Caringbah High School

Year 11 2019

Mathematics Advanced

Preliminary Course

Assessment Task 4

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black pen
- NESA-approved calculators may be used
- A reference sheet is provided
- In Questions 11-17, show relevant mathematical reasoning and/or calculations
- Answer each question in a separate answer booklet

Total marks – 85

Section I **10 marks**

Attempt Questions 1-10

Mark your answers on the answer sheet provided. Detach the sheet and write your name on it.

Section II **75 marks**

Attempt Questions 11-17

Write your answers in the answer booklets provided. Ensure your name or student number is clearly visible.

Name: _____ **Class:** _____

	Marker's Use Only								
Section I	**Section II**							**Total**	
Q 1-10	**Q11**	**Q12**	**Q13**	**Q14**	**Q15**	**Q16**	**Q17**		
/10	/11	/11	/11	/11	/11	/10	/10	/85	%

Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10

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1. Simplify fully $\frac{x^6}{y^6} \div x^2 y^{-3}$?

(A) $\frac{x^3}{y^2}$

(B) $\frac{x^4}{y^2}$

(C) $\frac{x^4}{y^3}$

(D) $\frac{x^4}{y^9}$

2. What is the minimum value of the expression $x^2 + 4x - 1 = 0$?

(A) -1

(B) -5

(C) -9

(D) -13

3. What are the solutions of the equation $\left|\frac{x}{2} - 1\right| = 3$?

(A) $x = -4$ or $x = -8$

(B) $x = 4$ or $x = -8$

(C) $x = -4$ or $x = 8$

(D) $x = 4$ or $x = 8$

4. The discrete random variable X denotes the number on the upper face when a fair die is rolled once. What is the probability that $4X+1$ is a perfect square?

