BAULKHAM HILLS HIGH SCHOOL

YEAR 11 PRELIMINARY EXAMINATION 2023
MATHEMATICS ADVANCED

MARKING COVER SHEET

FINAL MARK

Name:_____ **Teacher:**_____

Q	F1	T1	T2	C1	E1	S1	Total
MC	1,2,4,5,8,10 /6		7 /1	3 /1	6 /1	9 /1	**/10**
11					/2		**/2**
12					/2		**/2**
13	/2						**/2**
14			/3				**/3**
15				/2			**/2**
16		/2					**/2**
17		/3					**/3**
18	/2						**/2**
19	/2						**/2**
20					/3		**/3**
21						/4	**/4**
22				/6			**/6**
23						/3	**/3**
24				/3			**/3**
25	/2						**/2**
26	/2						**/2**
27	/2						**/2**
28	/3						**/3**
29						/4	**/4**
30		/5					**/5**
31			/4				**/4**
	/21	**/10**	**/8**	**/12**	**/8**	**/12**	**/71**

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BAULKHAM HILLS HIGH SCHOOL

2023

YEAR 11 PRELIMINARY EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes.
- Working time – 2 hours.
- Write using black or blue pen.
- NESA-approved calculators may be used.
- Answer all questions in the answer booklet provided.
- Show all necessary working in Questions 11-31.
- Marks may be deducted for careless or badly arranged work.
- A reference sheet is provided.

Total marks – 71
Exam consists of 23 pages.

This paper consists of TWO sections.

Section I– Pages 4-6 (10 marks)
Questions 1 - 10

- Attempt Questions 1 - 10

Allow about 15 minutes for this section.

Section II – Pages 7 – 22 (61 marks)

- Attempt questions 11 - 31

Allow about 1 hour 45 minutes for this section.

Section I - 10 marks

Use the multiple-choice answer sheet for questions 1-10.

1. Which of the following is equivalent to $\sqrt{5}-\sqrt{125}+3\sqrt{50}$?

A) $\sqrt{5}$

B) $6\sqrt{5}-15\sqrt{2}$

C) $-4\sqrt{5}+9\sqrt{2}$

D) $-4\sqrt{5}+3\sqrt{50}$

2. Given $f(x)=3x-4$ and $g(x)=-\frac{1}{x}$, what are the values of x when $f(x)=g(x)$?

A) $-\frac{1}{3}$ or 1

B) $\frac{1}{3}$ or -1

C) $\frac{1}{3}$ or 1

D) $-\frac{1}{3}$ or -1

3. What is the derivative of $f(x)=\sqrt{x^2-1}$?

A) $\frac{1}{2\sqrt{2x-1}}$

B) $\frac{1}{2\sqrt{x^2-1}}$

C) $\frac{2x}{\sqrt{x^2-1}}$

D) $\frac{x}{\sqrt{x^2-1}}$

4. Which of the following is the equivalent of the algebraic fraction $\dfrac{x+y}{x^{-2}-y^{-2}}$?

A) $\dfrac{x^2y^2}{x-y}$

B) $\dfrac{x^2y^2}{y-x}$

C) $\dfrac{x^2y^2}{x^2-y^2}$

D) $\dfrac{x^2y^2}{y^2-x^2}$

5. Which of the following functions is ***not*** many-to-one?

A) $f(x)=3-x^2$

B) $f(x)=\sin x\ (0\le x\le 2\pi)$

C) $f(x)=x^3-5$

D) $f(x)=|3x-2|$

6. Which of the following is an equivalent expression to $\log\dfrac{a}{b}+\log c$?

A) $\dfrac{\log ac}{\log b}$

B) $\log a+\log c-\log b$

C) $\log a\times\log c-\log b$

D) $\dfrac{\log a\times\log c}{\log b}$

7. Which of the following is equivalent to $\dfrac{1-\sin^2\theta\cos^2\theta}{\cos^2\theta}$?

A) $\tan^2\theta\cos^2\theta$

B) $\tan^2\theta-\cos^2\theta$

C) $\tan^2\theta+\cos^2\theta$

D) $\cos^2\theta-\tan^2\theta$

8. What are the solutions to $|2x+1|=7$?

A) $x=3, x=4$

B) $x=-3, x=4$

C) $x=3, x=-4$

D) $x=-3, x=-4$

9. A bag contains 4 red marbles and 6 green marbles. Ray randomly picks 3 marbles one after another (without replacement). What is the probability that at least one of these selected marbles is red?

A) $\frac{1}{6}$

B) $\frac{5}{6}$

C) $\frac{2}{5}$

D) $\frac{3}{5}$

10. What is the value of a given the lines $x+ay=8$ and $3x-2y=6$ are perpendicular?

A) $\frac{2}{3}$

B) $-\frac{2}{3}$

C) $\frac{3}{2}$

D) $-\frac{3}{2}$

End of Section I

Section II

61 marks

Attempt Questions 11-31
Allow about 1 hour and 45 minutes for this section

Instructions:

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing paper is available. If you use extra paper, write your name, your teacher's name on the paper and clearly indicate which question you are answering.

Question 11 (2 marks)

Solve the equation $\log_{10}(3x+1)=2$. **2**

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Question 12 (2 marks)

Solve $5^{-3x} = 125^{x+1}$.

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Question 13 (2 marks)

Solve the simultaneous equations.

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$$\begin{cases} 2x + y = 3 \\ x + 3y = -1 \end{cases}$$

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Question 14 (3 marks)

Solve $2\sin^2\theta = 1$ for $0 \le \theta \le 2\pi$. **3**

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Question 15 (2 marks)

Find from first principles the derivative of the curve $y = 3x^2 + 2$. **2**

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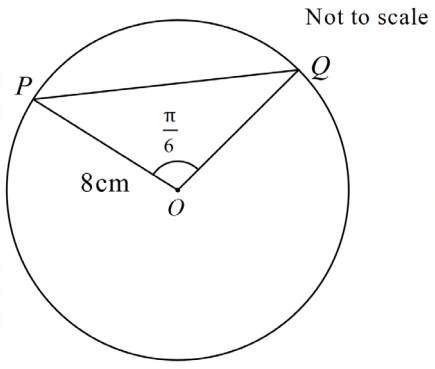
Question 16 (2 marks)

A chord PQ is drawn in a circle of radius 8 cm. The chord PQ subtends an angle of $\frac{\pi}{6}$ at the centre, O.



