Caringbah High School

Year 11 Mathematics 2014

Semester 2 Examination

Time Allowed: 2 hour + 5 minutes reading

Instructions

- **Start each question in a new booklet.**
- **Answers unsupported by working may not attract full marks.**
- **Marks may not be awarded for careless or badly arranged work.**
- **Approved calculators may be used.**
- **Attempts that show an understanding of concepts may attract some marks.**

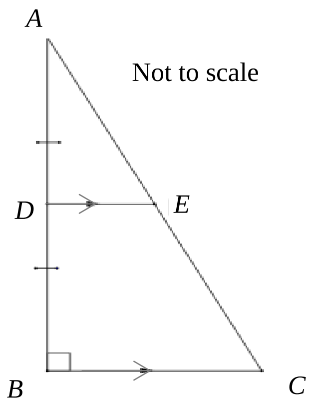
Question 1 (13 marks) **Marks**

a) Find the midpoint of the interval joining the points $A(5, -2)$ and $B(-3, -6)$. 1

b) State the gradient of the line $3y - 2x + 5 = 0$. 1

c) Sketch the graph of $y = 4 - x^2$, showing all intercepts on axes. 2

d)



The $\Delta\, ABC$ is right-angled at B.

$BC \parallel DE$.

$AD = DB$.

i) Show that $\Delta\, ADE$ is right-angled. 1

ii) Prove that $\Delta\, ABC \;///\; \Delta\, ADE$. 2

e) Sketch the graph of $y = 3^x$. 1

f) State the range of the function $xy = 4$. 1

g) If $f(x) = x^2 + 4x - 1$, find all possible values for x if $f(x) = 20$. 2

h) Find the derivative of $3x^2 - 2x + 1$. 2

Marks

Question 2 (13 marks)

a) The points $A(-3, 2)$, $B(5, 1)$, $C(7, -2)$ and D form a parallelogram.

i) Plot these points on a number plane and show the coordinates of D. 2

ii) Find the coordinates of the point E where AC intersects with BD. 1

b) Determine the natural domains of the following functions.

i) $y = \dfrac{x}{x+1}$ 1

ii) $y = \sqrt{x-4}$ 1

c) Find the distance between the points $P(5, 9)$ and $Q(-3, 4)$ correct to 3 significant figures. 2

d) Find the derivatives of the following functions

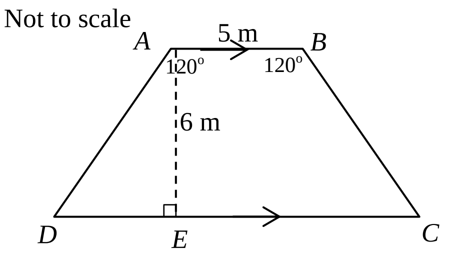
i) $y = 4x^{\frac{3}{5}}$ 2

ii) $y = \dfrac{5}{2x^4}$ 2

iii) $y = \sqrt[3]{4x-2}$ 2

Question 3 (13 marks) **Marks**

a)



ABCD is an isosceles trapezium with AB // DC and $\angle DAB = \angle CBA = 120^{o}$.

The perpendicular height *AE* is 6 m. *AB* = 5 m.

i) Show that $DE = 2\sqrt{3}$ m — 2

ii) Find the area of the trapezium. — 2

b) Sketch the graph of these parabolas and find the coordinates of their vertex.

i) $y = (x-2)(x+4)$ — 2

ii) $y = 3x(6-x)$ — 2

c) Find the derivatives of these functions.

i) $f(x) = (3x^2+5)^4$ — 2

ii) $f(x) = (2x^3-7)^3(4x-1)$, in simplest factorised form. — 3

Question 4 (13 marks)

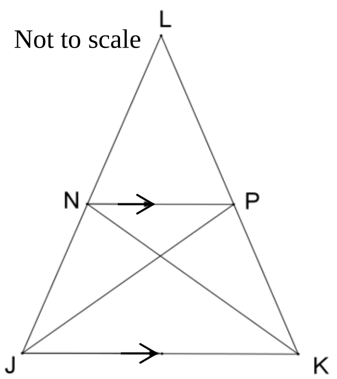
a) Find the equation of the line that passes through the points *A*(5, –2) and *B*(2, 1). — 3

b) Find the point of intersection of the straight lines $x + 2y = 5$ and $x - y = 2$. — 2

c) i) Find $\lim\limits_{x \to -2} \dfrac{x^2-25}{x-5}$ — 1

ii) Find $\lim\limits_{x \to 5} \dfrac{x^2-25}{x-5}$ — 1

d)



