

NAME: _____

CLASS: 11MTA___ or 11MTX___

Circle your Teacher's name

Sing	Li	Saxena	Saw	Batra
Singh	Pathuis	Tolhurst	Iacona	Lees

CHERRYBROOK TECHNOLOGY HIGH SCHOOL



2019

YEAR 11

AP2

Mathematics Advanced

General

Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and calculations

Total Marks:

75

Section I – 10 marks (pages 2–5)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 65 marks (pages 6–22)

- Attempt Questions 11–27
- Allow about 1 hour and 45 minutes for this section

Section I

10 marks

Attempt Questions 1–10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

1 Which of the following is equivalent to $\frac{3}{\sqrt{2}}$ with a rational denominator?

A $\frac{\sqrt{6}}{4}$

B $\frac{\sqrt{6}}{4}$

C $\frac{3\sqrt{2}}{2}$

D $\frac{3\sqrt{2}}{4}$

2 Which of the following is the closest to $\frac{2\pi}{7}$ in degrees?

A 0.9°

B 102.9°

C 51.4°

D 161.6°

3 It is given that $\ln a = \ln b + \ln c - \ln d$.

Which of the following statements is true?

A $a = b + c - d$

B $a = \frac{bc}{d}$

C $\ln a = \frac{\ln b \times \ln c}{\ln d}$

D $\ln a = \frac{b+c}{d}$

4 The solution to $|x - 3| = 6$ is

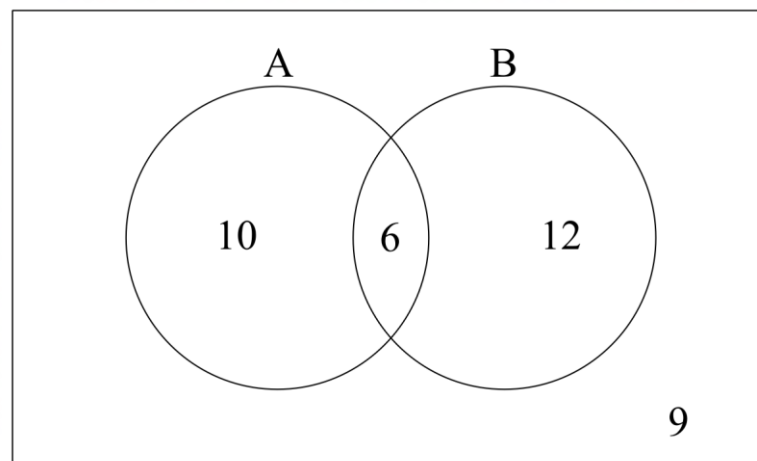
A $x = -3, 9$

B $x = 3, -9$

C $x = -3, -9$

D $x = 3, 9$

5 Consider the Venn diagram below.



According to the diagram, what is $P(A|B)$?

A $\frac{1}{3}$

B $\frac{5}{6}$

C $\frac{28}{37}$

D $\frac{3}{5}$

6 Which one of the following functions is *not* one-to-one?

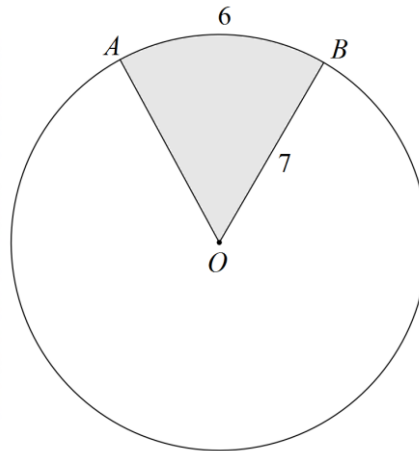
A $f(x) = 4 - x^2, x \geq 0$

B $f(x) = \sqrt{4 - x^2}$

C $f(x) = 1 - 4x$

D $f(x) = \frac{4}{x}$

- 7 The circle centred at O has a radius of 7. Arc AB has length 6, as shown in the diagram.



NOT TO
SCALE

What is the area of the shaded sector OAB ?

- A** 21π
- B** 21
- C** $\frac{343\pi}{12}$
- D** $\frac{343}{12}$
- 8 Given the equation $2x^2 - x + 5 = 0$ which statement is correct.
- A** The equation has equal roots.
- B** The equation has no real roots.
- C** The equation has unequal roots.
- D** The equation has rational roots.

- 9 The probability distribution of a random variable X is shown below:

x	0	1	2	3	4
$P(X = x)$	k	$2k$	$3k$	$2k$	k

What is the value of k ?

