## Section I (10 marks)

**1** Which of the following is equivalent to $\sqrt{243}+2\sqrt{75}$?

(A) $19\sqrt{3}$

(B) $81\sqrt{3}$

(C) $106\sqrt{3}$

(D) $2\sqrt{318}$

**2** What is the domain of the function $f(x)=\dfrac{1}{(x-3)(1-x)}$?

(A) $x\neq -1$ or $x\neq -3$

(B) $x\neq -1$ or $x\neq 3$

(C) $x\neq 1$ or $x\neq -3$

(D) $x\neq 1$ or $x\neq 3$

**3** What is the value of $\dfrac{dy}{dx}$ if $y=x^4+5x^{-1}$?

(A) $\dfrac{dy}{dx}=4x^3-5x^0$

(B) $\dfrac{dy}{dx}=4x^3+5x^0$

(C) $\dfrac{dy}{dx}=4x^3-5x^{-2}$

(D) $\dfrac{dy}{dx}=4x^3+5x^{-2}$

**4** Solve $|2-3x|\geq 5$

(A) $x\leq -1$ or $x\geq 2\dfrac{1}{3}$

(B) $x\geq -1$ or $x\leq 2\dfrac{1}{3}$

(C) $x\leq -2\dfrac{1}{3}$ or $x\geq 1$

(D) $x\geq -2\dfrac{1}{3}$ or $x\leq 1$

**5** Which type of function is $f(x) = 2x^3 - x$?

(A) Odd

(B) Even

(C) Neither odd or even

(D) Zero

**6** What is solution to the equation $2\sin\beta = -\sqrt{3}$ for $0^\circ \le \beta \le 360^\circ$?

(A) $\beta = 60^\circ$ or $300^\circ$

(B) $\beta = 120^\circ$ or $240^\circ$

(C) $\beta = 210^\circ$ or $330^\circ$

(D) $\beta = 240^\circ$ or $300^\circ$

**7** What is the solution to the equation $x^2 + 2x - 7 = 0$?

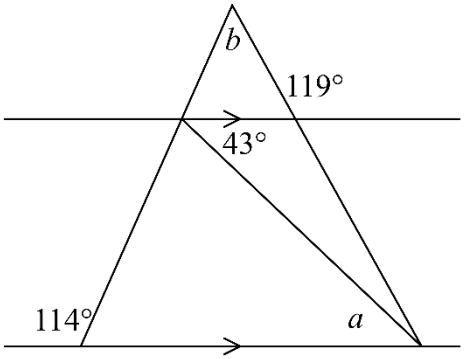
(A) $x = -1 \pm \sqrt{2}$

(B) $x = -2 \pm \sqrt{2}$

(C) $x = -2 \pm 2\sqrt{2}$

(D) $x = -1 \pm 2\sqrt{2}$

**8** What are the values of *a* and *b*?




Not to scale

(A) $a = 43^\circ$ and $b = 53^\circ$

(B) $a = 43^\circ$ and $b = 61^\circ$

(C) $a = 45^\circ$ and $b = 53^\circ$

(D) $a = 45^\circ$ and $b = 61^\circ$

**9** The line $4x - ky = 3$ passes through the point $(1, 2)$. What is the value of $k$?

(A) $-2$

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) $2$

**10** What is $pqx^2 - p^3q$ fully factorised?

(A) $pq(x+p)(x-p)$

(B) $p^2(x+q)(x-q)$

(C) $pq(x^2-p^2)$

(D) $p^2(x^2-q^2)$

## Section II (60 marks)

**Question 11** (15 marks) Use a SEPERATE sheet of paper **Marks**

(a) Express $0.1\dot{5}$ as a fraction in its simplest form. **2**

(b) Solve $\frac{1}{2}(y-3)-\frac{1}{3}(y-2)=3$ **2**

(c) The cost of a smart watch is \$979. This includes a 10% tax on the original price. Calculate the original price of the smart watch. **1**