In the diagram, *JKL* is an isosceles triangle where *JL* = *KL* and *JK* is parallel to *NP*.

i) Show that triangle *LNP* is an isosceles triangle. — 2

ii) Show that *JN* = *KP*. — 1

iii) Show that $\Delta NJK \equiv \Delta PKJ$. — 3

Question 5 (12 marks) **Marks**

a) State, with reasons, whether the function $y = \sin x$ is an odd or an even function. 1

b) Find the perpendicular distance from the point (2, 5) to the line $3x + 4y = 1$. 2

c) Find the equation of the line through the point (–2, 6) that is parallel to the line $2x - 4y + 3 = 0$. 2

d) Find the equation of the tangent to the curve $y = x^3 - 2x^2 + 3x - 1$ at the point (1, 1). 3

e) Show that the line $3x - 2y + 14 = 0$ is a tangent to the circle $(x-1)^2 + (y-2)^2 = 13$. 2

f) Shade the region defined by $y \leq x^3 + 1$ 2

Question 6 (12 marks)

a) Find the sum of the internal angles of a decagon. 1

b) The exterior angle of a regular polygon is 24°. 2

How many sides does the polygon have?

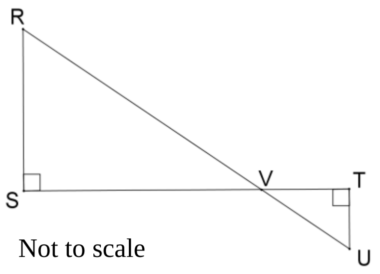
c) i) On the same set of axes, sketch the graphs of $y = |2x - 1|$ and $y = 3$. 2

ii) Find the points of intersection of these two graphs. 2

d) Differentiate from first principles $y = x^2 - 3x + 1$. 2

e) Differentiate $y = \dfrac{(3x-2)^3}{5x+1}$ 3

Question 7 (12 marks) **Marks**

a) 

Rudi used similar triangles to measure the width of a creek. The field diagram shown shows two points, R and S, on opposite banks of the creek. Using line of sight, he constructed a smaller similar triangle VTU on the land. 2

He measured the following distances.

SV = 25 m, VT 5 m, TU = 12 m and VU = 13 m.
Find the width of the creek RS.

b) Sketch the graph of $y = \sqrt{25 - x^2}$ 2

c) Find the equation of the line that passes through the origin and the point of intersection of the lines $2x + 3y - 9 = 0$ and $x - y - 2 = 0$. 4

d) A function is defined as $f(x) = \begin{cases} x^3 & x < 0 \\ x + 1 & x \geq 0 \end{cases}$

i) Explain why $f(x)$ is a not a continuous function. 1

ii) Evaluate $f(-2) + f(3)$. 1

iii) Sketch the graph of $y = f(x)$. 2

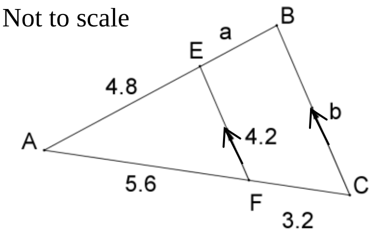
Question 8 (12 marks) **Marks**

a) i) Differentiate $y = \sqrt{x+3}$. 2

ii) Find the gradient of the curve $y = \sqrt{x+3}$ at the point (1, 2). 1

iii) Find the equation of the normal to the curve $y = \sqrt{x+3}$ at the point (1 , 2). 2

b) Not to scale



In this diagram *FE* // *CB*. 2

AE = 4·8, *AF* = 5·6, *FE* = 4·2 and *FC* = 3·2.