- A $\frac{1}{10}$
- B $\frac{1}{9}$
- C $\frac{1}{8}$
- D $\frac{1}{5}$
- 10 Which of the following is true if $f(x) = \sqrt{x}$ and $g(x) = 16 - x^2$
- A Domain of $f(g(x))$ is $[0,4]$ and range of $f(g(x))$ is $[0,4]$
- B Domain of $f(g(x))$ is $[-4,4]$ and range of $f(g(x))$ is $(0,4)$
- C Domain of $f(g(x))$ is $(-4,4)$ and range of $f(g(x))$ is $[0,4]$
- D Domain of $f(g(x))$ is $[-4,4]$ and range of $f(g(x))$ is $[0,4]$

Mathematics Advanced
Section II Answer Booklet

65 marks

Attempt Questions 11–27

Allow about 1 hour and 45 minutes for this section

Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
 - Your responses should include relevant mathematical reasoning and/or calculations.
 - Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.
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Question 11 (5 marks)

If $f(x) = x^2 + 2x - 8$

- (a) Find the x and y intercepts. **2**

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- (b) Find the axis of symmetry. **1**

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- (c) Find the turning point. State whether it is a maximum or a minimum. **2**

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Question 12 (6 marks)

The volume V of a gas varies inversely with the pressure P it is under.

- (a) If the volume of the gas is 120 cm^3 when it is under a pressure of 40 kg/cm^2 , find an equation that relates the variables V and P . **2**

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- (b) What amount of pressure is needed so that the volume of the gas is 80 cm^3 ? **1**

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- (c) Draw a graph of the relationship between V and P , using P as the independent variable and V as the dependent variable, where $P > 0$. Label the axes. **2**



- (d) Describe what would happen to the volume of the gas if the pressure approaches zero. **1**

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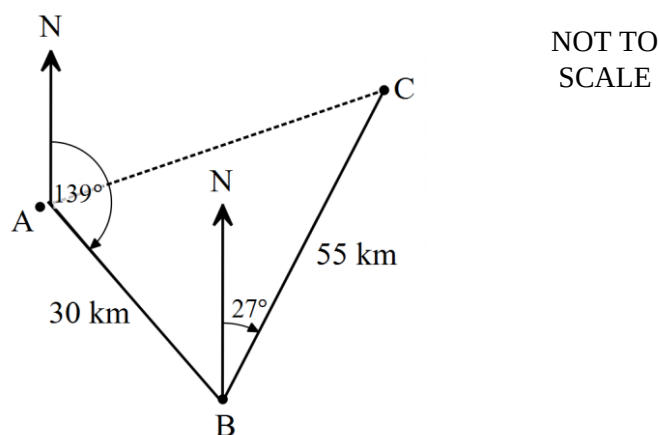
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Question 13 (4 marks)

A boat sails 30 kilometres from Island A to Island B on a bearing of 139° . It then sails 55 kilometres to Island C on a bearing of 27° from Island B.



Calculate the distance and bearing the boat must sail on to return to Island A from Island C. **4**

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Question 14 (2 marks)

Given that $\cos \theta = -\frac{5}{8}$ and $0 < \theta < \pi$, find the value of $\tan \theta$.

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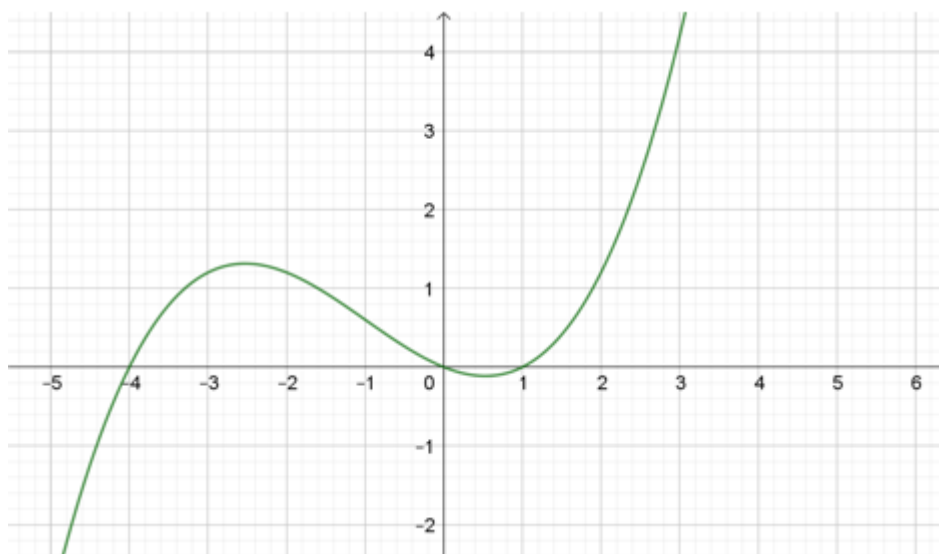
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Question 15 (2 marks)