(A) $\frac{1}{6}$

(B) $\frac{1}{3}$

(C) $\frac{1}{2}$

(D) $\frac{2}{3}$

5. What is the solution of the equation $\tan^2 x - 3 = 0$ for $0 \le x \le \pi$.

(A) $x = \frac{\pi}{6}$ only

(B) $x = \frac{\pi}{3}$ only

(C) $x = \frac{\pi}{6}$ or $x = \frac{5\pi}{6}$

(D) $x = \frac{\pi}{3}$ or $x = \frac{2\pi}{3}$

6. Which of the following statements about the graph of the function $y = 2\sin 4x$ is correct?

(A) The amplitude is 2 and the period is $\frac{\pi}{4}$

(B) The amplitude is 2 and the period is $\frac{\pi}{2}$

(C) The amplitude is 4 and the period is $\frac{\pi}{2}$

(D) The amplitude is 4 and the period is π

7. If $a = 10^x$, which of the following is an expression of $\log_{10} \sqrt{a}$?

(A) $\frac{x}{2}$

(B) x

(C) $\sqrt{x}$

(D) x^2

8. The lines $2x - ky = 0$ and $x - 3y = 0$ are perpendicular. What is the value of k ?

(A) $-\frac{1}{6}$

(B) $-\frac{2}{3}$

(C) $-\frac{3}{2}$

(D) –6

9. Which of the following is the derivative of $(x^3 + 4)^3$?

(A) $3(x^3 + 4)^2$

(B) $3(3x^2 + 4)^2$

(C) $9x^2(x^3 + 4)^2$

(D) $9x^2(3x^2 + 4)^2$

10. A function is given by $f(x) = \sqrt{9 - x^2}$. What is the domain of $f(x)$?

(A) $x \geq 3$

(B) $x \leq 3$

(C) $-3 \leq x \leq 3$

(D) $-9 \leq x \leq 9$

End of Section I- Multiple Choice Questions

Section II

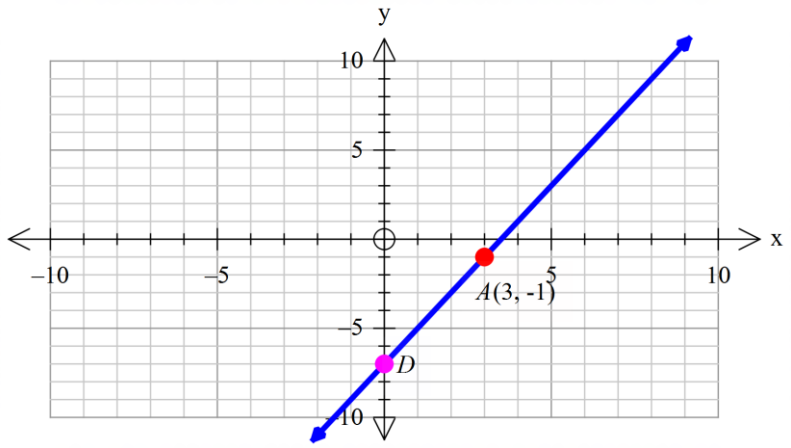
60 marks
Attempt Questions 11–17
Allow about 1 hours 45 minutes for this section

Answer each question in a SEPARATE writing booklet.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (11 marks) Use a SEPARATE writing booklet.

(a) $A(3,-1)$ is a vertex of the rectangle $ABCD$ whose side AD lies on the line, as shown in the diagram, has an equation $2x-y-7=0$. Find, in general, form the equation of the line that AB lies on. **3**



(b) A fair die is rolled once. *A* is the event *the score is even* and *B* is the event *the score is 5* or *6*. Determine, giving reasons, whether the events *A* and *B* are independent. **2**

(c) If $f(x)=\sqrt{4-x}$ and $g(x)=x^2$ find

(i) the composite function $f(g(x))$. **1**

(ii) the domain and range of $f(g(x))$. **2**

(d) Solve $2\sin^2 x+\sin x-1=0$ for $0°\le x\le 360°$ **3**

End of Question 11

Question 12 (11 marks) Use a SEPARATE writing booklet.

(a) Solve $\log_x 8 = 1.5$ **1**

(b) In Year 7 at Random High School, every student must do Art or Music. In a group of 100 students surveyed, 47 do Music and 59 do Art.

(i) Draw a Venn Diagram to represent the above information. **1**

(ii) Find the probability that this student does both Music and Art. **1**

(iii) A student is selected at random. Find the probability that this student does Art only. **1**

(c) Find any values of x where the graph of $f(x) = \dfrac{1}{x^2 - 4x}$ is not continuous. **2**

(d) Sketch the graph of the curve $y = \log_2(x+1)$ showing the intercepts on the axes and the equation of the asymptote. **3**

(e) In a game of chance, Chloe tosses 2 coins. She wins \$10 for 2 heads, \$5 for 2 tails and nothing for a head and a tail. Find the expected value of this game. **2**

End of Question 12

Question 13 (11 marks) Use a SEPARATE writing booklet.

(a) Differentiate:

(i) $\frac{2}{\sqrt{x}} - 3x^2$ **2**

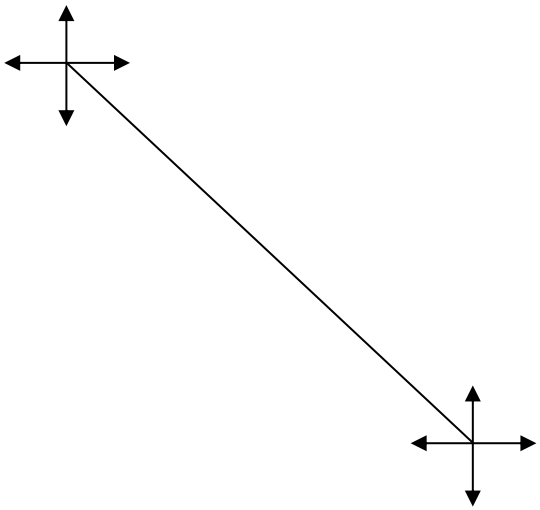
(ii) $(x^2 - 3x)^5$ **2**

(iii) $\frac{5 - x^3}{3x - 5}$ **3**

(b) Simplify $\frac{16}{2^{3x} \times 8^{1-x}}$ **1**

(c) A plane flies from Orange for 1800 km on a bearing of $300°$. It then turns and flies for 2500 km on a bearing of $205°$.

(i) Represent this question on a diagram by copying completely the diagram below in your booklet. **1**



(ii) How far is the plane from Orange, to the nearest km? **2**

End of Question 13

Question 14 (11 marks) Use a SEPARATE writing booklet.

