

Student Number/Name: _____



Caringbah High School

2020

Preliminary Course
Assessment Task 3

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- NESA-approved calculators may be used
- A reference sheet is provided
- In Questions 11 - 33, show relevant mathematical reasoning and/or calculations

Total marks – 75

Section I - 10 marks (pages 2 - 5)

- Attempt Questions 1 - 10
- Mark your answers on the answer sheet provided. Detach the sheet and write your name on it.
- Allow about 15 minutes for this section

Section II - 65 marks (pages 6 - 15)

- Attempt Questions 11 - 33
- Write your answers in the space provided

Marker's Use Only									
Section I	Section II							Total	
Q 1-10	Q11 - 15	Q16 - 19	Q20 - 23	Q24 - 26	Q27 - 29	Q30 - 31	Q32 - 33		
/10	/10	/11	/10	/10	/9	/7	/8	/75	%

Section I - 10 marks**Attempt Questions 1-10****Allow about 15 minutes for this section**Use the multiple-choice answer sheet for Questions 1–10

1. What is 0.0042359 written in scientific notation correct to four significant figures?

(A) 4.2×10^{-3}
(B) 4.236×10^{-3}
(C) 4.2×10^{-4}
(D) 4.236×10^{-4}

2. If a and b are two real numbers and $a, b \neq 0$, what is the gradient of the line joining $A(a, b)$ and $B(3a, -3b)$?

(A) $-\frac{2b}{a}$
(B) $-\frac{a}{2b}$
(C) $\frac{b}{a}$
(D) $\frac{a}{b}$

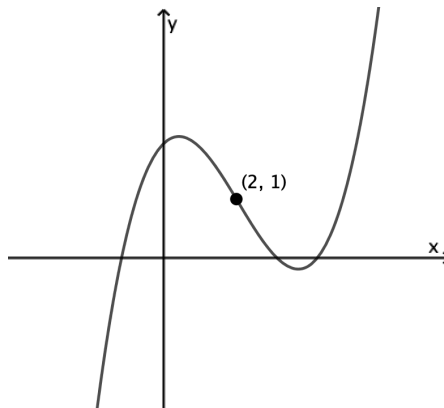
3. What are the solutions to $\sqrt{3} \tan x = -1$ for $0 \leq x \leq 2\pi$?

(A) $\frac{5\pi}{6}$ and $\frac{7\pi}{6}$
(B) $\frac{5\pi}{6}$ and $\frac{11\pi}{6}$
(C) $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$
(D) $\frac{2\pi}{3}$ and $\frac{5\pi}{3}$

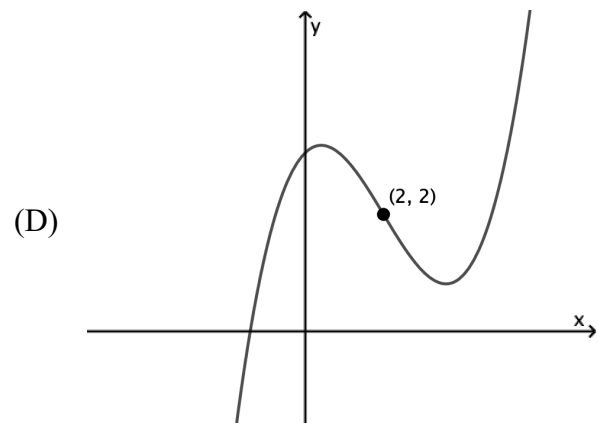
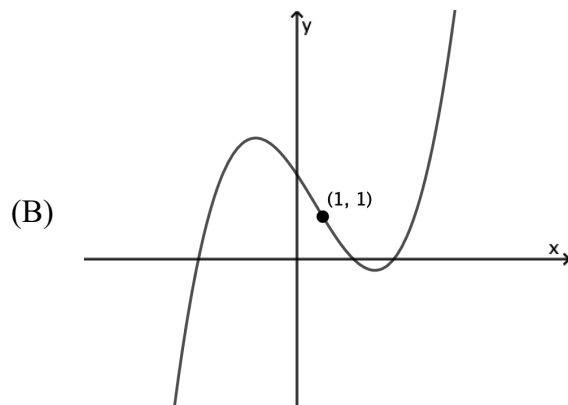
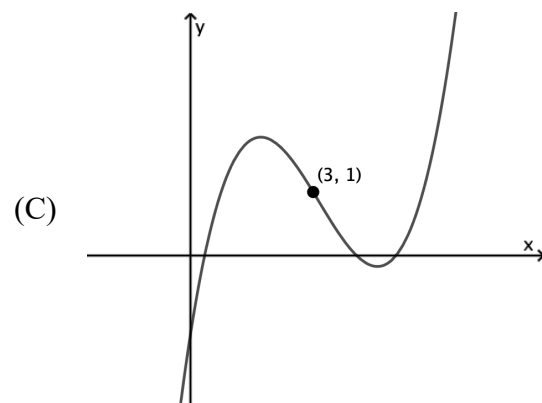
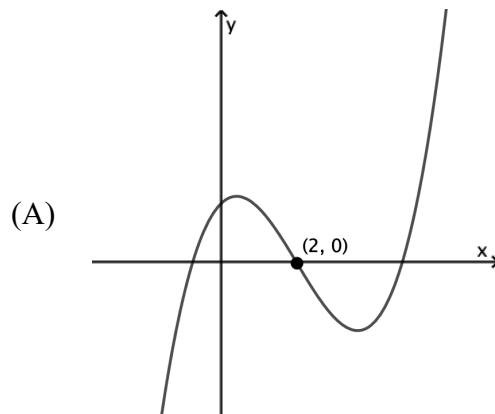
4. What is the domain of the function $f(x) = \sqrt{16 - x^2}$?