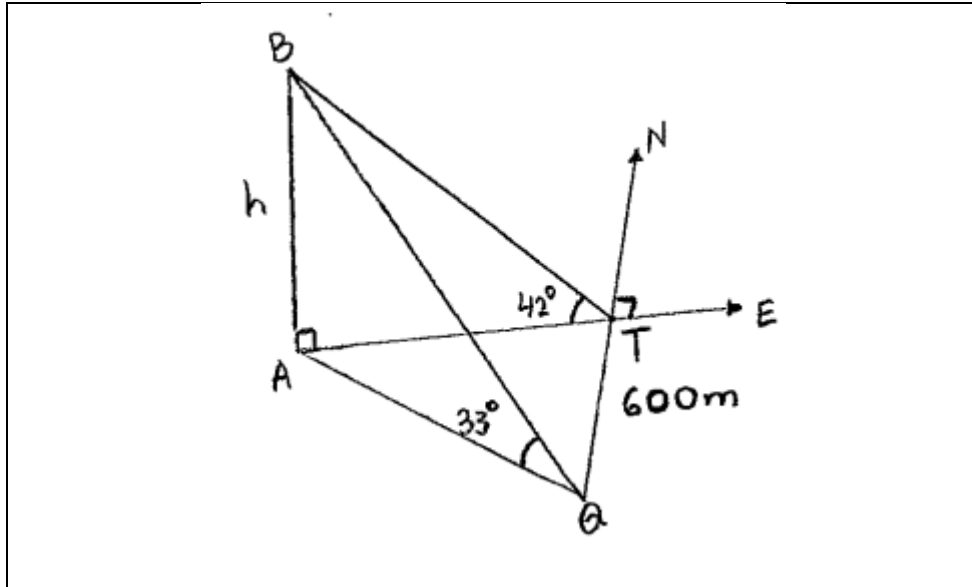
The graph of $y = f(x)$ is given below. Sketch the graph of $y = -f(-x)$

2



4

Let the height (h) of the building be AB where (A) is the foot of the building. Determine the height of the building to 2 decimal places

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Question 17 (4 marks)

For the given discrete probability distribution

X	1	2	3	4
$P(X = x)$	0.3	0.2	0.4	0.1

- (a) Find the expected value. 2

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- (b) Find the variance. 2

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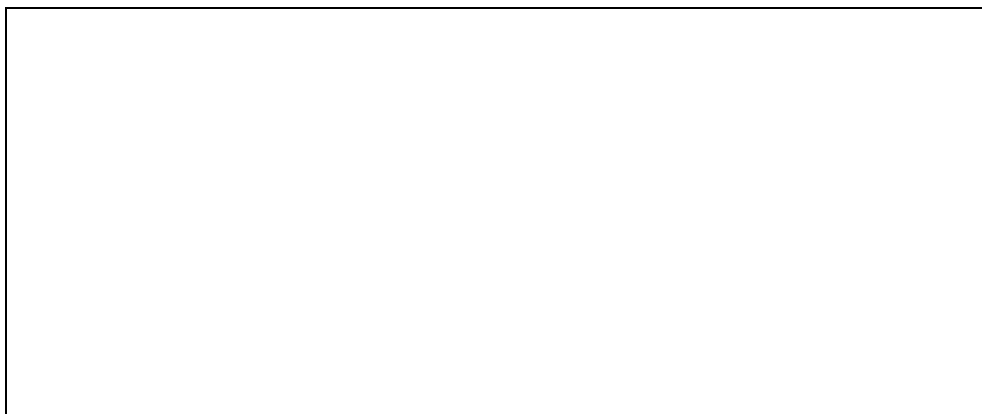
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Question 18 (3 marks)

In a class of 30 students, 13 students are in the soccer club, 14 students are in the chess club, and 8 students don't belong to either club.

- (a) Construct a Venn diagram to display this information.

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- (b) If a student is chosen at random, what is the probability that the student belongs to both the clubs?

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Question 19 (2 marks)

John enters a biathlon which includes cross-country skiing and rifle shooting. He has a 39% chance of winning the cross-country skiing and a 32% chance of winning both races. Find the probability that he wins the rifle shooting, if he has won the cross-country skiing race?

