



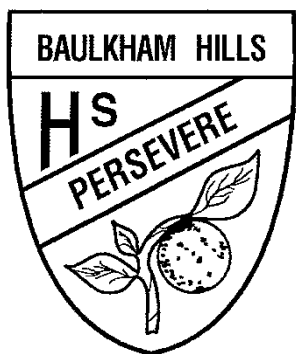
**BAULKHAM HILLS HIGH SCHOOL 2024
YEAR 11 ASSESSMENT TASK 3
MATHEMATICS ADVANCED**

MARKING COVER SHEET

FINAL MARK

Name: _____ **Teacher:** _____

Question	F1	T1	T2	C1	E1	S1	Total
Multiple Choice	1, 2, 4, 5, 6, 8, 9, 10 /8				3 /1	7 /1	10
11				/7			7
12	/4						4
13	/2						2
14					/2		2
15					/3		3
16	/2						2
17				/3			3
18		/3					3
19	/4						4
20				/2			2
21	/5						5
22	/4						4
23						/5	5
24			/3				3
25				/2			2
26		/3					3
27			/2				2
28		/5					5
29						/4	4
TOTAL	/29	/11	/5	/14	/6	/10	75



BAULKHAM HILLS HIGH SCHOOL

YEAR 11 ASSESSMENT TASK 3 2024

Name: _____

Teacher: _____

Mathematics Advanced

General Instructions

- Reading time - 10 minutes
- Working time - 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided separately
- In Questions 11 - 29, show relevant mathematical reasoning and/or calculations

Total marks: 75

Section I - 10 marks (pages 4 - 7)

- Attempt Questions 1 - 10
- Allow about 15 minutes for this section

Section II - 65 marks (pages 8 - 24)

- Attempt Questions 11 - 29
- Allow about 1 hour and 45 minutes for this section



Baulkham Hills High School

2024

Year 11- Assessment Task 3

Mathematics

Section I – Multiple Choice

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9
A ☐ B ☒ C ☐ D ☐

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ☒ B ☒ C ☐ D ☐

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

A ☒ B ☒ C ☐ D ☐
correct (arrow pointing to B)

Start
Here →

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|--|---|
| 1. A ☐ B ☐ C ☐ D ☐ | 6. A ☐ B ☐ C ☐ D ☐ |
| 2. A ☐ B ☐ C ☐ D ☐ | 7. A ☐ B ☐ C ☐ D ☐ |
| 3. A ☐ B ☐ C ☐ D ☐ | 8. A ☐ B ☐ C ☐ D ☐ |
| 4. A ☐ B ☐ C ☐ D ☐ | 9. A ☐ B ☐ C ☐ D ☐ |
| 5. A ☐ B ☐ C ☐ D ☐ | 10. A ☐ B ☐ C ☐ D ☐ |
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Section I

10 marks

Attempt Questions 1 - 10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10.

1 Which of the following is equivalent to $5x - (x + 3)(2x - 1)$?

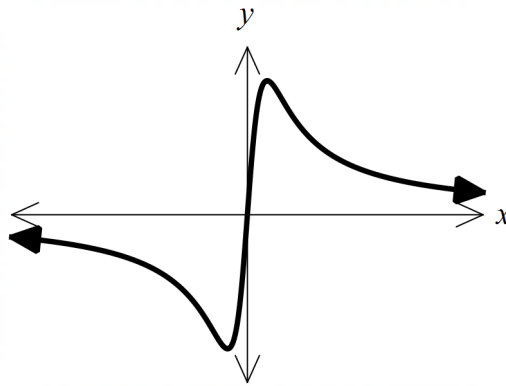
(A) $3 - 2x^2$

(B) $-2x^2 + 10x + 3$

(C) $2x^2 - 3$

(D) $2x^2 - 10x - 3$

2 The graph of $y = f(x)$ is shown below.



Which of the following best describes $y = f(x)$?

(A) One-to-one

(B) Many-to-one

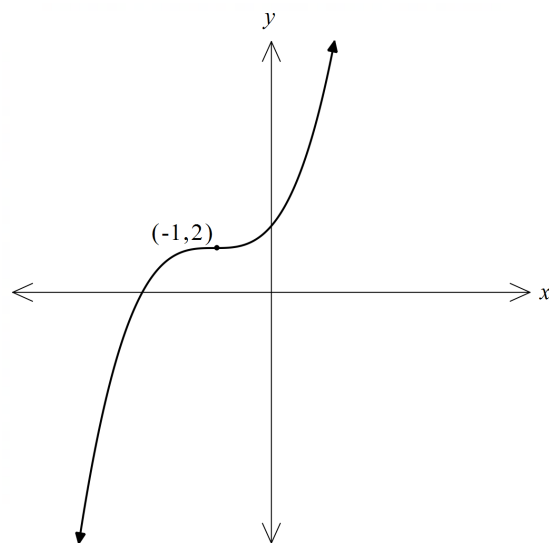
(C) Many-to-many

(D) One-to-many

- 3 Which of the following is the solution to $64^{4x} = 16^{3x-1}$?
- (A) $x = -3$
- (B) $x = -1$
- (C) $x = \frac{-1}{3}$
- (D) $x = 0$
- 4 What is the distance between the points $(-2, 5)$ and $(7, 1)$?
- (A) $\sqrt{41}$
- (B) $\sqrt{61}$
- (C) $\sqrt{97}$
- (D) $\sqrt{117}$
- 5 What is the vertex of $y = 4 - 6x - 3x^2$?
- (A) $(-2, 4)$
- (B) $(-1, 7)$
- (C) $(1, -5)$
- (D) $(2, -20)$
- 6 Which of the following is the solution to $|4x - 1| = 7$?
- (A) $x = 2$ or $x = -1.5$
- (B) $x = \pm 2$
- (C) $x = \pm 1.5$
- (D) $x = -2$ or $x = 1.5$

- 7 One hundred students in a school are surveyed as to how they get to school each morning. 45 students walked, 70 caught the bus and 26 students neither caught the bus nor walked. What is the probability that a student chosen at random walked and caught the bus?
- (A) 4%
- (B) 29%
- (C) 31.5%
- (D) 41%
- 8 The amount of rent paid per person varies inversely with the number of people sharing a house. If 8 people share a house, then each person needs to pay \$135. How much would the rent be per person if 10 people were to share the same house?
- (A) \$13.50
- (B) \$108
- (C) \$168.75
- (D) \$1350
- 9 Which of the following are solution/s to $\sqrt{1+4x-x^2} = x-1$?
- (A) $x = -1$
- (B) $x = 2 \pm \sqrt{5}$
- (C) $x = 0$ or $x = 3$
- (D) $x = 3$

10 The graph $y = f(x)$ is shown below.



Which of the following could be the equation of $y = f(x)$?