(d) Simplify $\frac{a^2+b^2+2ab}{-b^2+a^2}$ **2**

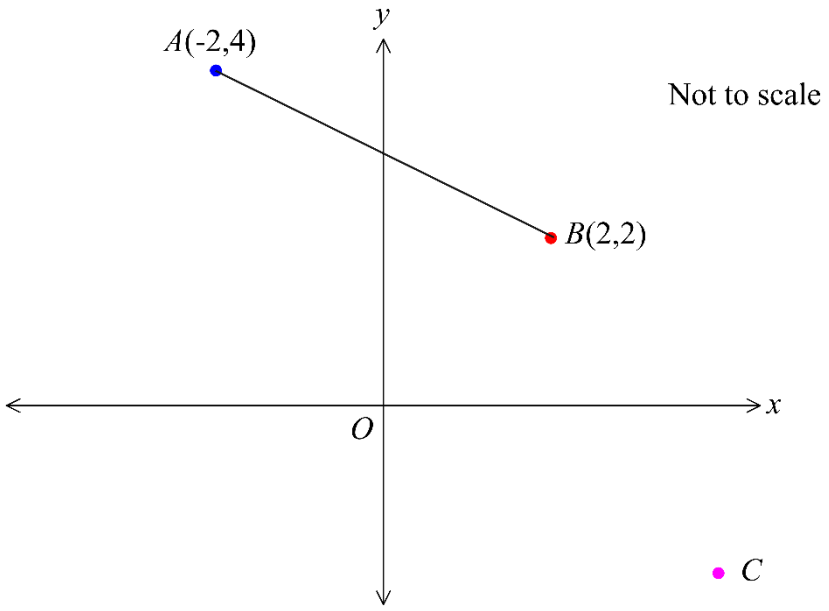
(e) Find the exact value of:

(i) $\tan 240^\circ$ **1**

(ii) $\sec 510^\circ$ **1**

(f) The diagram shows the points $A(-2,4)$, $B(2,2)$ and $O(0,0)$.

Point $C$ is the fourth vertex of the parallelogram $OABC$.




(i) Find the equation of the straight line that passes through the points A and B. **2**

(ii) Show that the length of $AB$ is $2\sqrt{5}$. **1**

(iii) Calculate the perpendicular distance from $O$ to the line $AB$. **1**

(iv) Calculate the area of parallelogram $OABC$. **1**

(v) What are the coordinates of $C$? **1**

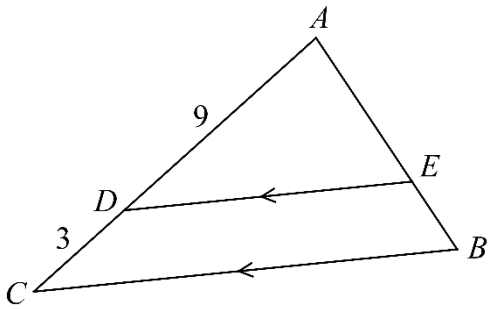
**Question 12** (13 marks) Use a SEPERATE sheet of paper **Marks**

(a) Simplify $\frac{x}{x^2-4}+\frac{3}{x-2}$ **2**

(b) Rationalise the denominator of $\frac{1}{\sqrt{5}-1}$ **2**

(c) Expand and then simplify $(1-y)^3+(1+y)^2$ **2**

(d) *ABC* is a triangle with *DE* drawn parallel to *BC*.



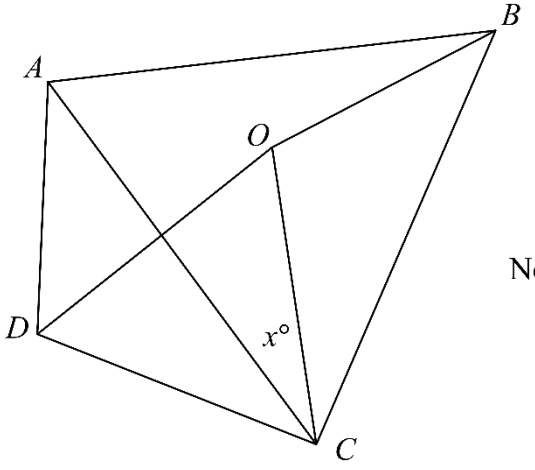

Not to scale

Given $AD=9$, $DC=3$ and $AB=8$.