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(The **biathlon** is a winter sport that combines cross-country skiing and rifle shooting. It is treated as a race where the contestant with the shortest total time wins.)

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Question 20 (6 marks)

- (a) Using the method of first principles, find $f'(x)$ if $f(x) = 3x^2 - 5$ **3**

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- (b) Hence, find the equation of the line that is tangent to $f(x)$ at $x = 2$. **3**

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Question 21 (6 marks)

Differentiate the following expressions with respect to x .

(a) $2x^2 + \frac{5}{x}$ 2

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(b) $(3x^2 - 5x)^5$ 2

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(c) $(3x - 4)(4x + 5)$ 2

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Question 22 (4 marks)

If $y = \frac{2x+4}{\sqrt{x+1}}$. Show that $\frac{dy}{dx} = \frac{x}{\sqrt{(1+x)^3}}$

4

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Question 23 (3 marks)

Velocity, V metres per second, of a particle at time t seconds is given by **3**

$$V = \frac{8t^3}{3} - 8t^2 + 12t + 8$$

Show that the acceleration is never less than 4 metres per second per second.

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Question 24 (2 marks)

Differentiate $y = -2e^{-3x} + 5x^3$ **2**

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Question 25 (3 marks)

The amount A of money in a bank account after n years grows with compound interest according to the formula $A = 820(1.035)^n$. **3**

Find how many years it will take for the amount in the bank to be \$2000?

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Question 26 (4 marks)

The pH value of a solution is given by the formula $\text{pH} = -\log_{10}[\text{H}^+]$, where $[\text{H}^+]$ is the concentration of hydrogen ions in moles per litre.

- (a) Find the pH value of a solution in a chemistry lab whose label indicates its concentration of hydrogen ions is 2.5×10^{-4} moles per litre. **1**
Give your answer to one decimal place.

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- (b) Find the concentration of hydrogen ions in moles per litre for vinegar if a test strip indicates it has a pH of 2.4. **3**
Give your answer as $a \times 10^b$ to two significant figures.

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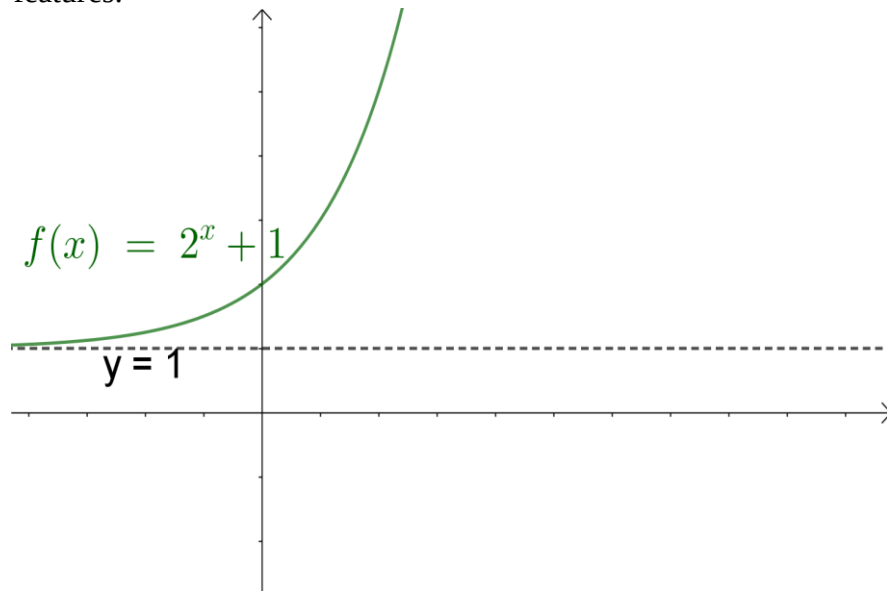
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Question 27 (5 marks)

(a) The graph of $f(x) = 2^x + 1$ is given below.

3

On the same set of axis sketch the graph of $g(x) = \log_2(x - 1)$ showing key features.



(b) Hence, state the domain and range of $g(x)$.

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End of examination

Section II extra writing space

If you use this space, clearly indicate which question you are answering

Section II extra writing space

If you use this space, clearly indicate which question you are answering

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SOLUTIONS

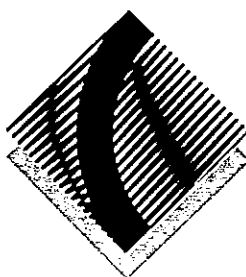
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Section I

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- 1 Which of the following is equivalent to $\frac{3}{\sqrt{2}}$ with a rational denominator?

