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Blacktown Boys' High School

Year 11 2020

Yearly Examination

Mathematics Advanced

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**General
Instructions**

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided for this paper
- In Questions in Section II, show all relevant mathematical reasoning and/or calculations

**Total marks:
70**

Section I – 10 marks (pages 3-6)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 8-21)

- Attempt Questions 11 – 28
- Allow about 1 hour 45 minutes for this section

Assessors: Villanueva

Section I

10 marks

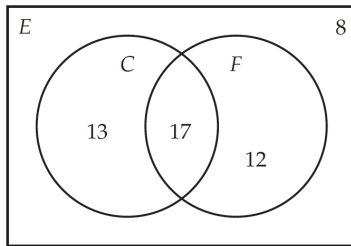
Attempt Questions 1–10

Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1–10.

Use the information and diagram provided below to answer questions 1 and 2.

The following Venn diagram shows $E = \{\text{people at a party}\}$, $C = \{\text{people who came in costume}\}$ and $F = \{\text{people who brought food}\}$.



1 One person from the people at the party is selected at random. Find $P(C)$.

- A. $\frac{13}{50}$
- B. $\frac{3}{5}$
- C. $\frac{13}{42}$
- D. $\frac{30}{42}$

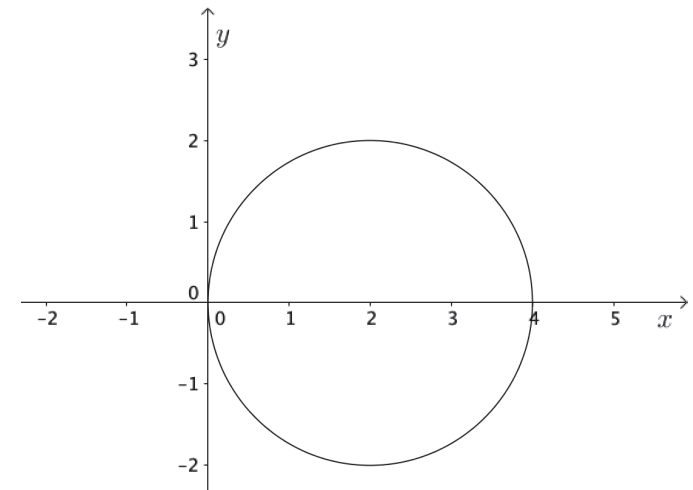
2 One person from the people at the party is selected at random. Find $P(C|F)$.

- A. $\frac{13}{50}$
- B. $\frac{30}{42}$
- C. $\frac{17}{29}$
- D. $\frac{17}{30}$

3 Which one of the following straight lines is perpendicular to the line $2x + y - 4 = 0$?

- A. $x - 2y - 4 = 0$
- B. $x + 2y - 4 = 0$
- C. $2x - y - 4 = 0$
- D. $2x + y + 4 = 0$

4 The equation of the circle below is:



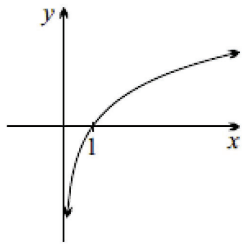
- A. $x^2 + (y - 2)^2 = 4$
- B. $(x - 2)^2 + y^2 = 16$
- C. $(x + 2)^2 + y^2 = 16$
- D. $(x - 2)^2 + y^2 = 4$

5 What is the natural domain of the function $y = \sqrt{1-x}$?

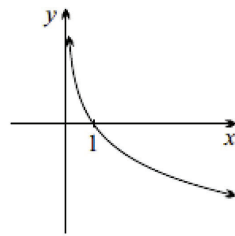
- A. $x \leq 1$
- B. $x > 1$
- C. $x \geq 1$
- D. $x < 1$

6 Which of the following could be the graph of $y = -\log_2(-x)$?

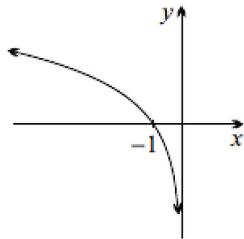
A.



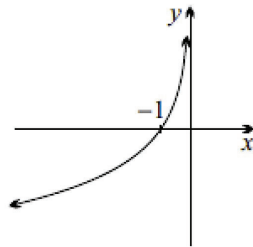
B.



C.



D.



