BAULKHAM HILLS HIGH SCHOOL

2020

YEAR 11 YEARLY EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided
- In Questions 11 – 31, show relevant mathematical reasoning and/or calculations

Total marks:
73

Section I – 10 marks (pages 2–5)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 63 marks (Answer Booklet pages 4–16)

- Attempt Questions 11 – 31
- Allow about 1 hour and 45 minutes for this section

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10.

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1 What is the value of $\sqrt{\frac{28-\pi^2}{5.3}}$ correct to 2 significant figures?

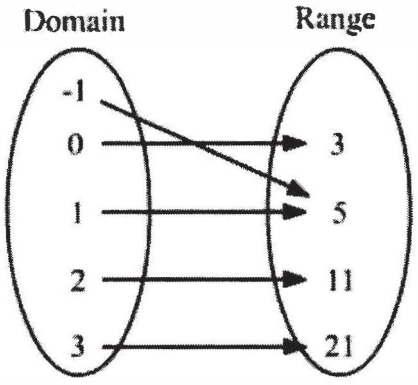
A. 1.8

B. 1.84

C. 1.85

D. 1.9

2 What type of relation is shown?



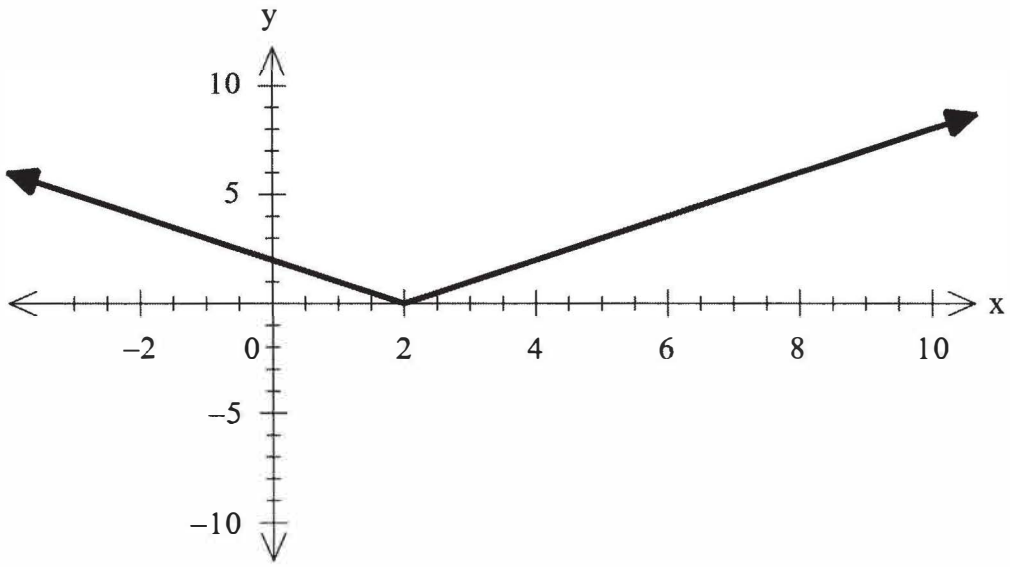
A. One to one

B. One to many

C. Many to one

D. Many to many

3 Which of the following statements is true about the following function?



A. The function is neither continuous nor differentiable at $x = 2$.

B. The function is continuous but not differentiable at $x = 2$.

C. The function is differentiable but not continuous at $x = 2$.

D. The function is both continuous and differentiable at $x = 2$.

4 Given that $\text{cosec}\beta = -\frac{9}{5}$ and that $\tan\beta < 0$, what is the value of $\cos\beta$?

A. $-\frac{9}{\sqrt{106}}$

B. $-\frac{\sqrt{56}}{9}$

C. $\frac{\sqrt{56}}{9}$

D. $\frac{9}{\sqrt{106}}$

5 If $\log_b x = 3$ and $\log_b y = 5$, what is the value of $\log_b(xy^2)$?

A. 8

B. 13

C. 28

D. 75

6 What is the minimum value of the function $f(x) = 2x^2 - 4x + 1$?

A. -2

B. -1

C. 1

D. 2

7 What is the natural domain of $f(x) = \frac{1}{\sqrt{x^2 - 9}}$?

A. $x < -3$ and $x > 3$

B. $x \leq -3$ and $x \geq 3$

C. $-3 < x < 3$

D. $-3 \leq x \leq 3$

8 Which of the following is an equivalent expression for $\frac{(1+\cos x)(1-\cos x)}{\sin^2 x}$?

A. $\tan^2 x$

B. $\cot^2 x$

C. $\operatorname{cosec}^2 x$

D. 1

9 If $f(x) = x^2 - 2x$ and $g(x) = \sqrt{x+1}$, what is $g\big(f(x)\big)$?

A. $\big(\sqrt{x+1}\big)(x^2 - 2x)$

B. $x + 1 - 2\sqrt{x+1}$

C. $\sqrt{x^2 - x + 1}$

D. $|x - 1|$

10 Which of the following functions is **not** even?

A. $f(x) = \sqrt{x^2 - 1}$

B. $f(x) = \dfrac{1}{1 + x^2}$

C. $f(x) = \operatorname{cosec} x$

D. $f(x) = \cos x$

End of Section I

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BAULKHAM HILLS HIGH SCHOOL
YEAR 11 Yearly Examination, 2020
MATHEMATICS ADVANCED

FINAL MARK

Name: _____

Teacher: _____

Question	F1	T1	S1	C1	E1	Total
MC	1,2,6,7,9,10 /6	4,8 /2		3 /1	5 /1	**/10**
Q.11-16 p 4-6	11,12,13 /6		16 /3	14a,14b,15 /6	14c /2	**/17**
Q.17-22 p 7-9	21 /2	19,22 /6	17,18 /6	20 /2		**/16**
Q.23-28 p 10-12	23,25,26 /8	27,28 /6			24 /3	**/17**
Q.29-31 p 13-16	30 /5	31 /5	29 /3			**/13**
Total	**/27**	**/19**	**/12**	**/9**	**/6**	**/73**

BAULKHAM HILLS HIGH SCHOOL

YEAR 11 Yearly Examination 2020

MATHEMATICS ADVANCED

Mathematics
Section I – Multiple Choice

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

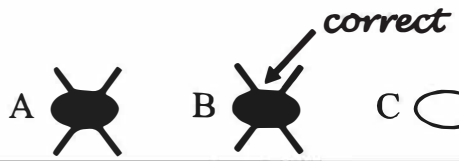
Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.



