Caringbah High School

Year 11 2015

Mathematics

Preliminary HSC Course

Semester 2 Exam

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In Questions 6 - 12, show relevant mathematical reasoning and/or calculations
- Marks may not be awarded for partial or incomplete answers

Total marks – 85

Section I **5 marks**
Attempt Questions 1-5
Mark your answers on the answer sheet provided. You may detach the sheet and write your name on it.

Section II **80 marks**
Attempt Questions 6 - 12
Write your answers on the answer sheets provided. Ensure your name or student number is clearly visible.

Name: _____ **Class:**

	Marker's Use Only								
Section I	**Section II**							**Total**	
Q 1-5	**Q6**	**Q7**	**Q8**	**Q9**	**Q10**	**Q11**	**Q12**		
/5	/12	/12	/12	/11	/11	/11	/11	/85	%

Question 1 - 5 (1 mark each) Answer on the page provided.

1) What is the value of $\sqrt{\dfrac{4^2+18^2}{23^2-18^2}}$, correct to 3 significant figures?

A) 1.28 B) 1.29 C) 1.287 D) 1.288

2) Which equation represents a line perpendicular to $3x-2y=5$?

A) $3x-2y=10$ B) $3x+2y=10$

C) $2x+3y=10$ D) $2x-3y=10$

3) If $\left(3+\sqrt{2}\right)^2 = a+\sqrt{b}$, which is the correct pairing for *a* and *b*, given that both *a* and *b* are rational?

A) $a=13,\ b=72$ B) $a=13,\ b=2$

C) $a=11,\ b=2$ D) $a=11,\ b=72$

4) Which of the expressions below is equivalent to $\cos(270°-\theta)$?

A) $\sin\theta$ B) $-\sin\theta$ C) $\cos\theta$ D) $-\cos\theta$

5) Which inequality defines the domain of the function $f(x)=\sqrt{1-x}+\sqrt{x+4}$?

A) $x\le -4$ and $x\ge 1$ B) $x\le -1$ and $x\ge 4$

C) $-4\le x\le 1$ D) $-1\le x\le 4$

End of multiple choice questions

Question 6 (12 marks) Start a NEW page. **Marks**

a) Insurance has increased by 10% to \$462. 1

What was the cost of insurance before the increase?

b) State the range of $y = 1 + 2^x$. 1

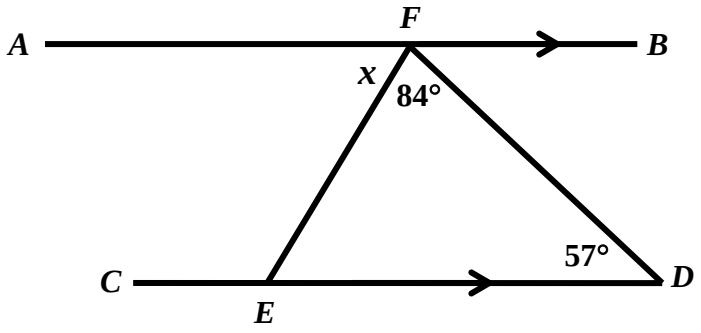
c) Solve $4(3x-2) - 3(x+4) = 13$. 2

d) Factorise fully $2x^2 + 8x + 8$. 2

e) Find the values of x for which $|3x+2| \leq 8$. 2

f) Evaluate $\lim_{x \to 0} \frac{x^3 - 2x}{x}$. 2

g) Find the value of x, giving reasons. 2



Question 7 (12 marks) Start a NEW page. **Marks**

a) Solve $x - \frac{x+4}{4} = 2$. 2

b) Sketch the graph of $y = \sqrt{9 - x^2}$ showing all important features. 2

c) The line $5x + ky = 8$ passes through the point $(-2, 3)$. 2

Find the value of k.