(i) Show that $\Delta ABC \,|||\, \Delta ADE$. **2**

(ii) Find the length of *BE*. **2**

(e) In the diagram, $\Delta ABC$ and $\Delta ODC$ are equilateral and $\angle OCA = x^\circ$.




Not to scale

(i) Show that $\angle ACD = \angle BCO$ **1**

(ii) Prove that $\Delta ACD \equiv \Delta BCO$ **2**

**Question 13** (16 marks) Use a SEPERATE sheet of paper **Marks**

(a) Solve the following equations.

(i) $x^2 - 16 = 0$ **1**

(ii) $25^k.(5^3)^4 = 1$ **2**

(b) Factorise completely

(i) $y^3 + 10y^2 + 25y$ **1**

(ii) $a^3 - \frac{1}{8}$ **1**

(c) Draw neat sketches of the following equations on a separate set of axes. Show clearly the essential features of each graph.

(i) $y = 4 - x^2$ **1**

(ii) $y = \sqrt{4 - x^2}$ **1**

(iii) $y = \frac{4}{x}$ **1**

(d) Find $\lim_{x \to 4} \frac{x^2 - x - 12}{x - 4}$ **2**

(e) The function $y = f(x)$ is defined as follows:

$$f(x) = \begin{cases} 3^x & \text{for } x > 0 \\ 1 & \text{for } x = 0 \\ x^{-1} & \text{for } x < 0 \end{cases}$$

Find the value of $f(3) + f(0) + f(-3)$. **2**

(f) The curve $y^2 = x + 9$ and the straight line $x - 3y + 9 = 0$ intersect at *A* and *B*.

(i) Find the coordinates of *A* and *B*. **2**

(ii) Find the values of *x* for which $24 + 2x - x^2 \leq 0$. **2**

**Question 14** (16 marks) Use a SEPERATE sheet of paper **Marks**

(a) Prove $\tan\theta - \tan\theta\sin^2\theta = \sin\theta\cos\theta$ **2**

(b) Solve the following pair of simultaneous equations. **2**

$$2x + y - 1 = 0$$
$$3x - y - 4 = 0$$

(c) Differentiate with respect to $x$.

(i) $\sqrt{x^4 + 7}$ **1**

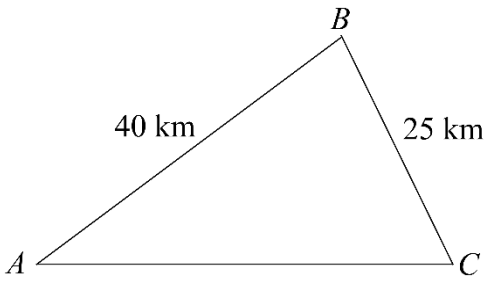
(ii) $x(1-x)^6$ **2**

(iii) $\dfrac{3x}{2-x}$ **2**

(d) Find the equation of the tangent to the curve $y = x^2 - 6x$ at $(-1, 7)$. **2**

(e) Solve the equation $2^x \times 4^{x+1} = \dfrac{1}{2}$ **1**

(f) Point $C$ is due east of $A$. Point $B$ is 40 km from $A$ and 25 km from $C$.
The bearing of $B$ from $C$ is $325^\circ$.




Not to scale

(i) What is the size of $\angle ACB$? **1**

(ii) What is the bearing of $B$ from $A$? **3**

**End of paper**

NAME……

CLASS 11MT……

Use the multiple-choice answer sheet.