Find:

(i) the length of minor arc PQ. **1**

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(ii) the area of the minor segment cut off by the chord PQ, correct to two decimal places. **1**

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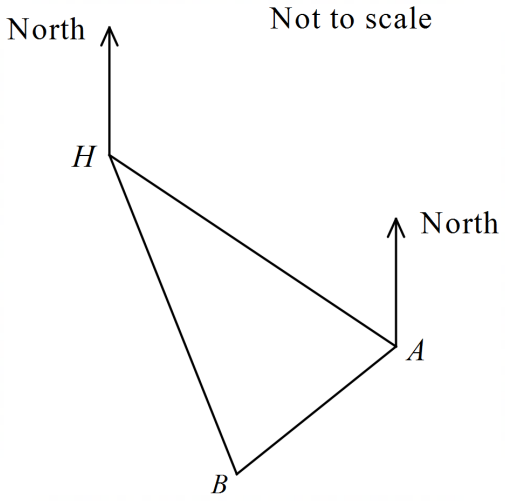
Question 17 (3 marks)

Ethan leaves his house H, and runs for 20 km on a bearing of 100° T, to town A. He then changes his course, and runs in a direction of 185° T, for 10 km to town B.

(i) Place the above information on the diagram below. **1**



(ii) Find the distance between his house H and town B correct to two decimal places. **2**

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Question 18 (2 marks)

Given $f(x) = 9 - x^4$ and $g(x) = \sqrt{x}$, find the composite function $f(g(x))$, in simplest form, and state its domain. **2**

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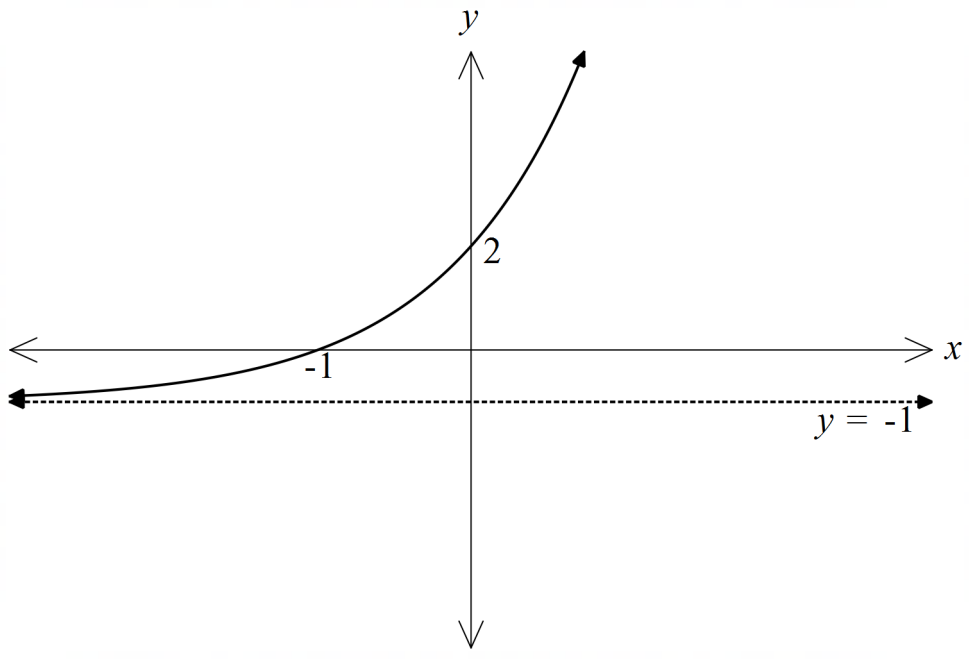
Question 19 (2 marks)

The graph of the function $y = f(x)$ is given below.

On the same diagram, sketch the function $y = -f(x)$, showing all important features. **2**



Question 20 (3 marks)

The population of rabbits in a certain bushland can be modelled using the formula $P(t) = 500e^{0.4t}$, where t is the time in years.

Find:

(i) the initial population of rabbits. **1**

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(ii) the rate of change when $t = 10$ years. Give your answer to the nearest whole number. **2**

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Question 21 (4 marks)

The discrete random variable X has the following probability distribution:

x	0	2	5
$P(X = x)$	$\frac{1}{6}$	$\frac{1}{3}$	a

(i) Find the value of a. **1**

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(ii) Calculate the mean $E(X)$. **1**

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(iii) Calculate the variance $Var(X)$. **1**

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(iii) Find the value of the standard deviation σ. **1**

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Question 22 (6 marks)

Differentiate with respect to x:

(i) $y = x^4 + \frac{1}{x}$ **2**

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(ii) $y = \frac{x+4}{x-1}$ **2**

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(iii) $y = xe^{7x}$ **2**

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Question 23 (3 marks)

a) In archery, Sally hits the target in 3 out of every 4 attempts, while Timothy hits it in 1 out of every 3 attempts.

If Sally and Timothy shoot one arrow each, find the probability that:

(i) both arrows hit the target. **1**

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(ii) at least one arrow hits the target. **1**

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b) In a particular archery competition, the winner is the first person to hit the target. **1**

If they take turns to shoot and Timothy shoots first, what is the probability that Timothy wins with his second arrow?

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Question 24 (3 marks)

Find the equation of the normal to the curve $y = 2x^2 + 5x - 6$ at the point where $x = 1$. **3**