(A) $y = (x-1)^3 + 2$

(B) $y = (x-1)^3 - 2$

(C) $y = (x+1)^3 - 2$

(D) $y = (x+1)^3 + 2$

Section II

65 marks

Attempt Questions 11 - 29

Allow about 1 hour and 45 minutes for this section

Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response. Your responses should include relevant mathematical reasoning.

Question 11 (7 marks)

Differentiate each of the following:

a) $y = 2x^4 - 5x^2 + 3$ 1

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b) $y = e^{4x-1}$ 1

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c) $y = \frac{8}{x^4}$ 1

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d) $y = \frac{2x}{3x-1}$

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e) $y = 4x\sqrt{6x-3}$

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Question 12 (4 marks)

a) Fully factorise $4x^3 - x^5$.

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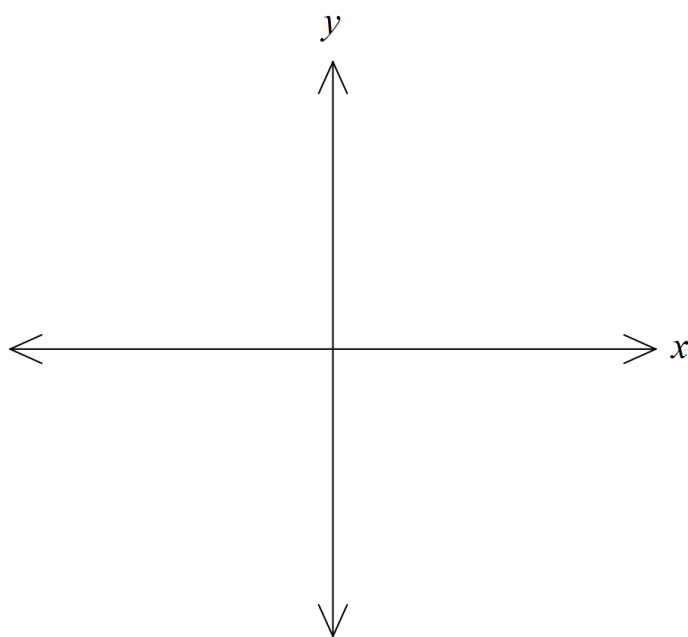
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b) Without the use of calculus, hence or otherwise sketch $y = 4x^3 - x^5$.

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Question 13 (2 marks)

Prove $\frac{2\sqrt{6}}{\sqrt{3}-1} = 3\sqrt{2} + \sqrt{6}$.

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Question 14 (2 marks)

Simplify $\frac{(5x^2y)^3 \times 6x^3y^4}{50x^4y^9}$

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Question 15 (3 marks)

If $\log_a 8 = 1.069$ and $\log_a 5 = 0.827$, without finding the value of a , evaluate:

a) $\log_a 40$.

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b) $\log_a \left(\frac{a}{5} \right)$.

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Question 16 (2 marks)

State the centre and radius of the circle $2x^2 + 2y^2 - 12y - 4 = 0$. **2**

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Question 17 (3 marks)

Find the equation of the tangent to the curve $y = 8x^3 - 6x^2 + x - 1$ at the point where $x = 1$. **3**

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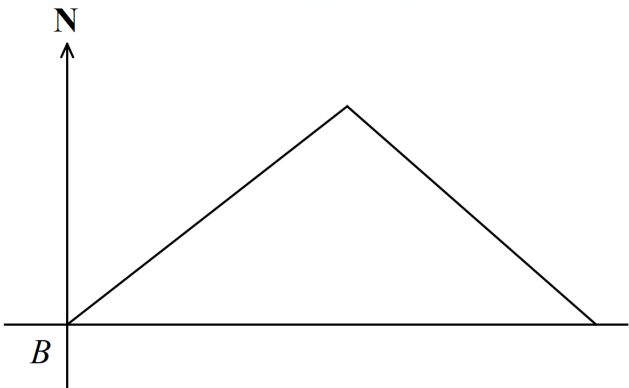
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Question 18 (3 marks)

Anika left base camp (B) and walked on a bearing of 052°T for 2.8 km to Josephine Falls (J). Big Rock (R) is due east of the base camp (B) and 5 km from Josephine Falls.

- a) Show the information given on the diagram below. 1



- b) Find the bearing of Josephine Falls from Big Rock to the nearest degree. 2

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Question 19 (4 marks)

- a) Show that $f(x) = \frac{1}{x^3 - 3x}$ is an odd function. 1

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- b) State the domain of $f(x) = \frac{1}{x^3 - 3x}$. 1

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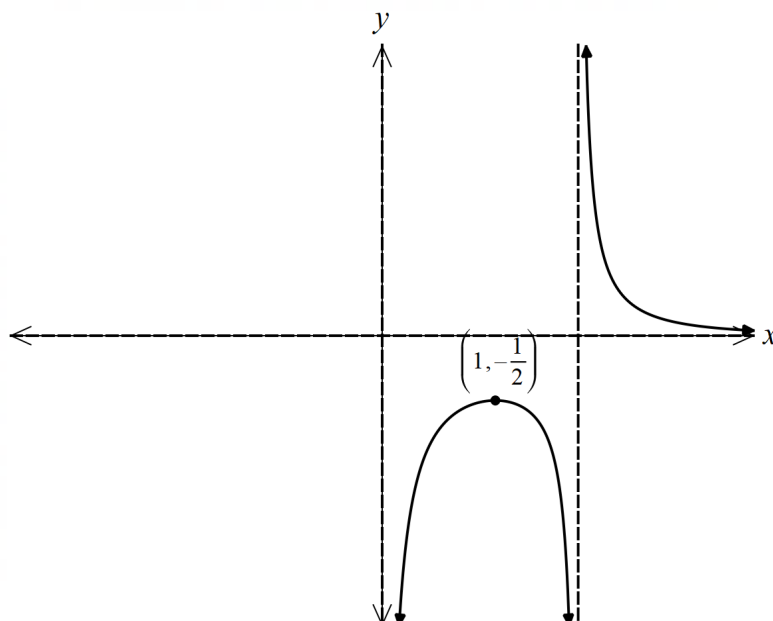
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c) Part of the graph of $y = f(x)$ is shown on the number plane below for $x > 0$.

Complete the graph over the natural domain of the function, showing important features.



Question 20 (2 marks)

Differentiate $y = x^2 + 6x$ from first principles.