Find values for a and b.

c) Sketch the graph of $y = \dfrac{-1}{x+2}$, showing all asymptotes. 2

d) The normal to the curve $y = ax^3 - 1$ at x = 1 meets the positive x-axis at an angle of 120°. Find the value of a. 3

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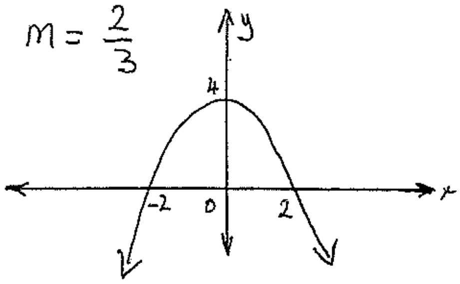
Year 11 Mathematics AT4 2014

Question 1

a) $(1, -4)$

b) $m = \frac{2}{3}$

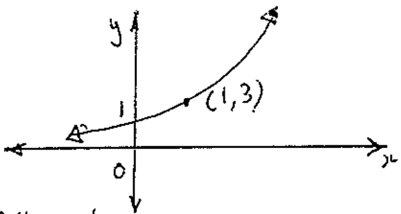
c)



d) $\angle ADE = \angle ABC$ corresponding angles, $DE \parallel BC$

$= 90^\circ$ – given $\therefore \angle ADE$ is right-angled

e)



f) All real y, $y \neq 0$

g) $x^2 + 4x - 1 = 20$

$x^2 + 4x - 21 = 0$

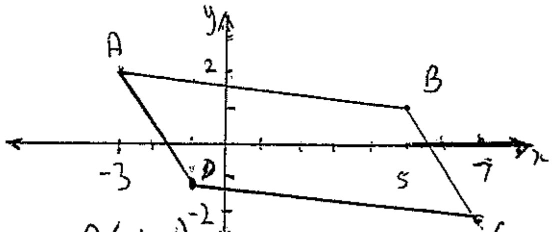
$(x+7)(x-3) = 0$

$x = 3$ or $x = -7$

h) $6x - 2$

Question 2

a) i)



$D(-1,-1)$

ii) $E(2, 0)$

b) i) D: All real x, $x \neq -1$

ii) D: $x \geq 4$

c) $d = \sqrt{(5+3)^2 + (9-4)^2}$

$= \sqrt{89}$

$= 9.43$

d) i) $y' = \frac{12}{5} x^{-\frac{2}{5}}$

ii) $y = \frac{5}{2} x^{-4}$ $\quad y' = -10x^{-5}$

$= -\frac{10}{x^5}$

iii) $y = (4x-2)^{\frac{1}{3}}$

$y' = \frac{4}{3}(4x-2)^{-\frac{2}{3}}$

$= \frac{4}{3\sqrt[3]{(4x-2)^2}}$

Question 3

a) i) $\angle DAE = 30^\circ$ $\quad \tan 30 = \frac{DE}{6}$

$\frac{1}{\sqrt{3}} = \frac{DE}{6}$

$DE = \frac{6}{\sqrt{3}}$

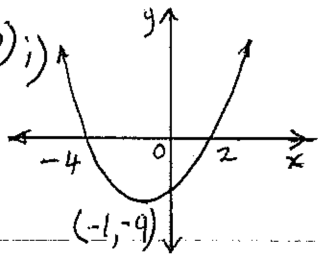
$= 2\sqrt{3}$

ii) $A = \frac{h}{2}(a+b)$

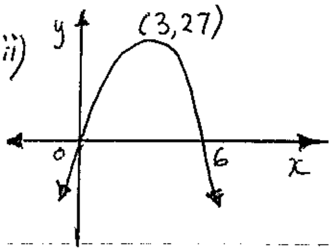
$= \frac{6}{2}\left(5 + (5 + 4\sqrt{3})\right) m^2$

$= 30 + 12\sqrt{3}\ m^2$

b) i)



ii)



c) i) $f'(x) = 24x(3x^2+5)^3$

ii)