- (A) $x < 4$
- (B) $x \leq 4$
- (C) $-4 < x < 4$
- (D) $-4 \leq x \leq 4$

5. The diagram below shows the graph of $y = f(x)$:

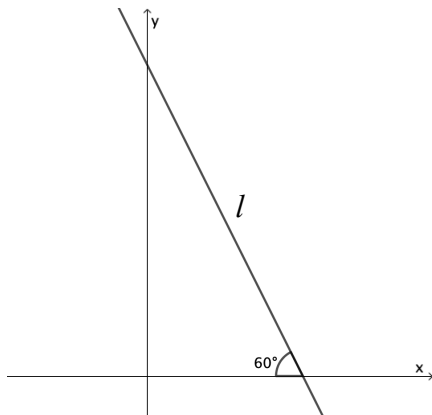


Which of these graphs represents $y = f(x - 1)$?



7. If $a = e^x$ what is the value of $\log_e \frac{1}{a^2}$?
- (A) $-x^2$
- (B) $-2x$
- (C) $2x$
- (D) x^2
6. What are the solutions of the equation $|2x - 3| = 6$?
- (A) $x = \frac{3}{2}$ or $x = -\frac{3}{2}$
- (B) $x = \frac{3}{2}$ or $x = \frac{9}{2}$
- (C) $x = \frac{9}{2}$ or $x = -\frac{3}{2}$
- (D) $x = \frac{3}{2}$ or $x = -\frac{9}{2}$
8. If $5\sqrt{2} - \sqrt{8} + \sqrt{32} = \sqrt{x}$, what is the value of x ?
- (A) 130
- (B) 98
- (C) 26
- (D) 7
9. Which of the following is the derivative of $(x^2 - 7)^4$?
- (A) $4(x^2 - 7)^3$
- (B) $4(2x - 7)^3$
- (C) $8x(x^2 - 7)^3$
- (D) $8x(2x - 7)^3$

10. The diagram shows the line l .



What is the slope of the line l ?

- (A) $\frac{1}{\sqrt{3}}$
- (B) $-\frac{1}{\sqrt{3}}$
- (C) $\sqrt{3}$
- (D) $-\sqrt{3}$

Section II

65 marks

Attempt Questions 11–33

Allow about 1 hours 45 minutes for this section

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
 - Your responses should include relevant mathematical reasoning and/or calculations.
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Question 11 (1 mark)

Express 495° in radians. Leave your answer in terms of π

1

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Question 12 (2 marks)

Rationalise the denominator of $\frac{-3}{\sqrt{4+5}}$

2

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Question 13 (2 marks)

Consider $f(x) = \frac{10}{x}$ and $g(x) = 2x + 8$ what are the values of x for which $f(x) = g(x)$?

2

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Question 14 (2 marks)

Simplify fully $\frac{6x+18}{9(x^2+4x+4)} \times \frac{x^2-4}{x+3}$

2

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Question 15 (3 marks)

Solve for x : $4^x - 9 \times 2^x + 8 = 0$

3

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Question 16 (2 marks)

Find the equation of the tangent to the curve $y = 5x^3 - 4x + 3$ at the point $(-1, 2)$.

2

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Question 17 (2 marks)

Two identical *biased* coins are tossed at the same time. The probability that both coins land on heads is 0.16. Find the probability that:

- (a) Both coins land on tails.

1

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- (b) At least one coin lands on heads.

1

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Question 18 (5 marks)

Differentiate the following with respect to x . Simplify your answers.

- (a) $y = (x\sqrt{x} + 1)^3$

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- (b) $y = \frac{e^{2x}}{x^2 - 3}$

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Question 19 (2 marks)

Show that $f(x) = 2x^3 + 4x$ is an odd function.

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Question 20 (2 marks)

Using the first principles formula, $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ to find $f'(x)$ when $f(x) = x^2 - 2x$

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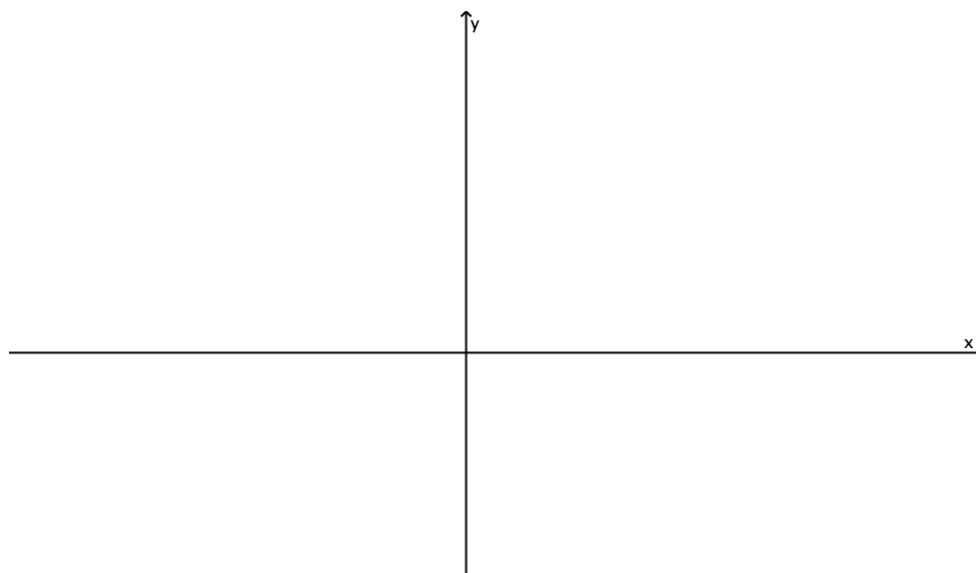
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Question 21 (2 marks)

Sketch the graph of $f(x) = 1 - \log_e x$ showing the x -intercept and the equation of the asymptote.

2



Question 22 (4 marks)

The point $P(4, 4)$ lies on the curve $y = -x^2 + 3x + 8$. The normal to the curve at P intersects the curve again at point Q .