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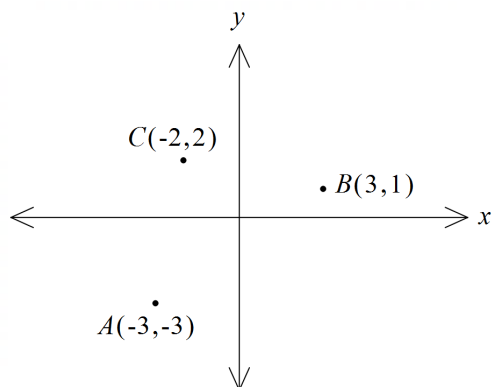
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Question 21 (5 marks)

The diagram shows the points $A(-3,-3)$, $B(3,1)$ and $C(-2,2)$.



- a) Find M , the midpoint of AB . 1

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- b) Find the gradient of AB . 1

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- c) AB is perpendicular to CM . State the gradient of CM . 1

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- d) It is known that the length of CM is $\sqrt{13}$ units. Hence, or otherwise find the area of $\triangle ABC$. 2

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Question 22 (4 marks)

If $f(x) = \sqrt{x-4}$ and $g(x) = x^2$, find:

a) $f(5)$

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b) $g(f(x))$

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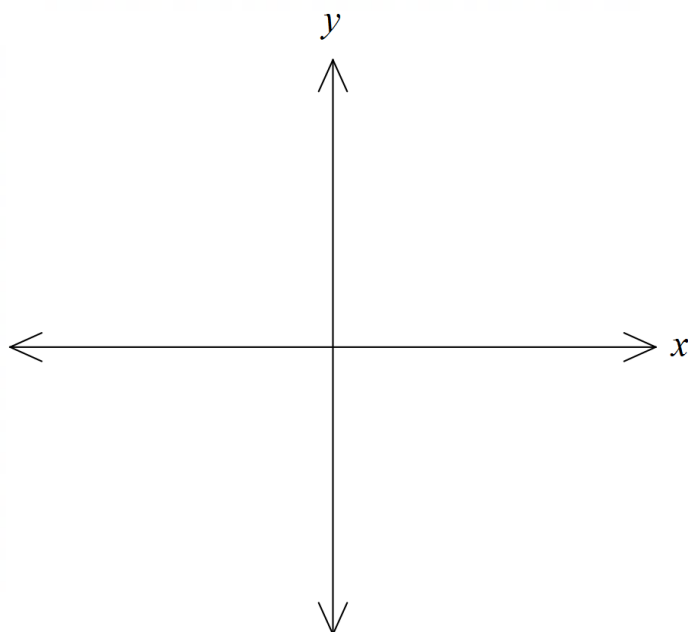
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c) Hence, sketch the graph of $y = g(f(x))$.

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Question 23 (5 marks)

The probability distribution of a discrete random variable is given by

x	0	1	2	3
$P(X = x)$	0.4	0.15	a	b

- a) The expected value of this probability distribution is 1.13. Show that $a = 0.37$ and $b = 0.08$. **2**

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- b) Show that $\text{Var}(X) = 1.0731$. **2**

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- c) Find $P(X = 0 | X \leq 2)$. **1**

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Question 24 (3 marks)

Solve $2\sin^2 t = 3\cos t$, where $0 \leq t \leq 2\pi$.

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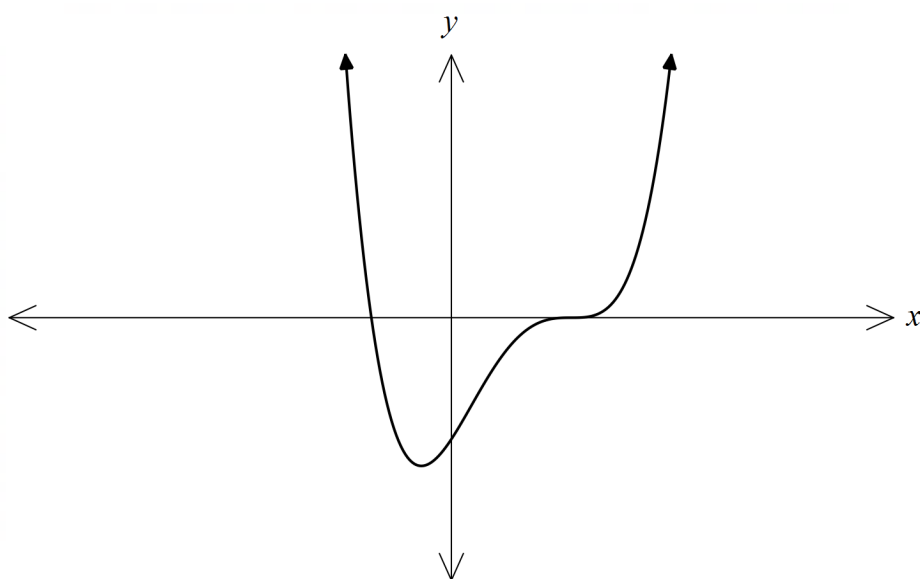
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Question 25 (2 marks)

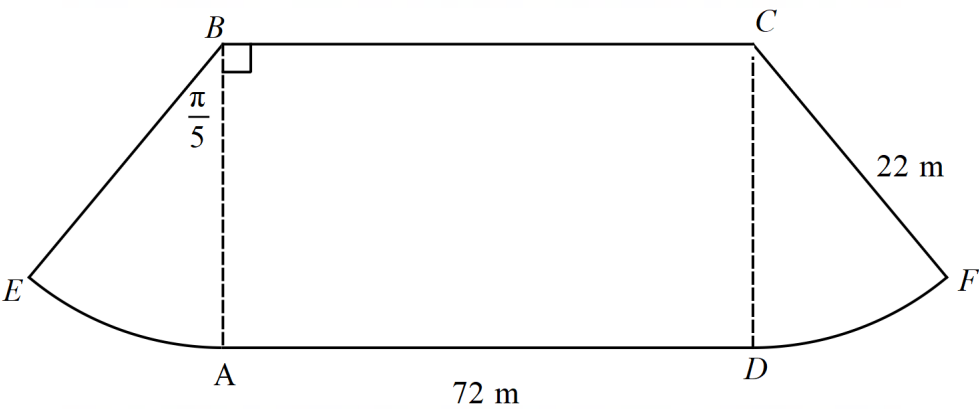
The function $y = f(x)$ is graphed below. On the same number plane, graph the derivative function $y = f'(x)$.

2



Question 26 (3 marks)

A stage is to be set up in a stadium consisting of a rectangle $ABCD$ with two identical circular sectors attached to AB and CD . Each of these sectors has a radius of 22 metres and subtends an angle of $\frac{\pi}{5}$ radians at the centre. The length of AD is 72 metres. This information is shown on the diagram below.