7 Simplify $\frac{3^{4x} \times 4^{-2y}}{3^{-2x} \times 4^{3y}}$ using index laws.

- A. $3^{-6x} 4^{-5y}$
- B. $3^{6x} 2^{-10y}$
- C. $3^{6x} 4^{5y}$
- D. $3^{2x} 2^{5y}$

8 Given that $f(x) = 2x + 3$ and $g(x) = 5x + b$. Find the value of b so that:

$$g(f(x)) = f(g(x))$$

- A. 2
- B. 4
- C. 8
- D. 12

9 It is given that $\ln m = n \ln p - \ln q - \ln e$, where $m, n, p, q > 0$. Which of the following statements is true?

- A. $m = \frac{p^n}{\ln eq}$
- B. $m = \frac{p^n}{eq}$
- C. $m = np - q - 1$
- D. $m = \frac{ep^n}{q}$

10 Which of the following functions has $3x^5 - 8$ as a derivative function?

- A. $15x^4$
- B. $15x^5 - 8x$
- C. $\frac{1}{2}x^6 - 8x$
- D. $2x^6$

End of Section I

Mathematics Advanced

Section II Answer Booklet

60 marks

Attempt Questions 11-28

Allow about 1 hour 45 minutes for this section

Instructions

- Answer the Questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your response should include relevant mathematical reasoning and/or calculations
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Question 11 (2 marks)

Rationalise the denominator and simplify $\frac{2\sqrt{3}}{2\sqrt{2}+3}$ completely.

2

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Question 12 (4 marks)

(a) Solve $2^{x-1} = 32$

1

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(b) Solve $\log_3 x = 4$

1

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(c) Given that $3 \log_x 2 - \log_x 128 = 4$, find the value of x .

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Question 13 (3 marks)

The probability distribution of random variable X is shown below.

x	1	2	3	4	5
$P(X = x)$	0.1	a	b	0.4	0.2

Find the values a and b if $E(X) = 3.5$

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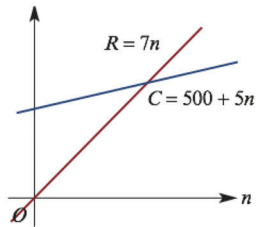
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Question 14 (1 mark)

A farmer grows and sells mushrooms. The graph below shows R the revenue, in dollars, earned by the farmer selling n kilograms of mushrooms, and the cost C , in dollars, of producing n kilograms of mushrooms. Calculate the number of kilograms of mushrooms the farmer must produce to break-even.



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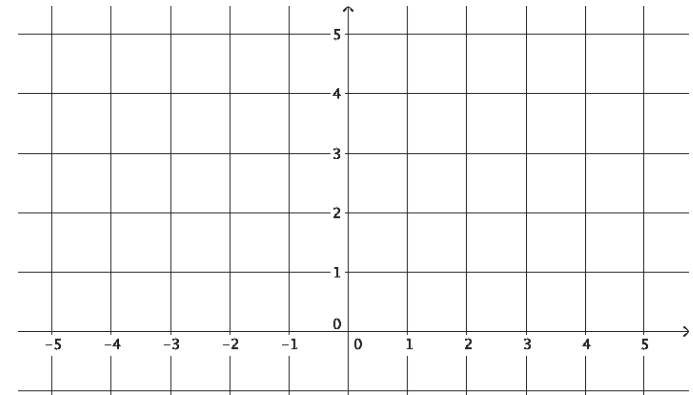
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Question 15 (4 marks)

- (a) Sketch the graph of $y = |3 - 2x|$ on the axis below showing all key features. 2



- (b) Sketch $y = 3$ on the same set of axes above. 1

- (c) Hence, solve for x for which $|3 - 2x| = 3$. 1

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Question 16 (2 marks)

- Solve $\frac{1}{2^x} - 5 = 10$. Round your answer to 3 significant figures. 2

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In a dog park there are seven Labradors, twelve German Shepherds and three Beagles. Two dogs are selected at random without replacement.

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Differentiate the following, with respect to x . Leave your answer in simplest form.

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Given that $f(x) = 3x^2 + 5x - 10$, show that $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = 6x + 5$

[illegible]

Show that $\frac{d}{dx}(x\sqrt{x+3}) = \frac{3(x+2)}{2\sqrt{x+3}}$

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

(a) Show that the gradient of the curve $y = \frac{3x^2 - 4}{3 - 2x}$ at the point $(2, -8)$

is 4.

[illegible]

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Question 22 (7 marks)

The displacement of an object in cm over time t seconds is given by $x = (4t^2 + 2)^3$.

- (a) Find the initial displacement. 1

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- (b) Find the initial velocity. 2

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- (c) What was the instantaneous velocity after 3 seconds? 1

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- (d) What was the average velocity for the first 3 seconds? 1

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- (e) What is the acceleration after 1 second? 2

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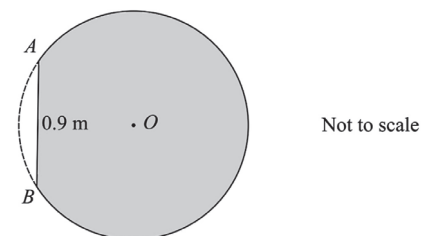
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Question 23 (4 marks)

A small segment of a circle has been removed as shown below. The circle has a centre O and radius 0.9 metres. The length of the straight edge AB is also 0.9 metres.



- (a) Explain why $\angle AOB = \frac{\pi}{3}$ 1

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- (b) What is the area of the shaded region? Answer correct to two decimal places. 3

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Question 24 (2 marks)

Three coins are tossed simultaneously. Determine the probability of getting at least 2 heads. Explain your answer.