(a) Show that the equation of the normal at P is $-x + 5y = 16$ **2**

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(b) Find the coordinates of Q . **2**

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Question 23 (2 marks)

Find the coordinates of the stationary point(s) on the curve $y = x(\sqrt{x} - 3)$ **2**

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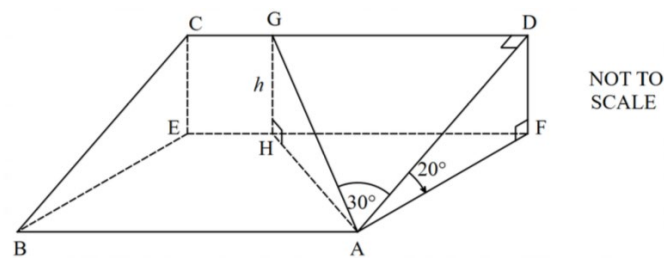
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Question 24 (4 marks)

‘Family hill’, a grassed spectating area at Shark Park makes an angle of 20° with the horizontal. In the representation below, the path AG makes an angle of 30° with the line of greatest slope, and $DF = GH = h$. This is shown in the diagram below.



- (a) Show that $AD = \frac{h}{\sin 20}$ 1

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- (b) Show that $AG = \frac{h}{\sin 20 \cos 30}$ 1

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- (c) Find the angle of inclination of the path AG to the horizontal. Give your answer correct to the nearest degree. 2

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Question 25 (4 marks)

(a) Show that $3 \cos x + 3 = -2 \sin^2 x$ can be written as $2 \cos^2 x - 3 \cos x - 5 = 0$ **2**

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(b) Hence or otherwise, solve $3 \cos x + 3 = -2 \sin^2 x$ for $0 \leq x \leq 2\pi$. **2**

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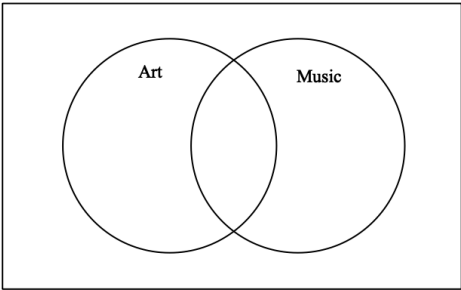
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Question 26 (3 marks)

A class has 25 students. 14 of them study Art and 12 study Music. 5 students study neither Art nor Music. Two students are chosen at random from the class.

(a) Draw a Venn diagram to represent the above information showing the number of students studying Art but not Music, and the number of students studying both Art and Music. **1**



(b) Find the probability that both students chosen study Art. **2**

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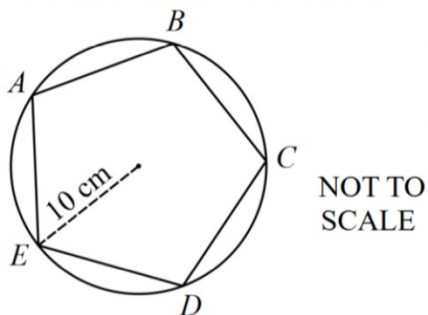
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Question 27 (4 marks)

4

A regular pentagon $ABCDE$ is inscribed in a circle of radius 10 cm, as shown below.



Find the perimeter of the pentagon, correct to 3 significant figures.

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Question 28 (1 mark)

1

Find the values of x where the graph of the function $f(x) = \frac{1}{x^2-4}$ is discontinuous.

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Question 29 (3 marks)

3

Solve the equation $\log_2(x + 1) + \log_2(x - 1) = 3$

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Question 30 (4 marks)

In a block of 40 apartments, when the weekly rent for each apartment is \$570, all 40 apartments are rented out. For each increase of \$20 in the weekly rent, the number of apartments rented out decreases by one. Each empty apartment incurs a weekly cost of \$70 for the block owner.

- (a) Show that if the weekly rent is increased by n lots of \$20 ($n = 0, 1, 2, 3 \dots$) then the total weekly profit $\$P$ from the apartments is given by $P = 22\,800 + 160n - 20n^2$ 2

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- (b) Find the maximum weekly profit. 2

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Question 31 (3 marks)

Prove that $\frac{1+\sin \theta}{1-\sin \theta} - \frac{1-\sin \theta}{1+\sin \theta} = 4 \sec \theta \tan \theta$ 3

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Question 32 (5 marks)

The energy, E Megajoules, radiated by an earthquake is given by the formula $E = k \times 10^{1.5M}$ where M is the magnitude of the earthquake on the Richter scale and k is a constant. An earthquake of magnitude 1 radiates 2 megajoules of energy.

- (a) Show that $\log_{10} E = \log_{10} 2 + 1.5(M - 1)$ **3**

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- (b) Find the magnitude of an earthquake which radiates 2×10^6 megajoules of energy **2**

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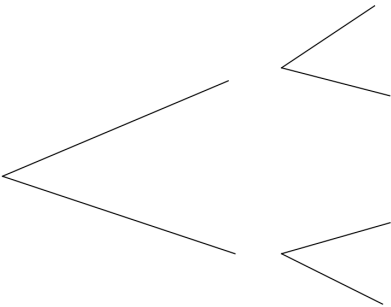
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Question 33 (3 marks)

On his way to school each day Ozzie catches a train and a bus. The train is late 10% of the time and the bus is late 20% of the time. If either the train or the bus is late then Ozzie is late for school.

- (a) Draw a probability tree diagram to represent the above information. **1**



- (b) Find the probability that Ozzie is late for school on any given day. **2**

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Extra Writing Space

If you use this space, clearly indicate which question you are answering.

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Student Name: _____

Multiple choice answer page. Fill in either A, B, C or D for questions 1-10.

This page must be handed in with your answer booklets

1.	
2.	
3.	
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6.	
7.	
8.	
9.	
10.	

