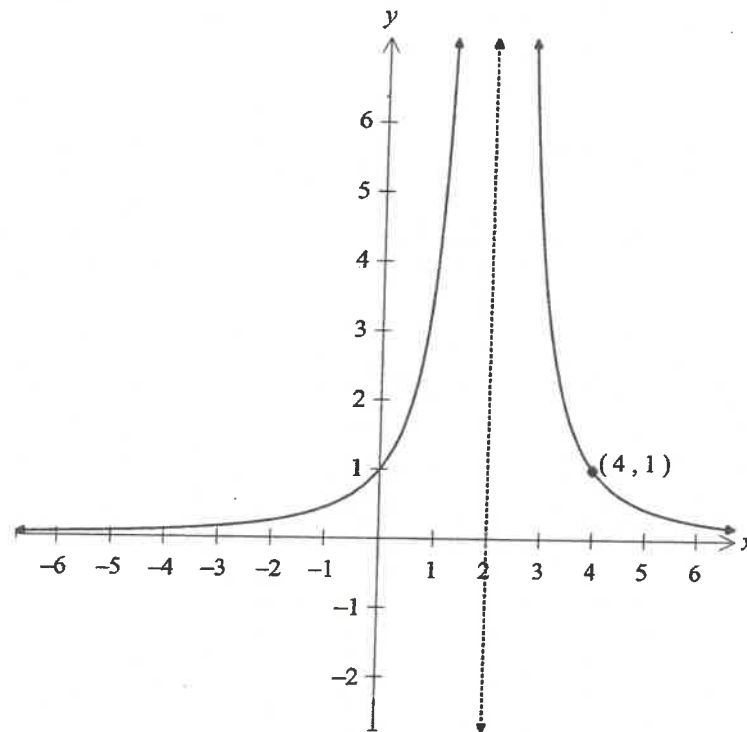
$$\sqrt{2}x = \frac{44}{\sqrt{3}}$$

$$\therefore \text{exact distance is } \frac{44}{\sqrt{6}} \text{ km}$$

$$x = \frac{44}{\sqrt{6}} \text{ km} \quad \left(\text{or } \frac{22\sqrt{6}}{3} \text{ km} \right)$$

- (e) Shown below is the graph of $y = f(x)$. Sketch the graph of $y = f(-x)$. Show all important features

2



Question 15 (15 marks)

- (a) Find the points of the curve $y = \frac{x^3}{3} + x^2 - 3x$ where the tangent is horizontal.

3

$$y = \frac{x^3}{3} + x^2 - 3x$$

$$y' = x^2 + 2x - 3 \quad \text{when } y' = 0$$

$$x^2 + 2x - 3 = 0$$

$$(x-1)(x+3) = 0$$

$$\therefore x = 1, -3$$

$$\text{when } x = 1$$

$$y = \frac{1^3}{3} + 1 - 3(1)$$

$$= -\frac{5}{3}$$

$$\text{when } x = -3$$

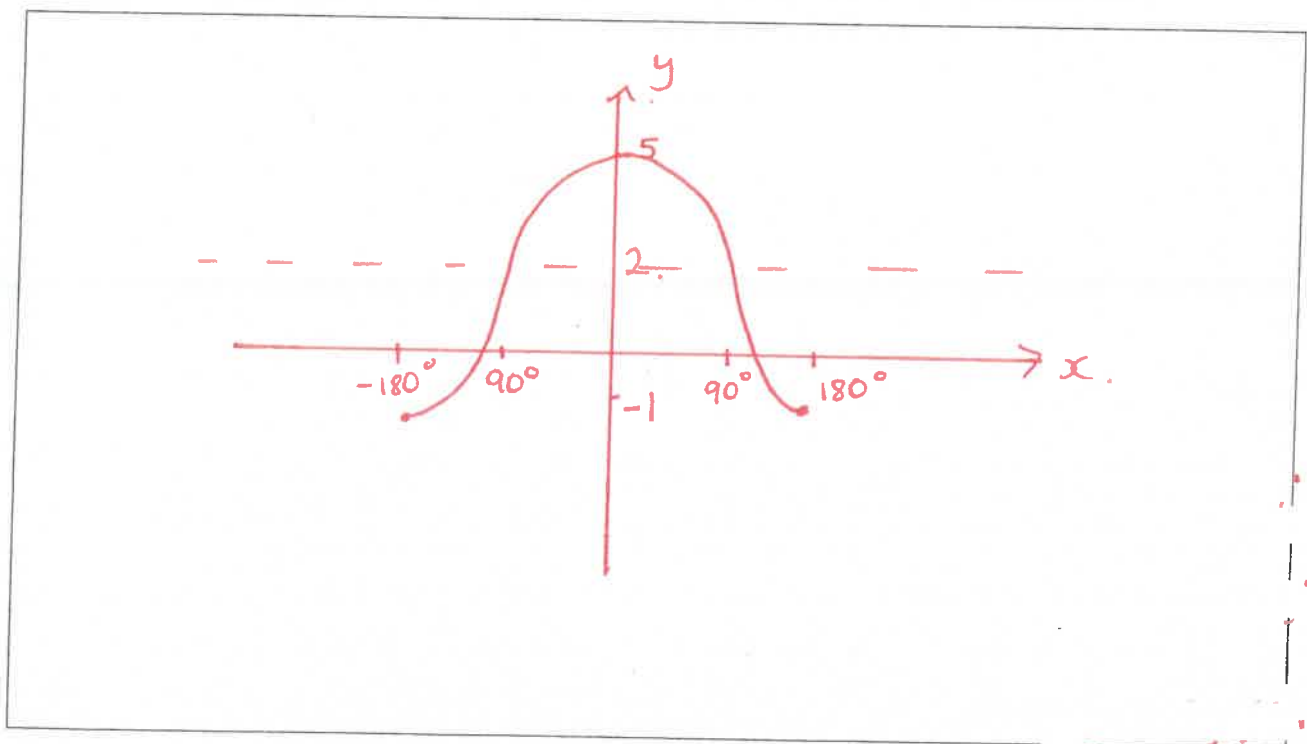
$$y = \frac{(-3)^3}{3} + (-3)^2 - 3(-3)$$