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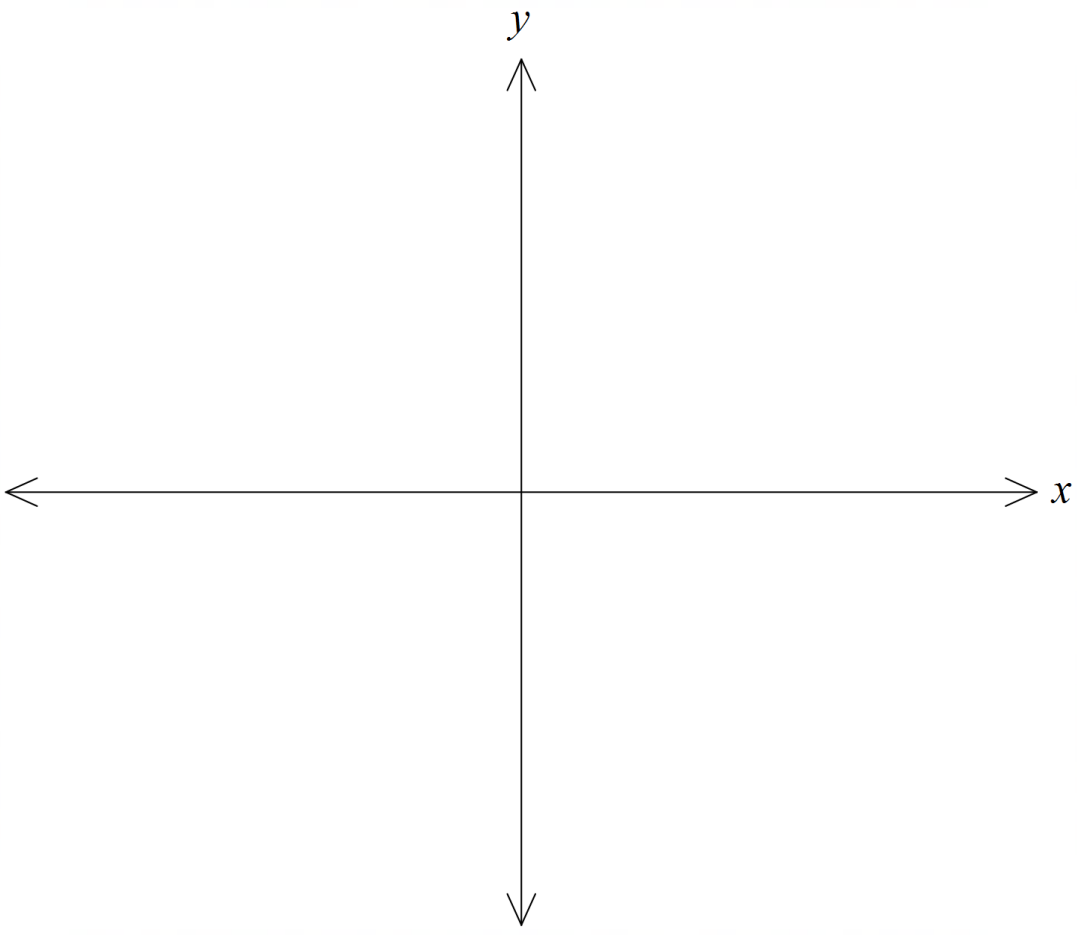
Question 25 (2 marks)

Sketch the graph of $y = (x-1)^3 + 2$. **2**



Question 26 (2 marks)

Find the values of k for which the equation $kx^2 + 2x + 8 = 0$ has:

(i) one real root **1**

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(ii) no real roots **1**

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Question 27 (2 marks)

Determine whether the function $f(x) = \dfrac{2x^3}{x^2 + 1}$ is even, odd or neither. Justify your answer. **2**

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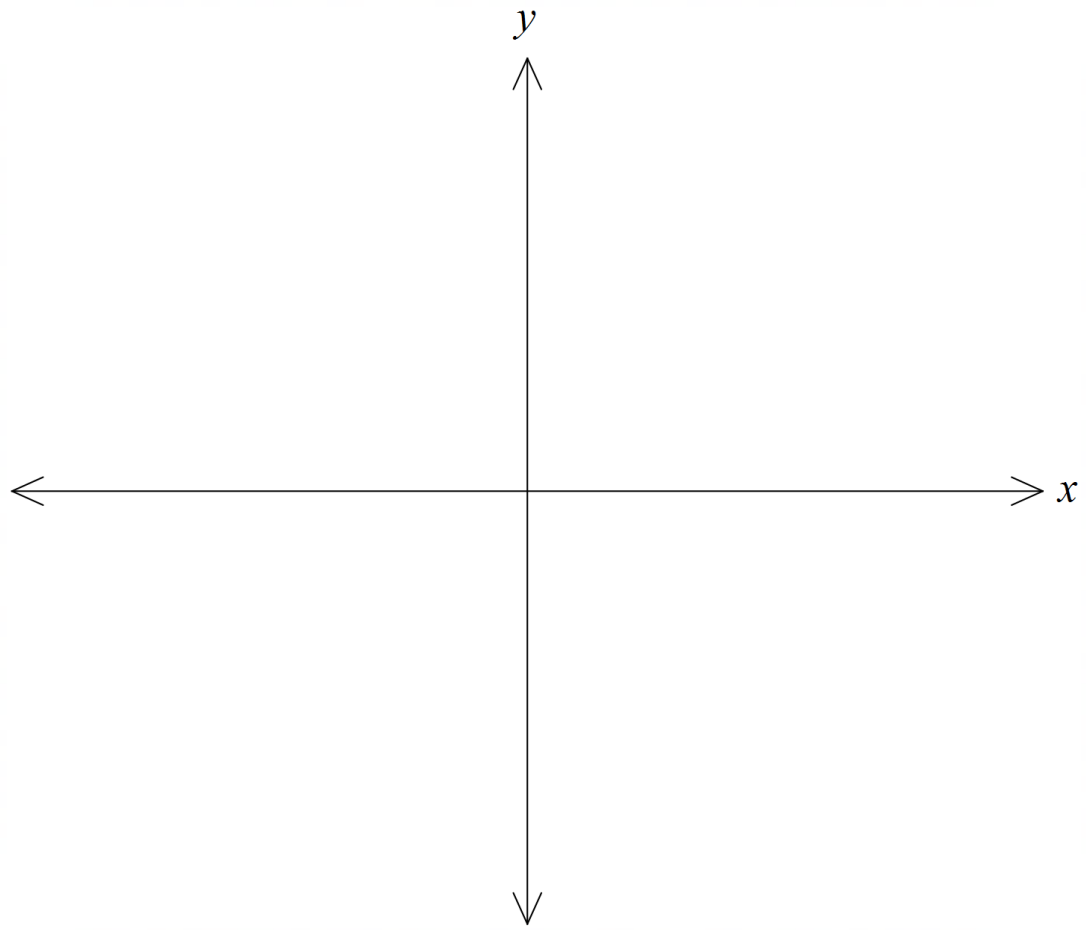
Question 28 (3 marks)

The function $y = f(x)$ is defined as follows:

$$f(x) = \begin{cases} 3x+1 & \text{for } x < 0 \\ x^2 & \text{for } x \geq 0 \end{cases}$$