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Question 25 (2 marks)

The Richter scale for earthquakes is logarithmic. What is the difference in magnitude between the earthquake in Coral Sea and Coffs Harbour? Give your answer correct to the nearest whole number.

Year	Location	Strength on Richter scale
2015	Coral Sea Qld	5.5
2018	Coffs Harbour NSW	4.2

2

Question 26 (3 marks)

Prove that $\frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = 2 \sec A$

3

Question 27 (5 marks)

- (a) Solve $2 \sin x - \sqrt{3} = 0$ for $0 \leq x \leq 2\pi$. Leave your solution in exact form.

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- (b) Solve $\tan^2 \theta - \tan \theta - 2 = 0$ for $0 \leq x \leq 2\pi$. Round your solution to 2 decimal places where necessary.

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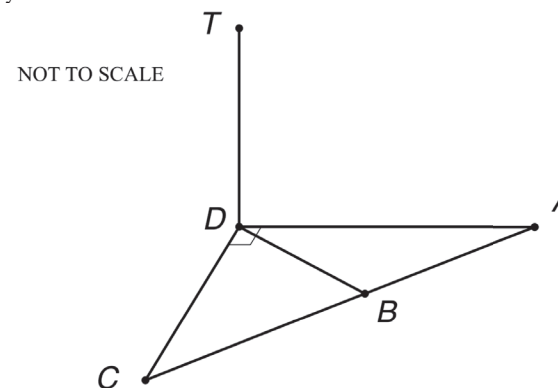
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Question 28 (3 marks)

Three surveyors A , B , and C are observing a tower of height h situated on flat ground. A is due east of the tower, C is due south of it and B is somewhere between A and C on the line AC . The tower's base is at D and its top is at T . The angles of elevation of the top of the tower from A , B , and C are 17° , 20° and 24° respectively. Note that $\angle ABD$ is obtuse.



Find the bearing of B from the base of the tower D .

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Student Name: _____

Multiple Choice Answer Sheet

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9
 A ☐ B ☒ C ☐ D ☐

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ☒ B ☒ C ☐ D ☐

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word 'correct' and drawing an arrow as follows.

A ☒ B ☒ C ☐ D ☐
 correct

**Start
Here** →

- | | | | | |
|-----|-------------------------|-------------------------|-------------------------|-------------------------|
| 1. | A ☐ | B ☐ | C ☐ | D ☐ |
| 2. | A ☐ | B ☐ | C ☐ | D ☐ |
| 3. | A ☐ | B ☐ | C ☐ | D ☐ |
| 4. | A ☐ | B ☐ | C ☐ | D ☐ |
| 5. | A ☐ | B ☐ | C ☐ | D ☐ |
| 6. | A ☐ | B ☐ | C ☐ | D ☐ |
| 7. | A ☐ | B ☐ | C ☐ | D ☐ |
| 8. | A ☐ | B ☐ | C ☐ | D ☐ |
| 9. | A ☐ | B ☐ | C ☐ | D ☐ |
| 10. | A ☐ | B ☐ | C ☐ | D ☐ |

**Blacktown Boys' High School****Year 11 2020****Yearly Examination****Mathematics Advanced****General
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**Total marks:
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Assessors: Villanueva

Section I

10 marks

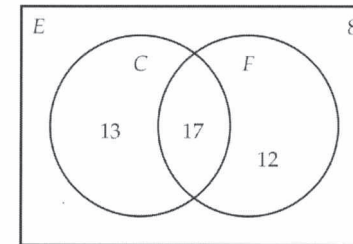
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The following Venn diagram shows $E = \{\text{people at a party}\}$,
 $C = \{\text{people who came in costume}\}$ and $F = \{\text{people who brought food}\}$.



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1 One person from the people at the party is selected at random. Find $P(C)$.

A. $\frac{13}{50}$ $P(C) = \frac{13+17}{13+17+12+8}$

☒ B. $\frac{3}{5}$ $= \frac{30}{50}$

C. $\frac{13}{42}$ $= \frac{3}{5}$

D. $\frac{30}{42}$

2 One person from the people at the party is selected at random. Find $P(C|F)$.

A. $\frac{13}{50}$ $P(C|F) = \frac{n(C \cap F)}{n(F)}$

B. $\frac{30}{42}$ $= \frac{17}{17+12}$

☒ C. $\frac{17}{29}$ $= \frac{17}{29}$

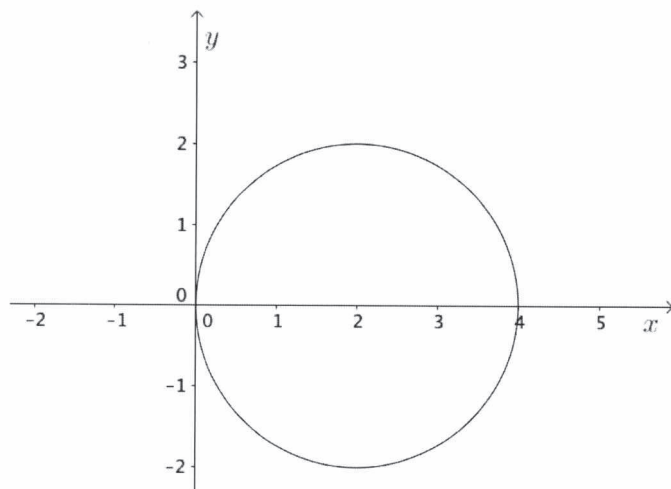
D. $\frac{17}{30}$

- 3 Which one of the following straight lines is perpendicular to the line $2x + y - 4 = 0$?