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

**Sample:** $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.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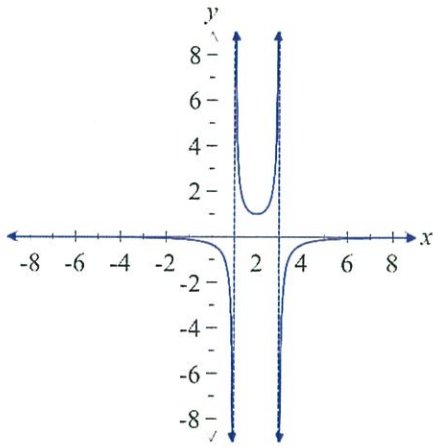
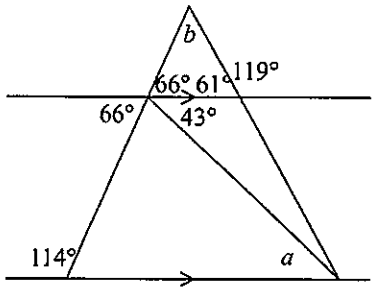
1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○
6. A ○ B ○ C ○ D ○
7. A ○ B ○ C ○ D ○
8. A ○ B ○ C ○ D ○
9. A ○ B ○ C ○ D ○
10. A ○ B ○ C ○ D ○

**CTHS AP2 Examination 2016**

**Year 11 Mathematics Yearly Examination**

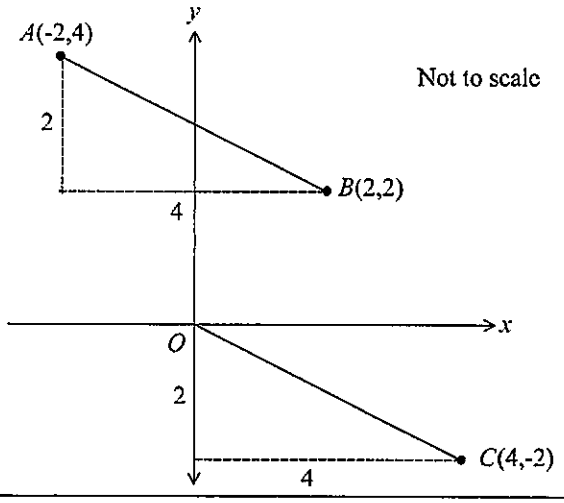
**Worked solutions and marking guidelines**

| **Section I** | | |
|---|---|---|
| | **Solution** | **Criteria** |
| 1 | $\sqrt{243}+2\sqrt{75}=\sqrt{81}\times\sqrt{3}+2\times\sqrt{25}\times\sqrt{3}$<br>$=9\sqrt{3}+10\sqrt{3}=19\sqrt{3}$ | 1 Mark: A |
| 2 | Domain:<br>$x\neq 1$ or $x\neq 3$<br>  <br> | 1 Mark: D |
| 3 | $y=x^4+5x^{-1}$<br>$\frac{dy}{dx}=4x^3-5x^{-2}$ | 1 Mark: C |
| 4 | $2-3x\geq 5$ or $2-3x\leq -5$<br>$-3x\geq 3$ $\quad -3x\leq -7$<br>$x\leq -1$ $\quad x\geq\frac{7}{3}$ or $2\frac{1}{3}$<br>Solution is $x\leq -1$ or $x\geq 2\frac{1}{3}$ | 1 Mark: A |
| 5 | $f(-x)=2(-x)^3-(-x)$<br>$=-(2x^3-x)$<br>$=-f(x)$<br>Therefore, the function is odd. | 1 Mark: A |
| 6 | $2\sin\beta=-\sqrt{3}$<br>$\sin\beta=-\frac{\sqrt{3}}{2}$<br>$\beta=240^\circ$ or $300^\circ$ | 1 Mark: D |

| | | |
|---|---|---|
| 7 | $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$<br>$=\dfrac{-2\pm\sqrt{2^2-4\times1\times-7}}{2\times1}$<br>$=\dfrac{-2\pm\sqrt{32}}{2}$<br>$=-1\pm2\sqrt{2}$ | 1 Mark: D |
| 8 |  <br>$a=43^\circ$ (alternate angles are equal, two parallel lines)<br>$b+66+61=180$ (angle sum of a triangle is 180)<br>$b=53^\circ$<br>$a=43^\circ$ and $b=53^\circ$ | 1 Mark: A |
| 9 | $(1,2)$ satisfies the equation $4x-ky=3$<br>$4\times1-k\times2=3$<br>$-2k=-1$<br>$k=\dfrac{1}{2}$ | 1 Mark: C |
| 10 | $pqx^2-p^3q=pq(x^2-p^2)$<br>$=pq(x+p)(x-q)$ | 1 Mark: A |