Start Here →

1.	A ○	B ○	C ○	D ○	**6.**	A ○	B ○	C ○	D ○
2.	A ○	B ○	C ○	D ○	**7.**	A ○	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ○	**8.**	A ○	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ○	**9.**	A ○	B ○	C ○	D ○
5.	A ○	B ○	C ○	D ○	**10.**	A ○	B ○	C ○	D ○

2020 Year 11 Yearly Examination

Mathematics Advanced

Section II Answer Booklet

63 marks
Attempt Questions 11 – 31
Allow about 1 hour and 45 minutes for this section

Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Question 11 (2 marks)

Express $\frac{x+1}{x^2-x} - \frac{x-1}{x^2+x}$ as a single fraction in its simplest form. **2**

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Question 12 (2 marks)

Express $\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}$ in the form $a + b\sqrt{6}$. **2**

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Question 13 (2 marks)

Solve $|x - 4| = 7$. **2**

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Question 14 (6 marks)

Differentiate with respect to x.

(a) $\dfrac{1}{\sqrt{x^3}}$ **2**

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(b) $(1-5x)^4$ **2**

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(c) $\dfrac{e^{2x}}{x^2+1}$ **2**

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Question 15 (2 marks)

Find the equation of the tangent to the curve $y = x^2 + 1$ at the point $(2, 5)$. **2**

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Question 16 (3 marks)

The table below shows a discrete probability distribution for a random variable X, where k is a constant.

x	0	1	2	3	4
$P(X=x)$	0.2	0.1	$2k$	$3k$	0.3

(a) Find the value of k. **2**

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(b) Find $P(X \geq 1)$. **1**

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Question 17 (4 marks)

A pair of fair 6-sided die are rolled. The number of sixes showing is represented by a random variable X with the following probability distribution.

x	0	1	2
$p(x)$	$\frac{25}{36}$	$\frac{5}{18}$	$\frac{1}{36}$

(a) Show that the expected value $E(X)$ is $\frac{1}{3}$. **1**

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(b) Hence, or otherwise, find $\text{Var}(X)$, correct to 2 decimal places. **2**

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(c) Hence, find the standard deviation, correct to 2 decimal places. **1**

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Question 18 (2 marks)

Two identical biased coins are tossed together, and the number of heads showing is recorded. After a large number of trials, it is observed that the probability that both coins land showing heads is 0.64. **2**

What is the experimental probability that both coins land showing tails?

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Question 19 (3 marks)

Solve $2\sin^2 x - 1 = 0$ for $0 \leq x \leq 2\pi$. **3**

Question 20 (2 marks)

If $f(x) = x^2 + x$, find its first derivative by using the first principles formula: **2**

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Question 21 (2 marks)

A circle is given by the equation $x^2 + y^2 + 6x - 8y = 0$. **2**

Find the centre and radius of this circle.

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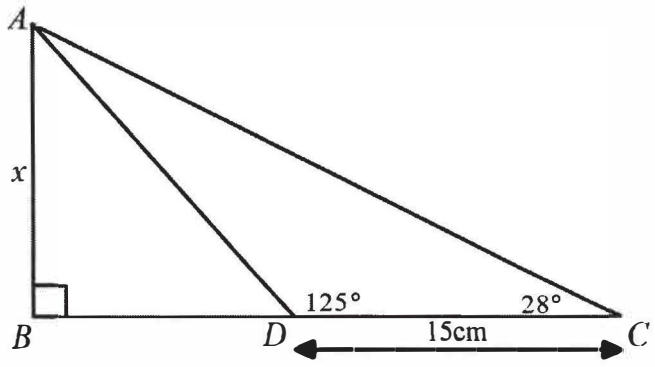
Question 22 (3 marks)

Find the value of x, correct to 2 decimal places. **3**



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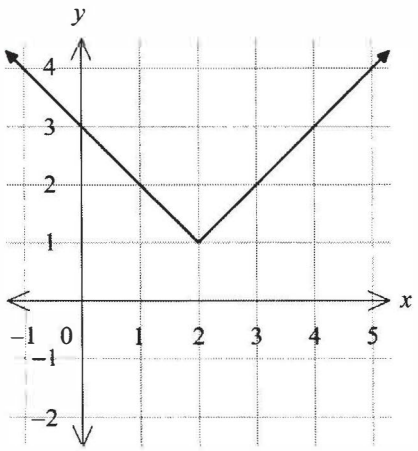
Question 23 (2 marks)

The graph of the function below is of the form $y = |x + b| + c$, where b and c are constants. **2**



What are the values of b and c?

