Caringbah High School

Student Name .....

2017 Year 11
Semester 2 Exam

Mathematics (2 Unit)

General Instructions

- Approved calculators may be used.
- Start each question in a new booklet.
- Answers not supported with working may not attract full marks.
- Marks may not be awarded for carelessly arranged work.
- Write using a black or blue pen
- A reference sheet with mathematical formulae is provided

Time Allowed

2 hours + 3 minutes reading time

Answer questions 1-5 on the multiple choice answer sheet provided

Multiple Choice (1 Mark each)

Question 1

If $x = a\left(b - \frac{1}{y}\right)$ then

A) $y = \frac{a}{b - x}$

B) $y = \frac{a}{ab - x}$

C) $y = \frac{1}{ab - x}$

D) $y = \frac{x}{a} - b$

Question 2

The solution to the inequality

$6 - x - x^2 \leq 0$ is

A) $-3 \leq x \leq 2$
B) $x \leq -2$ or $x \geq 3$
C) $x \leq -3$ or $x \geq 2$
D) $-2 \leq x \leq 3$

Question 3

What is the perpendicular distance between the lines $y = x + 5$ and $x - y = 3$

A) $4\sqrt{2}$
B) 4
C) $\sqrt{2}$
D) $2\sqrt{2}$

Question 4

Which of the following trigonometric expressions is equivalent to $\tan(90-x)^{\circ}$?

A) $\tan x^{\circ}$

B) $\cot x^{\circ}$

C) $-\tan x^{\circ}$

D) $-\cot x^{\circ}$

Question 5

The midpoint of $A(x,2), B(-5,y)$ is $M(8,-10)$
The value of x, y is,

A) $x=-21, y=22$

B) $x=21, y=22$

C) $x=-21, y=-22$

D) $x=21, y=-22$

Question 6 (12 Marks) Marks

a) Find the value of y such that

$$2\sqrt{50}-\sqrt{72}=\sqrt{y}$$ (2)

b) Express with a rational denominator

$$\frac{\sqrt{2}}{\sqrt{3}+1}$$ (2)

c) Solve the equation $2\cos\theta=-1$ for $0°\leq\theta\leq360°$ (2)

d) Find $\lim_{x\to-4}\frac{x^2+x-12}{x+4}$ (2)

e) Graph the region ($\frac{1}{3}$ page size) where the inequations hold simultaneously

$y>x^2$ and $y\geq x+6$ (3)

f) Show that $\cos\theta\tan\theta=\sin\theta$ (1)

Question 7 (12 Marks) Marks

a) State the domain and range of $y=\sqrt{x-2}$ (2)

b) If $f(x)=\begin{bmatrix} x^2-1 & x\geq 1 \\ 2-x & -1<x<1 \\ \frac{1}{x} & x\leq -1 \end{bmatrix}$

Find the value of $2f(3)-f(-3)$ (2)

c) Simplify $\frac{16}{2^{3x}\times 8^{1-x}}$ (2)

d) Draw separate sketches ($\frac{1}{3}$ page size) of the following. Show any intercepts with the axes , and any asymptotes.