**Section II, Question 11 (15 marks)**

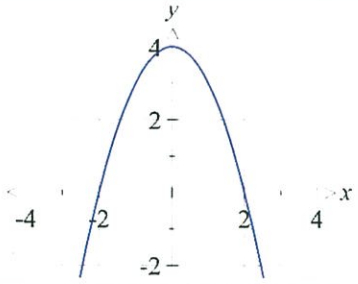
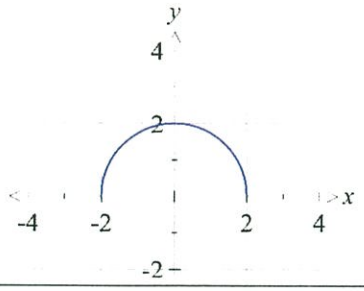
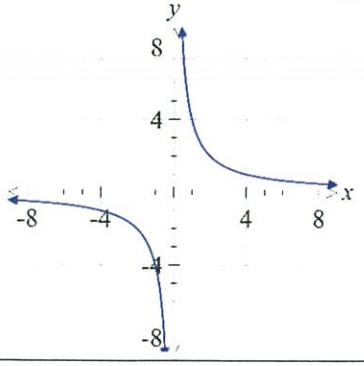
| | | |
|---|---|---|
| 11(a) | Let $x = 0.1\dot{5}$<br>$x = 0.1555555...$ (1)<br>$10x = 1.5555555...$ (2)<br>Equation (2) - (1)<br>$9x = 1.4$<br>$x = \frac{1.4}{9} = \frac{7}{45}$ | 2 Marks: Correct answer.<br><br>1 Mark: Makes some progress towards the solution. |
| 11(b) | $6\times\left[\frac{1}{2}(y-3)\right]-\left[\frac{1}{3}(y-2)\right]\times 6 = 3\times 6$<br>$3(y-3)-2(y-2)=18$<br>$3y-9-2y+4=18$<br>$y=23$ | 2 Marks: Correct answer.<br>1 Mark: Multiplies both sides by 6 or makes some progress. |
| 11(c) | Let \$$x$ be the original cost.<br>$x+$ 10% of $x = \$979$<br>$x+0.1x = \$979$<br>$1.1x = \$979$<br>$x = \$890$ | 1 Mark: Correct answer. |
| 11(d) | $\frac{a^2+b^2+2ab}{-b^2+a^2} = \frac{(a+b)^2}{a^2-b^2}$<br>$= \frac{(a+b)^2}{(a+b)(a-b)} = \frac{(a+b)}{(a-b)}$ | 2 Marks: Correct answer.<br>1 Mark: Factorises one term. |
| 11(e) (i) | $\tan 240^\circ = \tan(180^\circ + 60^\circ)$<br>$= \tan 60^\circ = \sqrt{3}$ | 1 Mark: Correct answer. |
| 11(e) (ii) | $\sec 510^\circ = \sec(360^\circ + 150^\circ) = \sec(150^\circ)$<br>$= \sec(180^\circ - 30^\circ)$<br>$= -\sec(30^\circ)$<br>$= -\frac{2}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}} = \frac{-2\sqrt{3}}{3}$ | 1 Mark: Correct answer. |
| 11(f) (i) | Gradient $AB = \frac{y_2-y_1}{x_2-x_1} = \frac{2-4}{2-(-2)} = \frac{-2}{4} = -\frac{1}{2}$ | 2 Marks: Correct answer.<br><br>1 Mark: Finds the gradient of $AB$ or shows some understanding of the problem. |