- a) Electrical wiring needs to be placed along the front edge of the stage to power equipment. Find the length of wiring $EADF$ that is required. Give your answer correct to 2 decimal places. 1

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- b) Calculate the area of the stage. Give your answer correct to 2 decimal places. 2

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Question 27 (2 marks)

Show that $\operatorname{cosec}^2 x(\tan^2 x - \sin^2 x) = \tan^2 x$

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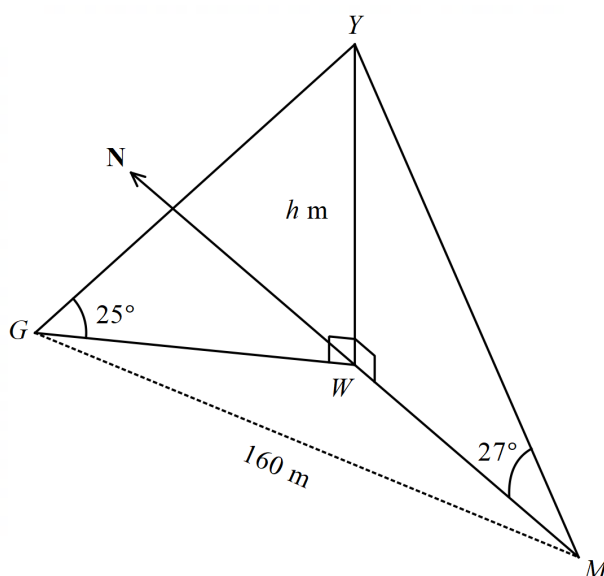
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Question 28 (5 marks)

Mridvika stands at a point (M) due south from the base of a windmill (W). The angle of elevation of the top of the windmill from where Mridvika stands is 27° . Gokul stands on a point (G) which is on a bearing of $300^\circ T$ from the windmill. The angle of elevation from where he stands is 25° . The distance between Mridvika and Gokul is 160 m (in a straight line). Let the height of the windmill be h metres. This information is shown in the diagram below.



- a) Show that $\angle MWG = 120^\circ$ 1

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- b) Show that $MW = h \tan 63^\circ$. 1

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c) Hence or otherwise, find the height (h) of the windmill.

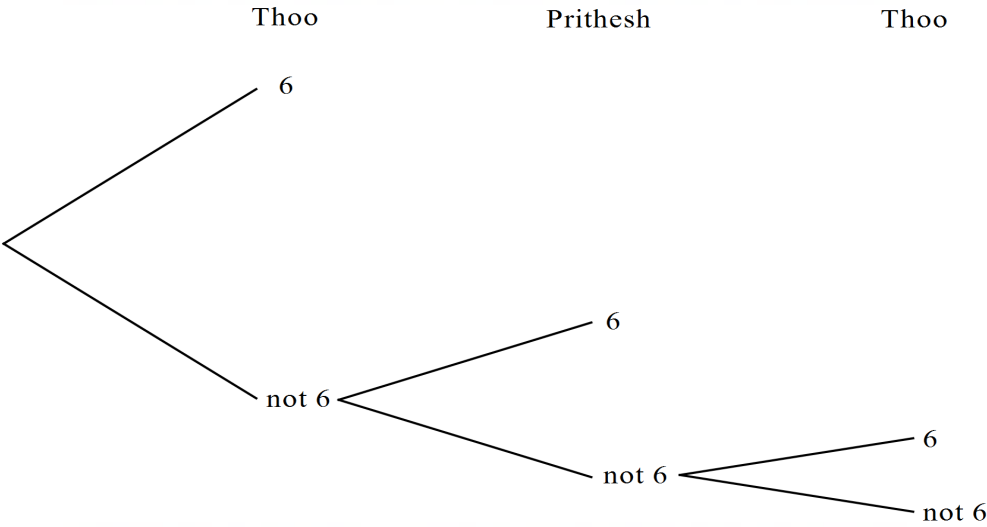
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Question 29 (4 marks)

Thoo and Prithesh are playing a game. Each person takes turns in rolling a fair six-side die with faces numbered 1, 2, 3, 4, 5, 6. The first player to roll a 6 wins and the game is over. Thoo takes his turn first.

- a) Complete the probability tree diagram of the first 3 turns. 1



- b) What is the probability that Thoo wins on his second turn? 1

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- c) How many turns must be taken in total so that there is a 95% chance of a winner being declared? 2

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End of Exam

Year 11 Mathematics Advanced

Assessment Task 3, 2024

Solutions

Multiple Choice Answer Key

Question	1	2	3	4	5	6	7	8	9	10
Answer	A	B	C	C	B	A	D	B	D	D

Question 1

$$\begin{aligned}5x - (x + 3)(2x - 1) &= 5x - (2x^2 - x + 6x - 3) \\&= 5x - 2x^2 + x - 6x + 3 \\&= 3 - 2x^2\end{aligned}$$

(A)

Question 2

There are many x values for one y value (fails the horizontal line test, passes the vertical line test).

$\therefore$ the function is many-to-one

(B)

Question 3

$$\begin{aligned}64^{4x} &= 16^{3x-1} \\(4^3)^{4x} &= (4^2)^{3x-1} \\4^{12x} &= 4^{6x-2} \\12x &= 6x - 2 \\6x &= -2 \\x &= -\frac{1}{3}\end{aligned}$$

(C)

Question 4

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$d = \sqrt{(7 - (-2))^2 + (1 - 5)^2}$$
$$= \sqrt{97}$$

(C)

Question 5

For the axis of symmetry:

$$x = \frac{-b}{2a}$$
$$= \frac{6}{2 \times (-3)}$$
$$= -1$$

$$\text{When } x = -1, y = 4 - 6(-1) - 3(-1)^2$$
$$= 7$$

$\therefore$ Vertex is $(-1, 7)$

(B)

Question 6

$$|4x - 1| = 7$$

$$4x - 1 = 7 \quad 4x - 1 = -7$$
$$4x = 8 \quad \text{or} \quad 4x = -6$$
$$x = 2 \quad \quad \quad x = -1.5$$

(A)

Question 7

	<i>Walked</i>	<i>Did not walk</i>	<i>Total</i>
<i>Caught Bus</i>	41	29	70
<i>Did not catch bus</i>	4	26	30
<i>Total</i>	45	55	100

$$P(\text{walked and bus}) = 41\% \quad \quad \quad (\text{D})$$

Question 8

Let r be the amount of rent per person and n be the number of people sharing the house

$$r \propto \frac{1}{n}$$

$$r = \frac{k}{n}$$

When $n = 8, r = 135$

$$135 = \frac{k}{8}$$

$$k = 1080$$

When $n = 10,$

$$r = \frac{1080}{10}$$

$$r = 108$$

(B)