d) Consider $f(x) = \frac{2}{x+1}$. Find $f\left(\frac{1}{a}\right)$ as a simple fraction. 2

e) Solve $\sin\theta = -\frac{\sqrt{3}}{2}$ for $0° \le \theta \le 360°$. 2

f) Factorise fully $x^3 - x^2 - x + 1$. 2

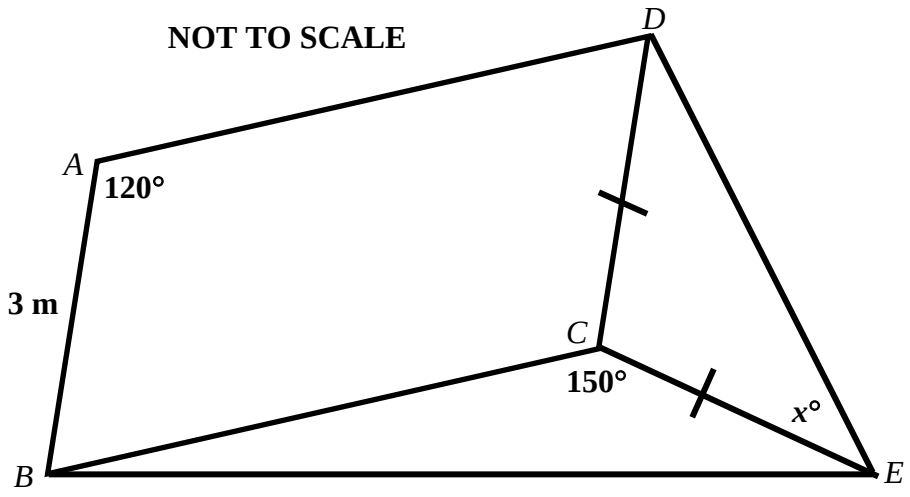
Question 8 (12 marks) Start a NEW page. **Marks**

a) Show that $f(x) = x^{2n}$ is an even function. 1

b) Find the domain of $y = \dfrac{\sqrt{x+1}}{x-1}$. 2

c) Shade the region defined by $x^2 + y^2 \leq 4$ 2

d)



ABCD is a parallelogram. $\angle BAD = 120°$ and $\angle BCE = 150°$.

CD = *CE* and *AB* = 3 metres.

Copy or trace this diagram onto your answer page.

i) Find the value of *x*, giving appropriate reasons for each step. 3

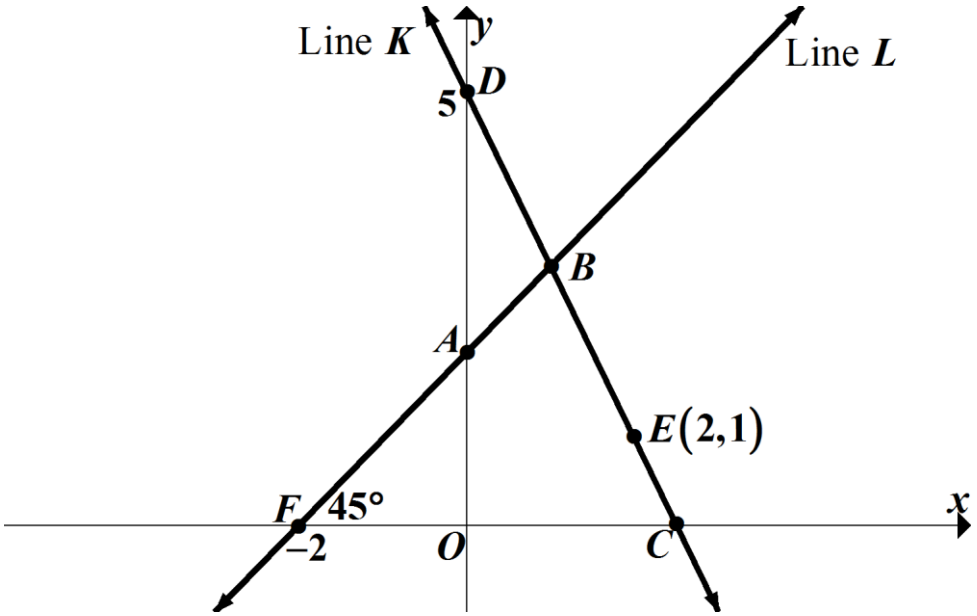
ii) Find the exact length of *DE* in simplest form. 2

e) If $g(x) = \dfrac{1}{x^2} - 9$, find the value(s) of x for which $g(x) = 0$. 2

Question 9 (11 marks) Start a NEW page. **Marks**

a) Simplify $\dfrac{(1-\cos\theta)(1+\cos\theta)}{\tan^2\theta}$. 2

b)



The questions below refer to the diagram drawn above.