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Question 24 (3 marks)

Solve $4^x - 10(2^x) + 24 = 0$. **3**

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Question 25 (2 marks)

Simplify $\frac{4^m-1}{2^m-1}$. **2**

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Question 26 (4 marks)

(a) Show that $\frac{2x+3}{x+2} = 2 - \frac{1}{x+2}$. **1**

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(b) Hence sketch the graph of $y = \frac{2x+3}{x+2}$, showing all intercepts and asymptotes. **3**

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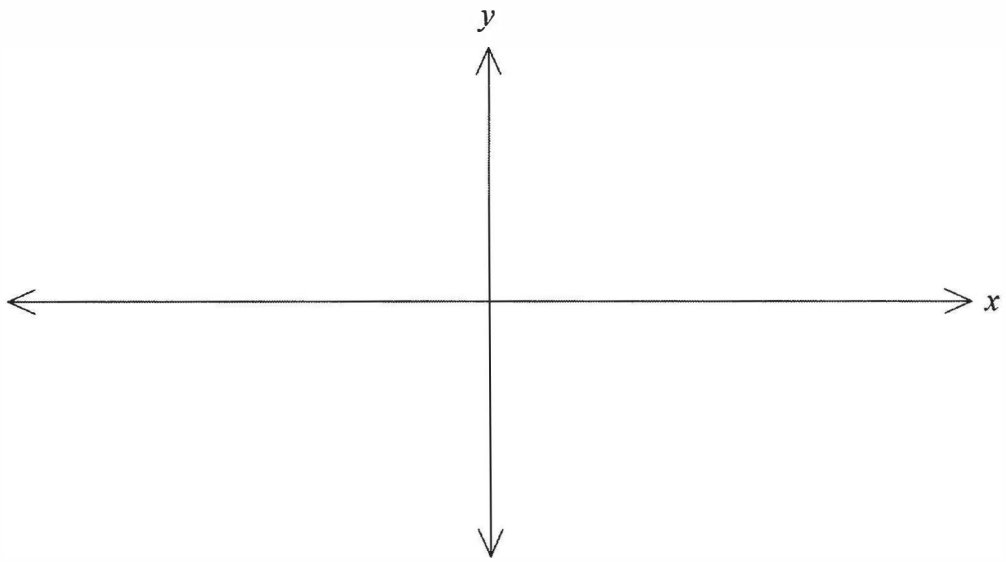

Question 27 (2 marks)

Express $2\sin(180° - \alpha) + \cos(90° - \alpha)$ in terms of $\sin\alpha$. **2**

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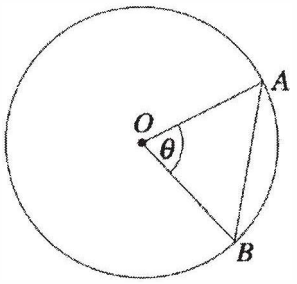
Question 28 (4 marks)

The diagram shows a circle with centre O and radius 2 centimetres. The points A and B lie on the circumference of the circle and $\angle AOB = \theta$.



NOT TO SCALE

(a) There are two possible values of θ in radians, for which the area of ΔAOB is $\sqrt{3}$ square centimetres. One value is $\frac{\pi}{3}$. Find the other value. **2**

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(b) Suppose that $\theta = \frac{\pi}{3}$. Find the exact area, in square centimetres, of the minor segment bounded by the chord AB and the arc AB. **2**

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Question 29 (3 marks)

For events A and B from a given sample space, $P(A|B) = \frac{1}{5}$ and $P(B|A) = \frac{1}{3}$. Let $P(A \cap B) = p$.

(a) Find $P(A)$ in terms of p. **1**

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(b) Find $P(\bar{A} \cap \bar{B})$ in terms of p. **2**

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Question 30 (5 marks)

Consider the polynomial $g(x) = (2 - x)(x + 2)^2$.

(a) What is the degree of the polynomial? **1**

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(b) Draw a neat sketch of $g(x) = (2 - x)(x + 2)^2$, showing all x and y intercepts. **2**

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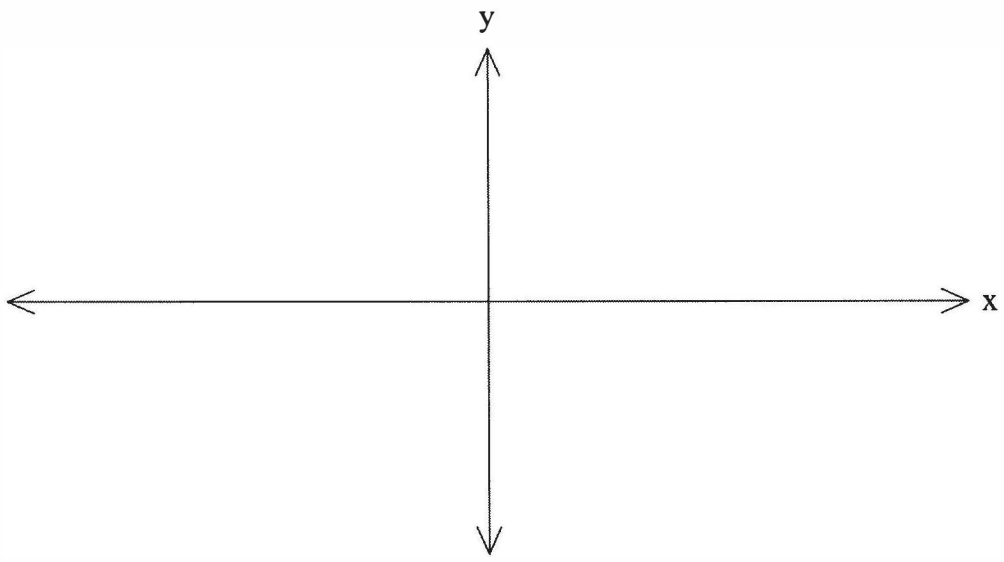

(c) For what values of x is $g(x) \leq 0$? **2**

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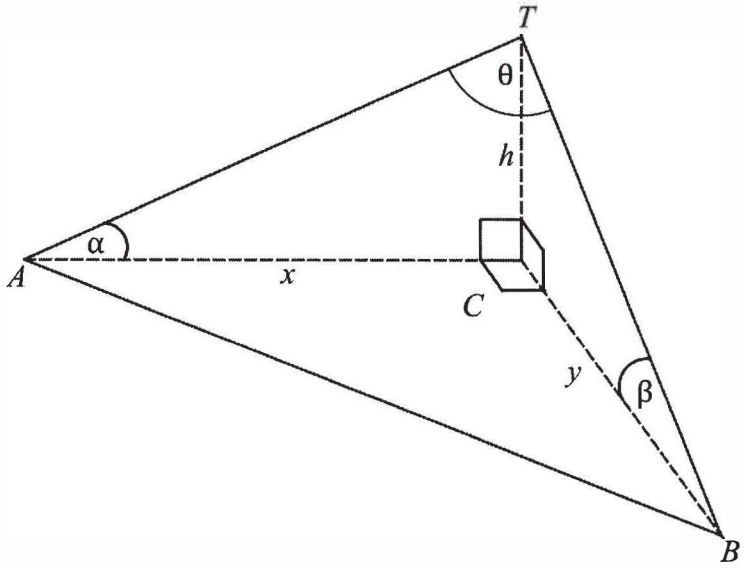
Question 31 (5 marks)

In the diagram $AC = x, BC = y$ and $TC = h$.