Question 9

$$\sqrt{1+4x-x^2} = x-1$$

will have solutions when

$$x-1 \geq 0$$

$$x \geq 1$$

$$1+4x-x^2 = (x-1)^2$$

$$1+4x-x^2 = x^2-2x+1$$

$$2x^2-6x=0$$

$$2x(x-3)=0$$

$$x=0 \text{ or } x=3$$

but solutions exist when $x \geq 1$

$\therefore x = 3$ only

(D)

Question 10

Since $y = x^3$ has been translated 1 unit to the left and 2 units up, equation is

$$y = (x + 1)^3 + 2$$

(D)

Question 11(a)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$y = 2x^4 - 5x^2 + 3$$

$$y' = 8x^3 - 10x$$

Question 11(b)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$y = e^{4x-1}$$

$$y' = 4e^{4x-1}$$

Question 11(c)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$y = \frac{8}{x^4}$$

$$= 8x^{-4}$$

$$y' = -32x^{-5}$$

$$= \frac{-32}{x^5}$$

Question 11(d)

Criteria	Marks
• Provides correct solution	2
• Attempts to use quotient rule	1

Sample answer:

$$\begin{aligned}y &= \frac{2x}{3x-1} \\ \frac{d}{dx} \left(\frac{u}{v} \right) &= \frac{vu' - uv'}{v^2} \\ u &= 2x \quad v = 3x-1 \\ u' &= 2 \quad v' = 3 \\ y' &= \frac{(3x-1) \times 2 - 2x(3)}{(3x-1)^2} \\ &= \frac{6x-2-6x}{(3x-1)^2} \\ &= \frac{-2}{(3x-1)^2}\end{aligned}$$

Question 11(e)

Criteria	Marks
• Provides correct expression in simplified index form	2
• Attempts to use the product rule	1

Sample answer:

$$y = 4x\sqrt{6x-3}$$

$$= 4x(6x-3)^{\frac{1}{2}}$$

$$\frac{d}{dx}(uv) = uv' + vu'$$

$$u = 4x \quad v = (6x-3)^{\frac{1}{2}}$$

$$u' = 4 \quad v' = \frac{1}{2}(6x-3)^{-\frac{1}{2}} \times 6$$

$$= 3(6x-3)^{-\frac{1}{2}}$$

$$y' = 4x \times 3(6x-3)^{-\frac{1}{2}} + 4 \times (6x-3)^{\frac{1}{2}}$$

$$= \frac{12x}{(6x-3)^{\frac{1}{2}}} + 4(6x-3)^{\frac{1}{2}}$$

$$= \frac{12x}{\sqrt{6x-3}} + 4\sqrt{6x-3}$$

Question 12(a)

Criteria	Marks
• Provides correct solution	2
• Correctly factorises with a common factor	1

Sample answer:

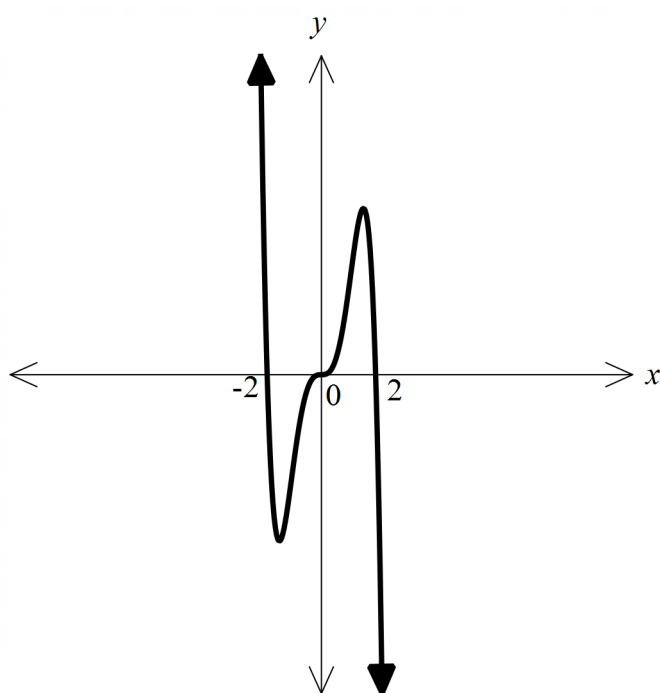
$$4x^3 - x^5 = x^3(4 - x^2)$$

$$= x^3(2-x)(2+x)$$

Question 12(b)

Criteria	Marks
Provides correct solution	2
- Correct graph without x-intercepts labelled OR - Graph contains correct x-intercepts and passes through $(0,0)$ without a horizontal point of inflection OR - Graph contains correct x-intercepts and horizontal point of inflection at $(0,0)$ but as $x \rightarrow \infty, y \rightarrow \infty$ and as $x \rightarrow -\infty, y \rightarrow -\infty$	1

Sample answer:



Question 13

Criteria	Marks
Provides correct solution	2
Correctly multiplies by the conjugate	1

Sample answer:

$$\begin{aligned}
 LHS &= \frac{2\sqrt{6}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} \\
 &= \frac{2\sqrt{18} + 2\sqrt{6}}{2} \\
 &= \frac{6\sqrt{2} + 2\sqrt{6}}{2} \\
 &= 3\sqrt{2} + \sqrt{6}
 \end{aligned}$$

Question 14

Criteria	Marks
• Provides correct solution	2
• Correctly applies one index law	1

Sample answer:

$$\begin{aligned}\frac{(5x^2y)^3 \times 6x^3y^4}{50x^4y^9} &= \frac{125x^6y^3 \times 6x^3y^4}{50x^4y^9} \\ &= 15x^5y^{-2} \\ &= \frac{15x^5}{y^2}\end{aligned}$$

Question 15(a)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$\begin{aligned}\log_a 40 &= \log_a (8 \times 5) \\ &= \log_a 8 + \log_a 5 \\ &= 1.069 + 0.827 \\ &= 1.896\end{aligned}$$

Question 15(b)

Criteria	Marks
• Provides correct solution	2
• Correctly applies one logarithm law	1

Sample answer:

$$\begin{aligned}\log_a \left(\frac{a}{5} \right) &= \log_a a - \log_a 5 \\ &= 1 - 0.827 \\ &= 0.173\end{aligned}$$

Question 16

Criteria	Marks
Provides correct solution	2
- Correctly completes the square to find the equation of the circle correctly OR - Correctly interprets the centre and radius from an incorrect equation if the centre is not the origin.	1