(a) Using first principles formula $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ to find $f'(x)$ when $f(x) = x^2 + 3$ **2**

(b) Solve the equation $\log_2(x+1) - \log_2(x-1) = 2$ **2**

(c) Using the equation $y = (2x-3)^3$, find

(i) the equation of the tangent to the curve at the point where $x = 1$. **3**

(ii) the angle of inclination of this tangent line, correct to the nearest degree. **1**

(d) Show that $\frac{1+\cos\theta}{\sin\theta} + \frac{\sin\theta}{1+\cos\theta} = 2\operatorname{cosec}\theta$ **3**

End of Question 14

Question 15 (11 marks) Use a SEPARATE writing booklet.

(a) Sketch the graphs on separate Cartesian planes (one-third page size), showing distinct features and intercepts.

(i) $y = 2^x + 1$ **2**

(ii) $y = |x - 2|$ **2**

(b) Find the exact value of a and b if $\dfrac{\sqrt{3}-4}{2+3\sqrt{3}} = a + b\sqrt{3}$. **3**

(c) Solve

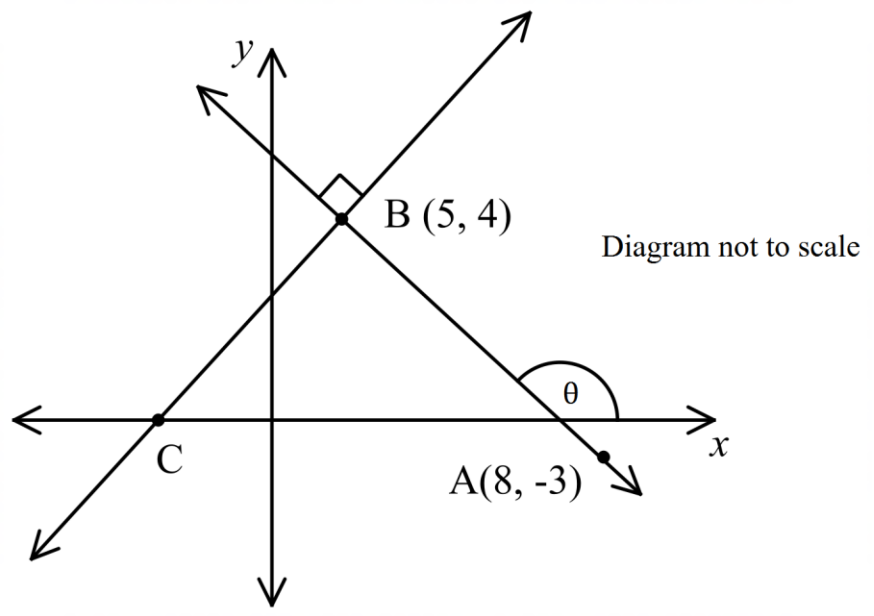
(i) $3x^2 - 5x + 2 \geq 0$ **2**

(ii) $|4t - 5| = t + 2$ **2**

End of Question 15

Question 16 (10 marks) Use a SEPARATE writing booklet.

(a) The points $A(8,-3)$ and $B(5,4)$ are shown in the diagram below. The line through AB makes an angle of θ with the positive x-axis and the point C lies on the x-axis.



(i) Find the gradient of the line AB **1**

(ii) Find the value of θ to the nearest degree. **1**

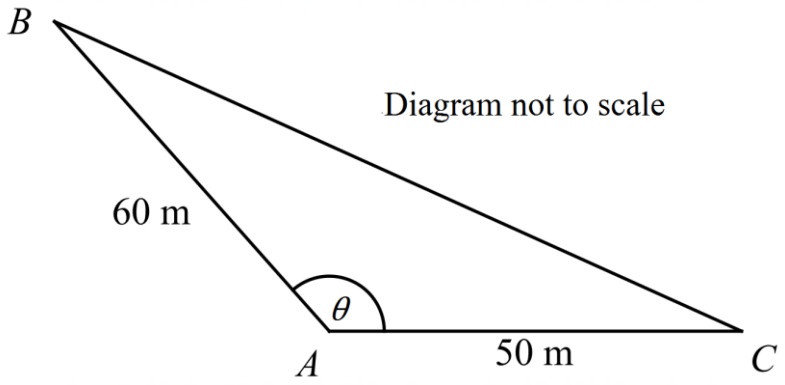
(iii) Find the coordinates of C given $AB \perp BC$. **2**

(iv) Find the coordinates of M, the midpoint of AB. **1**

(v) Find the equation of the line, in general form, through M parallel to the line BC. **2**

Q16 continued on page 11

(b) Two sides of a triangular field are $AB = 60$ m, $AC = 50$ m and the included angle $\angle BAC = \theta$ is obtuse.