(A) $x - 2y - 4 = 0$ $2x + y - 4 = 0$
 B. $x + 2y - 4 = 0$ $y = -2x + 4$
 C. $2x - y - 4 = 0$ $m_1 = -2$
 D. $2x + y + 4 = 0$ $\therefore m_2 = \frac{1}{2}$

Option A $x - 2y - 4 = 0$
 $2y = x - 4$
 $y = \frac{x}{2} - 2$
 $\therefore m = m_2$

- 4 The equation of the circle below is:

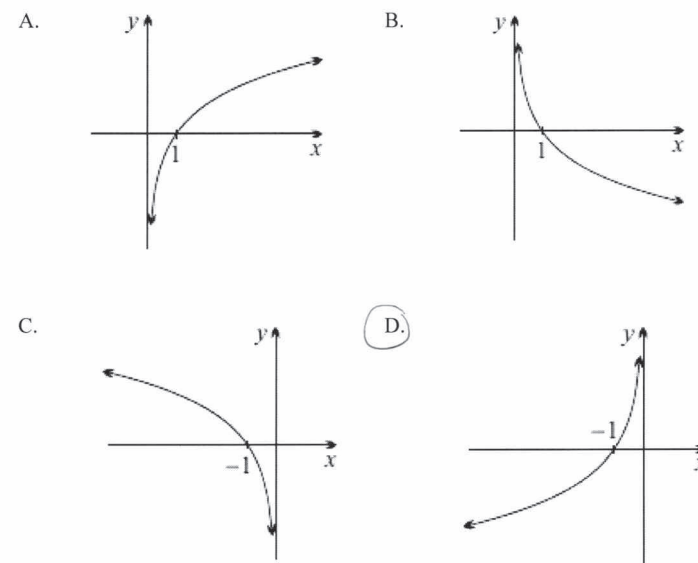


- A. $x^2 + (y - 2)^2 = 4$
 B. $(x - 2)^2 + y^2 = 16$
 C. $(x + 2)^2 + y^2 = 16$
 (D) $(x - 2)^2 + y^2 = 4$

- 5 What is the natural domain of the function $y = \sqrt{1 - x}$?

(A) $x \leq 1$ $1 - x \geq 0$
 B. $x > 1$ $-x \geq -1$
 C. $x \geq 1$ $x \leq 1$
 D. $x < 1$

- 6 Which of the following could be the graph of $y = -\log_2(-x)$?



- 7 Simplify $\frac{3^{4x} \times 4^{-2y}}{3^{-2x} \times 4^{3y}}$ using index laws.

A. $3^{-6x} 4^{-5y}$ $3^{4x} \div 3^{-2x} = 3^{6x}$
 (B) $3^{6x} 2^{-10y}$ $4^{-2y} \div 4^{3y} = 4^{-5y}$
 C. $3^{6x} 4^{5y}$ $= (2^2)^{-5y}$
 D. $3^{2x} 2^{5y}$ $= 2^{-10y}$

- 8 Given that $f(x) = 2x + 3$ and $g(x) = 5x + b$. Find the value of b so that:

$$g(f(x)) = f(g(x))$$

A. 2

B. 4

C. 8

☒ D. 12

$$5(2x+3)+b = 2(5x+b)+3$$

$$10x+15+b = 10x+2b+3$$

$$12 = b$$

- 9 It is given that $\ln m = n \ln p - \ln q - \ln e$, where $m, n, p, q > 0$. Which of the following statements is true?

A. $m = \frac{p^n}{\ln eq}$

$$\ln m = \ln\left(\frac{p^n}{eq}\right)$$

☒ B. $m = \frac{p^n}{eq}$

$$m = \frac{p^n}{eq}$$

C. $m = np - q - 1$

D. $m = \frac{ep^n}{q}$

- 10 Which of the following functions has $3x^5 - 8$ as a derivative function?