Sample answer:

$$2x^2 + 2y^2 - 12y - 4 = 0$$

$$x^2 + y^2 - 6y - 2 = 0$$

$$x^2 + y^2 - 6y = 2$$

$$x^2 + y^2 - 6y + 9 = 2 + 9$$

$$x^2 + (y - 3)^2 = 11$$

Circle: Centre = (0,3), radius = $\sqrt{11}$

Question 17

Criteria	Marks
Provides correct solution	3
Correctly does two of the following: differentiates to find the gradient function; finds $\frac{dy}{dx}$ when $x = 1$; finds the equation of the tangent	2
Correctly finds one of the following: differentiates to find the gradient function; finds $\frac{dy}{dx}$ when $x = 1$; finds the equation of the tangent	1

Sample answer:

$$y = 8x^3 - 6x^2 + x - 1$$

$$\frac{dy}{dx} = 24x^2 - 12x + 1$$

$$\text{when } x = 1, \frac{dy}{dx} = 24(1)^2 - 12(1) + 1$$

$$m = 13$$

$$\begin{aligned} y &= 8(1)^3 - 6(1)^2 + 1 - 1 \\ &= 2 \end{aligned}$$

$$\text{Equation of tangent} \rightarrow y - y_1 = m(x - x_1)$$

$$y - 2 = 13(x - 1)$$

$$y - 2 = 13x - 13$$

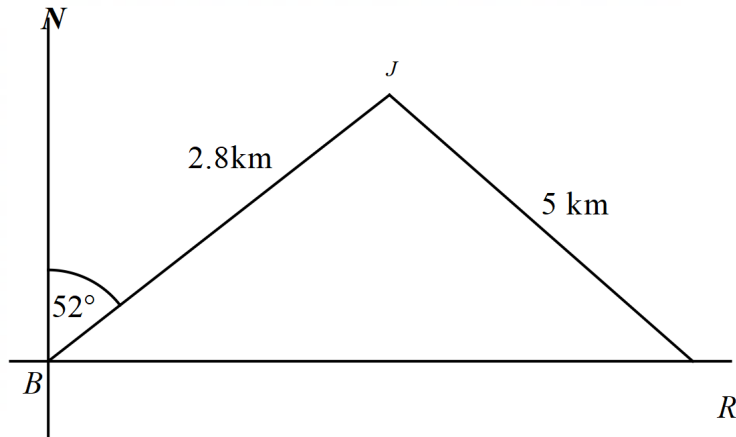
$$y = 13x - 11$$

$$13x - y - 11 = 0$$

Question 18(a)

Criteria	Marks
Provides correct solution	1

Sample answer:



Question 18(b)

Criteria	Marks
Provides correct solution	2
- Correctly finds $\angle ARB$ OR - Correctly finds the bearing of Anika from Big Rock from an incorrect $\angle ARB$	1

Sample answer:

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{2.8}{\sin R} = \frac{5}{\sin 38^\circ}$$

$$2.8 \sin 38^\circ = 5 \sin R$$

$$\sin R = \frac{2.8 \sin 38^\circ}{5}$$

$$\angle ARB = 20.16778236^\circ$$

$\therefore$ Bearing of Anika from Big Rock is $270^\circ + 20^\circ = 290^\circ T$ (nearest degree)

Question 19(a)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$f(x) = \frac{1}{x^3 - 3x}$ will be an odd function if $f(-x) = -f(x)$

$$\begin{aligned} f(-x) &= \frac{1}{(-x)^3 - 3(-x)} \\ &= \frac{1}{-x^3 + 3x} \\ &= \frac{1}{-(x^3 - 3x)} \\ &= -\frac{1}{x^3 - 3x} \\ &= -f(x) \end{aligned}$$

$\therefore f(x) = \frac{1}{x^3 - 3x}$ is an odd function

Question 19(b)

Criteria	Marks
• Provides correct solution	1

Sample answer:

For the domain:

$$\begin{aligned} x^3 - 3x &\neq 0 \\ x(x^2 - 3) &\neq 0 \\ x(x - \sqrt{3})(x + \sqrt{3}) &\neq 0 \\ x &\neq 0, \pm\sqrt{3} \end{aligned}$$

Domain is $x \in (-\infty, -\sqrt{3}) \cup (-\sqrt{3}, 0) \cup (0, \sqrt{3}) \cup (\sqrt{3}, \infty)$ or

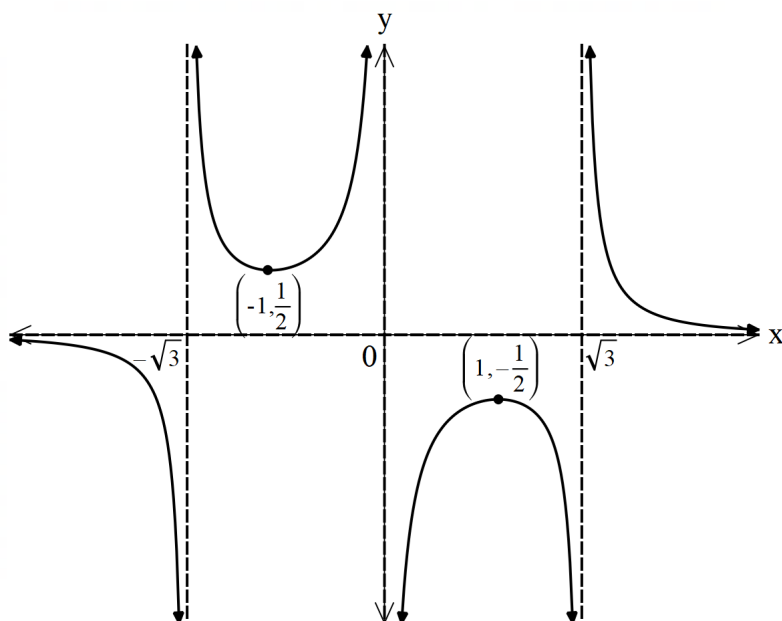
$$\{x \in \mathbb{R}, x \neq 0, \pm\sqrt{3}\} \text{ or}$$

all real x ; $x \neq 0, \pm\sqrt{3}$

Question 19(c)

Criteria	Marks
Provides correct solution	2
- Correctly draws the graph with point symmetry about the origin without labelling asymptotes and minimum turning point OR - Attempts to draw a graph with point symmetry about the origin and labels asymptotes and minimum turning point (ie incorrect scale)	1

Sample answer:



Question 20

Criteria	Marks
Provides correct solution	2
Correctly substitutes into the formula and expands the numerator	1

Sample answer:

$$\begin{aligned}
 \frac{dy}{dx} &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{(x+h)^2 + 6(x+h) - (x^2 + 6x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 + 6x + 6h - x^2 - 6x}{h} \\
 &= \lim_{h \rightarrow 0} \frac{2xh + h^2 + 6h}{h} \\
 &= \lim_{h \rightarrow 0} (2x + h + 6) \\
 &= 2x + 6
 \end{aligned}$$