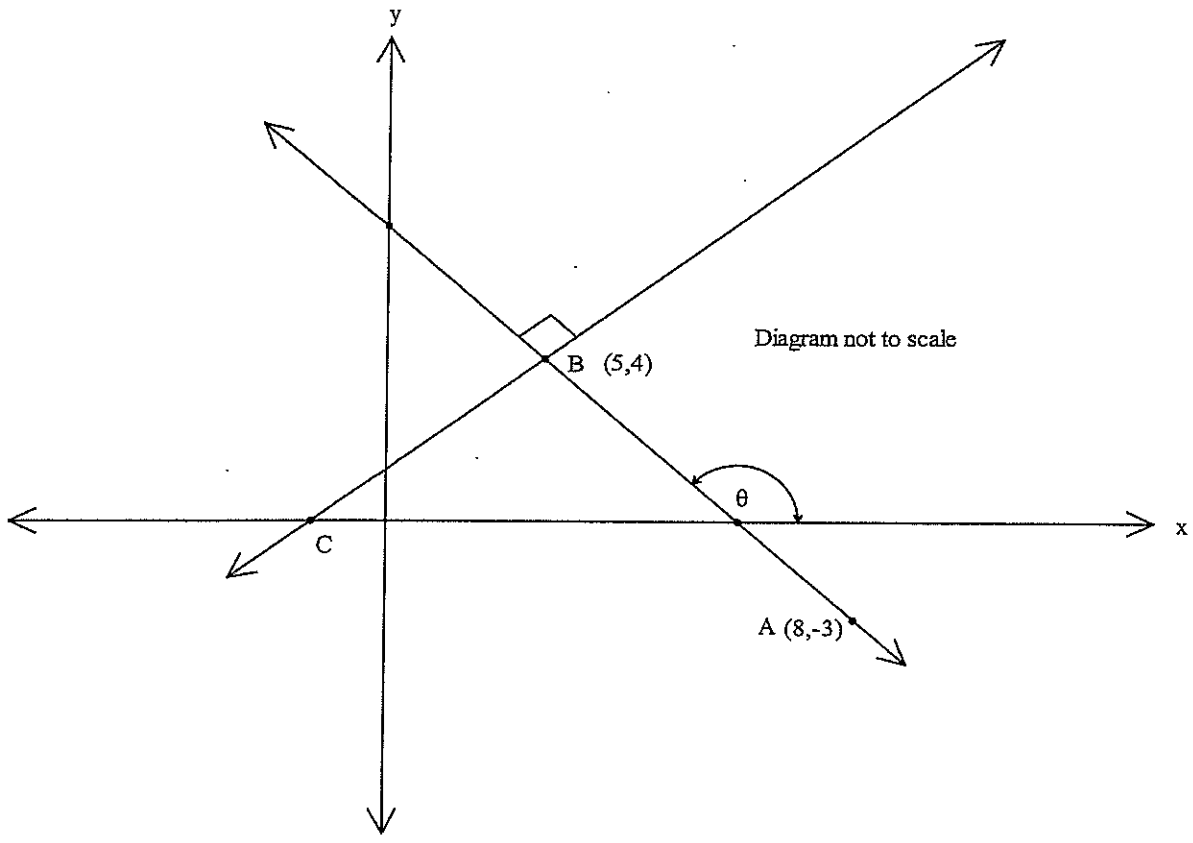
i) $y=2x-1$ (1)

ii) $y=-\sqrt{4-x^2}$ (2)

iii) $y=2^x+1$ (3)

Question 8 (12 Marks) Marks

a) The points $A(8,-3)$ and $B(5,4)$ are shown in the diagram below. The line through AB makes an angle of θ with the positive x axis and the point C lies on the x axis.



i) Find the gradient of the line AB (1)

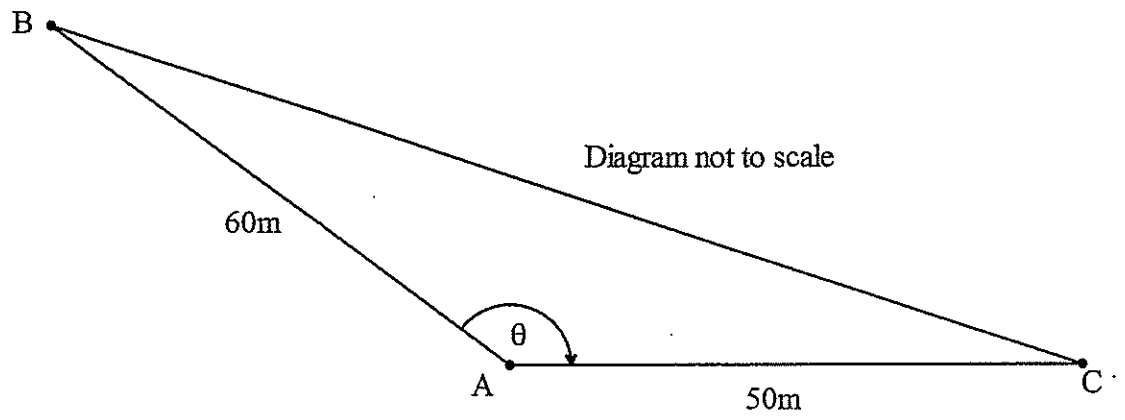
ii) Find the value of θ to the nearest degree. (1)

iii) Find the coordinates of C given $AB \perp BC$ (2)

iv) Find the coordinates of M, the midpoint of AB (1)

v) Find the equation of the line through M parallel to the line BC in general form (2)

b) Two sides of a triangular field are $AB = 60$ m, $AC = 50$ m and the included angle $\angle BAC = \theta$ is obtuse.