A. $15x^4$

B. $15x^5 - 8x$

☒ C. $\frac{1}{2}x^6 - 8x$

D. $2x^6$

End of Section I

Mathematics Advanced

Section II Answer Booklet

60 marks

Attempt Questions 11-28

Allow about 1 hour 45 minutes for this section

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Question 11 (2 marks)

Rationalise the denominator and simplify $\frac{2\sqrt{3}}{2\sqrt{2}+3}$ completely.

$$\frac{2\sqrt{3}}{2\sqrt{2}+3} \times \frac{2\sqrt{2}-3}{2\sqrt{2}-3} = \frac{4\sqrt{6}-6\sqrt{3}}{8-9}$$

$$= \frac{4\sqrt{6}-6\sqrt{3}}{-1}$$

$$= 6\sqrt{3}-4\sqrt{6}$$

Question 12 (4 marks)

(a) Solve $2^{x-1} = 32$

$$2^{x-1} = 2^5 \quad \text{OR} \quad \ln 2^{x-1} = \ln 32$$

$$x-1 = 5 \quad x-1 = \ln 32 / \ln 2$$

$$x = 6 \quad x = 5+1$$

$$x = 6$$

(b) Solve $\log_3 x = 4$

$$x = 3^4$$

$$x = 81$$

(c) Given that $3 \log_x 2 - \log_x 128 = 4$, find the value of x .

$$\log_x x^3 - \log_x 128 = 4$$

$$\log_x x^{\frac{2^3}{128}} = 4$$

$$\log_x x^{\frac{1}{16}} = 4$$

$$x^4 = \frac{1}{16}$$

$$x = \pm \frac{1}{2} \quad x = \frac{1}{2}, x > 0$$

Question 13 (3 marks)

The probability distribution of random variable X is shown below.

x	1	2	3	4	5
$P(X=x)$	0.1	a	b	0.4	0.2

Find the values a and b if $E(X) = 3.5$

$$0.1 + a + b + 0.4 + 0.2 = 1$$

$$a + b = 0.3 \quad (1)$$

$$(1 \times 0.1) + (2 \times a) + (3 \times b) + (4 \times 0.4) + (5 \times 0.2) = 3.5$$

$$0.1 + 2a + 3b + 1.6 + 1 = 3.5$$

$$2a + 3b = 0.8 \quad (2) \quad (2) - (1a)$$

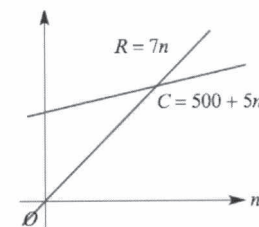
$$-2a + 2b = 0.6 \quad (1a)$$

$$b = 0.2$$

$$a + 0.2 = 0.3 \quad \therefore a = 0.1, b = 0.2$$

Question 14 (1 mark)

A farmer grows and sells mushrooms. The graph below shows R the revenue, in dollars, earned by the farmer selling n kilograms of mushrooms, and the cost C , in dollars, of producing n kilograms of mushrooms. Calculate the number of kilograms of mushrooms the farmer must produce to break-even.



$$7n = 500 + 5n$$

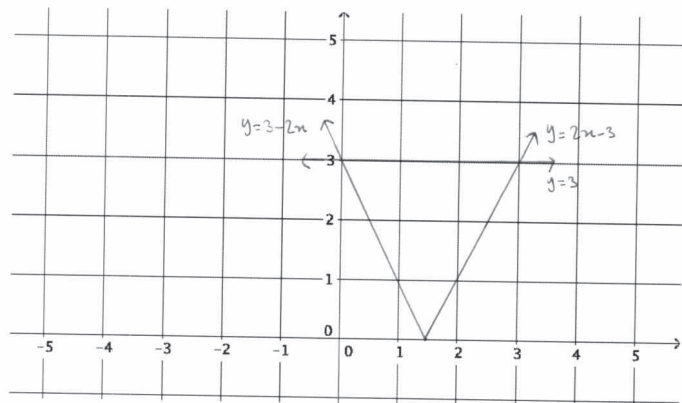
$$2n = 500$$

$$n = 250$$

$\therefore$ needs to produce 250 kg of mushrooms to break-even.

Question 15 (4 marks)

- (a) Sketch the graph of $y = |3 - 2x|$ on the axis below showing all key features.



- (b) Sketch $y = 3$ on the same set of axes above.

- (c) Hence, solve for x for which $|3 - 2x| = 3$.

$x = 0, x = 3$

Question 16 (2 marks)

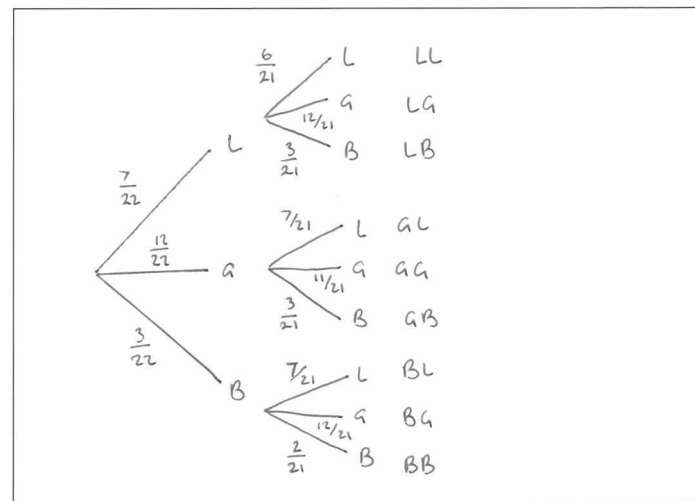
Solve $\frac{1}{2^x} - 5 = 10$. Round your answer to 3 significant figures.