Copy the diagram onto your answer page.

i) Show that Line K has equation $2x+y-5=0$. 2

ii) Give appropriate reasons to explain why Line L has equation $y=x+2$. 2

iii) Line K and Line L intersect at the point B. Find the coordinates of B. 2

iv) Find the coordinates of the point C. 1

v) Hence or otherwise, find the area of quadrilateral $OABC$. 2

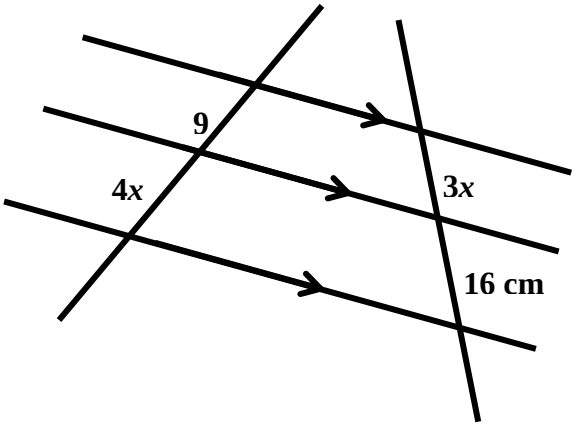
Question 10 (11 marks) Start a NEW page. **Marks**

a) Determine the sum of the interior angles of a regular polygon if each exterior angle is 24°. 2

b) The line $kx + 3y + 7 = 0$ is perpendicular to the line $y = \frac{5}{2}x + 4$. 2

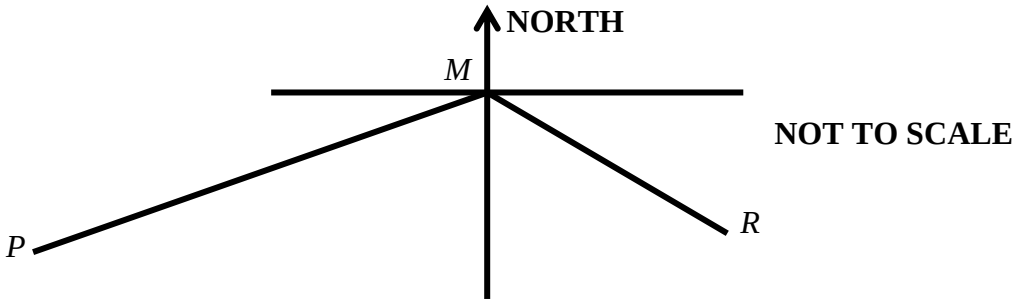
Find the value of k.

c) Using the diagram below, find the value of x. (All measurement are in cm) 2



d) From a town M, town R is 109 km away on a bearing of 110°.

Town P is 220 km away from town M on a bearing of 230°.



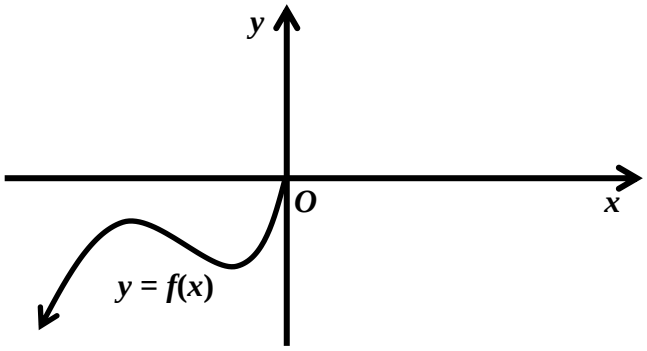
i) Copy the above diagram onto your answer sheet, add the given information and find the size of $\angle PMR$. 1