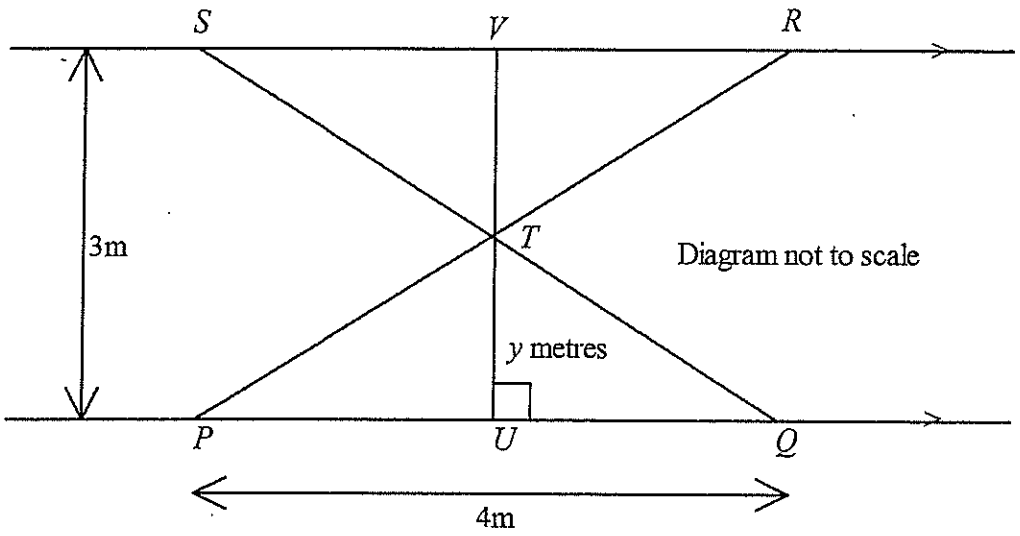


i) If the area of the triangle is 750m^2, find the size of $\angle BAC$ (2)

ii) Find the length of the side BC (1 decimal place) (2)

c) Make b the subject if $\frac{1}{x} = \frac{1}{a} + \frac{1}{b}$ (1)

Question 9 (11 Marks) Marks



a) In the diagram PQ and SR are parallel railings 3metres apart. The points P and Q are fixed 4 metres apart on the lower railing. Two crossbars PR and QS intersect at T as shown in the diagram. The line through T perpendicular to PQ intersects PQ at U and SR at V. The length UT is y metres.

i. By using similar triangles, or otherwise, show that $\frac{SR}{PQ}=\frac{VT}{UT}$ (3)

ii. Show that $SR=\frac{12}{y}-4$ (2)

iii. Find the total area, A of ΔPTQ and ΔRTS in terms of y in simplest form (3)