Student Number/Name: Solutions.



Caringbah High School

2020

Preliminary Course
Assessment Task 3

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- NESA-approved calculators may be used
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Total marks – 75

Section I - 10 marks (pages 2 - 5)

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Marker's Use Only									
Section I	Section II							Total	
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/10	/10	/11	/10	/10	/9	/7	/8	/75	%

Section I - 10 marks**Attempt Questions 1-10****Allow about 15 minutes for this section**

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(B) 4.236×10^{-3}

(C) 4.2×10^{-4}

(D) 4.236×10^{-4}

B

2. If a and b are two real numbers and $a, b \neq 0$, what is the gradient of the line joining $A(a, b)$ and $B(3a, -3b)$?

(A) $-\frac{2b}{a}$

(B) $-\frac{a}{2b}$

(C) $\frac{b}{a}$

(D) $\frac{a}{b}$

$$m = \frac{-3b - b}{3a - a}$$

$$= \frac{-4b}{2a}$$

$$= \frac{-2b}{a}$$

A

3. What are the solutions to $\sqrt{3} \tan x = -1$ for $0 \leq x \leq 2\pi$?

(A) $\frac{5\pi}{6}$ and $\frac{7\pi}{6}$

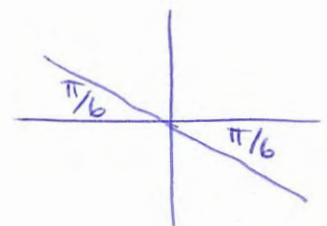
(B) $\frac{5\pi}{6}$ and $\frac{11\pi}{6}$

(C) $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$

(D) $\frac{2\pi}{3}$ and $\frac{5\pi}{3}$

$$\tan x = -\frac{1}{\sqrt{3}}$$

$$x = \frac{5\pi}{6}, \frac{11\pi}{6}$$



B

4. What is the domain of the function $f(x) = \sqrt{16 - x^2}$?

(A) $x < 4$

(B) $x \leq 4$

(C) $-4 < x < 4$

(D) $-4 \leq x \leq 4$

$$16 - x^2 = 0$$

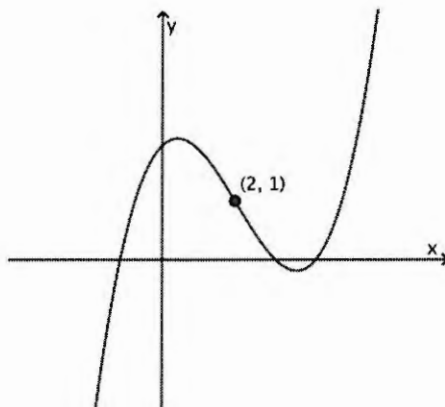
$$x^2 = 16$$

$$x = \pm 4$$

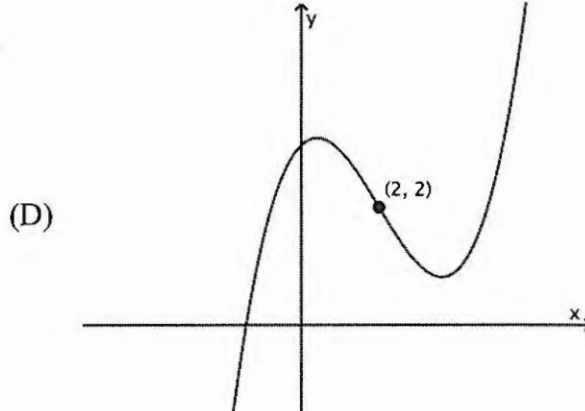
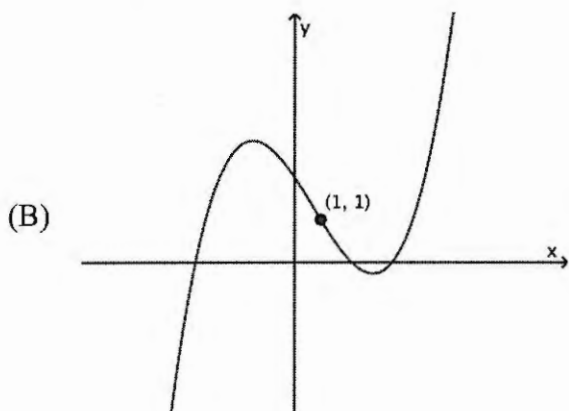
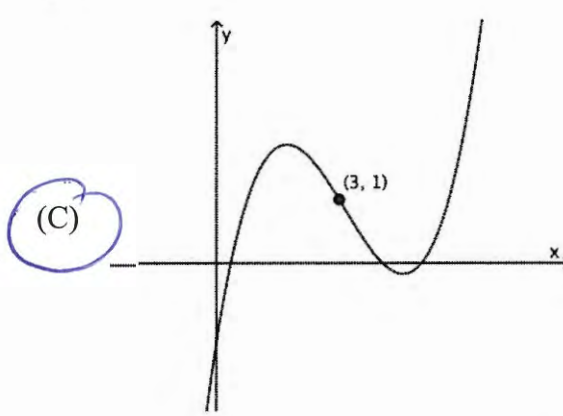
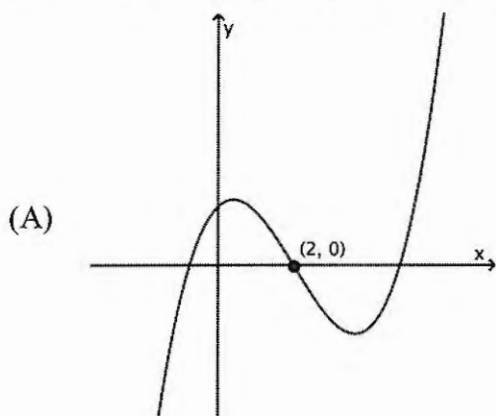
$$-4 \leq x \leq 4$$

D

5. The diagram below shows the graph of $y = f(x)$:



Which of these graphs represents $y = f(x - 1)$?