$\frac{1}{2^x} = 15$
 $2^{-x} = 15$
 $\ln 2^{-x} = \ln 15$
 $-x \ln 2 = \ln 15$
 $-x = \frac{\ln 15}{\ln 2}$
 $x = -\frac{\ln 15}{\ln 2}$

Question 17 (5 marks)

In a dog park there are seven Labradors, twelve German Shepherds and three Beagles. Two dogs are selected at random without replacement.

- (a) Draw a tree diagram, including a list of all the outcomes, to represent the information.



- (b) What is the probability of selecting two Beagles?

$$\frac{3}{22} \times \frac{2}{21} = \frac{1}{77}$$

- (c) What is the probability that neither of the dogs selected are Labradors?

$$\begin{aligned} &= P(GG) + P(GB) + P(BG) + P(BB) \\ &= \left(\frac{12}{22} \times \frac{11}{21}\right) + \left(\frac{12}{22} \times \frac{3}{21}\right) + \left(\frac{3}{22} \times \frac{12}{21}\right) + \left(\frac{3}{22} \times \frac{2}{21}\right) \\ &= \frac{5}{11} \end{aligned}$$

Question 18 (4 marks)

Differentiate the following, with respect to x . Leave your answer in simplest form.

(a) $y = 4x^3 - \frac{7}{x} + 3$

2

$$y = 4x^3 - 7x^{-1} + 3$$

$$\frac{dy}{dx} = 12x^2 + 7x^{-2}$$

$$\frac{dy}{dx} = 12x^2 + \frac{7}{x^2}$$

(b) $y = 8e^{-\frac{x}{4}} - 5x$

2

$$y = 8e^{-\frac{1}{4}x} - 5x$$

$$\frac{dy}{dx} = \left(-\frac{1}{4}\right) 8e^{-\frac{1}{4}x} - 5$$

$$\frac{dy}{dx} = -2e^{-\frac{x}{4}} - 5$$

Question 19 (2 marks)

Given that $f(x) = 3x^2 + 5x - 10$, show that $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = 6x + 5$

2

$$f(x+h) = 3(x+h)^2 + 5(x+h) - 10$$

$$= 3(x^2 + 2xh + h^2) + 5x + 5h - 10$$

$$= 3x^2 + 6xh + 3h^2 + 5x + 5h - 10$$

$$\lim_{h \rightarrow 0} \frac{3x^2 + 6xh + 3h^2 + 5x + 5h - 10 - (3x^2 + 5x - 10)}{h}$$

$$\lim_{h \rightarrow 0} \frac{6xh + 3h^2 + 5h}{h}$$

$$= \lim_{h \rightarrow 0} 6x + 3h + 5$$

$$= 6x + 5$$

Question 20 (3 marks)

Show that $\frac{d}{dx}(x\sqrt{x+3}) = \frac{3(x+2)}{2\sqrt{x+3}}$

3

$$u = x \quad v = (x+3)^{1/2}$$

$$u' = 1 \quad v' = \frac{1}{2}(x+3)^{-1/2}$$

$$\frac{d}{dx} = \frac{x}{2\sqrt{x+3}} + \sqrt{x+3}$$

$$= \frac{x + 2(x+3)}{2\sqrt{x+3}}$$

$$= \frac{x + 2x + 6}{2\sqrt{x+3}}$$

$$= \frac{3x + 6}{2\sqrt{x+3}}$$

$$= \frac{3(x+2)}{2\sqrt{x+3}}$$

$$= \text{RHS}$$

Question 21 (4 marks)

- (a) Show that the gradient of the curve $y = \frac{3x^2 - 4}{3 - 2x}$ at the point $(2, -8)$

is 4.

$$\frac{dy}{dx} = \frac{v u' - u v'}{v^2}$$

$$u = 3x^2 - 4 \quad v = 3 - 2x$$

$$u' = 6x \quad v' = -2$$

$$\frac{dy}{dx} = \frac{6x(3-2x) - (-2)(3x^2-4)}{(3-2x)^2}$$

$$\text{at } x=2 \quad \frac{dy}{dx} = \frac{6(2)(3-2(2)) - (-2)(3(2)^2-4)}{(3-2(2))^2}$$

$$= 4$$

- (b) Hence, find the equation of the tangent at the point $(2, -8)$.

$$y + 8 = 4(x - 2)$$

$$y + 8 = 4x - 8$$

$$y = 4x - 16$$

$$4x - y - 16 = 0$$

3

Question 22 (7 marks)

The displacement of an object in cm over time t seconds is given by $x = (4t^2 + 2)^3$.