(i) Sketch the function $y = f(x)$, showing all important features. **2**



(ii) Find the value of $f(-c)$ where $c > 0$. **1**

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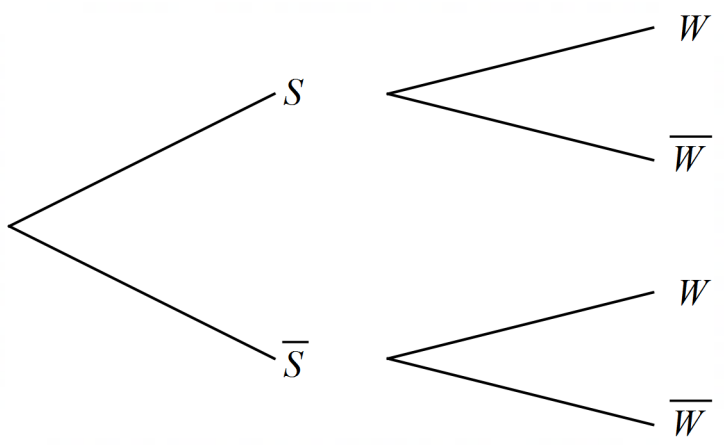
Question 29 (4 marks)

The Matildas have a probability of 60% of winning the World Cup (W) when Sam is playing on the team (S). When Sam is not playing, the Matildas have a probability of 55% of winning the World Cup. The probability of the Matildas having Sam playing at any one time is 95%.

(i) Complete the probability tree diagram below. **1**



(ii) Calculate the probability of the Matildas winning the World Cup. **2**

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(iii) Given that the Matildas won the World Cup, find the probability that Sam was playing. **1**

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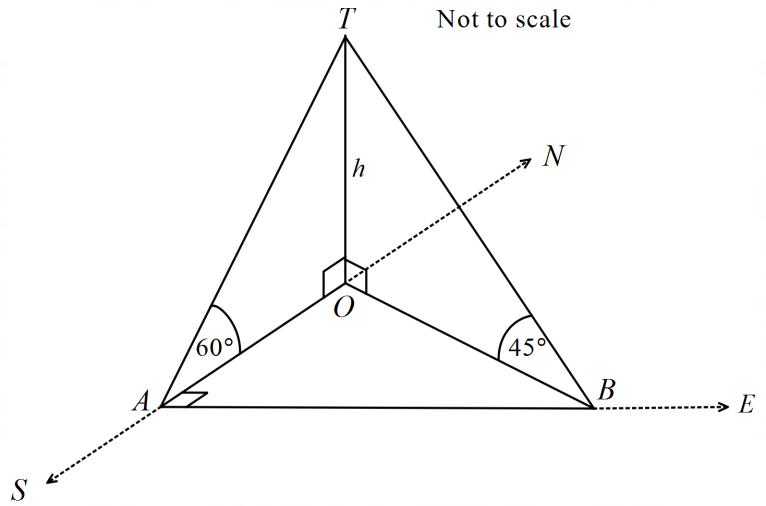
Question 30 (5 marks)

Alex stands at a point A, which is due south of a tower OT of height h m.
The angle of elevation of the top of the tower from A is $60°$. Alex then walks 20 m due east to point B, from where she measures the angle of elevation of the top of the tower to be $45°$.



(i) Express the length of OA in terms of h. **1**

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(ii) Show that $h = 10\sqrt{6}$ m. **2**

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(iii) Calculate the bearing of B from the base of the tower correct to the nearest degree. **2**

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Question 31 (4 marks)

(i) Show that $\cot\theta + \tan\theta = \operatorname{cosec}\theta \sec\theta$. **2**

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(ii) Hence or otherwise solve the following equation. **2**

$$\frac{\sec\theta}{\cot\theta + \tan\theta} - \frac{1 - \cot\theta}{\operatorname{cosec}\theta} = \frac{1}{2} \text{ for } 0° \le \theta \le 360°.$$

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End of Exam

BAULKHAM HILLS HIGH SCHOOL

YEAR 11 PRELIMINARY EXAMINATION 2023

Name: _____

Teacher: _____

Mathematics Advanced
Section I – Multiple Choice

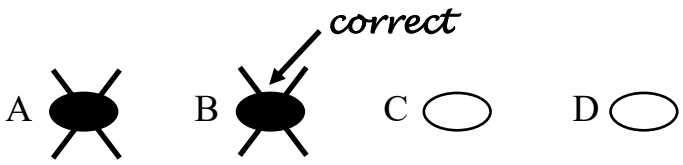
Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.



Start Here ➔

1.	A ○	B ○	C ○	D ○	**6.**	A ○	B ○	C ○	D ○
2.	A ○	B ○	C ○	D ○	**7.**	A ○	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ○	**8.**	A ○	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ○	**9.**	A ○	B ○	C ○	D ○
5.	A ○	B ○	C ○	D ○	**10.**	A ○	B ○	C ○	D ○

Solutions

BAULKHAM HILLS HIGH SCHOOL

2023

YEAR 11 PRELIMINARY EXAMINATION

Mathematics Advanced

General Instructions
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- Working time – 2 hours.
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Total marks – 71
Exam consists of 23 pages.

This paper consists of TWO sections.

Section I – Pages 4-6 (10 marks)
Questions 1 - 10
- Attempt Questions 1 - 10

Allow about 15 minutes for this section.

Section II – Pages 7 – 22 (61 marks)
- Attempt questions 11 - 31

Allow about 1 hour 45 minutes for this section.

Section I - 10 marks

Use the multiple-choice answer sheet for questions 1-10.