C

7. If $a = e^x$ what is the value of $\log_e \frac{1}{a^2}$?

- (A) $-x^2$
- ☒ (B) $-2x$
- (C) $2x$
- (D) x^2

$$\begin{aligned}\log_e \frac{1}{e^{2x}} \\&= \log_e 1 - \log_e e^{2x} \\&= 0 - 2x\end{aligned}$$

B

6. What are the solutions of the equation $|2x - 3| = 6$?

- (A) $x = \frac{3}{2}$ or $x = -\frac{3}{2}$
- (B) $x = \frac{3}{2}$ or $x = \frac{9}{2}$
- ☒ (C) $x = \frac{9}{2}$ or $x = -\frac{3}{2}$
- (D) $x = \frac{3}{2}$ or $x = -\frac{9}{2}$

$$\begin{aligned}2x - 3 &= 6 \\x &= \frac{9}{2}\end{aligned}$$

$$\begin{aligned}2x - 3 &= -6 \\x &= -\frac{3}{2}\end{aligned}$$

C

8. If $5\sqrt{2} - \sqrt{8} + \sqrt{32} = \sqrt{x}$, what is the value of x ?

- (A) 130
- ☒ (B) 98
- (C) 26
- (D) 7

$$\begin{aligned}5\sqrt{2} - 2\sqrt{2} + 4\sqrt{2} \\&= 7\sqrt{2} \\&= \sqrt{49 \times 2} \\&= \sqrt{98}\end{aligned}$$

B

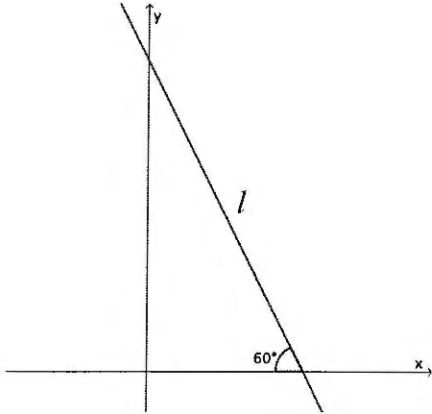
9. Which of the following is the derivative of $(x^2 - 7)^4$?

- (A) $4(x^2 - 7)^3$
- (B) $4(2x - 7)^3$
- ☒ (C) $8x(x^2 - 7)^3$
- (D) $8x(2x - 7)^3$

$$\begin{aligned}y' &= 4(x^2 - 7)^3 \times 2x \\&= 8x(x^2 - 7)^3\end{aligned}$$

C

10. The diagram shows the line l .



What is the slope of the line l ?

- (A) $\frac{1}{\sqrt{3}}$
- (B) $-\frac{1}{\sqrt{3}}$
- (C) $\sqrt{3}$
- (D) $-\sqrt{3}$

$$\tan \theta = m$$

$$\tan (120^\circ) = -\sqrt{3}$$

Section II

65 marks

Attempt Questions 11–33

Allow about 1 hours 45 minutes for this section

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (1 mark)

Express 495° in radians. Leave your answer in terms of π

1

$$2.75\pi \quad \text{or} \quad \frac{11\pi}{4} \quad \checkmark$$

Question 12 (2 marks)

Rationalise the denominator of $\frac{-3}{\sqrt{4}+5}$

2

$$= \frac{-3}{\sqrt{4}+5} \times \frac{\sqrt{4}-5}{\sqrt{4}-5}$$

$$= \frac{-3\sqrt{4}+15}{4-25}$$

$$= \frac{-3\sqrt{4}+15}{-21} \quad \checkmark$$

$$= \frac{\sqrt{4}-5}{7} \quad \checkmark$$

$$\text{OR} \quad \frac{-5+\sqrt{4}}{7} \quad \checkmark \quad \text{OR} \quad -\frac{5-\sqrt{4}}{7}$$

Question 13 (2 marks)

Consider $f(x) = \frac{10}{x}$ and $g(x) = 2x + 8$ what are the values of x for which $f(x) = g(x)$?

2

$$\frac{10}{x} = 2x + 8$$

$$x = -5 \quad \text{or}$$

$$10 = 2x^2 + 8x$$

$$x = 1 \quad \checkmark$$

$$2x^2 + 8x - 10 = 0$$

$$x^2 + 4x - 5 = 0 \quad \checkmark$$

$$(x+5)(x-1) = 0$$

Question 14 (2 marks)

Simplify fully $\frac{6x+18}{9(x^2+4x+4)} \times \frac{x^2-4}{x+3}$

2

$$\frac{6\cancel{(x+3)}}{9(x+2)^2} \times \frac{(x-2)\cancel{(x+2)}}{\cancel{x+3}}$$

$$= \frac{2(x-2)}{3(x+2)}$$

Question 15 (3 marks)

Solve for x : $4^x - 9 \times 2^x + 8 = 0$

3

$2^{2x} - 9 \times 2^x + 8 = 0$	$2^x = 8$	$2^x = 1$
let $u = 2^x$	$2^x = 2^3$	$2^x = 2^0$
$u^2 - 9u + 8 = 0$	$x = 3$	$x = 0$
$(u-8)(u-1) = 0$		
$u = 8, u = 1$		

Question 16 (2 marks)

Find the equation of the tangent to the curve $y = 5x^3 - 4x + 3$ at the point $(-1, 2)$.

2

$y' = 15x^2 - 4$	$y - 2 = 11(x + 1)$
at $x = -1$,	$y = 11x + 13$
$m = 15 - 4$	
$= 11$	OR
	$11x - y + 13 = 0$

Question 17 (2 marks)

Two identical *biased* coins are tossed at the same time. The probability that both coins land on heads is 0.16. Find the probability that:

- (a) Both coins land on tails.