- (a) Find the initial displacement.

$$t=0, \quad x = (4(0)^2 + 2)^3$$

$$x = 8$$

- (b) Find the initial velocity.

$$\dot{x} = 8t \times 3(4t^2 + 2)^2$$

$$\dot{x} = 24t(4t^2 + 2)^2$$

$$t=0, \quad \dot{x} = 24(0)(4(0)^2 + 2)^2$$

$$\dot{x} = 0$$

- (c) What was the instantaneous velocity after 3 seconds?

$$\dot{x} = 24(3)(4(3)^2 + 2)^2$$

$$= 103968 \text{ cm/s}$$

- (d) What was the average velocity for the first 3 seconds?

$$\text{Average} = \frac{(4 \times 3^2 + 2)^3 - 8}{3 - 0}$$

$$= 18288 \text{ cm/s}$$

- (e) What is the acceleration after 1 second?

$$u = 24t \quad v = (4t^2 + 2)^2$$

$$u' = 24 \quad v' = 8t \times 2(4t^2 + 2)$$

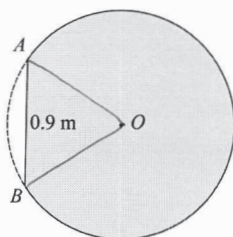
$$\ddot{x} = 24t \times 16t(4t^2 + 2) + 24(4t^2 + 2)^2$$

$$= 384t^2(4t^2 + 2) + 24(4t^2 + 2)^2$$

$$t=1, \quad \ddot{x} = 3168 \text{ cm/s}^2$$

Question 23 (4 marks)

A small segment of a circle has been removed as shown below. The circle has a centre O and radius 0.9 metres. The length of the straight edge AB is also 0.9 metres.



Not to scale

- (a) Explain why $\angle AOB = \frac{\pi}{3}$

1

$$OA = OB = AB = 0.9 \text{ m}$$

Equal sides in a triangle $\rightarrow$ equilateral

All angles are 60°

$$\therefore \angle AOB = \frac{\pi}{3}$$

- (b) What is the area of the shaded region? Answer correct to two decimal places.

3

$$\text{Area of minor segment} = \frac{1}{2} r^2 (\theta - \sin \theta)$$

$$= \frac{1}{2} (0.9)^2 \left(\frac{\pi}{3} - \sin \frac{\pi}{3} \right)$$

$$= \frac{0.81}{2} \left(\frac{\pi}{3} - \frac{\sqrt{3}}{2} \right)$$

$$\text{Area of circle} = \pi (0.9)^2$$

$$\text{Shaded region} = \text{circle} - \text{minor segment}$$

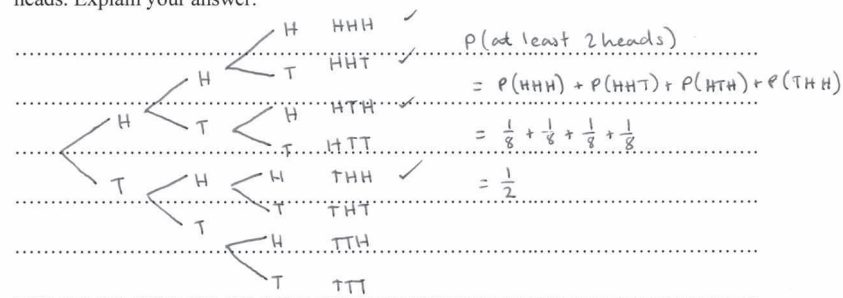
$$= \pi (0.9)^2 - \left(\frac{0.81}{2} \left(\frac{\pi}{3} - \frac{\sqrt{3}}{2} \right) \right)$$

$$= 2.47 \text{ m}^2$$

Question 24 (2 marks)

Three coins are tossed simultaneously. Determine the probability of getting at least 2 heads. Explain your answer.

2



Question 25 (2 marks)

The Richter scale for earthquakes is logarithmic. What is the difference in magnitude between the earthquake in Coral Sea and Coffs Harbour? Give your answer correct to the nearest whole number.

2

Year	Location	Strength on Richter scale
2015	Coral Sea Qld	5.5
2018	Coffs Harbour NSW	4.2

$$5.5 - 4.2 = 1.3$$

$$10^{1.3} = 19.9526 \dots$$

$\therefore$ difference in magnitude is 20 (nearest whole)

Question 26 (3 marks)

Prove that $\frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = 2 \sec A$

3

$$\begin{aligned} \text{LHS} &= \frac{\cos^2 A + (1 + \sin A)^2}{(1 + \sin A) \cos A} \\ &= \frac{\cos^2 A + 1 + 2\sin A + \sin^2 A}{(1 + \sin A) \cos A} \\ &= \frac{\cos^2 A + \sin^2 A + 1 + 2\sin A}{(1 + \sin A) \cos A} \\ &= \frac{1 + 1 + 2\sin A}{(1 + \sin A) \cos A} \\ &= \frac{2(1 + \sin A)}{(1 + \sin A) \cos A} \\ &= \frac{2}{\cos A} \\ &= 2 \sec A \\ &= \text{RHS} \end{aligned}$$

Question 27 (5 marks)

(a) Solve $2 \sin x - \sqrt{3} = 0$ for $0 \leq x \leq 2\pi$. Leave your solution in exact form.