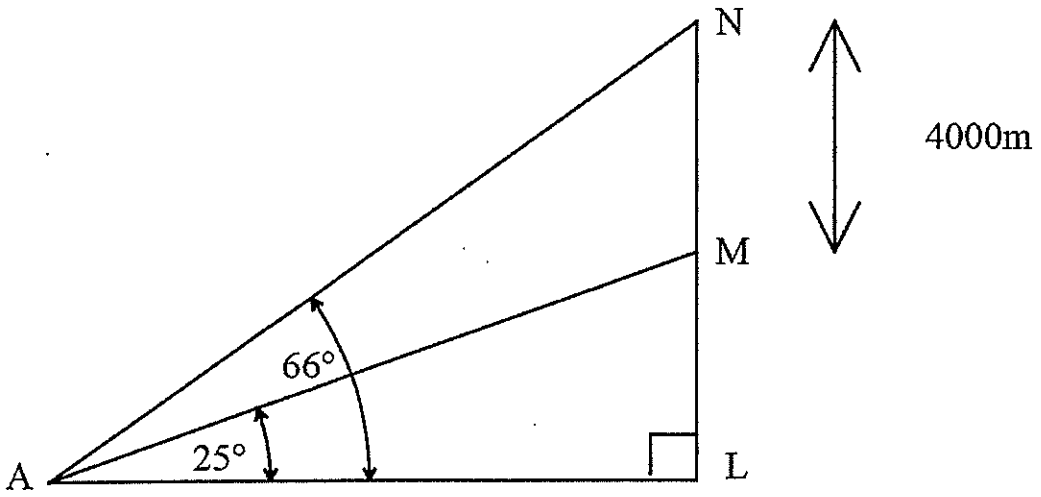
b) Solve for x

$$\frac{1}{2\sqrt{5}+\sqrt{x}}+\frac{1}{2\sqrt{5}-\sqrt{x}}=2\sqrt{5}$$

(3)

Question 10 (11 Marks) Marks

a) A rocket launched vertically from L is observed on from A. Soon after launch when at position M its angle of elevation is $25°$. After it climbs 4000 metres from this position to N its angle of elevation is $66°$



i) Find $\angle ANL$ (1)

ii) Find length of AM (to nearest metre) (2)

iii) Find how far the observer is from the launch pad (to nearest metre) (2)

Question 10 (continued)

b)

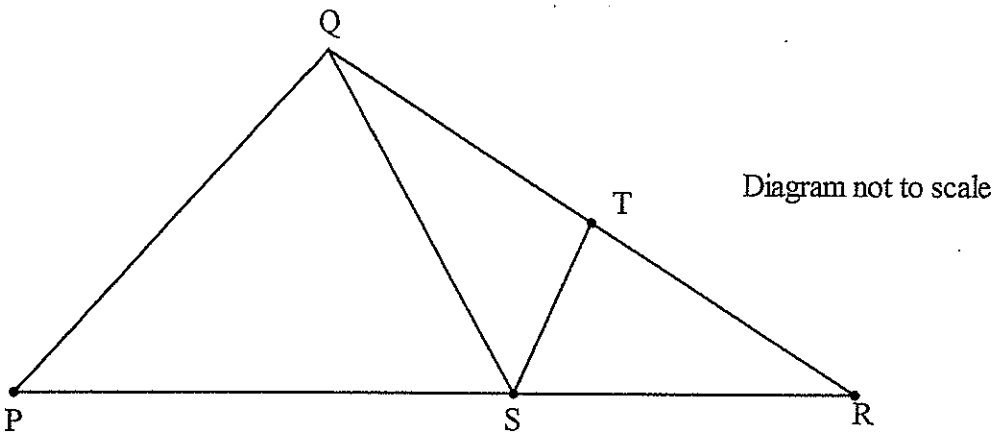


Diagram not to scale

In ΔPQR, T lies on side QR and S lies on side PR such that $QT = TR$, $QS = QP$, $ST \perp QT$

i) Copy the diagram into your answer booklet and show all the given information. (1)

ii) Prove that $\Delta QTS \equiv \Delta RTS$ (2)

iii) Prove that $\angle QPS = 2\angle TQS$ (3)

Question 11 (11 Marks) Marks

a. Consider the circle $(x-1)^2+(y-2)^2=18$

i) Find the centre of the circle and its radius (2)

ii) Show that the line $y=x-5$ is tangent to the circle (3)

b. Consider the function $g(x)=\dfrac{2x}{x^2-1}$

i) State the domain of this function (1)

ii) Determine with justification whether the function is odd, even, or neither (2)

c. Given $f(x)=\sqrt{5x-25}-\sqrt{x-1}$ find the value of x for which $f(x)=2$. (3)

Question 12 (11 Marks) Marks

a. A regular polygon has an exterior angle of $20°$.
How many sides does the polygon have? (1)

b. Prove that $\cos^2(90^\circ-\theta)\cot\theta=\sin\theta\cos\theta$ (2)

c. Show that $\lim\limits_{x\to\infty}\dfrac{4x^2-x^3+2}{3x^3-x^2+1}=\dfrac{-1}{3}$ (2)

d. Find the equation of the line passing through the point of intersection of the lines