$u = (2x^3-7)^3 \quad v = (4x-1)$

$u' = 18x^2(2x^3-7)^2 \quad v' = 4$

$f'(x) = 18x^2(2x^3-7)^2(4x-1) + 4(2x^3-7)^3$

$= 2(2x^3-7)^2\left[9x^2(4x-1) + 2(2x^3-7)\right]$

$= 2(2x^3-7)^2(40x^3 - 9x^2 - 14)$

Question 4

a) $m = \frac{3}{-3}$ $\quad$ Eqn $(y-1) = -1(x-2)$

$= -1$ $\quad y - 1 = -x + 2$

$x + y - 3 = 0$

b) $x + 2y = 5$ ①

$x - y = 2$ ②

① − ② $3y = 3$

$y = 1 \quad x = 3$

Q4c i) $\frac{4-25}{-2-5} = 3$

ii) $\lim_{x \to 5} \frac{\cancel{(x-5)}(x+5)}{\cancel{(x-5)}} = 10$

d) i) $\angle LNP = \angle LJK$ corres. $\angle$s, $NP \parallel JK$.

$\angle LPN = \angle LKJ$ " ", $NP \parallel JK$.

$\angle LJK = \angle LKJ$ angles opp. equal sides of an isos. Δ.

$\therefore \angle LNP = \angle LPN$

$\therefore \Delta LNP$ is isosceles

ii) $JL = KL$ given

$LN = LP$ equal sides of isos. Δ ①

$NJ = JL - LN$

$PK = KL - LP$ $\quad JL - LN = KL - LP$ from ①

$\therefore NJ = PK$.

iii) In ΔNJK and ΔPKJ

S $\quad JK$ is common

A $\quad \angle NJK = \angle PKJ$ angles opp. equal sides of isos ΔLJK.

S $\quad NJ = PK$ from ii)

$\therefore \Delta NJK \equiv \Delta PKJ$ SAS

Question 5

a) $\sin(-\theta) = -\sin\theta \therefore \sin x$ is odd

NB. You must reference the question. ie use $\sin x$.

b) $d = \left| \frac{(3 \times 2) + (4 \times 5) - 1}{\sqrt{3^2 + 4^2}} \right|$

$= 5$

c) $m = \frac{1}{2}$ $\quad (y-6) = \frac{1}{2}(x+2)$

$2y - 12 = x + 2$

$x - 2y + 14 = 0$

d) $y' = 3x^2 - 4x + 3$

$x = 1 \quad m = 2$

Eqn of tangent

$(y-1) = 2(x-1)$

$y - 1 = 2x - 2$

$2x - y - 1 = 0$

e) Circle centre $(1,2)$ rad $= \sqrt{13}$

Using $\perp$ dis from centre

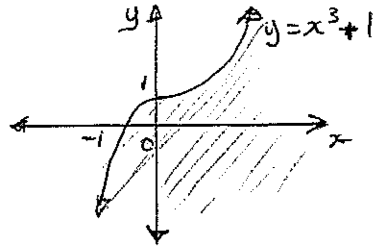
$d = \left| \frac{(3 \times 1) - (2 \times 2) + 14}{\sqrt{3^2 + (-2)^2}} \right|$

$= \left| \frac{13}{\sqrt{13}} \right|$

$= \sqrt{13}$ (= radius)

$\therefore 3x - 2y + 14 = 0$ is a tangent.

f)

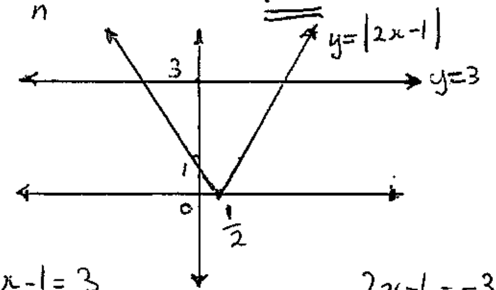


Question 6

a) $180° \times 8 = 1440°$

b) $\frac{360}{n} = 24 \quad \therefore n = 15$

c)



ii) $2x - 1 = 3$ $\quad$ or $\quad 2x - 1 = -3$

$x = 2 \qquad x = -1$

$y = 3 \qquad y = 3$

d) $f(x) = x^2 - 3x + 1$

$f(x+h) = (x+h)^2 - 3(x+h) + 1$

$= x^2 + 2xh + h^2 - 3x - 3h + 1$

$f'(x) = \lim_{h \to 0} \frac{(\cancel{x^2} + 2xh + h^2 - \cancel{3x} - 3h + \cancel{1}) - (\cancel{x^2} - \cancel{3x} + \cancel{1})}{h}$