1. Which of the following is equivalent to $\sqrt{5}-\sqrt{125}+3\sqrt{50}$?

$= \sqrt{5} - 5\sqrt{5} + 3\sqrt{50}$

$= -4\sqrt{5} + 3\sqrt{50}$

A) $\sqrt{5}$

B) $6\sqrt{5}-15\sqrt{2}$

C) $-4\sqrt{5}+9\sqrt{2}$

(D) $-4\sqrt{5}+3\sqrt{50}$

2. Given $f(x)=3x-4$ and $g(x)=-\frac{1}{x}$, what are the values of x when $f(x) = g(x)$?

$3x-4 = -\frac{1}{x}$

$3x^2-4x+1=0$

$(3x-1)(x-1)=0$

$x=\frac{1}{3}$ or $x=1$

A) $-\frac{1}{3}$ or 1

B) $\frac{1}{3}$ or -1

(C) $\frac{1}{3}$ or 1

D) $-\frac{1}{3}$ or -1

3. What is the derivative of $f(x)=\sqrt{x^2-1}$?

$y' = \frac{1}{2}(x^2-1)^{\frac{1}{2}-1} \times 2x$

$= \frac{x}{\sqrt{x^2-1}}$

A) $\frac{1}{2\sqrt{2x-1}}$

B) $\frac{1}{2\sqrt{x^2-1}}$

C) $\frac{2x}{\sqrt{x^2-1}}$

(D) $\frac{x}{\sqrt{x^2-1}}$

4. Which of the following is the equivalent of the algebraic fraction $\frac{x+y}{x^{-2}-y^{-2}}$?

A) $\frac{x^2y^2}{x-y}$

(B) $\frac{x^2y^2}{y-x}$

C) $\frac{x^2y^2}{x^2-y^2}$

D) $\frac{x^2y^2}{y^2-x^2}$

$\frac{x+y}{\frac{1}{x^2}-\frac{1}{y^2}} = \frac{x+y}{\frac{y^2-x^2}{x^2y^2}}$

$= \frac{x^2y^2(x+y)}{(y+x)(y-x)}$

$= x^2y^2$

5. Which of the following functions is *not* many-to-one? $y=x$

A) $f(x)=3-x^2$

B) $f(x)=\sin x\ (0\le x\le 2\pi)$

(C) $f(x)=x^3-5$

D) $f(x)=|3x-2|$

6. Which of the following is an equivalent expression to $\log\frac{a}{b}+\log c$?

A) $\frac{\log ac}{\log b}$

(B) $\log a+\log c-\log b$

C) $\log a\times\log c-\log b$

D) $\frac{\log a\times\log c}{\log b}$

$\log a - \log b + \log c$

7. Which of the following is equivalent to $\frac{1-\sin^2\theta\cos^2\theta}{\cos^2\theta}$?

A) $\tan^2\theta\cos^2\theta$

B) $\tan^2\theta-\cos^2\theta$

(C) $\tan^2\theta+\cos^2\theta$

D) $\cos^2\theta-\tan^2\theta$

$= \frac{1}{\cos^2\theta} - \sin^2 x$

$= \sec^2 x - \sin^2 x$

$= \tan^2 x + 1 - \sin^2 x$

$= \tan^2 x + \cos^2 x$

8. What are the solutions to $|2x+1|=7$?

A) $x=3, x=4$

B) $x=-3, x=4$

(C) $x=3, x=-4$

D) $x=-3, x=-4$

$2x+1=7$ or $2x+1=-7$

$x=3$ or $x=-4$

9. A bag contains 4 red marbles and 6 green marbles. Ray randomly picks 3 marbles one after another (without replacement). What is the probability that at least one of these selected marbles is red?

A) $\frac{1}{6}$

(B) $\frac{5}{6}$

C) $\frac{2}{5}$

D) $\frac{3}{5}$

$P(G)\times P(G)\times P(G) = \frac{6}{10}\times\frac{5}{9}\times\frac{4}{8}$

$= \frac{1}{6}$

$P(\text{at least one red}) = 1-\frac{1}{6}$

$= \frac{5}{6}$

10. What is the value of a given the lines $x+ay=8$ and $3x-2y=6$ are perpendicular?

A) $\frac{2}{3}$

B) $-\frac{2}{3}$

(C) $\frac{3}{2}$

D) $-\frac{3}{2}$

$y = -\frac{x}{a}+\frac{8}{a}$ $\quad y=\frac{3x}{2}-3$

$-\frac{1}{a}\times\frac{3}{2} = -1$

$a = \frac{3}{2}$

End of Section I

Section II

61 marks

Attempt Questions 11-31
Allow about 1 hour and 45 minutes for this section

Instructions:

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing paper is available. If you use extra paper, write your name, your teacher's name on the paper and clearly indicate which question you are answering.

Question 11 (2 marks)

Solve the equation $\log_{10}(3x+1)=2$. 2

* 2 - correct answer

$3x+1=10^2$ * 1 - obtains $3x+1=100$

$x=\frac{100-1}{3}$

$=\frac{99}{3}$

$=33$

Question 12 (2 marks)

Solve $5^{-3x}=125^{x+1}$. 2

$5^{-3x}=(5^3)^{x+1}$ * 2 - correct answer

* 1 - obtains $-3x=3(x+1)$

$\therefore -3x=3(x+1)$

$=3x+3$

$6x=-3$

$x=\frac{-1}{2}$

Question 13 (2 marks)

Solve the simultaneous equations. 2

$\begin{cases} 2x+y=3 & (1) \\ x+3y=-1 & (2) \end{cases}$

* 2 - correct answer

* 1 - obtains correct $x=2$ or correct $y=-1$

$2x+y=3$

$-\ 2(x+3y)=2x-1$

$-5y=5$

$y=-1$

$x=-1+3$

$=2$

$y=-1, x=2$

Question 14 (3 marks)

Solve $2\sin^2\theta = 1$ for $0 \le \theta \le 2\pi$. 3

$\sin^2\theta = \frac{1}{2}$

$\sin\theta = \pm\frac{1}{\sqrt{2}}$

$\sin\theta = \frac{1}{\sqrt{2}}$ $\sin\theta = -\frac{1}{\sqrt{2}}$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}$ $\theta = \frac{5\pi}{4}, \frac{7\pi}{4}$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

* 3 – correct solution

* 2 – obtains $\sin\theta = \frac{1}{\sqrt{2}}$ and $\theta = \frac{\pi}{4}, \frac{3\pi}{4}$ OR obtains $\sin\theta = -\frac{1}{\sqrt{2}}$ and $\theta = \frac{5\pi}{4}, \frac{7\pi}{4}$

* 1 – obtains either $\sin\theta = \frac{1}{\sqrt{2}}$ or $\sin 2\theta = \frac{1}{\sqrt{2}}$

Question 15 (2 marks)

Find from first principles the derivative of the curve $y = 3x^2 + 2$. 2

$f(x) = 3x^2 + 2$

$f(x+h) = 3(x+h)^2 + 2$

$= 3x^2 + 6xh + h^2 + 2$

$f(x+h) - f(x) = 3x^2 + 6xh + h^2 + 2 - (3x^2 + 2)$

$= 6xh + h^2$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{6xh + h^2}{h}$

$= \lim_{h \to 0} (6x + h)$

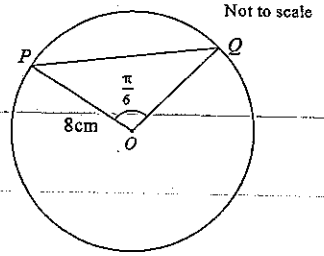
$= 6x$

* 2 – correct solution

* 1 – obtains $f(x+h) - f(x)$ in simplest form.