1

$$\begin{array}{l|l}
 P(HH) = 0.16 & P(T) = 0.6 \\
 = P(H) \times P(H) & P(TT) = 0.6 \times 0.6 \\
 \therefore P(H) = 0.4 & = 0.36 \checkmark
 \end{array}$$

- (b) At least one coin lands on heads.

1

$$\begin{aligned}
 P(\text{at least one head}) &= 1 - P(TT) \\
 &= 1 - 0.36 \\
 &= 0.64 \checkmark
 \end{aligned}$$

Question 18 (5 marks)

Differentiate the following with respect to x . Simplify your answers.

- (a) $y = (x\sqrt{x} + 1)^3$

2

$$\begin{aligned}
 \frac{dy}{dx} &= 3(x\sqrt{x} + 1)^2 \times \frac{3}{2}x^{1/2} \checkmark \\
 &= \frac{9\sqrt{x}}{2} (x\sqrt{x} + 1)^2 \checkmark
 \end{aligned}$$

- (b) $y = \frac{e^{2x}}{x^2 - 3}$

3

$$\begin{aligned}
 u &= e^{2x} & v &= x^2 - 3 \\
 u' &= 2e^{2x} \checkmark & v' &= 2x \\
 \frac{dy}{dx} &= \frac{2e^{2x}x^2 - 6e^{2x} - 2e^{2x}x}{(x^2 - 3)^2} = \frac{2e^{2x}(x^2 - x - 3)}{(x^2 - 3)^2} \checkmark
 \end{aligned}$$

Question 19 (2 marks)

Show that $f(x) = 2x^3 + 4x$ is an odd function.

2

$$\begin{aligned}
 f(-x) &= 2(-x)^3 + 4(-x) \\
 &= -2x^3 - 4x \checkmark \\
 -f(x) &= -2x^3 - 4x \\
 &= f(-x) \therefore f(x) \text{ is odd. } \checkmark
 \end{aligned}$$

Question 20 (2 marks)

Using the first principles formula, $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ to find $f'(x)$ when $f(x) = x^2 - 2x$

2

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{(x+h)^2 - 2(x+h) - x^2 + 2x}{h} \checkmark \\
 &= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - 2x - 2h - x^2 + 2x}{h} \\
 &= \lim_{h \rightarrow 0} \frac{2xh + h^2 - 2h}{h} \\
 &= \lim_{h \rightarrow 0} 2x + h - 2 \\
 &= 2x - 2 \checkmark
 \end{aligned}$$

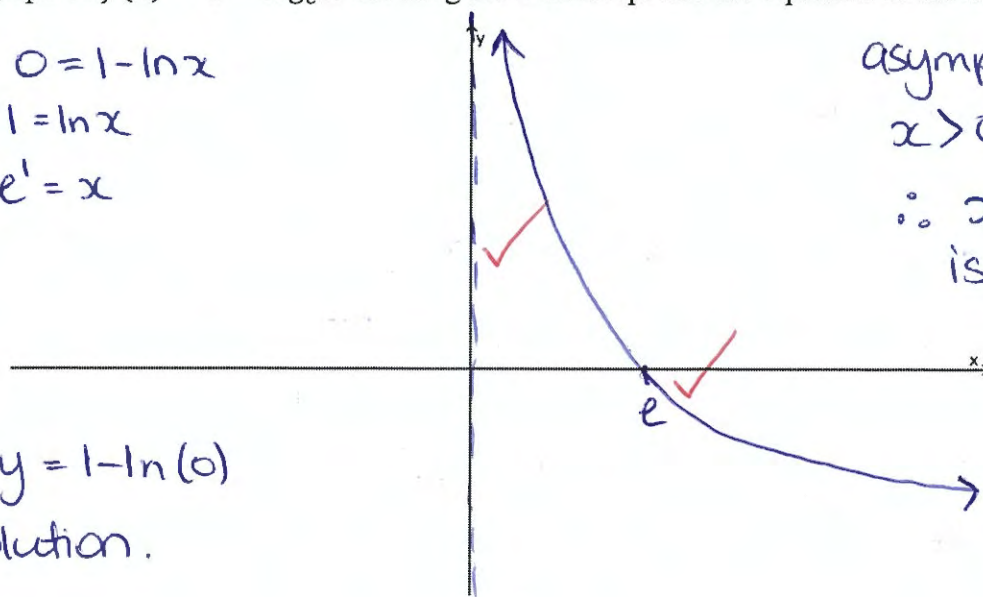
Question 21 (2 marks)

2

Sketch the graph of $f(x) = 1 - \log_e x$ showing the x -intercept and the equation of the asymptote.

$x\text{-int: } 0 = 1 - \ln x$
 $1 = \ln x$
 $e^1 = x$

$y\text{-int: } y = 1 - \ln(0)$
 no solution.



asymptote:
 $x > 0$
 $\therefore x = 0$
 is asymptote.

Question 22 (4 marks)

The point $P(4, 4)$ lies on the curve $y = -x^2 + 3x + 8$. The normal to the curve at P intersects the curve again at point Q .

- (a) Show that the equation of the normal at P is $-x + 5y = 16$

2

$$\frac{dy}{dx} = -2x + 3$$

$$m_T = -2(4) + 3$$

$$= -5$$

$$m_N = \frac{1}{5} \checkmark$$

$$y - 4 = \frac{1}{5}(x - 4)$$

$$5y - 20 = x - 4$$

$$-x + 5y = 16 \checkmark$$

- (b) Find the coordinates of Q .