$4x-2y+3=0$ and $x+4y+6=0$ and which has a gradient of $\dfrac{2}{3}$. (3)

e. A large tank can be filled by 2 similar small pumps and 1 larger pump working together in 1 hour and 12 minutes. The larger pump B alone takes 1 hour less than the smaller pump A alone to fill the tank. Find out how long each pump takes, given that $\dfrac{V}{P}=T$ where

$V=$ Volume of the tank, $P=$ Pump, $T=$ Time (3)

END OF EXAM

Solution Year 11 2unit Semester 2 2017

Question 6

a) $2\sqrt{50} - \sqrt{72} = \sqrt{y}$

$10\sqrt{2} - 6\sqrt{2} = \sqrt{y}$

$4\sqrt{2} = \sqrt{y}$

$\therefore y = 32$

b) $\frac{\sqrt{2}}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$

$= \frac{\sqrt{6}-\sqrt{2}}{2}$

c) $2\cos\theta = 1$

$\cos\theta = -\frac{1}{2}$

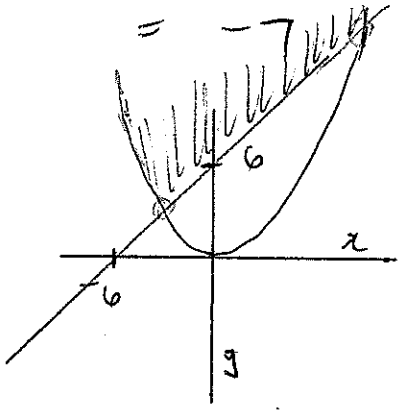
$\theta = 120°, 240°$

d) $\lim_{x \to -4} \frac{x^2+x-12}{x+4}$

$= \frac{(x+4)(x-3)}{x+4}$

$= -7$

e)



f) $\cos\theta \tan\theta = \sin\theta$

L.H.S

$\cos\theta \cdot \frac{\sin\theta}{\cos\theta}$

$= R.H.S.$

Question 7

a) $x \neq 2$.

b) $2(8) - -\frac{1}{3}$

$= 16\frac{1}{3}$.

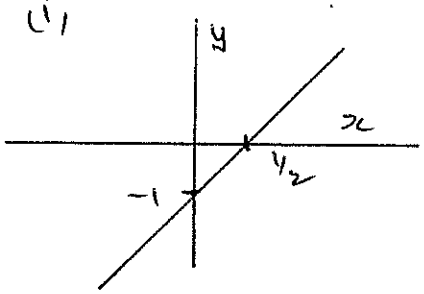
c) $\frac{16}{2^{3x} \times 8^{1-x}}$

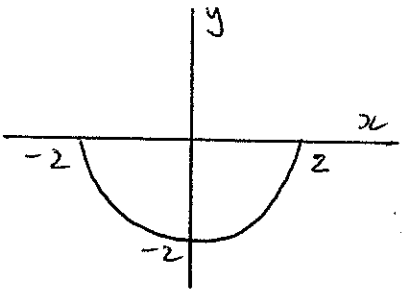
$= \frac{2^4}{2^{3x} \times 2^{3-3x}}$

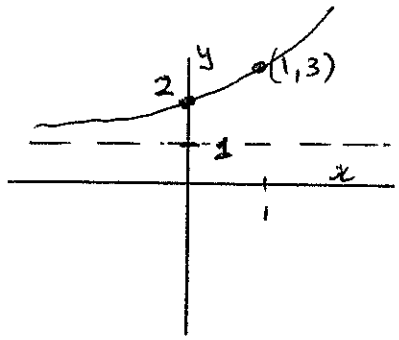
$= \frac{2^4}{2^3}$

$= 2$.

d) (i)







Multiple Choice

Q1 B

2 C

3 A

4 B

5 D

Question 8

(a) (i) $\frac{-7}{3}$

(ii) $\tan^{-1}(-7/3)$

$= 113^0$

(iii) B(5,4) C(x,0)

$\frac{4}{5-x} = \frac{3}{7}$

$28 = 15 - 3x$

$x = -\frac{13}{3}$.