$\angle ATB = \theta, \angle TAC = \alpha$ and $\angle TBC = \beta$. There are three right angles at C as shown.



(a) Show that $AT = \sqrt{x^2 + h^2}$ and write a similar expression for BT. **1**

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(b) Show that $\cos\theta = \dfrac{h^2}{\sqrt{(x^2+h^2)(y^2+h^2)}}$. **2**

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Question 31 continues on page 19

(c) Hence show that $\sin\alpha\sin\beta = \cos\theta$. **2**

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End of paper

Solutions

BAULKHAM HILLS HIGH SCHOOL

2020

YEAR 11 YEARLY EXAMINATION

Mathematics Advanced

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- Reading time – 10 minutes
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Total marks:
73

Section I – 10 marks (pages 2–5)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 63 marks (Answer Booklet pages 4-16)

- Attempt Questions 11 – 31
- Allow about 1 hour and 45 minutes for this section

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10.

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1 What is the value of $\sqrt{\frac{28-\pi^2}{5.3}}$ correct to 2 significant figures?

(A.) 1.8

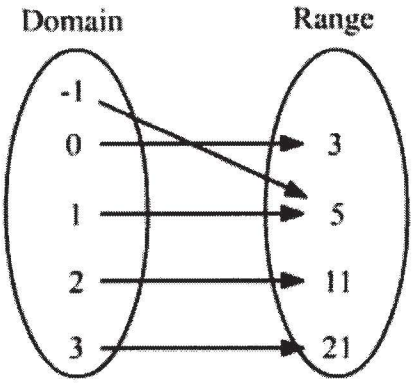
B. 1.84

C. 1.85

D. 1.9

$= 1.849548...$
≈ 1.8

2 What type of relation is shown?



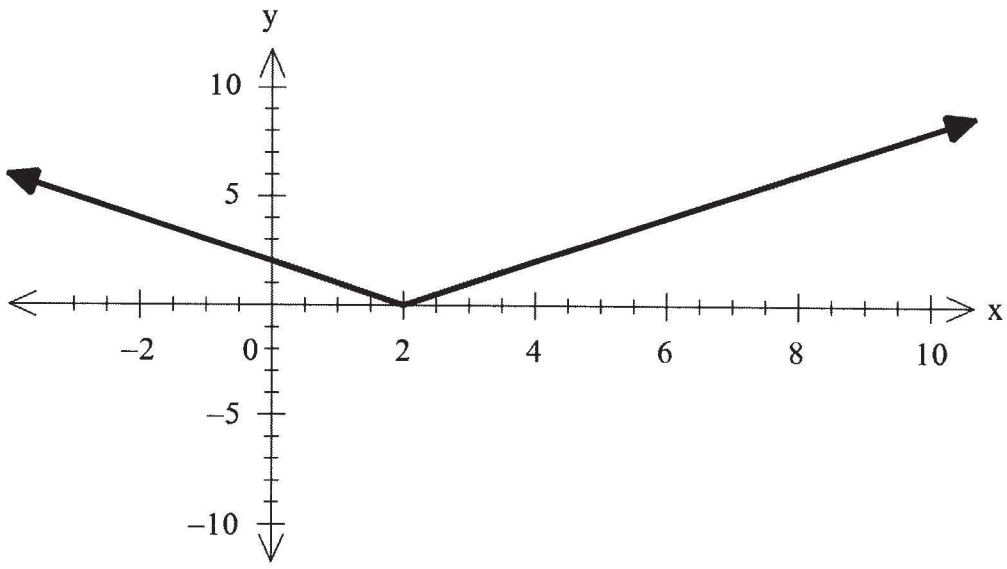
A. One to one

B. One to many

(C.) Many to one

D. Many to many

3 Which of the following statements is true about the following function?



A. The function is neither continuous nor differentiable at $x = 2$.

B. The function is continuous but not differentiable at $x = 2$.

C. The function is differentiable but not continuous at $x = 2$.

D. The function is both continuous and differentiable at $x = 2$.

4 Given that $\text{cosec}\beta = -\frac{9}{5}$ and that $\tan\beta < 0$, what is the value of $\cos\beta$?

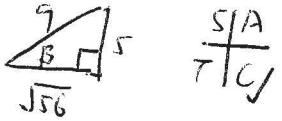
A. $-\frac{9}{\sqrt{106}}$

B. $-\frac{\sqrt{56}}{9}$

C. $\frac{\sqrt{56}}{9}$

D. $\frac{9}{\sqrt{106}}$

$\sin\beta = \frac{-5}{9}$



S|A
T|C

$\therefore \cos\beta = \frac{\sqrt{56}}{9}$

5 If $\log_b x = 3$ and $\log_b y = 5$, what is the value of $\log_b(xy^2)$?

A. 8

B. 13

C. 28

D. 75

$= \log_b x + 2\log_b y$

$= 3 + 2(5)$

$= 13$

6 What is the minimum value of the function $f(x) = 2x^2 - 4x + 1$?

A. -2

B. -1

C. 1

D. 2

axis of symmetry: $= \frac{-b}{2a}$

$= \frac{4}{4}$

$= 1$

$\therefore f(1) = 2(1)^2 - 4(1) + 1$

$= -1$

7 What is the natural domain of $f(x) = \frac{1}{\sqrt{x^2-9}}$?

A. $x < -3$ and $x > 3$

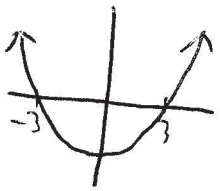
B. $x \leq -3$ and $x \geq 3$

C. $-3 < x < 3$

D. $-3 \leq x \leq 3$

$x^2 - 9 > 0$

$(x+3)(x-3) > 0$



8 Which of the following is an equivalent expression for $\frac{(1+\cos x)(1-\cos x)}{\sin^2 x}$?

A. $tan^2 x$

B. $cot^2 x$

C. $cosec^2 x$

(D.) 1

$= \frac{1-\cos^2 x}{\sin^2 x}$

$= \frac{\sin^2 x}{\sin^2 x}$

$= 1$

9 If $f(x) = x^2 - 2x$ and $g(x) = \sqrt{x+1}$, what is $g(f(x))$?

A. $(\sqrt{x+1})(x^2-2x)$

B. $x + 1 - 2\sqrt{x+1}$

C. $\sqrt{x^2 - x + 1}$