Question 16 (2 marks)

A chord PQ is drawn in a circle of radius 8 cm. The chord PQ subtends an angle of $\frac{\pi}{6}$ at the centre, O.



Find:

(i) the length of minor arc PQ. 1

$l = r\theta$

$= 8 \times \frac{\pi}{6}$

$= \frac{4\pi}{3}$ or 4.19 cm (2dp)

(ii) the area of the minor segment cut off by the chord PQ, correct to two decimal places. 1

Area = minor sector − ΔPOQ

$= \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$

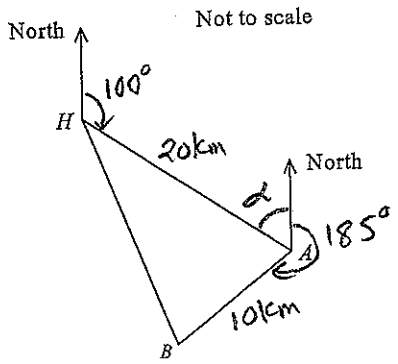
$= \frac{1}{2} \times 8^2 \times \frac{\pi}{6} - \frac{1}{2} \times 8^2 \times \sin\frac{\pi}{6}$

$= 0.76$ (2dp)

Question 17 (3 marks)

Ethan leaves his house H, and runs for 20 km on a bearing of 100° T, to town A. He then changes his course, and runs in a direction of 185° T, for 10 km to town B.

(i) Place the above information on the diagram below. 1



(ii) Find the distance between his house H and town B correct to two decimal places. 2

$\alpha = 180° - 100°$

$= 80°$

$\angle HAB = 360° - 185° - 80°$

$= 95°$

$HB^2 = 20^2 + 10^2 - 2 \times 20 \times 10 \cos 95°$

$HB = 23.13$ km (2 dp)

* 2 – correct solution

* 1 – obtains $\angle HAB = 95°$

Question 18 (2 marks)

Given $f(x) = 9 - x^4$ and $g(x) = \sqrt{x}$, find the composite function $f(g(x))$, in simplest form, and state its domain. 2

* 2 – correct solution

* 1 – obtains correct $f(g(x))$ or obtains correct domain

$f(g(x)) = 9 - (\sqrt{x})^4$

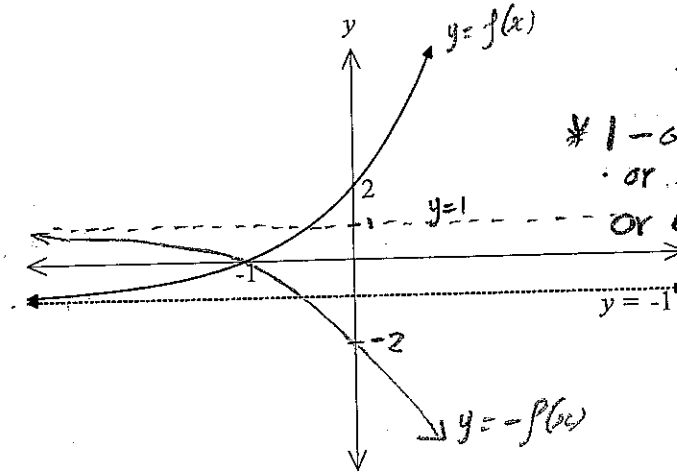
$= 9 - x^2$

domain : $x \geq 0$

Question 19 (2 marks)

The graph of the function $y = f(x)$ is given below.

On the same diagram, sketch the function $y = -f(x)$, showing all important features. 2



* 2 – correct graph + asymptote + x & y intercepts

* 1 – obtains correct graph or correct x & y intercepts or correct asymptote.

Question 20 (3 marks)

The population of rabbits in a certain bushland can be modelled using the formula $P(t) = 500e^{0.4t}$, where t is the time in years.

Find:

(i) the initial population of rabbits. 1

at $t=0$

$P(0) = 500 \times e^{0.4 \times 0}$

$= 500$

(ii) the rate of change when $t = 10$ years. Give your answer to the nearest whole number. 2

$P(t) = 500e^{0.4t}$ * 2 - correct solution

$P'(t) = 500e^{0.4t} \times 0.4$ *1 - obtains correct $P'(t)$

$= 200e^{0.4t}$

$P'(10) = 200e^{0.4 \times 10}$

$= 200e^{4}$

$= 10919.63$

$= 10920$ (nearest whole number)

Question 21 (4 marks)

The discrete random variable X has the following probability distribution:

x	0	2	5
$P(X=x)$	$\frac{1}{6}$	$\frac{1}{3}$	a

(i) Find the value of a. 1

$\frac{1}{6} + \frac{1}{3} + a = 1$

$a = \frac{1}{2}$

(ii) Calculate the mean $E(X)$. 1

$E(X) = \sum x P(x)$

$= 0 \times \frac{1}{6} + 2 \times \frac{1}{3} + 5 \times \frac{1}{2}$

$= \frac{19}{6}$

(iii) Calculate the variance $Var(X)$. 1

$Var(x) = E(x^2) - (E(x))^2$

$= 0^2 \times \frac{1}{6} + 2^2 \times \frac{1}{3} + 5^2 \times \frac{1}{2} - \left(\frac{19}{6}\right)^2$

$= \frac{137}{36}$ or 3.81 (2dp)

(iii) Find the value of the standard deviation σ. 1

$\sigma = \sqrt{\frac{137}{36}}$

$= \frac{\sqrt{137}}{6}$ or 1.95 (2dp)

Question 22 (6 marks)

Differentiate with respect to x:

(i) $y = x^4 + \frac{1}{x}$ 2

$= x^4 + x^{-1}$

$y' = 4x^3 - x^{-2}$

$= 4x^3 - \frac{1}{x^2}$

* 2 – correct solution

* 1 – obtains $4x^3$ or $-\frac{1}{x^2}$

(ii) $y = \frac{x+4}{x-1}$ 2

$y' = \frac{(x+4)'(x-1) - (x+4)(x-1)'}{(x-1)^2}$

$= \frac{x-1-(x+4)}{(x-1)^2}$

$= \frac{-5}{(x-1)^2}$

* 2 – correct solution

* 1 – obtains either correct numerator or obtains correct denominator

(iii) $y = xe^{7x}$ 2

$y' = x'e^{7x} + x(e^{7x})'$

$= e^{7x} + xe^{7x} \times 7$

$= e^{7x} + 7xe^{7x}$

* 2 – correct solution

* 1 – obtains either e^{7x} or $7xe^{7x}$

Question 23 (3 marks)

a) In archery, Sally hits the target in 3 out of every 4 attempts, while Timothy hits it in 1 out of every 3 attempts.