(iv) $M(13/2, 1/2)$

(v) $y - 4 = \frac{-7}{3}(x-5)$

$3y - 12 = -7x + 35$

$7x + 3y - 47 = 0$

(b) (i) $750 = \frac{1}{2} \times 60 \times 50 \times \sin\theta$

$\frac{1}{2} = \sin\theta$

$\theta = 150^0$

(ii) $BC^2 = 60^2 + 50^2 - 2 \times 60.50 \cos 150^0$

$BC \doteq 106.3$.

(c) $\frac{1}{x} = \frac{1}{a} + \frac{1}{b}$

$\frac{1}{x} - \frac{1}{a} = \frac{1}{b}$

$\frac{a-x}{ax} = \frac{1}{b}$

$b = \frac{ax}{a-x}$

Question 9

(a) (i) In Δ's SRT & PQT

$\angle RST = \angle TQP$ (alt $\angle$'s, SR $\parallel$ PQ)

$\angle STR = \angle PTQ$ (vert opp $\angle$'s)

$\Delta SRT \,|||\, \Delta PQT$ (equiangular)

hence corresponding heights and sides are in the same ratio.

$\therefore \frac{SR}{PQ} = \frac{VT}{UT}$

(ii) $SR = \frac{VT \times PQ}{UT}$

$= \frac{(3-y) \times 4}{y}$

$= \frac{12 - 4y}{y}$

$= \frac{12}{y} - 4$

(iii) $A = \frac{1}{2} SR \times VT + \frac{1}{2} PQ \times UT$

$= \frac{1}{2}\left(\frac{12}{y} - 4\right)(3-y) + \frac{1}{2}(4)(y)$

$= \frac{1}{2}\left(\frac{36}{y} - 12 - 12 + 4y\right) + 2y$.

$= \frac{18}{y} - 12 + 2y + 2y$

$= \frac{18}{y} - 12 + 4y$.

Question 9 cont'd

(b) $\frac{1}{2\sqrt{5}+\sqrt{x}} + \frac{1}{2\sqrt{5}-\sqrt{x}} = 2\sqrt{5}$

$\frac{2\sqrt{5}-\sqrt{x}}{20-x} + \frac{2\sqrt{5}+\sqrt{x}}{20-x} = 2\sqrt{5}$

$4\sqrt{5} = 40\sqrt{5} - 2\sqrt{5}x$

$-36\sqrt{5} = -2\sqrt{5}x$

$x = 18$

Question 10.

(a) (i) $\angle ANL = 24^\circ$

(ii) $\frac{AM}{\sin 24^\circ} = \frac{4000}{\sin 41^\circ}$

$AM = \frac{4000 \sin 24^\circ}{\sin 41^\circ}$

$AM = 2480\text{m}.$

(iii) $\cos 25^\circ = \frac{AL}{AM}$

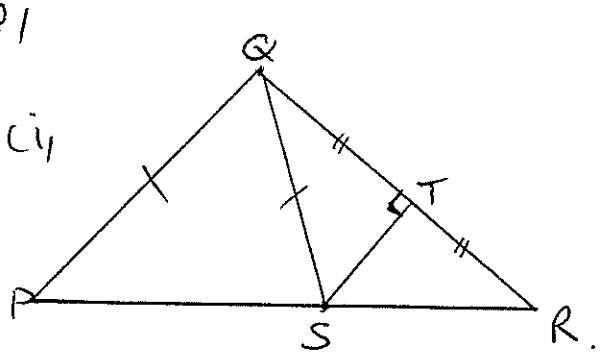
$\cos 25^\circ = \frac{AL}{2480}$

$AL = 2480 \cos 25^\circ$

$AL = 2248\text{m}.$

(b)

(i)



(ii) $QT = TR$ given

$\angle QTS = \angle STR = 90^\circ$ (given)

ST common side

$\Delta QTS \equiv \Delta RTS$ (S.A.S)

(iii) $QS = SR$ (corr sides in $\equiv$ Δ's from (ii))

Let $\angle TSR = \phi$

$\angle TSQ = \phi$

] (corr $\angle$'s in $\equiv \Delta$'s)