(D.) $|x-1|$

$g(f(x)) = \sqrt{(x^2-2x)+1}$

$= \sqrt{(x-1)^2}$

$= |x-1|$

10 Which of the following functions is **not** even?

A. $f(x) = \sqrt{x^2-1}$

B. $f(x) = \frac{1}{1+x^2}$

(C.) $f(x) = \text{cosec}x$

D. $f(x) = \cos x$

End of Section I

2020 Year 11 Yearly Examination

Mathematics Advanced

Section II Answer Booklet

63 marks
Attempt Questions 11 – 31
Allow about 1 hour and 45 minutes for this section

Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Question 11 (2 marks)

Express $\frac{x+1}{x^2-x} - \frac{x-1}{x^2+x}$ as a single fraction in its simplest form. **2**

$= \frac{x+1}{x(x-1)} - \frac{x-1}{x(x+1)}$

$= \frac{(x+1)(x-1) - (x-1)(x-1)}{x(x-1)(x+1)}$

$= \frac{x^2+2x+1 - x^2+2x-1}{x(x-1)(x+1)}$

$= \frac{4x}{x(x+1)(x-1)}$

$= \frac{4}{(x+1)(x-1)}$

2MK: Provides correct solution.
1MK: Obtains correct expression with the same denominator

Question 12 (2 marks)

Express $\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}$ in the form $a + b\sqrt{6}$. **2**

$\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}} \times \frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = \frac{18-2(6\sqrt{6})+12}{18-12}$

$= \frac{30-12\sqrt{6}}{6}$

$= 5-2\sqrt{6}$

2MKs: Provides correct solution.
1MK: Attempts to rationalise by using the conjugate

Question 13 (2 marks)

Solve $|x-4| = 7$. **2**

$x-4 = 7$, $x-4 = -7$

$x = 11$ $x = -3$

$\therefore x = 11$ or -3

2MKs: Provides correct solution.
1MK: Correctly solves one case.

Question 14 (6 marks)

Differentiate with respect to x.

(a) $\dfrac{1}{\sqrt{x^3}}$ **2**

$\frac{d}{dx}\left(\frac{1}{\sqrt{x^3}}\right) = \frac{d}{dx}\left(x^{-\frac{3}{2}}\right)$

$= -\frac{3}{2}x^{-\frac{5}{2}}$ or $\frac{-3}{2\sqrt{x^5}}$

2M1C: Provides correct solution.

1M1C: Obtains correct index form for the expression.

(b) $(1-5x)^4$ **2**

$\frac{d}{dx}(1-5x)^4 = 4(1-5x)^3 \times -5$

$= -20(1-5x)^3$

2M1C: Provides correct solution

1M1C: -Attempts to use the chain rule
-Obtains $4(1-5x)^3$

(c) $\dfrac{e^{2x}}{x^2+1}$ **2**

$u = e^{2x}$ $\quad v = x^2+1$

$u' = 2e^{2x}$ $\quad v' = 2x$

$\therefore \frac{dy}{dx} = \frac{(x^2+1)(2e^{2x}) - e^{2x}(2x)}{(x^2+1)^2}$

$= \frac{2e^{2x}(x^2-x+1)}{(x^2+1)^2}$

2M1C: Provides correct solution

1M1C: Attempts to use the quotient rule

Question 15 (2 marks)

Find the equation of the tangent to the curve $y = x^2 + 1$ at the point $(2, 5)$. **2**

$y' = 2x$

When $x = 2$, $y' = 2(2)$

$= 4$

$\therefore$ Eqn of tangent: $y - 5 = 4(x - 2)$

$y - 5 = 4x - 8$

$\therefore 4x - y - 3 = 0$ or $y = 4x - 3$

2M/K: Provides correct solution

1M/K: Obtains correct gradient at $x = 2$ is 4.

Question 16 (3 marks)

The table below shows a discrete probability distribution for a random variable X, where k is a constant.

x	0	1	2	3	4
$P(X=x)$	0.2	0.1	$2k$	$3k$	0.3

(a) Find the value of k. **2**

$0.2 + 0.1 + 2k + 3k + 0.3 = 1$

$\therefore 5k = 0.4$

$k = 0.08$

2M/K: Provides correct solution.

1M/K: Obtains a correct probability statement to solve for k.

(b) Find $P(X \geq 1)$. **1**

$1 - 0.2 = 0.8$

1M/K: Provides correct solution.