$$= 9$$

$\therefore$ points are $(1, -\frac{5}{3})$
and $(-3, 9)$

- (b) Sketch the graph $y = 3 \cos(x) + 2$ in the domain $[-180^\circ, 180^\circ]$ in the space below.

3



- (c) Solve
- $\log_2 x + \log_2 (x+2) = 3$
- .

3

$$\log_2 (x(x+2)) = 3$$

$$x^2 + 2x = 2^3$$

$$x^2 + 2x - 8 = 0$$

$$(x-2)(x+4) = 0$$

$$\therefore x = 2, -4 \text{ but } x > 0$$

$$\therefore x = 2$$

- (d) Jenna tosses three coins. Two of the coins are unbiased however one coin has heads on both sides. Given that she obtains at least one tail, what is the probability that she obtains at least two heads?

3

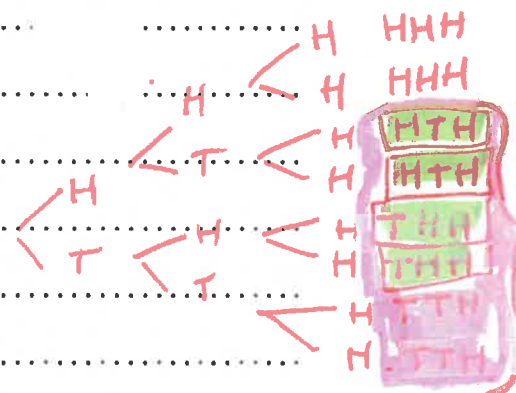
$$P(\text{at least 2H} | \text{at least 1T}) = \frac{P(\text{at least 2H} \cap \text{at least 1T})}{P(\text{at least 1T})}$$

$$= \frac{4}{8}$$

$$= \frac{2}{3}$$

$$\text{OR: } P(\text{at least 2H} | \text{at least 1T}) = \frac{4}{6}$$

$$= \frac{2}{3}$$



- (e) Solve
- $\tan^2 \frac{x}{2} = 3$
- in the domain
- $[-2\pi, 2\pi]$
- .

3

$$\tan\left(\frac{x}{2}\right) = \pm\sqrt{3}$$

$$[-2\pi, 2\pi]$$

$$\frac{x}{2} = \frac{\pi}{3}, \frac{\pi}{3}, \frac{-\pi}{3}, \frac{-\pi}{3}$$

$$-2\pi \leq x \leq 2\pi$$

$$-\pi \leq \frac{x}{2} \leq \pi$$

$$= \frac{\pi}{3}, \frac{2\pi}{3}, \frac{-\pi}{3}, \frac{-2\pi}{3}$$

$$\text{vs } \frac{\pi}{2}$$

$$\therefore x = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{-2\pi}{3}, \frac{-4\pi}{3}$$

$$\text{rel } < = \frac{\pi}{3}$$

Question 16 (15 marks)

- (a) Construct a probability distribution table given the probability distribution function

$$p(x) = \begin{cases} \frac{5-x}{9}, & \text{if } x = 1, 2, 3 \\ 0, & \text{for all other values of } x. \end{cases}$$

2

x	1	2	3	sum
$p(x)$	$\frac{4}{9}$	$\frac{3}{9}$	$\frac{2}{9}$	1 (check)
$x p(x)$	$\frac{4}{9}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{16}{9} \leftarrow \mu \text{ or } E(x)$
$x^2 p(x)$	$\frac{4}{9}$	$\frac{4}{3}$	2	$\frac{34}{9} \leftarrow E(x^2)$

Using your table above, calculate the expected value $E(X)$.