2

① $-x + 5y = 16$ intersects $y = -x^2 + 3x + 8$ ②

② $\rightarrow$ ①

$$-x + 5(-x^2 + 3x + 8) = 16$$

$$-x - 5x^2 + 15x + 40 = 16$$

$$5x^2 - 14x - 24 = 0$$

$$(x - 4)(5x + 6) = 0 \checkmark$$

$$x = 4 \text{ and } x = -\frac{6}{5}$$

$$\text{when } x = -\frac{6}{5}$$

$$y = \frac{74}{25} \checkmark$$

Question 23 (2 marks)

Find the coordinates of the stationary point(s) on the curve $y = x(\sqrt{x} - 3)$

2

$$\frac{dy}{dx} = \frac{3}{2}x^{1/2} - 3 \checkmark$$

$$y = x^{3/2} - 3x$$

$$\frac{3\sqrt{x} - 3}{2} = 0$$

$$\frac{3\sqrt{x}}{2} = 3$$

$$3\sqrt{x} = 6$$

$$\sqrt{x} = 2$$

$$\therefore x = 4$$

$$\text{When } x = 4$$

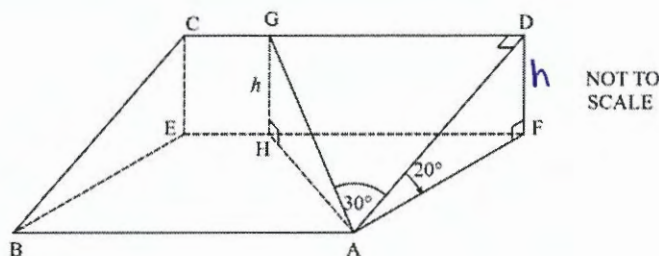
$$y = 4(2 - 3)$$

$$= -4$$

$$(4, -4) \checkmark$$

Question 24 (4 marks)

'Family hill', a grassed spectating area at Shark Park makes an angle of 20° with the horizontal. In the representation below, the path AG makes an angle of 30° with the line of greatest slope, and $DF = GH = h$. This is shown in the diagram below.



(a) Show that $AD = \frac{h}{\sin 20}$

1

$$\sin 20^\circ = \frac{h}{AD}$$

$$AD = \frac{h}{\sin 20^\circ} \checkmark$$

(b) Show that $AG = \frac{h}{\sin 20 \cos 30}$

1

$$\cos 30^\circ = \frac{AD}{AG}$$

$$AG = \frac{AD}{\cos 30^\circ}$$

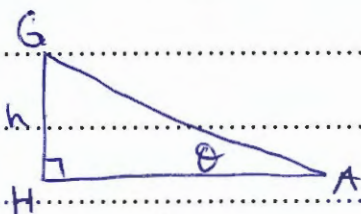
$$AD = \frac{h}{\sin 20}$$

$$AG = \frac{h}{\sin 20} \div \cos 30$$

$$AG = \frac{h}{\sin 20 \cos 30} \checkmark$$

(c) Find the angle of inclination of the path AG to the horizontal. Give your answer correct to the nearest degree.

2



$$\sin \theta = \frac{h}{AG}$$

$$\sin \theta = \cancel{h} \times \frac{\sin 20 \cos 30}{\cancel{h}} \checkmark$$

$$\sin \theta = \sin 20 \cos 30$$

$$\theta \doteq 17^\circ \checkmark$$

Question 25 (4 marks)

- (a) Show that $3 \cos x + 3 = -2 \sin^2 x$ can be written as $2 \cos^2 x - 3 \cos x - 5 = 0$

2

$$3 \cos x + 3 = -2(1 - \cos^2 x) \quad \checkmark$$

$$3 \cos x + 3 = -2 + 2 \cos^2 x$$

$$2 \cos^2 x - 3 \cos x - 5 = 0 \quad \checkmark$$

- (b) Hence or otherwise, solve $3 \cos x + 3 = -2 \sin^2 x$ for $0 \leq x \leq 2\pi$.

2

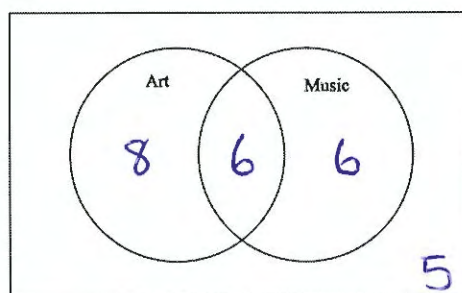
let $u = \cos x$	$(2u-5)(u+1) = 0$
$2u^2 - 3u - 5 = 0$	$u = 5/2 \quad u = -1 \quad \checkmark$
P: -10	$\cos x = 5/2 \rightarrow \text{no sol.}$
S: -3	$\cos x = -1$
F: -5, 2	$x = \pi \quad \checkmark$

Question 26 (3 marks)

A class has 25 students. 14 of them study Art and 12 study Music. 5 students study neither Art nor Music. Two students are chosen at random from the class.

- (a) Draw a Venn diagram to represent the above information showing the number of students studying Art but not Music, and the number of students studying both Art and Music.

1



- (b) Find the probability that both students chosen study Art.

2

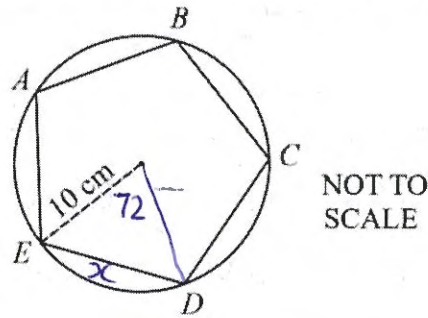
$$P(AA) = \frac{14}{25} \times \frac{13}{24} \quad \checkmark$$

$$= \frac{91}{300} \quad \text{OR} \quad 0.303 \quad \checkmark$$

Question 27 (4 marks)

4

A regular pentagon $ABCDE$ is inscribed in a circle of radius 10 cm, as shown below.