Question 21(a)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$
$$M = \left(\frac{(-3) + (3)}{2}, \frac{(-3) + 1}{2} \right)$$
$$= (0, -1)$$

Question 21(b)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m_{AB} = \frac{1 - (-3)}{3 - (-3)}$$
$$= \frac{2}{3}$$

Question 21(c)

Criteria	Marks
• Provides correct solution	1

Sample answer:

If two lines are perpendicular, then $m_1 m_2 = -1$

$$\therefore m_{CM} = \frac{-3}{2}$$

Question 21(d)

Criteria	Marks
• Provides correct solution	2
• Correctly finds the distance of AB or area of $\triangle ABC$	1

Sample answer:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AB = \sqrt{(3 - (-3))^2 + (1 - (-3))^2}$$

$$= \sqrt{52}$$

$$\therefore \text{Area of } \triangle ABC = \frac{1}{2} \times \sqrt{52} \times \sqrt{13}$$

$$= 13$$

Question 22(a)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$f(x) = \sqrt{x-4}$$

$$f(5) = \sqrt{5-4}$$

$$= 1$$

Question 22(b)

Criteria	Marks
• Provides correct solution	1

$$g(f(x)) = g(\sqrt{x-4})$$

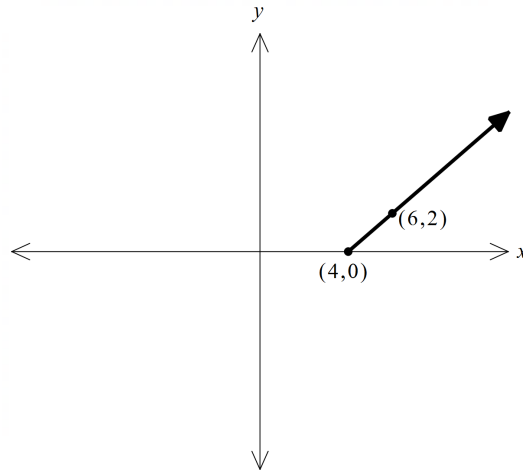
$$= (\sqrt{x-4})^2$$

$$= x-4 \text{ for } x \geq 4$$

Question 22(c)

Criteria	Marks
• Provides correct solution	2
• Draws a straight line through (4, 0) with positive gradient, ignoring the domain	1

Sample answer:



Question 23(a)

Criteria	Marks
• Provides correct solution	2
• Correctly forms a pair of simultaneous equations	1

Sample answer:

$$0.4 + 0.15 + a + b = 1$$

$$a + b = 0.45 \rightarrow (1)$$

$$0 \times 0.4 + 1 \times 0.15 + 2a + 3b = 1.13$$

$$2a + 3b = 0.98 \rightarrow (2)$$

$$2 \times (1) \rightarrow 2a + 2b = 0.9 \rightarrow (3)$$

$$(2) - (3) \rightarrow b = 0.08$$

$$\text{sub } b = 0.08 \text{ into } (1) \rightarrow a + 0.08 = 0.45$$

$$a = 0.37$$

Question 23(b)

Criteria	Marks
• Provides correct solution	2
• Correctly finds $E(X^2)$ or $(x - \mu)^2 p(x)$ for each score OR	1
• Correctly finds $E(X^2) - [E(X)]^2$ or $\sum (x - \mu)^2 p(x)$ from previous error	

Sample answer:

$$\text{Var}(X) = E(X^2) - [E(X)]^2$$

$$\begin{aligned} E(X^2) &= 0^2 \times 0.4 + 1^2 \times 0.15 + 2^2 \times 0.37 + 3^2 \times 0.08 \\ &= 3.2 \end{aligned}$$

$$\begin{aligned} \text{Var}(X) &= 3.2 - 1.13^2 \\ &= 1.0731 \end{aligned}$$

Question 23(c)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$\begin{aligned} P(X = 0 \mid X \leq 2) &= \frac{P(X = 0)}{P(X \leq 2)} \\ &= \frac{0.4}{0.92} \\ &= 0.4347826087 \\ &= 43.5\% \text{ (1d.p.)} \end{aligned}$$

Question 24

Criteria	Marks
• Provides correct solution	3
• Correctly factorises the quadratic equation and substitutes $\sin^2 t = 1 - \cos^2 t$	2
• Substitutes $\sin^2 t = 1 - \cos^2 t$	1

Sample answer:

$$2 \sin^2 t = 3 \cos t$$

$$2(1 - \cos^2 t) = 3 \cos t \quad \text{since } \sin^2 \theta + \cos^2 \theta = 1$$

$$2 - 2 \cos^2 t = 3 \cos t$$

$$2 \cos^2 t + 3 \cos t - 2 = 0$$

$$(\cos t + 2)(2 \cos t - 1) = 0$$

$$\cos t + 2 = 0 \quad \text{or} \quad 2 \cos t - 1 = 0$$

$$\cos t = -2 \quad \text{or} \quad \cos t = \frac{1}{2}$$

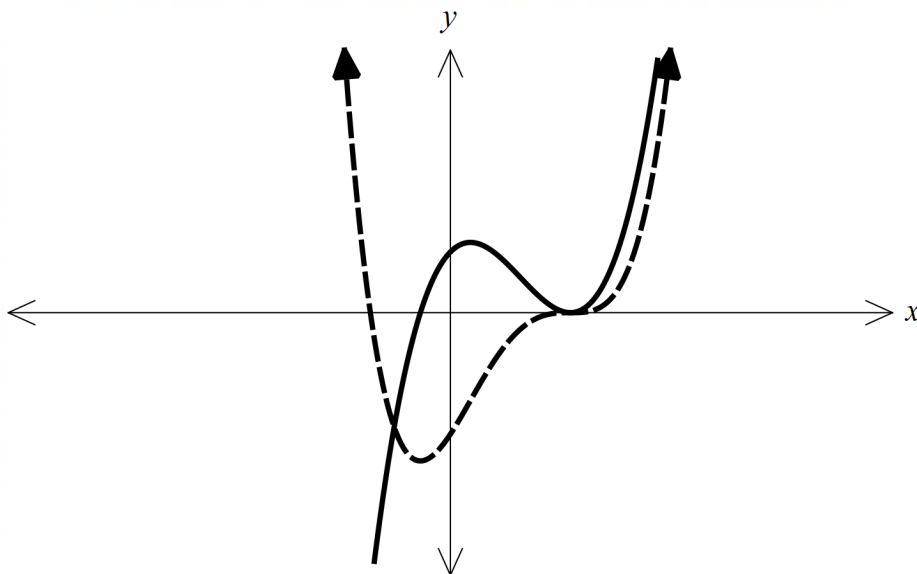
$$\text{No solutions} \quad \text{or} \quad t = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$$

$$\therefore t = \frac{\pi}{3} \text{ or } \frac{5\pi}{3}$$

Question 25

Criteria	Marks
• Provides correct solution	2
• Correctly finds x intercepts	1

Sample answer:



Question 26(a)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$\begin{aligned}EADF &= 2 \times \frac{\pi}{5} \times 22 + 72 \\&= 99.64601535 \\&= 99.65\text{m (2d.p.)}\end{aligned}$$

Question 26(b)

Criteria	Marks
• Provides correct solution	2
• Correctly finds the area of the sector of the circle	1

Sample answer:

$$\begin{aligned}\text{Area} &= 2 \times \frac{1}{2} \times 22^2 \times \frac{\pi}{5} + 72 \times 22 \\&= 1888.106169 \\&= 1888.1\text{m}^2 \text{ (2d.p.)}\end{aligned}$$

Question 27

Criteria	Marks
• Provides correct solution	2
• Correctly uses a trigonometric identity	1

Sample answer:

$$\begin{aligned}LHS &= \cos \operatorname{ec}^2 x (\tan^2 x - \sin^2 x) \\&= \frac{1}{\sin^2 x} \left(\frac{\sin^2 x}{\cos^2 x} - \sin^2 x \right) \\&= \frac{1}{\cos^2 x} - 1 \\&= \sec^2 x - 1 \\&= 1 + \tan^2 x - 1 \\&= \tan^2 x \\&= RHS\end{aligned}$$

Question 28(a)

Criteria	Marks
Provides correct solution	1

Sample answer:

$$\begin{aligned}\angle MWG &= 300^\circ - 180^\circ \\ &= 120^\circ\end{aligned}$$

Question 28(b)

Criteria	Marks
Provides correct solution	1

$$\begin{aligned}\tan 27^\circ &= \frac{h}{MW} \\ MW &= \frac{h}{\tan 27^\circ} \\ MW &= h \cot 27^\circ \\ &= h \tan 63^\circ \quad (\text{since } \cot \theta = \tan(90^\circ - \theta))\end{aligned}$$

Question 28(c)

Criteria	Marks
Provides correct solution	3
Correctly substitutes into the cosine rule	2
Writes an equation for GW	1

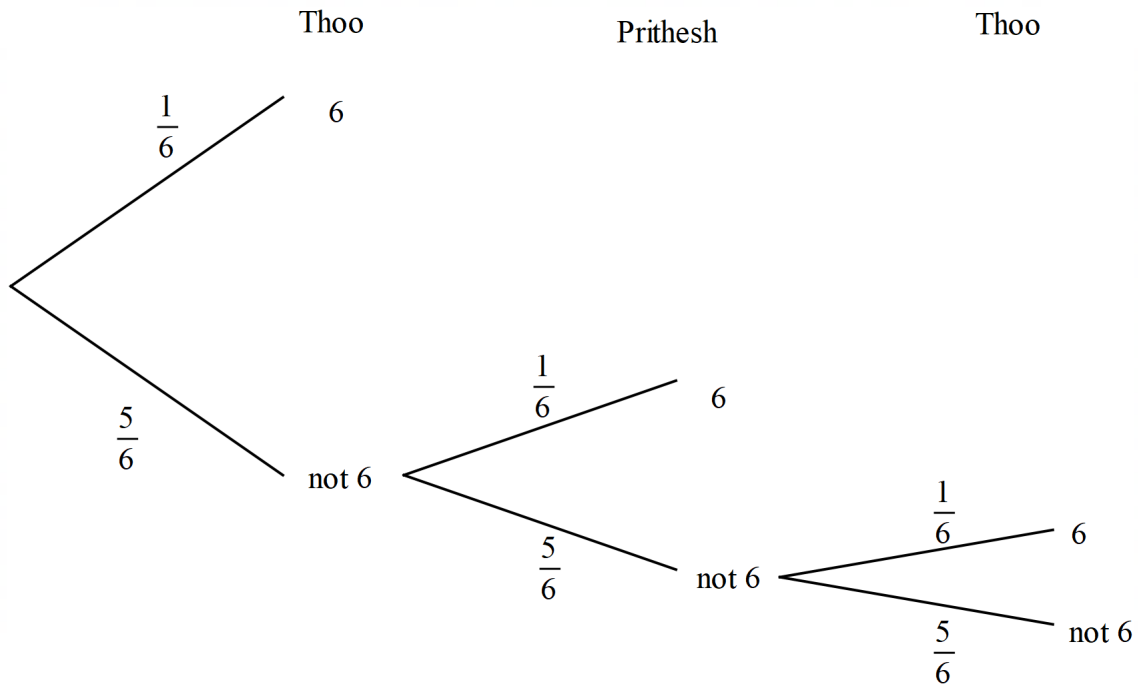
Sample answer:

$$\begin{aligned}\tan 25^\circ &= \frac{h}{GW} \\ GW &= \frac{h}{\tan 25^\circ} \\ &= h \tan 65^\circ \\ 160^2 &= (h \tan 63^\circ)^2 + (h \tan 65^\circ)^2 - 2 \times h \tan 63^\circ \times h \tan 65^\circ \cos 120^\circ \\ 160^2 &= h^2 (\tan^2 63^\circ + \tan^2 65^\circ - 2 \tan 63^\circ \tan 65^\circ \cos 120^\circ) \\ h^2 &= \frac{160^2}{(\tan^2 63^\circ + \tan^2 65^\circ - 2 \tan 63^\circ \tan 65^\circ \cos 120^\circ)} \\ &= 2022.183712 \\ h &= 44.96869703 \\ &= 45\text{m (to the nearest metre)}\end{aligned}$$

Question 29(a)

Criteria	Marks
Provides correct solution	1

Sample answer:



Question 29 (b)

Criteria	Marks
Provides correct solution	1

Sample answer:

$$\begin{aligned}
 P(\text{Thoo wins on 2nd turn}) &= \left(\frac{5}{6}\right)^2 \times \frac{1}{6} \\
 &= \frac{25}{216}
 \end{aligned}$$

Question 29 (c)

Criteria	Marks
• Provides correct solution	2
• Correctly writes an equation to find n	1

Sample answer:

$$P(\text{winner}) = 1 - P(\text{not a winner})$$

$$0.95 = 1 - \left(\frac{5}{6}\right)^n$$

$$-0.05 = -\left(\frac{5}{6}\right)^n$$

$$0.05 = \left(\frac{5}{6}\right)^n$$

$$\ln 0.005 = n \ln \left(\frac{5}{6}\right)$$

$$n = \frac{\ln 0.005}{\ln \left(\frac{5}{6}\right)}$$

$$n = 16.43103715$$

$\therefore$ 17 turns must be taken in total