Question 17 (4 marks)

A pair of fair 6-sided die are rolled. The number of sixes showing is represented by a random variable X with the following probability distribution.

x	0	1	2
$p(x)$	$\frac{25}{36}$	$\frac{5}{18}$	$\frac{1}{36}$

(a) Show that the expected value $E(X)$ is $\frac{1}{3}$. **1**

$E(X) = 2 \times \frac{1}{36} + 1 \times \frac{5}{18} + 0 = \frac{1}{3}$ //

1mk: Provides correct proof.

(b) Find Var(X), correct to 2 decimal places. **2**

$Var(X) = (4 \times \frac{1}{36} + 1 \times \frac{5}{18} + 0) - (\frac{1}{3})^2$

$= 0.2777...$

≈ 0.28

2mk: Provides correct solution.

1mk: Attempts to find variance using a correct method.

(c) Hence, find the standard deviation, correct to 2 decimal places. **1**

$\sqrt{0.2777} = 0.527...$

≈ 0.53

1mk: Provides correct solution.

Question 18 (2 marks)

Two identical biased coins are tossed together, and the number of heads showing is recorded. After a large number of trials, it is observed that the probability that both coins land showing heads is 0.64. **2**

What is the experimental probability that both coins land showing tails?

$P(H) \times P(H) = 0.64$

$\therefore P(H) = 0.8$

2mk: Provides correct solution.

$\therefore P(T) = 0.2$

1mk: Obtains correct P(Head)

$P(T) \times P(T) = 0.2 \times 0.2 = 0.04$

Question 19 (3 marks)

Solve $2\sin^2 x - 1 = 0$ for $0 \leq x \leq 2\pi$. **3**

$\sin^2 x = \frac{1}{2}$

$\sin x = \pm\sqrt{\frac{1}{2}}$

$= \pm\frac{1}{\sqrt{2}}$

$\therefore$ x is in all 4 quadrants

$\therefore$ $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

3M/C: Provides 4 correct solutions.
2M/C: Provides 2 correct solutions
1M/C: Obtains an equation for $\sin x$, ie $\sin x = \pm\frac{1}{\sqrt{2}}$

Question 20 (2 marks)

If $f(x) = x^2 + x$, find its first derivative by using the first principles formula: **2**

$$f'(x) = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$$

$f'(x) = \lim_{h\to 0} \frac{(x+h)^2 + (x+h) - (x^2+x)}{h}$

$= \lim_{h\to 0} \frac{(x+h)^2 + (x+h) - x^2 - x}{h}$

$= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 + x + h - x^2 - x}{h}$

$= \lim_{h\to 0} \frac{h(2x+h+1)}{h}$

$= 2x + (0) + 1$

$= 2x + 1$

2M/C: Provides correct solution.
1M/C: Substitutes correctly into the first principles formula
Note: limit notation enforced.

Question 21 (2 marks)

A circle is given by the equation $x^2 + y^2 + 6x - 8y = 0$. **2**

Find the centre and radius of this circle.

$(x+3)^2 + (y-4)^2 = 9+16$

$\therefore (x+3)^2 + (y-4)^2 = 25$

$\therefore$ Centre $(-3, 4)$.

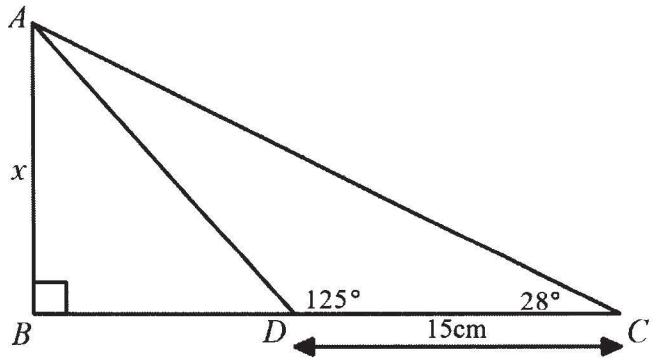
& Radius is 5.

2mk: Provides correct centre and radius.

1mk: Provides correct centre or radius.

Question 22 (3 marks)

Find the value of x, correct to 2 decimal places. **3**



In $\triangle ADC$: $\frac{AD}{\sin 28} = \frac{15}{\sin 27}$

$\therefore y = \frac{15 \sin 28}{\sin 27}$

In $\triangle ABD$: $\frac{x}{y} = \sin 55$

$x = \left(\frac{15 \sin 28}{\sin 27}\right) \sin 55$

$= 12.706\ldots$

$\hat{=} 12.71$ cm

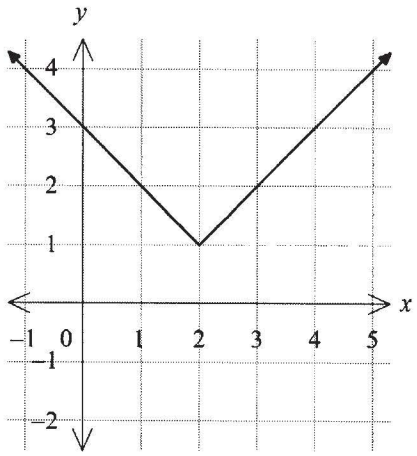
3mk: Provides correct solution.

2mk: Obtains a correct trig ratio expression involving x.

1mk: Obtains a correct sine rule equation.

Question 23 (2 marks)

The graph of the function below is of the form $y = |x + b| + c$, where b and c are constants as shown. **2**



What are the values of b and c?

2MK: Provides two correct solutions

$b = -2, \quad c = 1$

1MK: Identifies one correct constant.

Question 24 (3 marks)

Solve $4^x - 10(2^x) + 24 = 0$. **3**

Let $u = 2^x$

$\therefore \quad u^2 - 10u + 24 = 0$

$(u-4)(u-6) = 0$

$\therefore u = 4$ or 6

$\therefore 2^x = 4 \quad / \quad 2^x = 6$

$x = 2 \quad / \quad x = \log_2 6$

3MK: Correctly finds both solutions.

2MK: Correctly finds one solution.