(i) If the area of the triangle is 750 m^2, find the size of $\angle BAC$ **1**

(ii) Find the length of the side BC, correct to 1 decimal place. **2**

End of Question 16

Question 17 (10 marks) Use a SEPARATE writing booklet.

(a) Consider the circle $x^2 - 2x + y^2 - 4y = 13$

(i) Find the centre and its radius **2**

(ii) Show that the line $y = x - 5$ is a tangent to the circle. **3**

(b) Consider the function $f(x) = \dfrac{2x}{x^2 - 1}$. **1**

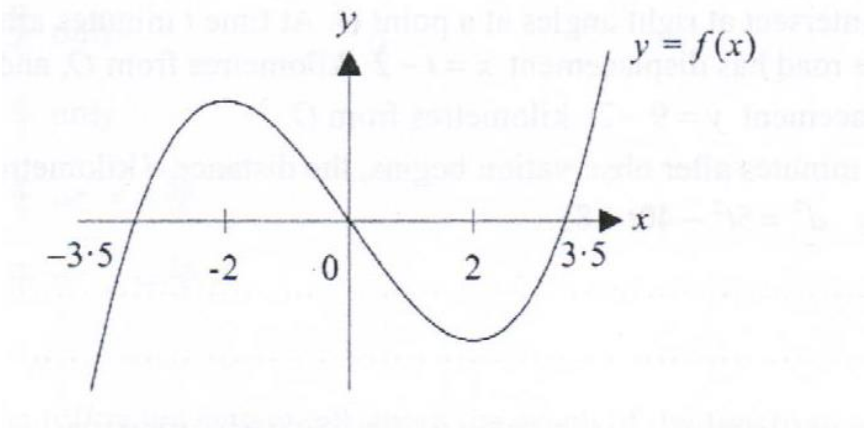
Determine with justification whether the function is odd, even or neither.

(c) A discrete random variable X with mean, $\mu = 2.1$, has probability distribution

x	1	2	3
$P(X = x)$	0.2	a	b

Find the values of the constants a and b. **3**

(d) Using the curve given, sketch $f(-x)$. **1**



End of Question 17

End of Examination!

Student Name: _____

Multiple choice answer page. Fill in either A, B, C or D for questions 1-10.

This page must be handed in with your answer booklets

1.	
2.	
3.	
4.	
5.	

6.	
7.	
8.	
9.	
10.	

Yr 11 Advanced Exam 2019 AT4

Multiple choice

① $\frac{x^6}{y^6} \div x^2y^{-3} = x^6y^{-6} \div x^2y^{-3}$

$= x^{6-2}y^{-6-(-3)}$

$= x^4y^{-6+3}$

$= x^4y^{-3}$

$= \frac{x^4}{y^3}$ (C)

② $x^4+4x-1=0$

$x^2+4x+4=1+4$

$(x+2)^2=5$

$(x+2)^2-5=0$

∴ min value is -5 (B)

③ $\left|\frac{x}{2}-1\right|=3$.

$\frac{x}{2}-1=3$ OR $\frac{x}{2}-1=-3$

$\frac{x}{2}=4$ $\frac{x}{2}=-2$

$x=8$ $x=-4$

∴ $x=-4, 8$ (C)

④

X	1	2	3	4	5	6
4X+1	5	9	13	17	21	25
		*				*

∴ P(perfect square) $= \frac{2}{6}$

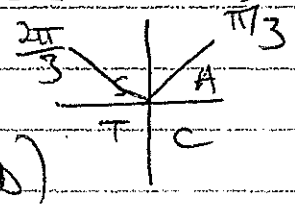
$= \frac{1}{3}$ (B)

⑤ $\tan^2x-3=0$ $0 \le x \le \pi$

$\tan^2x=3$

$\tan x = \pm\sqrt{3}$

∴ $x=\frac{\pi}{3}, \frac{2\pi}{3}$ (D)



⑥ $y=2\sin\frac{1}{2}x$

amplitude $=2$, period $=\frac{2\pi}{\frac{1}{2}}=4\pi$.