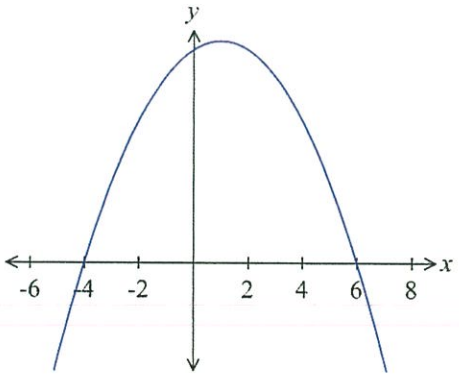
| | | |
|---|---|---|
| | Equation of $AB$ $\quad y-y_1=m(x-x_1)$<br>$y-2=-\frac{1}{2}(x-2)$<br>$-2y+4=x-2$<br>$x+2y-6=0$ | |
| 11(f)<br>(ii) | Distance $AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$<br>$=\sqrt{(2--2)^2+(2-4)^2}$<br>$=\sqrt{20}=2\sqrt{5}$ | 1 Mark: Correct answer. |
| 11(f)<br>(iii) | $d=\left\|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right\|$<br>$=\left\|\frac{1\times 0+2\times 0-6}{\sqrt{1^2+2^2}}\right\|$<br>$=\left\|\frac{-6}{\sqrt{5}}\right\|=\frac{6}{\sqrt{5}}$ or $\frac{6\sqrt{5}}{5}$ | 1 Mark: Correct answer. |
| 11(f)<br>(iv) | $A=bh=2\sqrt{5}\times\frac{6}{\sqrt{5}}=12$ square units | 1 Mark: Correct answer. |
| 11(f)<br>(v) | $OC$ and $AB$ are parallel and have the same gradient.<br><br>$C(4,-2)$<br>  <br> | 1 Mark: Correct answer. |

**Question 12 (13 marks)**

| | | |
|---|---|---|
| 12(a) | $\frac{x}{x^2-4}+\frac{3}{x-2}=\frac{x}{(x-2)(x+2)}+\frac{3}{(x-2)}$<br>$=\frac{x+3(x+2)}{(x-2)(x+2)}$<br>$=\frac{4x+6}{(x-2)(x+2)}=\frac{2(2x+3)}{x^2-4}$ | 2 Marks: Correct answer.<br>1 Mark: Finds a common denominator or recognises the difference of two squares. |
| 12(b) | $\frac{1}{\sqrt{5}-1}=\frac{1}{\sqrt{5}-1}\times\frac{\sqrt{5}+1}{\sqrt{5}+1}$<br>$=\frac{\sqrt{5}+1}{5-1}=\frac{\sqrt{5}+1}{4}$ | 2 Marks: Correct answer.<br>1 Mark: Uses the conjugate. |
| 12(c) | $(1-y)^3+(1+y)^2=(1-y)(1-2y+y^2)+(1+2y+y^2)$<br>$=1-2y+y^2-y+2y^2-y^3+1+2y+y^2$<br>$=2-y+4y^2-y^3$ | 2 Marks: Correct answer.<br>1 Mark: Expands one binomial. |
| 12(d) (i) | In $\Delta ABC$ and $\Delta ADE$<br>$\angle CAB=\angle DAE$ (common angle)<br>$\angle BCA=\angle EDA$ (corresponding angles are equal, parallel lines)<br>$\angle CBA=\angle DEA$ (corresponding angles are equal, parallel lines)<br>$\Delta ABC$ is similar to $\Delta ADE$ (equiangular) | 2 Marks: Correct answer.<br>1 Mark: One or more correct statements. |
| 12(d) (ii) | $\frac{AE}{AB}=\frac{AD}{AC}$ (matching sides in similar triangles)<br>$\frac{AE}{8}=\frac{9}{3+9}$<br>$AE=\frac{9}{12}\times 8=6$<br>$BE=AB-AE=8-6=2$ | 2 Marks: Correct answer.<br>1 Mark: Uses the corresponding sides in similar triangles. |
| 12(e) (i) | $\angle OCD=60^\circ$ (all angles are equal, equilateral triangle $\Delta ODC$)<br>$\angle ACD=(60-x)^\circ$ (adjacent angles)<br>$\angle ACB=60^\circ$ (all angles are equal, equilateral triangle $\Delta ABC$)<br>$\angle BCO=(60-x)^\circ$ (adjacent angles)<br>$\therefore \angle ACD=\angle BCO$ (both equal to $(60-x)^\circ$) | 1 Mark: Correct answer. |
| 12(e) (ii) | In $\Delta ACD$ and $\Delta BCO$<br>$\angle ACD=\angle BCO$ (from part (i))<br>$AC=BC$ (all sides are equal, equilateral triangle $\Delta ABC$)<br>$CD=CO$ (all sides are equal, equilateral triangle $\Delta ODC$)<br>$\therefore \Delta ACD\equiv\Delta BCO$ (SAS) | 2 Marks: Correct answer.<br>1 Mark: Correct argument insufficient justification. |