2

$$\begin{aligned} \sin x &= \frac{\sqrt{3}}{2} \\ x &= 60^\circ, 180^\circ - 60^\circ \\ x &= \pi/3, \pi - \pi/3 \\ x &= \pi/3, 2\pi/3 \end{aligned}$$

(b) Solve $\tan^2 \theta - \tan \theta - 2 = 0$ for $0 \leq \theta \leq 2\pi$. Round your solution to 2 decimal places where necessary.

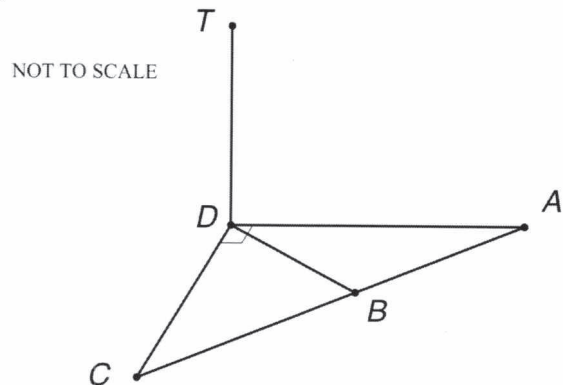
3

$$\begin{aligned} (\tan \theta + 1)(\tan \theta - 2) &= 0 \\ \tan \theta &= -1 \quad \tan \theta = 2 \\ \theta &= 135^\circ, 315^\circ \\ \theta &= 3\pi/4, 7\pi/4 \\ \therefore \theta &= 3\pi/4, 7\pi/4 \end{aligned}$$

Radian mode
 $\theta = \tan^{-1} 2$
 $\theta = 1.1071 \dots, \pi + 1.1071 \dots$
 $\theta \approx 1.11, 4.25$

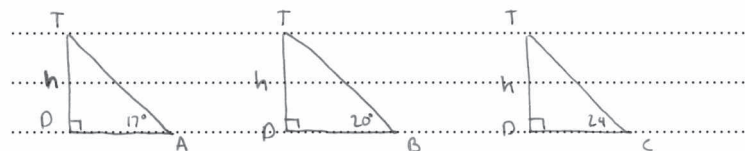
Question 28 (3 marks)

Three surveyors A , B , and C are observing a tower of height h situated on flat ground. A is due east of the tower, C is due south of it and B is somewhere between A and C on the line AC . The tower's base is at D and its top is at T . The angles of elevation of the top of the tower from A , B , and C are 17° , 20° and 24° respectively. Note that $\angle ABD$ is obtuse.

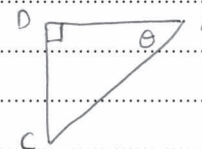


Find the bearing of B from the base of the tower D .

3



$$\begin{aligned} \tan 17^\circ &= \frac{h}{AD} & \tan 20^\circ &= \frac{h}{BD} & \tan 24^\circ &= \frac{h}{CD} \\ AD &= \frac{h}{\tan 17^\circ} & BD &= \frac{h}{\tan 20^\circ} & CD &= \frac{h}{\tan 24^\circ} \end{aligned}$$

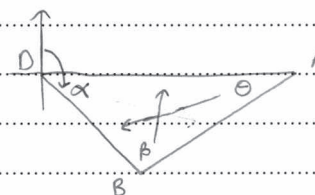


Find $\angle DAC$

$$\begin{aligned} \tan \theta &= \frac{CD}{AD} \\ \tan \theta &= \frac{\frac{h}{\tan 24^\circ}}{\frac{h}{\tan 17^\circ}} = \frac{\tan 17^\circ}{\tan 24^\circ} \\ \theta &= \tan^{-1} \left(\frac{\tan 17^\circ}{\tan 24^\circ} \right) \end{aligned}$$

$$\theta = 34.47669\dots$$

$$\theta = 34^\circ 28' 36.12''$$



$$\frac{\sin \theta}{BD} = \frac{\sin \alpha}{AB} = \frac{\sin \beta}{AD}$$

AB unknown

$$\frac{\sin \theta}{h/\tan 20^\circ} = \frac{\sin \beta}{h/\tan 17^\circ}$$

$$\sin \theta \tan 20^\circ = \sin \beta \tan 17^\circ$$

$$\sin \beta = \frac{\sin \theta \tan 20^\circ}{\tan 17^\circ}$$

$$\beta = 42.36906\dots, 137.6309\dots$$

$$\therefore \beta = 137^\circ 37' 51.36''$$

$$\alpha = 180^\circ - \beta - \theta$$

$$= 7.89236\dots$$

$$= 7^\circ 53' 32.52''$$

$$\begin{aligned} \therefore \text{Bearing} &= 90^\circ + \alpha \\ &= 97^\circ 53' 32.52'' \\ &= 97^\circ 54' \text{ (nearest min)} \end{aligned}$$

End of paper

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

[illegible]