2

$$E(X) = \sum x p(x)$$

$$= \frac{16}{9}$$

(working may be done on table)

Hence find the variance and standard deviation for the probability distribution function.

2

$$\text{Var } X = E(X^2) - \mu^2$$

$$= \frac{34}{9} - \left(\frac{16}{9}\right)^2$$

$$= \frac{50}{81}$$

$$\sigma = \sqrt{\text{Var}(X)}$$

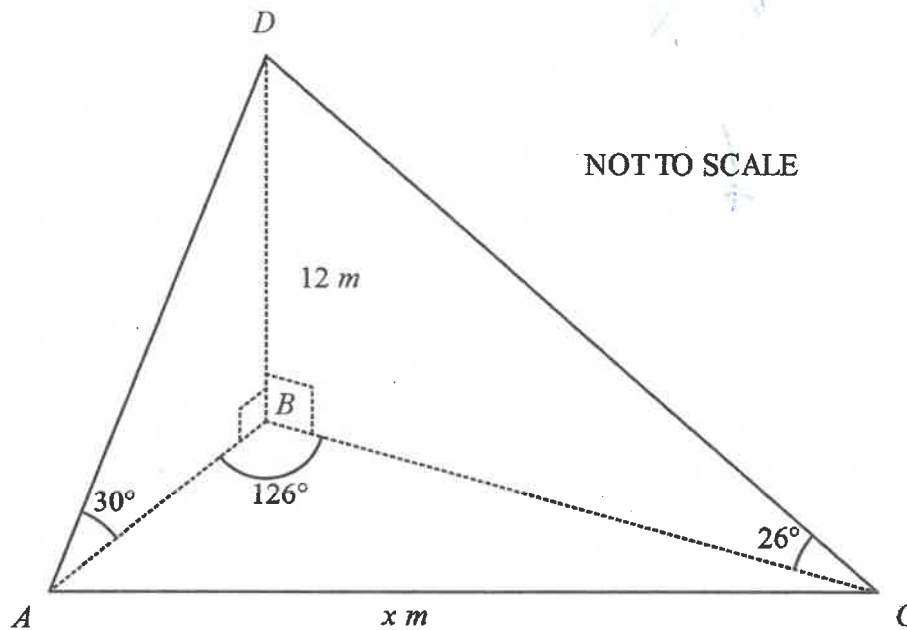
$$= \sqrt{\frac{50}{81}}$$

$$= 0.7856\dots$$

$$= 0.8 \text{ (1dp)}$$

- (b) In the diagram BD represents a vertical tower of height 12m standing on level ground. From A and C at ground level, the angles of elevation to the top of the tower are 30° and 26° respectively. Let x represent the distance between A and C . Solve for x , correct to 2 decimal places.

3



$$\tan 30 = \frac{12}{AB}$$

$$\tan 26 = \frac{12}{BC}$$

$$AB = \frac{12}{\tan 30}$$

$$BC = \frac{12}{\tan 26}$$

$$= \frac{12}{\frac{1}{\sqrt{3}}}$$

$$= 12\sqrt{3}$$

$$x^2 = (12\sqrt{3})^2 + \left(\frac{12}{\tan 26}\right)^2 - 2(12\sqrt{3})\left(\frac{12}{\tan 26}\right)\cos 126$$

$$= 1638.499 \dots$$

$$x = \sqrt{1638.499 \dots}$$

$$= 40.478$$

$$= 40.48 \text{ m (2dp)}$$

(c) Simplify $\frac{1}{\sin x - 1} - \frac{1}{\sin x + 1}$

3

$$\begin{aligned} \frac{1}{\sin x - 1} - \frac{1}{\sin x + 1} &= \frac{\sin x + 1 - (\sin x - 1)}{(\sin x - 1)(\sin x + 1)} \\ &= \frac{2}{\sin^2 x - 1} \\ &= \frac{2}{-\cos^2 x} \\ &= -2 \sec^2 x. \end{aligned}$$

(d) Solve the following pair of simultaneous equations:

$$\begin{cases} e^{x+y} = \frac{1}{e} \\ e^{3x+2y} = 1 \end{cases}$$

2

$$\begin{aligned} e^{x+y} &= e^{-1} \\ x+y &= -1 \quad \text{--- (1)} \\ e^{3x+2y} &= 1 \\ 3x+2y &= 0 \quad \text{--- (2)} \\ \text{using (1) } x &= -1-y \quad \text{--- (1A)} \\ \text{sub (1A) into (2)} \\ 3(-1-y) + 2y &= 0 \\ -3 - 3y + 2y &= 0 \\ y &= -3 \\ \text{sub } y = -3 \text{ into (1A)} \\ x &= -1 - (-3) \\ &= 2 \\ \therefore x &= 2, y = -3. \end{aligned}$$