$\angle SQT = 90 - \phi$ base $\angle$'s isos ΔQSR

$\angle QSP = 180 - 2\phi$ supplement of $\angle QSR$

$\angle QSP = \angle QPS$ Base $\angle$'s isos ΔQPS

$\therefore \angle QPS = 2\angle TQS.$

Question 11

(a) $(x-1)^2 + (y-2)^2 = 18$

(i) $C(1,2)$ $\quad r = 3\sqrt{2}$

(ii) $(1,2)$ $\quad x - y - 5 = 0$

$$PD = \frac{|1 - 2 - 5|}{\sqrt{2}}$$

$$= \frac{6}{\sqrt{2}}$$

$$= \frac{6\sqrt{2}}{2}$$

$$= 3\sqrt{2}.$$

$\therefore$ equal to radius and $\perp$

thus $y = x - 5$ is tangent to the circle

(b) (i) $g(x) = \dfrac{2x}{x^2 - 1}$

all $\mathbb{R}x$, $x \neq \pm 1$

(ii) $g(-x) = \dfrac{-2x}{x^2 - 1}$

$g(x) = -g(-x)$

$\therefore$ odd function

(c)

$$f(x) = \sqrt{5x - 25} - \sqrt{x - 1}$$

$$2 = \sqrt{5x - 25} - \sqrt{x - 1}$$

$$4 = 5x - 25 - 2\sqrt{(5x-25)(x-1)} + x - 1$$

$$30 - 6x = -2\sqrt{(5x-25)(x-1)}$$

$$(3x - 15)^2 = 5x^2 - 30x + 25$$

$$4x^2 - 60x + 200 = 0$$

$$x^2 - 15x + 50 = 0$$

$$(x - 5)(x - 10) = 0$$

$$x = 5, \quad x = 10$$

Soln $x = 10$ only.

Question 12

(a) $\frac{360}{20}$

$= 18$

(b) $\cos^2(90-\theta)\cot\theta = \sin\theta\cos\theta$

L.H.S $\sin^2\theta \cdot \frac{\cos\theta}{\sin\theta}$

$\sin\theta\cos\theta$

(c) $\lim_{x\to\infty} \frac{4x^2 - x^3 + 2}{3x^3 - x^2 + 1}$

$$\frac{\frac{4x^2}{x^3} - \frac{x^3}{x^3} + \frac{2}{x^3}}{\frac{3x^3}{x^3} - \frac{x^2}{x^3} + \frac{1}{x^3}}$$

$$\frac{\frac{4}{x} - 1 + \frac{2}{x^3}}{3 - \frac{1}{x} + \frac{1}{x^3}}$$

now $\lim_{x\to\infty} \frac{1}{x} = 0$

$$\frac{0 - 1 + 0}{3 - 0 + 0}$$

$= -\frac{1}{3}$.

(d) $4x - 2y + 3 + K(x + 4y + 6) = 0$

$4x - 2y + 3 + Kx + 4Ky + 6K = 0$

$x(4+K) - y(2-4K) + 3 + 6K = 0$

$y = \frac{x(4+K)}{2-4K} + \frac{3+6K}{2-4K}$

(d) cont'd

$\frac{4+K}{2-4K} = \frac{2}{3}$

$12 + 3K = 4 - 8K$

$11K = -8$

$K = -\frac{8}{11}$

Eqn of line

$4x - 2y + 3 - \frac{8}{11}(x + 4y + 6) = 0$

$44x - 22y + 33 - 8x - 32y - 48 = 0$

$36x - 54y - 15 = 0$.

$12x - 18y - 5 = 0$

(e) $\frac{V}{2A+B} = \frac{6}{5}$ — (3)

$\frac{V}{B} = T - 1$ — (1)

$\frac{V}{A} = T$ — (2)

from (1) $\frac{V}{T-1} = B$

from (2) $\frac{V}{T} = A$

} sub into (3)

$\frac{V}{\frac{2V}{T} + \frac{V}{T-1}} = \frac{6}{5}$

$\frac{T(T-1)}{[2(T-1)+T]} = \frac{6}{5}$

$5T^2 - 23T + 12 = 0$

$(5T - 3)(T - 4) = 0$

$T = 4 \quad T = \frac{3}{5}$

$\therefore$ Pump B = 3 hrs

Pump A = 4 hrs