ii) Find the distance between the two towns R and P (nearest km). 2

e) Sketch $y = \log_2(x - 2)$, showing any important features. 2

Question 11 (11 marks) Start a NEW page. **Marks**

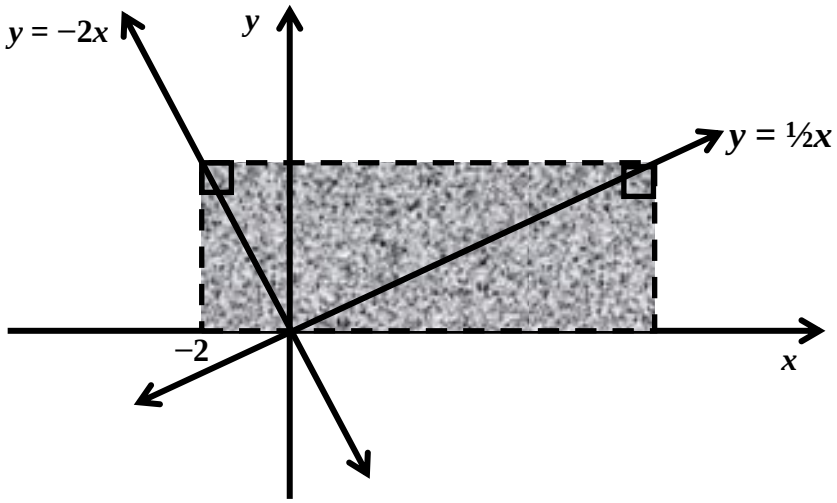
a) $y = f(x)$ has been drawn for $x \leq 0$.



Copy the diagram and complete the graph of $y = f(x)$ if 1

$f(x)$ is an odd function.

b) Calculate the shaded area. 2



c) Sketch $y = \dfrac{x^2 - 1}{x - 1}$ showing any important features. 2

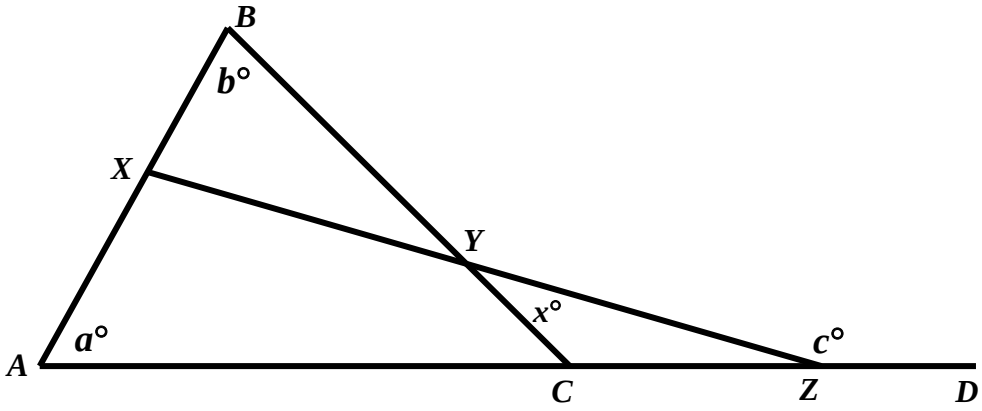
d) Find the centre and radius of $x^2 + y^2 - 6y - 7 = 0$. 3

e) If $\tan\theta = \dfrac{3}{7}$ and $\sin\theta < 0$, find the value of $\cos\theta$ in exact form. 3