(B)

⑦ $a=10^x$

$\sqrt{a}=10^{x/2}$

$\log_{10}\sqrt{a}=\log_{10}10^{x/2}$

∴ $\frac{x}{2}=\log_{10}\sqrt{a}$ (A)

⑧ $2x-ky=0$ $x-3y=0$

$ky=2x$ $3y=x$

$y=\frac{2}{k}x$ $y=\frac{1}{3}x$.

∴ $\frac{2}{k}\times\frac{1}{3}=-1$

$\frac{2}{3k}=-1$

$2=-3k$

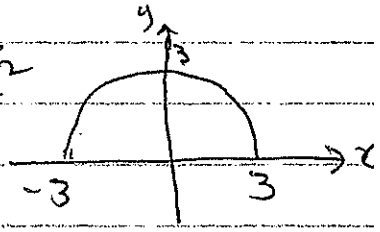
$k=\frac{-2}{3}$ (B)

⑨ $\frac{d}{dx}(x^3+4)^3$

$= 3(x^3+4)^2(3x^2)$

$= 9x^2(x^3+4)^2$ (C)

⑩ $f(x)=\sqrt{9-x^2}$



Domain: $-3 \le x \le 3$ (C)

① C	⑤ D	⑨ C	Mult.	
② B	⑥ B	⑩ C	choice	
③ C	⑦ A			
④ B	⑧ B			

Question (11)

(a) AD: $2x-y-7=0$. $A(3,-1)$

$y=2x-7$

$m_{AD}=2$, $m_{AB}=-\frac{1}{2}$.

$y-y_1=m(x-x_1)$

$y+1=-\frac{1}{2}(x-3)$

$2y+2=-x+3$

$x+2y-1=0$.

(b) $P(A)\times P(B)=\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}=P(A\cap B)$

OR

$P(A|B)=\frac{1}{6}\div\frac{1}{3}=\frac{1}{2}=P(A)$.

$\therefore$ A & B are independent.

(c) $f(x)=\sqrt{4-x}$ $g(x)=x^2$

(i) $f(g(x))=f(x^2)$

$=\sqrt{4-x^2}$

(ii) Domain: $-2\le x\le 2$

Range: $0\le y\le 2$.

(d) $2\sin^2x+\sin x-1=0$, $0°\le x\le 360°$

Let $m=\sin x$

$2m^2+m-1=0$.

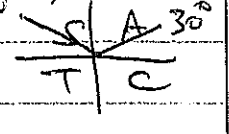
$(m+1)(2m-1)=0$

$m=-1,\frac{1}{2}$

But $m=\sin x$

$\sin x=-1$ $\sin x=\frac{1}{2}$

$x=270°$ $x=30°, 150°$



$\therefore x=30°, 150°, 270°$

Question (12)

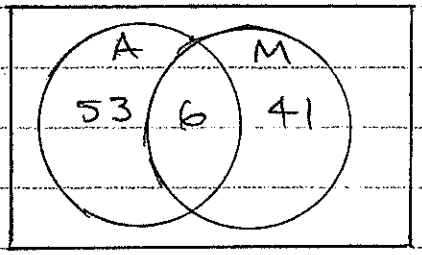
(a) $\log_x 8=1.5$

$(x^{3/2})^{2/3}=(8)^{2/3}$

$x=\sqrt[3]{8^2}$

$x=4$

(b) (i)