**Question 13 (16 marks)**

| | | |
|---|---|---|
| 13(a) (i) | $x^2 - 16 = 0$<br>$(x+4)(x-4) = 0$<br>$\therefore\ x = \pm 4$ | 1 Mark: Correct answer. |
| 13(a) (ii) | $25^k.(5^3)^4 = 1$<br>$(5^2)^k \times (5^3)^4 = 5^0$<br>$2k + 12 = 0$<br>$2k = -12$<br>$k = -6$ | 2 Marks: Correct answer.<br><br>1 Mark: Makes some progress. |
| 13(b) (i) | $y^3 + 10y^2 + 25y = y(y^2 + 10y + 25)$<br>$= y(y+5)^2$ | 1 Mark: Correct answer. |
| 13(b) (ii) | $a^3 - \frac{1}{8} = a^3 - \left(\frac{1}{2}\right)^3$<br>$= (a - \frac{1}{2})(a^2 + \frac{1}{2}a + \frac{1}{4})$ | 1 Mark: Correct answer. |
| 13(c) (i) |  | 1 Mark: Correct answer. |
| 13(c) (ii) |  | 1 Mark: Correct answer. |
| 13(c) (iii) |  | 1 Mark: Correct answer. |

| | | |
|---|---|---|
| 13(d) | $\lim_{x\to 4}\dfrac{x^2-x-12}{x-4}=\lim_{x\to 4}\dfrac{(x-4)(x+3)}{(x-4)}$<br>$=\lim_{x\to 4}(x+3)=7$ | 2 Marks: Correct answer.<br>1 Mark: Factorises the quadratic. |
| 13(e) | $f(3)+f(0)+f(-3)=3^3+1-\dfrac{1}{3}$<br>$=27\dfrac{2}{3}$ | 2 Marks: Correct answer.<br>1 Mark: Finds the value of one function. |
| 13(f) (i) | $y^2=x+9$ (1)<br>$x-3y+9=0$ (2)<br>Make $x$ the subject of equation (1)<br>$x=y^2-9$ (3)<br>Substitute $y^2-9$ for $x$ into equation (2)<br>$y^2-9-3y+9=0$<br>$y(y-3)=0$<br>$\therefore y=0$ or $y=3$<br>Substitute $y=0$ into equation (3)<br>$x=0^2-9=-9$<br>Substitute $y=3$ into equation (3)<br>$x=3^2-9=0$<br>Points $A$ and $B$ are $(0,3)$ and $(-9,0)$. | 2 Marks: Correct answer.<br>1 Mark: Finds one of the solutions or shows some understanding. |
| 13(f) (ii) | $24+2x-x^2\le 0$<br>$(6-x)(4+x)\le 0$<br>  <br><br>$\therefore x\le -4$ or $x\ge 6$ | 2 Marks: Correct answer.<br>1 Mark: Factorises the quadratic. |