If Sally and Timothy shoot one arrow each, find the probability that:

(i) both arrows hit the target. 1

$= P(\text{Sally hits target}) \times P(\text{Tim hits the target})$

$= \frac{3}{4} \times \frac{1}{3}$

$= \frac{1}{4}$

(ii) at least one arrow hits the target. 1

$P(\text{both Sally and Tim miss the target})$

$= P(\text{Sally misses the target}) \times P(\text{Tim misses the target})$

$= \frac{1}{4} \times \frac{2}{3}$

$= \frac{1}{6}$

$P(\text{at least one arrow hits the target}) = 1 - P(\text{both misses the target})$

$= 1 - \frac{1}{6}$

$= \frac{5}{6}$

b) In a particular archery competition, the winner is the first person to hit the target. 1

If they take turns to shoot and Timothy shoots first, what is the probability that Timothy wins with his second arrow?

$P(\text{Tim misses the target with 1st arrow}) \times P(\text{Sally misses the target with 1st arrow}) \times P(\text{Tim hits target with 2nd arrow})$

$= \frac{2}{3} \times \frac{1}{4} \times \frac{1}{3}$

$= \frac{1}{18}$

Question 24 (3 marks)

Find the equation of the normal to the curve $y = 2x^2 + 5x - 6$ at the point where $x = 1$. 3

$f(1) = 2\times1^2 + 5\times1 - 6$
$= 1$

$f'(x) = 4x + 5$

$f'(1) = 4\times1 + 5$
$= 9$

Equation of the normal to the curve at $x = 1$

$y - 1 = -\frac{1}{9}(x - 1)$

$y = -\frac{x}{9} + \frac{10}{9}$

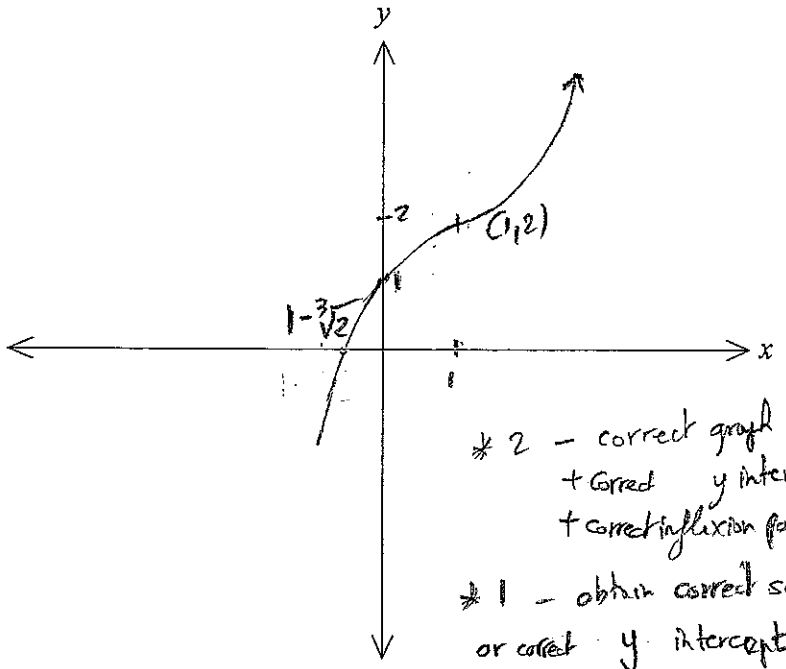
* 3 – correct solution

* 2 – find gradient and y value of normal

* 1 – correct value of $f'(1)$

Question 25 (2 marks)

Sketch the graph of $y = (x-1)^3 + 2$. 2



* 2 – correct graph
+ correct y intercepts
+ correct inflexion point (1,2)

* 1 – obtain correct shape of graph
or correct y intercept
or Correct inflexion point (1,2)

Question 26 (2 marks)

Find the values of k for which the equation $kx^2 + 2x + 8 = 0$ has:

(i) one real root 1

$b^2 - 4ac = 0$

$2^2 - 4k\times8 = 0$

$k = \frac{4}{4\times8}$

$= \frac{1}{8}$

(ii) no real roots 1

$b^2 - 4ac < 0$

$2^2 - 4k\times8 < 0$

$k > \frac{1}{8}$

Question 27 (2 marks)

Determine whether the function $f(x) = \frac{2x^3}{x^2+1}$ is even, odd or neither. Justify your answer. 2

$f(-x) = \frac{2(-x)^3}{(-x)^2+1}$

$= \frac{-2x^3}{x^2+1}$

$= -f(x)$

$\therefore$ It's odd function

* 2 – correct solution

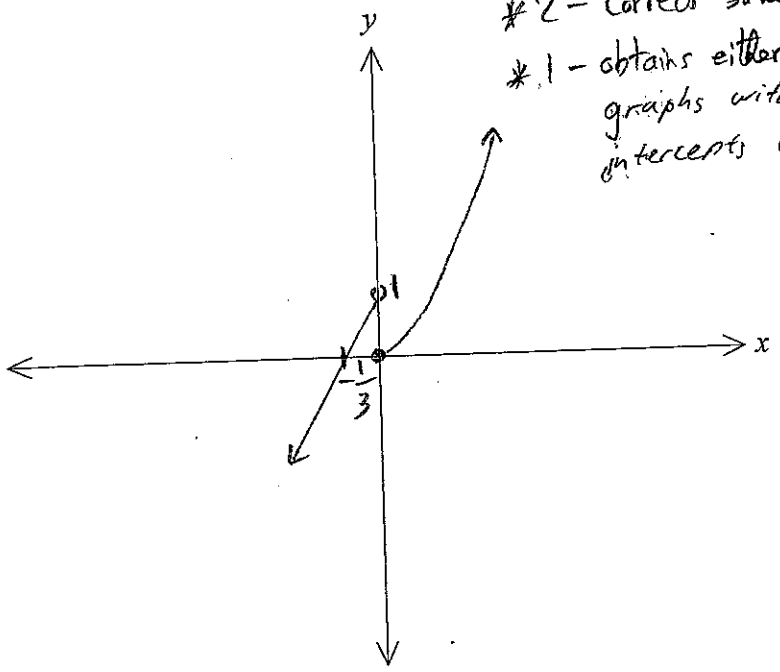
* 1 – obtains correct $f(-x)$

Question 28 (3 marks)