47+59=106

(ii) $P(\text{both})=\frac{6}{100}=\frac{3}{50}$

(iii) $P(\text{Art only})=\frac{53}{100}$

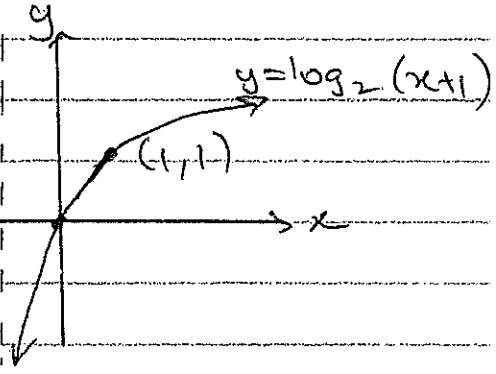
(c) $f(x)=\frac{1}{x^2-4x}$

Not continuous: $x^2-4x=0$

$x(x-4)=0$

$x=0,4$

(d) $y=\log_2(x+1)$



(e)

x	0	1	2
$P(X=x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

y	\$0	\$5	\$10
$P(Y=y)$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$

$E(Y)=\$0\times\frac{1}{2}+\$5\times\frac{1}{4}+\$10\times\frac{1}{4}$

$=\$3.75$

(13)(a)(i) $\frac{d}{dx}\left(\frac{2}{\sqrt{x}} - 3x^2\right)$

$= \frac{d}{dx}\left(2x^{-1/2} - 3x^2\right)$

$= -x^{-3/2} - 6x$

$= -\frac{1}{\sqrt{x^3}} - 6x$

(ii) $\frac{d}{dx}(x^2 - 3x)^5$

$= 5(x^2-3x)^4(2x-3)$

$= (10x-6)(x^2-3x)^4$

(iii) $\frac{d}{dx}\left(\frac{5-x^3}{3x-5}\right)$

$= \frac{(3x-5)(-3x^2) - (5-x^3)(3)}{(3x-5)^2}$

$= \frac{-9x^3 + 15x^2 - 15 + 3x^3}{(3x-5)^2}$

$= \frac{-6x^3 + 15x^2 - 15}{(3x-5)^2}$

(b) $\frac{16}{2^{3x} \times 8^{1-x}} = \frac{2^4}{2^{3x} \times 2^{3(1-x)}}$

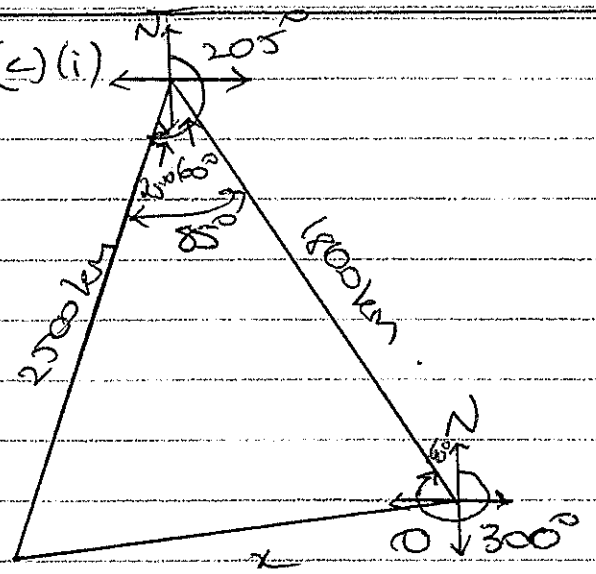
$= \frac{2^4}{2^{3x} \times 2^{3-3x}}$

$= \frac{2^4}{2^{3x+3-3x}}$

$= \frac{2^4}{2^3}$

$= 2$

(c)(i)



(ii) $x^2 = 2500^2 + 1800^2 - 2(2500)(1800)\cos 85°$

$x^2 = 8705598.315$

$x = 2950.5$

$\doteq 2951$ km (nearest km)

Question (14)

(a) $f(x) = x^2 + 3$

$f(x+h) = (x+h)^2 + 3$

$= x^2 + 2xh + h^2 + 3$

$f'(x) = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 3 - (x^2+3)}{h}$

$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 3 - x^2 - 3}{h}$

$= \lim_{h \to 0} \frac{2xh + h^2}{h}$

$= \lim_{h \to 0} \frac{h(2x+h)}{h}$

$= 2x$

Question (14) cont.