Question 12 (11 marks) Start a NEW page. **Marks**

a) Sketch $y = |2x - 1|$ showing any important features. 2

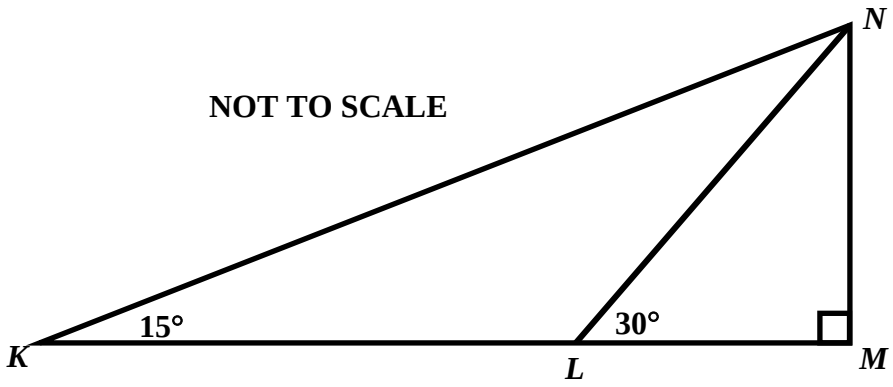
b)



AD, *XZ*, *AB* and *BC* are straight lines. If $\angle CYZ = x°$, find an expression for x in terms of a, b and c. 3

c) Factorise $16x^3 - 250$. 2

d)



i) Explain why $KL = LN$. 1

ii) If $NM = 1$, show that $\tan 15° = 2 - \sqrt{3}$. 3

END OF EXAM

Name: _____

Multiple choice answer page. Fill in either A, B, C or D for questions 1-5.

This page must be handed in with your answer booklets

1.	
2.	
3.	
4.	
5.	

CHS YEAR 11 MATHEMATICS (2U) 2015

SEMESTER 2 EXAM SOLUTIONS

Multiple Choice Section:

1.B 2.C 3.D 4.B 5.C

Question 1.

1.287841832 = 1.29 (3 sig fig) ------$\boxed{B}$

Question 2.

$2x + 3y = 10$ ---------$\boxed{C}$

Question 3.

$$\left(3+\sqrt{2}\right)^2 = 9 + 6\sqrt{2} + 2$$
$$= 11 + \sqrt{72}$$ ---------$\boxed{D}$

Question 4.

$$\cos(270° - \theta) = \cos\left(180° + \left(90^\circ - \theta\right)\right)$$
$$= -\cos\left(90^\circ - \theta\right)$$
$$= -\sin\theta$$ --------$\boxed{B}$

Question 5.

$\sqrt{1-x} \rightarrow 1 - x \geq 0 \rightarrow x \leq 1$

$\sqrt{x+4} \rightarrow x + 4 \geq 0 \rightarrow x \geq -4$

$\therefore \quad -4 \leq x \leq 1$ ----------$\boxed{C}$

Question 6.

a) 110 % = \$462

$\therefore$ 100 % = \$462 ÷ 1.1 = \$420

b) $R: y > 1$

c) $12x - 8 - 3x - 12 = 13$

$\therefore 9x = 33 \quad \rightarrow \quad x = \dfrac{11}{3}$

d) $2x^2 + 8x + 8 = 2\left(x^2 + 4x + 4\right)$
$= 2(x+2)^2$

e) $-8 \leq 3x + 2 \leq 8$

$-10 \leq 3x \leq 6$

$-\dfrac{10}{3} \leq x \leq 2$

f) $\lim\limits_{x \to 0} \dfrac{x^3 - 2x}{x} = \lim\limits_{x \to 0} \left(x^2 - 2\right)$

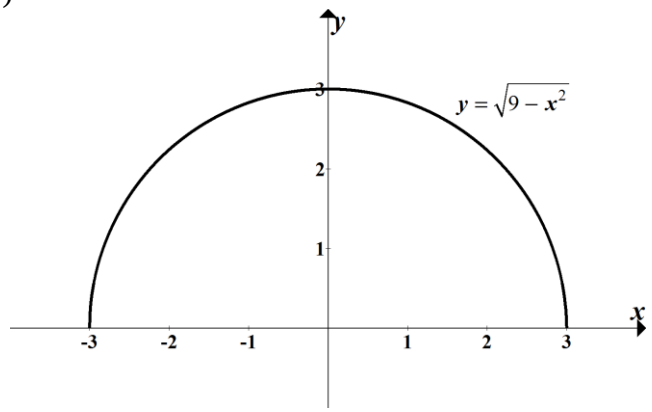
$= -2$

g) $\angle FED = 39° \left(\angle \text{ sum of } \Delta FED\right)$
$\therefore x = \angle AFE = 39° \left(\text{alt } \angle\text{'s and } AB \| CD\right)$