Find the perimeter of the pentagon, correct to 3 significant figures.

$$x^2 = 10^2 + 10^2 - 2 \times 10 \times 10 \times \cos 72^\circ \checkmark$$

$$= 138.196 \dots$$

$$x \doteq 11.755 \checkmark$$

$$\therefore P = 5 \times 11.755 \checkmark$$

$$P \doteq 58.8 \text{ cm} \checkmark$$

Question 28 (1 mark)

1

Find the values of x where the graph of the function $f(x) = \frac{1}{x^2-4}$ is discontinuous.

$$x^2 - 4 = 0$$

$$x = \pm 2 \checkmark$$

Question 29 (3 marks)

3

Solve the equation $\log_2(x+1) + \log_2(x-1) = 3$

$$\log_2((x+1)(x-1)) = 3$$

$$\log_2(x^2-1) = 3 \checkmark$$

$$2^3 = x^2 - 1$$

$$x^2 = 9$$

$$x = \pm 3 \checkmark$$

however,

when $x = -3$,

$\log_2(x+1)$ AND $\log_2(x-1)$
are undefined.

$$\therefore \underline{x = 3} \checkmark$$

Question 30 (4 marks)

In a block of 40 apartments, when the weekly rent for each apartment is \$570, all 40 apartments are rented out. For each increase of \$20 in the weekly rent, the number of apartments rented out decreases by one. Each empty apartment incurs a weekly cost of \$70 for the block owner.

- (a) Show that if the weekly rent is increased by n lots of \$20 ($n = 0, 1, 2, 3 \dots$) then the total weekly profit \$ P from the apartments is given by $P = 22\,800 + 160n - 20n^2$ 2

$$\begin{aligned}
 P &= (570 + 20n) \times (40 - n) - 70n \\
 &= 22800 - 570n + 800n - 20n^2 - 70n \\
 &= 22800 + 160n - 20n^2
 \end{aligned}$$

- (b) Find the maximum weekly profit. 2

$$\begin{aligned}
 &\text{Concave down parabola } \therefore \text{Vertex is maximum} \\
 x &= \frac{-b}{2a} = \frac{-160}{-40} = 4 \\
 P &= 22800 + (160 \times 4) - (20 \times 16) = \$23120
 \end{aligned}$$

Question 31 (3 marks)

Prove that $\frac{1+\sin\theta}{1-\sin\theta} - \frac{1-\sin\theta}{1+\sin\theta} = 4 \sec\theta \tan\theta$ 3

$$\begin{aligned}
 \text{LHS} &= \frac{1+\sin\theta}{1-\sin\theta} - \frac{1-\sin\theta}{1+\sin\theta} \\
 &= \frac{(1+\sin\theta)^2 - (1-\sin\theta)^2}{(1+\sin\theta)(1-\sin\theta)} \\
 &= \frac{1 + 2\sin\theta + \sin^2\theta - 1 + 2\sin\theta - \sin^2\theta}{1 - \sin^2\theta} \\
 &= \frac{4\sin\theta}{\cos^2\theta} = 4\tan\theta \times \frac{1}{\cos\theta} \\
 &= 4\sec\theta \tan\theta = \text{RHS}
 \end{aligned}$$

Question 32 (5 marks)

The energy, E Megajoules, radiated by an earthquake is given by the formula $E = k \times 10^{1.5M}$ where M is the magnitude of the earthquake on the Richter scale and k is a constant. An earthquake of magnitude 1 radiates 2 megajoules of energy.

- (a) Show that $\log_{10} E = \log_{10} 2 + 1.5(M - 1)$

3

$$\begin{array}{l|l}
 E = k \times 10^{1.5M} & E = 2 \times 10^{1.5(M-1)} \\
 2 = k \times 10^{1.5} & \log_{10} \left(\frac{E}{2} \right) = 1.5(M-1) \checkmark \\
 \frac{2}{10^{1.5}} = k & \log_{10} E - \log_{10} 2 = 1.5(M-1) \\
 k = 2 \times 10^{-1.5} \checkmark & \log_{10} E = \log_{10} 2 + 1.5(M-1) \checkmark \\
 E = 2 \times 10^{-1.5} \times 10^{1.5M} &
 \end{array}$$

- (b) Find the magnitude of an earthquake which radiates 2×10^6 megajoules of energy

2

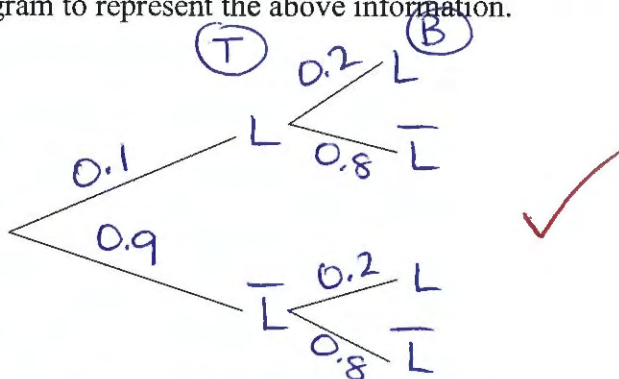
$$\begin{array}{l|l}
 \log_{10}(2 \times 10^6) = \log_{10} 2 + 1.5(M-1) & M-1 = 13.95 \\
 \log_{10} \left(\frac{2 \times 10^6}{2} \right) = 1.5(M-1) & M = 14.95 \checkmark \\
 20.93 = 1.5(M-1) \checkmark &
 \end{array}$$

Question 33 (3 marks)

On his way to school each day Ozzie catches a train and a bus. The train is late 10% of the time and the bus is late 20% of the time. If either the train or the bus is late then Ozzie is late for school.

- (a) Draw a probability tree diagram to represent the above information.

1



- (b) Find the probability that Ozzie is late for school on any given day.

2

$$\begin{aligned}
 P(L) &= P(TL) + P(BL) \\
 &= 1 - P(\bar{T}\bar{L}) \\
 &= 1 - (0.9 \times 0.8) \checkmark = 0.28 \\
 &\quad \text{or } \frac{7}{25} \checkmark
 \end{aligned}$$

End of Exam ☺