1MK: - Correctly obtains a factorised expression

- Correctly substitutes into the quadratic formula

Question 25 (2 marks)

Simplify $\frac{4^m-1}{2^m-1}$. **2**

$= \frac{(2^m)^2-1}{2^m-1}$

$= \frac{(2^m+1)(2^m-1)}{2^m-1}$

$= 2^m+1$

2mk: Provides correct solution.
1mk: Rewrites 4^m as $(2^m)^2$

Question 26 (4 marks)

(a) Show that $\frac{2x+3}{x+2} = 2-\frac{1}{x+2}$. **1**

RHS: $\frac{2(x+2)-1}{x+2}$

$= \frac{2x+4-1}{x+2}$

$= \frac{2x+3}{x+2}$

= LHS as required

1mk: Provides correct proof.

(b) Hence sketch the graph of $y = \frac{2x+3}{x+2}$, showing all intercepts and asymptotes. **3**

When $x=0$, $y = \frac{3}{2}$

When $y=0$, $0 = 2-\frac{1}{x+2}$

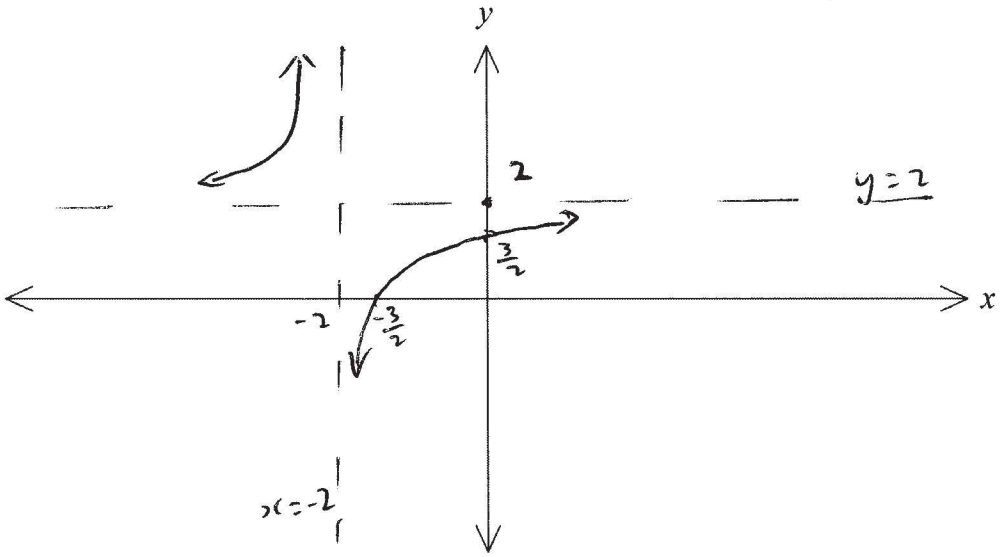
$\frac{1}{x+2} = 2$

$1 = 2x+4$

$-3 = 2x$

$x = -\frac{3}{2}$

3mk: Provides correct sketch.
2mk: Provides curve with correct intercepts and asymptotes
1mk: Provides curve with correct intercepts or asymptotes.



Question 27 (2 marks)

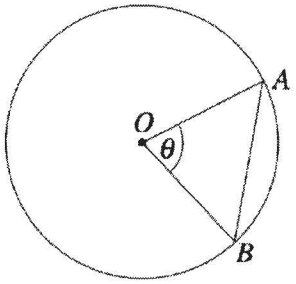
Express $2\sin(180° - \alpha) + \cos(90° - \alpha)$ in terms of $\sin\alpha$. **2**

$= 2\sin\alpha + \sin\alpha$ — 2MK: Provides correct solution.

$= 3\sin\alpha$ — 1MK: Simplifies one correct identity

Question 28 (4 marks)

The diagram shows a circle with centre O and radius 2 centimetres. The points A and B lie on the circumference of the circle and $\angle AOB = \theta$.



NOT TO SCALE

(a) There are two possible values of θ in radians, for which the area of ΔAOB is $\sqrt{3}$ square centimetres. One value is $\frac{\pi}{3}$. Find the other value. **2**

$A = \frac{1}{2}ab\sin C$ — 2MK: Provides correct solution.

$\sqrt{3} = \frac{1}{2}(2)(2)\sin\theta$ — 1MK: Obtains correct equation for area of a triangle with substitution.

$\frac{\sqrt{3}}{2} = \sin\theta$

$\therefore \theta = \frac{\pi}{3}, \frac{2\pi}{3}$ — Bold answer

(b) Suppose that $\theta = \frac{\pi}{3}$. Find the exact area, in square centimetres, of the minor segment bounded by the chord AB and the arc AB. **2**

$A_{segment} = A_{sector} - A_{triangle}$ — 2MK: Provides correct solution in exact form.

$= \frac{1}{2}r^2\theta - \sqrt{3}$ — 1MK: Obtains correct equation for area of minor segment with substitution.

$= \frac{1}{2}(2)^2\left(\frac{\pi}{3}\right) - \sqrt{3}$

$= \frac{2\pi}{3} - \sqrt{3}\ cm^2$

Question 29 (3 marks)

For events A and B from a given sample space, $P(A|B) = \frac{1}{5}$ and $P(B|A) = \frac{1}{3}$. Let $P(A \cap B) = p$.

(a) Find $P(A)$ in terms of p. **1**

$P(A) = \frac{P(A \cap B)}{P(B|A)}$

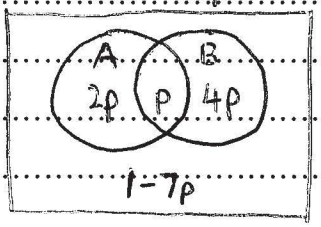
$= \frac{p}{(\frac{1}{3})}$

$= 3p$

1Mk: Provides correct solution

(b) Find $P(\bar{A} \cap \bar{B})$ in terms of p. **2**

$P(B) = 5p$



$\therefore P(\bar{A} \cap \bar{B}) = 1 - 7p$

2Mk: Provides correct solution.