Question 7.

a) $4x - x - 4 = 8$
$3x = 12 \quad \rightarrow \quad x = 4$

b)



c) $\therefore \ 5 \times -2 \ + \ 3k = 8$ by substitution

$3k = 18 \quad \rightarrow \quad k = 6$

d) $f\left(\frac{1}{a}\right) = \frac{2}{\frac{1}{a}+1}$

$= \frac{2}{\frac{1+a}{a}}$

$= \frac{2a}{1+a}$

e) Basic angle = 60°

$\therefore \theta = 180° + 60°$ and $\theta = 360° - 60°$

$\therefore \theta = 240°$ and $\theta = 300°$

f) $x^2(x-1) - (x-1) = (x^2-1)(x-1)$

$= (x+1)(x-1)(x-1)$

$= (x-1)^2(x+1)$

Question 8.

a) For an even function $f(x) = f(-x)$

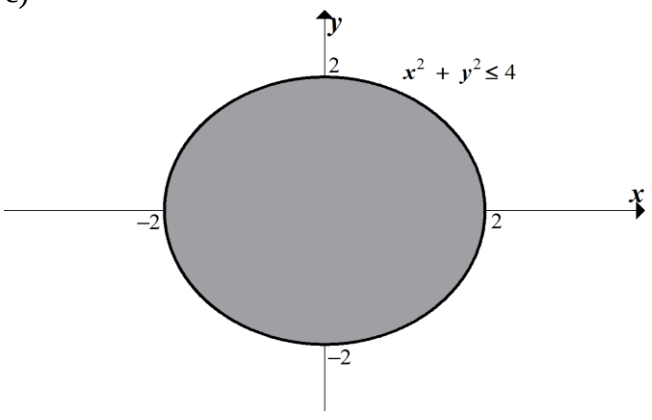
$f(-x) = \left[(-x)^2\right]^n$

$= \left[x^2\right]^n$

$= x^{2n} = f(x)$

b) D: $x \geq -1$, excluding $x = 1$

c)



d) i) $\angle BCD = 120°$ (opp $\angle$'s in ||gram are =)

$\therefore \angle DCE = 90°$ ($\angle$'s at a point $= 360°$)

$\therefore x = \frac{180° - 90°}{2}$ ($\angle$'s opp = sides in isos ΔCDE)

$\therefore x = 45°$ ($\angle$ sum ΔCDE)

ii) $\sin 45° = \frac{3}{DE}$

$\therefore DE = \frac{3}{1/\sqrt{2}} = 3\sqrt{2}$

e) $\therefore \frac{1}{x^2} - 9 = 0$

$\therefore \frac{1}{x^2} = 9 \quad \rightarrow \quad x^2 = \frac{1}{9}$

$\therefore x = \pm\frac{1}{3}$

Question 9.

a) $\frac{(1-\cos\theta)(1+\cos\theta)}{\tan^2\theta} = \sin^2\theta \times \cot^2\theta$

$= \sin^2\theta \times \frac{\cos^2\theta}{\sin^2\theta}$

$= \cos^2\theta$

b) i) Use $D(0,5)$ and $E(2,1)$

$m = \frac{1-5}{2-0} = -2$

$\therefore y - 5 = -2(x-0)$

$y - 5 = -2x$

$\therefore 2x + y - 5 = 0$

ii) $m = \tan 45° = 1$

$\therefore A$ has coordinates $(0,2)$

$\therefore$ the $y-$intercept is 2

hence using $y = mx + b$ the equation is

$y = x + 2$

9b) iii) $2x + y - 5 = 0$

$y = x + 2$

$2x + (x+2) - 5 = 0 \quad \rightarrow \quad x = 1$

when $x = 1, y = 3 \quad \rightarrow \quad B(1,3)$

iv) Use Line K and put $y = 0$.