**Question 14 (16 marks)**

| | | |
|---|---|---|
| 14(a) | $\text{LHS} = \tan\theta - \tan\theta\sin^2\theta$<br>$= \tan\theta(1-\sin^2\theta)$<br>$= \frac{\sin\theta}{\cos\theta}\times\cos^2\theta$<br>$= \sin\theta\cos\theta = \text{RHS}$ | 2 Marks: Correct answer.<br><br>1 Mark: Uses relevant trig identity. |
| 14(b) | $2x+y-1=0 \quad (1)$<br>$3x-y-4=0 \quad (2)$<br>Equation (1) + (2)<br>$5x-5=0$<br>$x=1$<br>Substitute $x=1$ into equation (1)<br>$2\times1+y-1=0$ or $y=-1$<br>Solution is $x=1$ and $y=-1$. | 2 Marks: Correct answer.<br><br>1 Mark: Finds one of the solutions. |
| 14(c) (i) | $\frac{d}{dx}\sqrt{x^4+7} = \frac{d}{dx}\left(x^4+7\right)^{\frac{1}{2}} = \frac{1}{2}\left(x^4+7\right)^{-\frac{1}{2}}\times4x^3$<br>$= \frac{2x^3}{\sqrt{x^4+7}}$ | 1 Mark: Correct answer. |
| 14(c) (ii) | $\frac{d}{dx}x(1-x)^6 = x\times6(1-x)^5\times-1+(1-x)^6\times1$<br>$= (1-x)^5\left[-6x+1(1-x)\right]$<br>$= (1-x)^5(1-7x)$ | 2 Marks: Correct answer.<br>1 Mark: Uses the product rule with one correct value. |
| 14(c) (iii) | $\frac{d}{dx}\left(\frac{3x}{2-x}\right) = \frac{(2-x)\times3-3x\times-1}{(2-x)^2}$<br>$= \frac{6-3x+3x}{(2-x)^2} = \frac{6}{(2-x)^2}$ | 2 Marks: Correct answer.<br>1 Mark: Uses the quotient rule with one correct value. |
| 14(d) | $y=x^2-6x, \ \frac{dy}{dx}=2x-6$<br>At $(-1,7)$ $\frac{dy}{dx}=2\times-1-6=-8$<br>Equation of the tangent at $(-1,7)$ with $m=-8$<br>$y-y_1=m(x-x_1)$<br>$y-7=-8(x-(-1))$<br>$y-7=-8x-8$<br>$8x+y+1=0$ | 2 Marks: Correct answer.<br><br>1 Mark: Finds the derivative or shows some understanding. |

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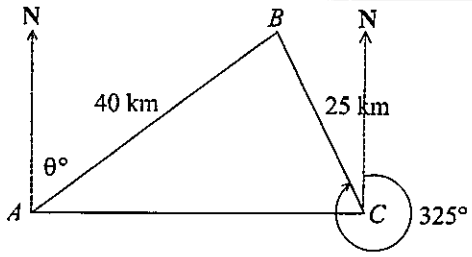
|---|---|---|
| 14(e) | $2^x \times 4^{x+1} = \frac{1}{2}$ or $2^x \times 2^{2(x+1)} = 2^{-1}$<br>$2^{x+2(x+1)} = 2^{-1}$<br>$x + 2x + 2 = -1$<br>$3x = -3$<br>$x = -1$ | 1 Mark: Correct answer. |
| 14(f) (i) | $\angle ACB = 325° - 270°$<br>$= 55°$<br>  | 1 Mark: Correct answer. |
| 14(f) (ii) | In $\Delta ABC$ use sine rule.<br>$\frac{\sin \angle BAC}{25} = \frac{\sin 55°}{40}$<br>$\sin \angle BAC = \frac{25 \times \sin 55°}{40}$<br>$\angle BAC = 30.79514161 \approx 31°$<br>Bearing $= 90° - 31° = 059°$ | 3 Marks: Correct answer.<br>2 Marks: Finds $\angle BAC$.<br>1 Mark: Uses the sine rule with one correct value. |