(b) $\log_2(x+1) - \log_2(x-1) = 2$

$\log_2\left(\frac{x+1}{x-1}\right) = 2$

$\frac{x+1}{x-1} = 2^2$

$\frac{x+1}{x-1} = 4, \quad x \neq 1$

$x+1 = 4x-4$

$3x = 5$

$x = 5/3$

(c) $y = (2x-3)^3, \quad x=1$

(i) $y' = 3(2x-3)^2(2)$

$= 6(2x-3)^2$

when $x=1$, $y' = 6(2-3)^2$

$m = 6$

when $x=1$, $y = -1$

$y - y_1 = m(x-x_1)$

$y+1 = 6(x-1)$

$y+1 = 6x-6$

$y = 6x-7$

(ii) $m = \tan\theta$

$\tan\theta = 6$

$\theta = 81^\circ$

(d) $\frac{1+\cos\theta}{\sin\theta} + \frac{\sin\theta}{1+\cos\theta} = 2\operatorname{cosec}\theta$

LHS:

$\frac{1+\cos\theta}{\sin\theta} + \frac{\sin\theta}{1+\cos\theta}$

$= \frac{(1+\cos\theta)^2 + \sin^2\theta}{\sin\theta(1+\cos\theta)}$

$= \frac{1+2\cos\theta+\cos^2\theta+\sin^2\theta}{\sin\theta(1+\cos\theta)}$

$= \frac{1+2\cos\theta+1}{\sin\theta(1+\cos\theta)}$

$= \frac{2+2\cos\theta}{\sin\theta(1+\cos\theta)}$

$= \frac{2(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$

$= \frac{2}{\sin\theta}$

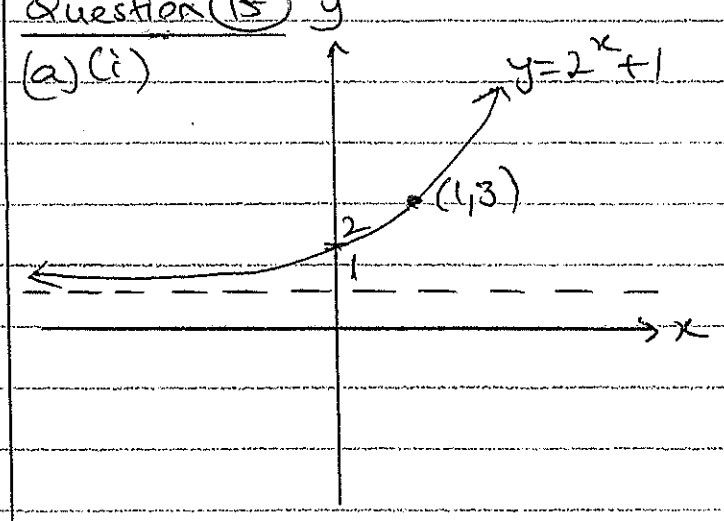
$= 2 \times \frac{1}{\sin\theta}$

$= 2\operatorname{cosec}\theta$

$=$ RHS

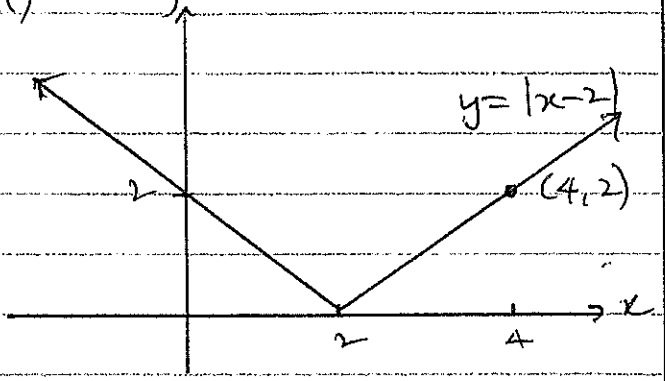
Question (15)

(a)(i)



Question 15 cont.

(a)(ii)



(b) $\frac{\sqrt{3}-4}{2+3\sqrt{3}} = a+b\sqrt{3}$

LHS:

$\frac{\sqrt{3}-4}{2+3\sqrt{3}} \times \frac{2-3\sqrt{3}}{2-3\sqrt{3}}$

$= \frac{2\sqrt{3}-9-8+12\sqrt{3}}{4-27}$

$= \frac{14\sqrt{3}-17}{-23}$

$= \frac{-14\sqrt{3}+17}{23}$.