$\therefore \; 2x - 5 = 0 \quad \rightarrow \quad x = \dfrac{5}{2}$

$\therefore \; C\left(\dfrac{5}{2}, 0\right)$

v) Area of OABC = area FBC − area

Area of quad $OABC$ = area ΔFBC − area ΔOAF

$= \dfrac{1}{2} \times 4.5 \times 3 - \dfrac{1}{2} \times 2 \times 2$

$= 4\dfrac{3}{4} u^2$

Question 10.

a) Number of sides $(n) = \dfrac{360^\circ}{24} = 15$

$\therefore$ interior angle sum $= (180 - 24) \times 15$

$= 2340°$

b) Gradients multiply to equal −1.

$\therefore \; -\dfrac{k}{3} \times \dfrac{5}{2} = -1$

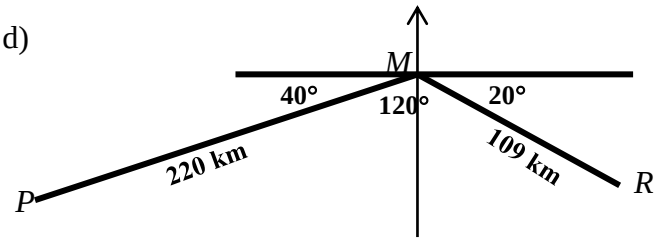
$\therefore \; k = \dfrac{6}{5}$

c) $\dfrac{9}{4x} = \dfrac{3x}{16}$ (transversals cut || lines in the same ratio)

$\therefore \; 12x^2 = 144 \quad \rightarrow \quad x^2 = 12$

$\therefore \; x = \sqrt{12} \quad (\text{as } x > 0)$

d)

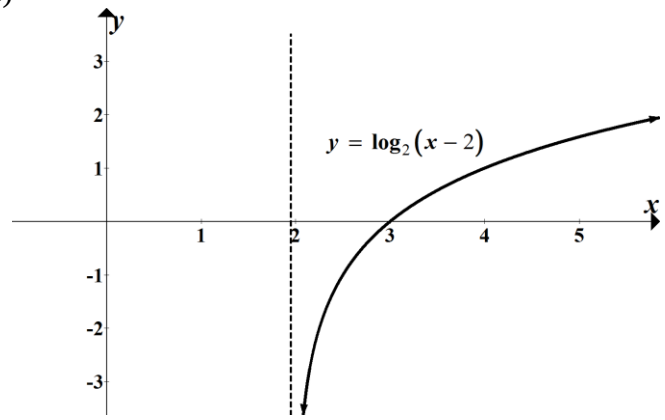


i) $\angle PMR = 230° - 110^\circ = 120^\circ$

ii) $PR^2 = 220^2 + 109^2 - 2 \times 220 \times 109 \times \cos 120^\circ$

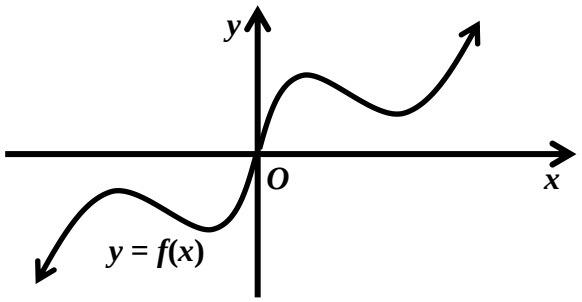
$\therefore \; PR = 290 \text{km} \quad (\text{nearest km})$

e)



Question 11.