$= \lim_{h \to 0} \frac{\cancel{h}(2x + h - 3)}{\cancel{h}}$

$= 2x - 3$

e) $y' = \frac{vu' - uv'}{v^2}$ $\qquad u = (3x-2)^3 \quad v = (5x+1)$

$u' = 9(3x-2)^2 \quad v' = 5$

$= \frac{9(3x-2)^2(5x+1) - 5(3x-2)^3}{(5x+1)^2}$

$= \frac{(3x-2)^2[9(5x+1) - 5(3x-2)]}{(5x+1)^2}$

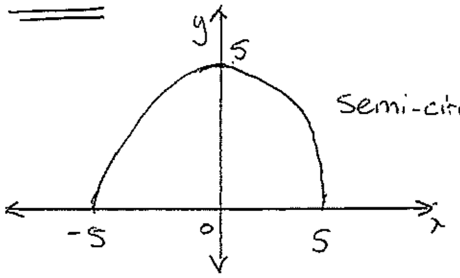
$= \frac{(3x-2)^2(30x+19)}{(5x+1)^2}$

Question 7

a) $\frac{x}{25} = \frac{12}{5}$

$\underline{\underline{x = 60}}$

b)



Semi-circle.

c) $L_1 + kL_2 = 0$

$(2x+3y-9) + k(x-y-2) = 0$

$x=0, y=0 \quad -9-2k=0$

$k = -4\frac{1}{2}$

$(2x+3y-9) - \frac{9}{2}(x-y-2) = 0$

$\underline{\underline{x - 3y = 0}}$

(Alternatively, solving simultaneously to find point of intersection $(3,1)$ giving $m = \frac{1}{3}$)

d) i) at $x=0$ $f(0) = 1$

but as $x \to 0$ from the negative.

$f(x) \to 0^3 = 0$

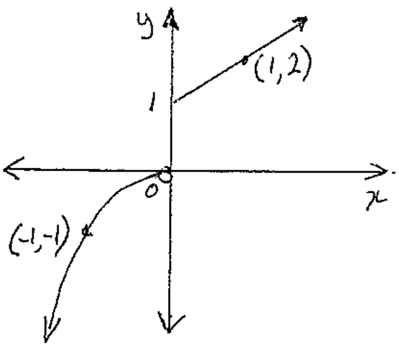
$\therefore$ There is a gap in the graph at $x=0$

ii) $f(-2) + f(3) = -8 + 4$

$= -4$

NB: You must reference the question on the paper.

iii)



Question 8

a) i) $y = (x+3)^{\frac{1}{2}}$

$y' = \frac{1}{2}(x+3)^{-\frac{1}{2}}$

$= \frac{1}{2\sqrt{x+3}}$

ii) $x = 1 \quad m = \frac{1}{4}$

iii) $m_N = -4$ Eq'n of normal

$(y-2) = -4(x-1)$

$y - 2 = -4x + 4$

$\underline{\underline{4x + y - 6 = 0}}$

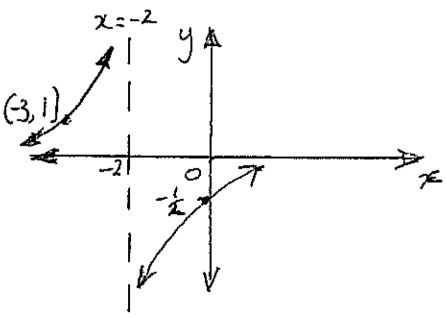
b) $\frac{a}{4\cdot8} = \frac{3\cdot2}{5\cdot6}$ ratio of intercepts on parallel lines.

$a = \underline{\underline{2\cdot74}}$ (2 d.p)

$\frac{b}{4\cdot2} = \frac{8\cdot8}{5\cdot6}$ ratio of sides of similar Δs.

$b = \underline{\underline{6\cdot6}}$

c)



d) $y' = 3ax^2$

at $x=1$ $m_T = 3a$. $m_N = -\frac{1}{3a}$

$m_N = \tan 120°$

$= -\sqrt{3}$

$\frac{-1}{3a} = -\sqrt{3}$

$a = \frac{1}{3\sqrt{3}}$ // $\left(\frac{\sqrt{3}}{9}\right)$