1Mk: Obtains $P(B) = 5p$ or equivalent merit.

Question 30 (5 marks)

Consider the polynomial $g(x) = (2 - x)(x + 2)^2$.

(a) What is the degree of the polynomial? **1**

3

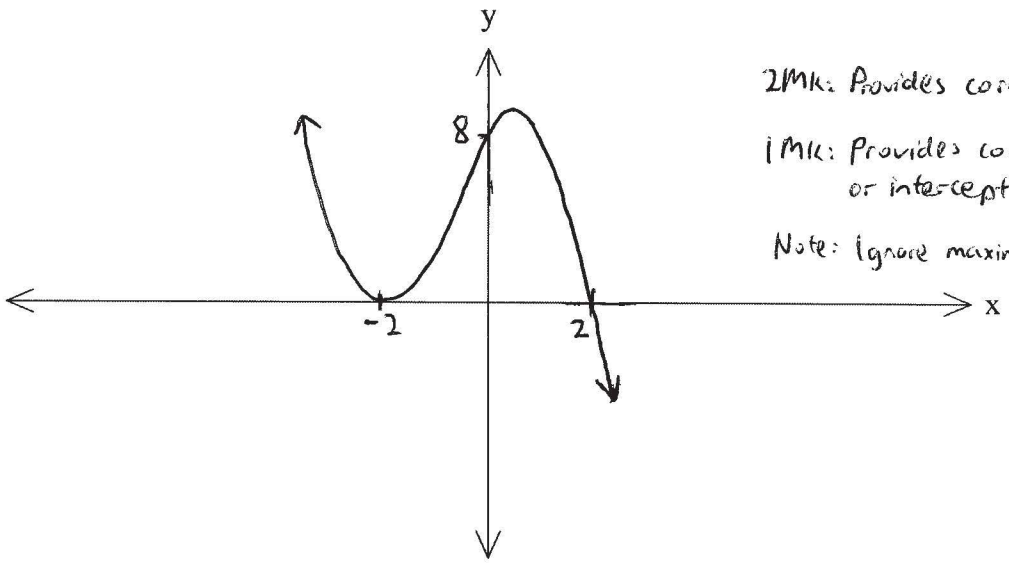
1Mk: Provides correct answer.

(b) Draw a neat sketch of $g(x) = (2 - x)(x + 2)^2$, showing all x and y intercepts. **2**

x-int: $x = 2, -2$

y-int: $y = (2)(2)^2$

$= 8$



2Mk: Provides correct sketch.

1Mk: Provides correct shape or intercepts.

Note: Ignore maximum turning point.

(c) For what values of x is $g(x) \leq 0$? **2**

From graph: $x = -2, x \geq 2$

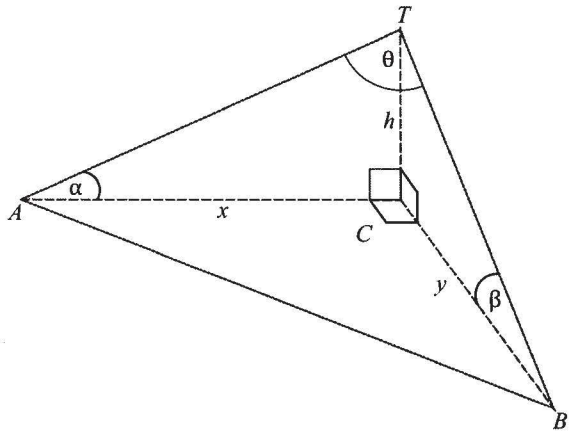
2Mk: Provides correct solution.

1Mk: Obtains part of the correct x-values.

Question 31 (5 marks)

In the diagram $AC = x, BC = y$ and $TC = h$.

$\angle ATB = \theta, \angle TAC = \alpha$ and $\angle TBC = \beta$. There are three right angles at C as shown.



(a) Show that $AT = \sqrt{x^2 + h^2}$ and write a similar expression for BT. **1**

In ΔATC: $AT^2 = x^2 + h^2$ 1MK: Provides correct proof.

$\therefore AT = \sqrt{x^2+h^2}$ // & $BT = \sqrt{y^2+h^2}$

(b) Show that $\cos\theta = \dfrac{h^2}{\sqrt{(x^2+h^2)(y^2+h^2)}}$. **2**

In ΔATB: $(AB)^2 = (AT)^2 + (BT)^2 - 2(AT)(BT)\cos\theta$

$x^2 + y^2 = (x^2+h^2) + (y^2+h^2) - 2\sqrt{x^2+h^2}\sqrt{y^2+h^2}\cos\theta$

$\therefore 0 = 2h^2 - 2\sqrt{(x^2+h^2)(y^2+h^2)}\cos\theta$

$\cos\theta = \dfrac{h^2}{\sqrt{(x^2+h^2)(y^2+h^2)}}$ as required. 2MK: Provides correct proof.

1MK: Obtains correct equation for cosine rule in ΔATB.

(c) Hence show that $\sin\alpha\sin\beta = \cos\theta$. **2**

In ΔATC: $\sin\alpha = \dfrac{h}{AT}$ & In ΔBTC: $\sin\beta = \dfrac{h}{BT}$

$= \dfrac{h}{\sqrt{x^2+h^2}}$ (from (i)) $= \dfrac{h}{\sqrt{y^2+h^2}}$ (from (i))

$\therefore \sin\alpha\sin\beta = \dfrac{h^2}{\sqrt{x^2+h^2}\sqrt{y^2+h^2}}$ 2MK: Provides correct proof.

$= \dfrac{h^2}{\sqrt{(x^2+h^2)(y^2+h^2)}}$ 1MK: Obtains $\sin\alpha$ or $\sin\beta$ correctly

$= \cos\theta$

End of paper