a)



b) When $x = -2, \; y = 4$

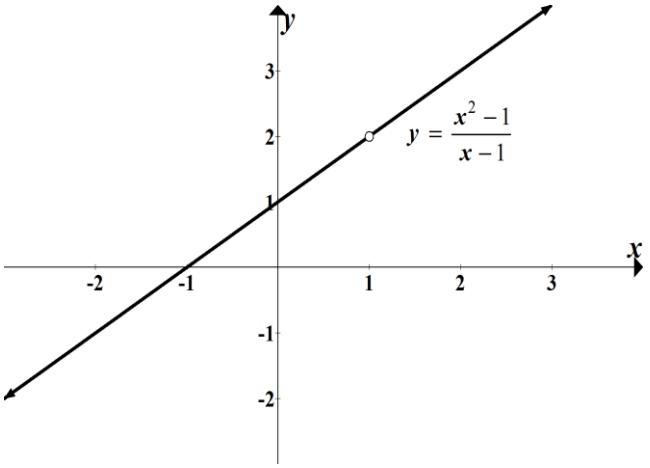
$\therefore$ width = 4

When $y = 4, \; x = 8$

$\therefore$ length = 8 + 2 = 10

hence the area = 40 u²

c)



d) $x^2 + y^2 - 6y + (-3)^2 = 7 + (-3)^2$

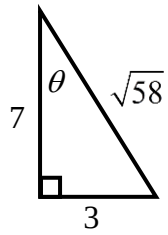
$x^2 + (y-3)^2 = 16$

$\therefore$ centre is $(0,3)$; radius = 4

e) $\tan\theta > 0 \rightarrow \theta$ is in the 1st or $\boxed{\text{3rd}}$ quad

$\sin\theta < 0 \rightarrow \theta$ is in the $\boxed{\text{3rd}}$ or 4th quad

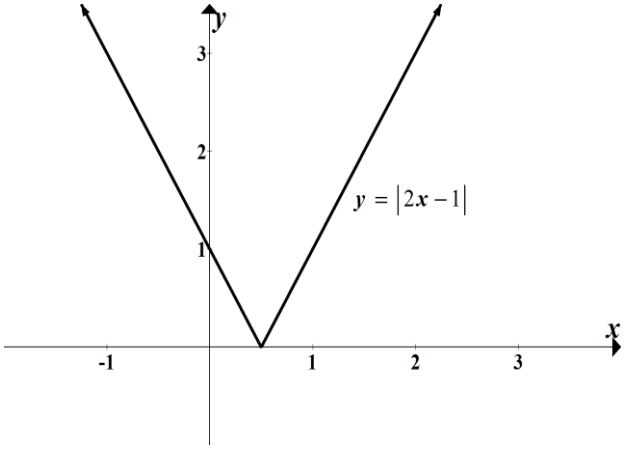
$\therefore$ θ is in the 3rd quadrant and hence $\cos\theta < 0$.



$\therefore \cos\theta = -\frac{7}{\sqrt{58}}$

Question 12.

a)



b) $\angle BCA = 180° - (a+b); (\angle \text{sum } \Delta ABC)$

$\angle YZC = 180° - c; (\angle \text{sum straight line})$

$\therefore 180° - (a+b) = x + (180° - c); (\text{exterior } \angle \Delta YZC)$

$\therefore x = c - a - b$

c) $16x^3 - 250 = 2(8x^3 - 125)$

$= 2((2x)^3 - 5^3)$

$= 2(2x-5)(4x^2 + 10x + 25)$

d) i) $\angle NLM = \angle NKL + \angle KNL$ (ext $\angle$ of ΔNKL)

$\therefore 30° = 15° + \angle KNL \rightarrow \angle KNL = 15°$

hence ΔKLN is isosceles $\rightarrow KL = LN$

(sides opposite equal angles)

ii) Using ΔNML with $NM = 1$:

$\sin 30° = \frac{1}{NL} \rightarrow NL = 2$

$\therefore \boxed{KL = 2}$ from (i)

Also $\tan 30° = \frac{1}{LM} \rightarrow \boxed{LM = \sqrt{3}}$

Now using ΔKNM with $NM = 1$:

$\tan 15° = \frac{1}{KM} = \frac{1}{KL + LM}$

$\therefore \tan 15° = \frac{1}{2+\sqrt{3}}$

$= \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$

$= \frac{2-\sqrt{3}}{4-3}$

$= 2 - \sqrt{3}$