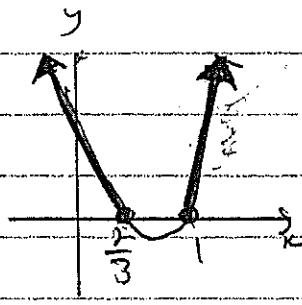
$\therefore a = \frac{-14}{23}, \quad b = \frac{17}{23}$

(c)(i) $3x^2-5x+2 \geq 0$.

$(x-1)(3x-2) \geq 0$.

$x = 1, \ ^2/_3$

$\therefore x \leq \frac{2}{3}, \quad x \geq 1$



(ii) $|4t-5| = t+2$

$4t-5 = t+2$ OR $4t-5 = -(t+2)$

$3t = 7$ $\quad$ $4t-5 = -t-2$

$t = \frac{7}{3}$ $\quad$ $5t = 3$

$t = \frac{3}{5}$

$\therefore t = \frac{3}{5}, \frac{7}{3}$

Question 16

(a)(i) A(8,-3) B(5,4)

$m = \frac{4+3}{5-8}$

$= \frac{7}{-3}$

$= -\frac{7}{3}$

(ii) $\tan\theta = -\frac{7}{3}$

$\theta = 113^\circ$

(iii) $m = \frac{3}{7}$ B(5,4)

$y-y_1 = m(x-x_1)$

$y-4 = \frac{3}{7}(x-5)$

x-int, $y=0$

$-4 = \frac{3}{7}(x-5)$

$-28 = 3x-15$

$3x = -13$

$x = -13/3$

$\therefore C(\frac{-13}{3}, 0)$

(iv) A(8,-3) B(5,4)

midpoint $= (\frac{8+5}{2}, \frac{-3+4}{2})$

$= (\frac{13}{2}, \frac{1}{2})$

Question 16 cont.

(a)(v) $M\left(\frac{13}{2}, \frac{1}{2}\right)$ $m = \frac{3}{7}$

$y - \frac{1}{2} = \frac{3}{7}\left(x - \frac{13}{2}\right)$

$7y - \frac{7}{2} = 3x - \frac{39}{2}$

$7y = 3x - 16$

$3x - 7y - 16 = 0.$

(b)(i) $750 = \frac{1}{2} \times 60 \times 50 \sin\theta$

$750 = 1500 \sin\theta$

$\sin\theta = \frac{1}{2}$

$\theta = 30°$

But θ is obtuse, $\theta = 150°$

(ii) $BC^2 = 60^2 + 50^2 - 2(60)(50) \cos 150°$

$BC^2 = 11296.15242$

$BC = 106.3\text{ m}$ (1dp)

Question 17

(a) $x^2 - 2x + y^2 - 4y = 13$

(i) $x^2 - 2x + 1 + y^2 - 4y + 4 = 13 + 1 + 4$

$(x-1)^2 + (y-2)^2 = 18$

∴ centre $(1, 2)$, radius $= \sqrt{18}$

$= 3\sqrt{2}$

(ii) $x^2 - 2x + y^2 - 4y = 13 \ldots (1)$

$y = x - 5 \ldots (2)$

sub (2) into (1)

$x^2 - 2x + (x-5)^2 - 4(x-5) = 13$

$x^2 - 2x + x^2 - 10x + 25 - 4x + 20 = 13$

$2x^2 - 16x + 45 = 13$

$2x^2 - 16x + 32 = 0.$

$\Delta = b^2 - 4(2)(32)$

$\Delta = 0$

∴ line is a tangent to the circle.

(b) $f(x) = \frac{2x}{x^2 - 1}$

$f(-x) = \frac{2(-x)}{(-x)^2 - 1}$

$= \frac{-2x}{x^2 - 1}$

$= -f(x)$

∴ $f(x)$ is an odd fnc.

(c) $0.2 + 2a + 3b = 2.1$

$2a + 3b = 1.9 \ldots (1)$

$0.2 + a + b = 1$

$a + b = 0.8 \ldots (2)$

(1) − 2×(2)

$2a + 3b = 1.9$

$2a + 2b = 1.6$ −

$b = 0.3$

sub $b = 0.3$ into (2)

$a + 0.3 = 0.8$

$a = 0.5$

∴ $a = 0.5$, $b = 0.3$

(c)