The function $y = f(x)$ is defined as follows:

$$f(x) = \begin{cases} 3x+1 & \text{for } x < 0 \\ x^2 & \text{for } x \geq 0 \end{cases}$$

(i) Sketch the function $y = f(x)$, showing all important features. 2



*2 – Correct solution
*1 – obtains either correct graphs with correct intercepts labelled.

(ii) Find the value of $f(-c)$ where $c > 0$. 1

$c > 0$ then $-c < 0$

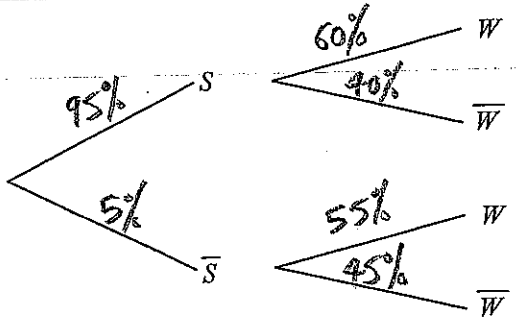
$f(-c) = 3(-c)+1$

$= -3c+1$

Question 29 (4 marks)

The Matildas have a probability of 60% of winning the World Cup (W) when Sam is playing on the team (S). When Sam is not playing, the Matildas have a probability of 55% of winning the World Cup. The probability of the Matildas having Sam playing at any one time is 95%.

(i) Complete the probability tree diagram below. 1



(ii) Calculate the probability of the Matildas winning the World Cup. 2

P(Sam plays) × P(Matildas win) + P(Sam not plays) × P(Matildas wins)

$= 95\% \times 60\% + 5\% \times 55\%$

$= 0.5975$

*2 – correct solution
*1 – obtains either $95\% \times 60\%$ or $5\% \times 55\%$

(iii) Given that the Matildas won the World Cup, find the probability that Sam was playing. 1

P(Sam playing given Matildas won)

$= \dfrac{95\% \times 60\%}{95\% \times 60\% + 5\% \times 55\%}$

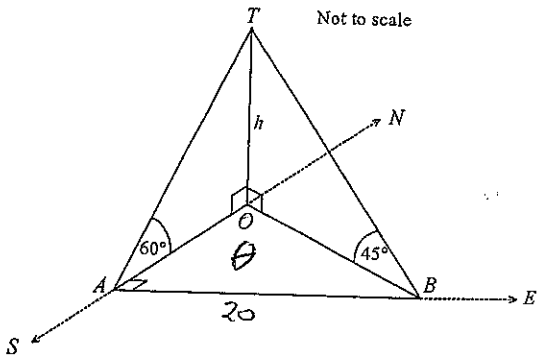
$= 0.954$ (3dp)

* Must show working

Question 30 (5 marks)

Alex stands at a point A, which is due south of a tower OT of height h m.

The angle of elevation of the top of the tower from A is 60°. Alex then walks 20 m due east to point B, from where she measures the angle of elevation of the top of the tower to be 45°.



(i) Express the length of OA in terms of h. 1

$\tan 60° = \frac{h}{OA}$

$OA = \frac{h}{\tan 60°}$

$= \frac{h}{\sqrt{3}}$

(ii) Show that $h = 10\sqrt{6}$ m. 2

$OB = h$

$OA^2 + AB^2 = OB^2$

$\left(\frac{h}{\sqrt{3}}\right)^2 + 20^2 = h^2$

$\frac{h^2}{3} - 400 = h^2$

$\frac{2h^2}{3} = 400$

$h = 10\sqrt{6}$

* 2 – correct solution

* 1 – obtain $OB = h$ or setting Pythagoras formula correctly

(iii) Calculate the bearing of B from the base of the tower correct to the nearest degree. 2

$\sin \angle AOB = \frac{AB}{OB}$

$\sin\theta = \frac{20}{h}$

$= \frac{20}{10\sqrt{6}}$

$\therefore \theta = 54°44'$

Bearing of B from the base of the tower

$= 180° - \theta$

$= 125°$ (nearest deg)

* 2 – correct solution

* 1 – obtain correct $\sin\theta$ or θ

Question 31 (4 marks)

(i) Show that $\cot\theta + \tan\theta = \operatorname{cosec}\theta \sec\theta$. 2

* 2 – correct solution

$\cot\theta + \tan\theta = \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}$ * 1 – obtain $\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}$

$= \frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta}$ * 1 – obtains $\cos^2\theta + \sin^2\theta = 1$

$= \frac{1}{\sin\theta\cos\theta}$

$= \frac{1}{\sin\theta} \times \frac{1}{\cos\theta}$

$= \operatorname{cosec}\theta \sec\theta$

(ii) Hence or otherwise solve the following equation. 2

$\frac{\sec\theta}{\cot\theta + \tan\theta} - \frac{1 - \cot\theta}{\operatorname{cosec}\theta} = \frac{1}{2}$ for $0° \le \theta \le 360°$.

$\frac{\sec\theta}{\cos\theta + \tan\theta} - \frac{1 - \cot\theta}{\operatorname{cosec}\theta} = \frac{\sec\theta}{\operatorname{cosec}\theta \sec\theta} - \frac{1 - \cot\theta}{\operatorname{cosec}\theta}$

$= \frac{1}{\operatorname{cosec}\theta} - \frac{1 - \cot\theta}{\operatorname{cosec}\theta}$

$= \frac{\cot\theta}{\operatorname{cosec}\theta}$

$= \frac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}}$

$= \cos\theta$

$\cos\theta = \frac{1}{2}$

$\theta = 60°, 300°$

* 2 – correct solution

* 1 – obtains LHS = $\cos\theta$

* 1 – obtains either $\theta = 60°$ or $\theta = 300°$

End of Exam