A $\frac{\sqrt{6}}{4}$

B $\frac{\sqrt{6}}{4}$

C $\frac{3\sqrt{2}}{2}$

D $\frac{3\sqrt{2}}{4}$

$$\frac{3}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$$

- 2 Which of the following is the closest to $\frac{2\pi}{7}$ in degrees?

A 0.9°

B 102.9°

C 51.4°

D 161.6°

$$\frac{2\pi}{7} = 51.4^\circ$$

- 3 It is given that $\ln a = \ln b + \ln c - \ln d$.

Which of the following statements is true?

A $a = b + c - d$

B $a = \frac{bc}{d}$

C $\ln a = \frac{\ln b \times \ln c}{\ln d}$

D $\ln a = \frac{b+c}{d}$

$$\ln a = \ln \frac{bc}{d}$$

$$a = \frac{bc}{d}$$

4 The solution to $|x-3|=6$ is

A $x = -3, 9$

B $x = 3, -9$

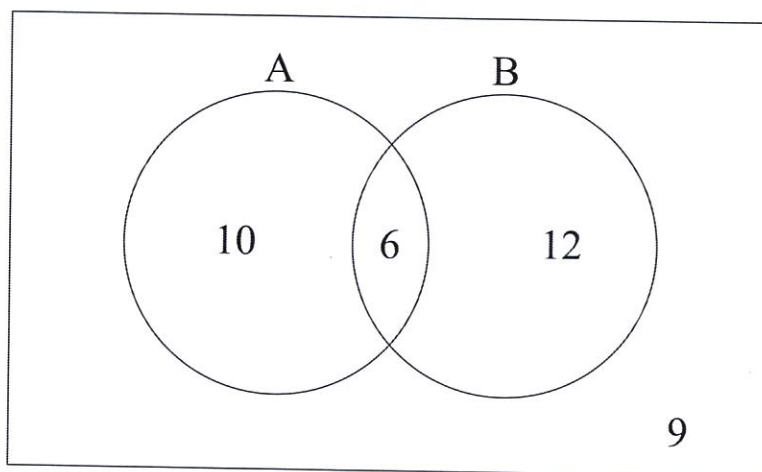
C $x = -3, -9$

D $x = 3, 9$

$$\begin{aligned} x-3 &= 6 \\ x &= 9 \end{aligned}$$

$$\begin{aligned} \text{or } -(x-3) &= 6 \\ x-3 &= -6 \\ x &= -6+3 \\ &= -3 \end{aligned}$$

5 Consider the Venn diagram below.



According to the diagram, what is $P(A|B)$?

A $\frac{1}{3}$

B $\frac{5}{6}$

C $\frac{28}{37}$

D $\frac{3}{5}$

$$\frac{P(A \cap B)}{P(B)} = \frac{6}{18} = \frac{1}{3}$$

6 Which one of the following functions is *not* one-to-one?

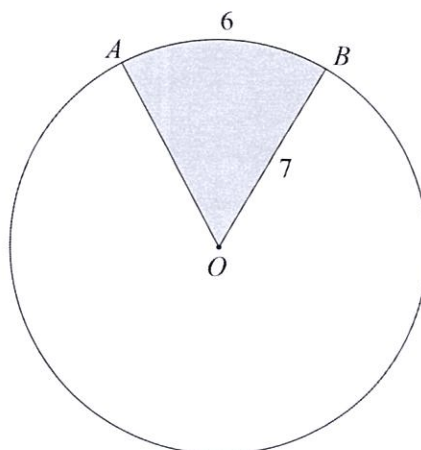
A $f(x) = 4 - x^2, x \geq 0$

B $f(x) = \sqrt{4 - x^2}$

C $f(x) = 1 - 4x$

D $f(x) = \frac{4}{x}$

- 7 The circle centred at O has a radius of 7. Arc AB has length 6, as shown in the diagram.



NOT TO
SCALE

What is the area of the shaded sector OAB ?

A 21π

B 21

C $\frac{343\pi}{12}$

D $\frac{343}{12}$

$$\begin{aligned} l &= r\theta \\ 6 &= \theta \times 7 \\ \theta &= \frac{6}{7} \\ A &= \frac{1}{2} r^2 \times \theta \\ &= \frac{1}{2} \times 7^2 \times \frac{6}{7} \\ &= 21 \end{aligned}$$

- 8 Given the equation $2x^2 - x + 5 = 0$ which statement is correct.

A The equation has equal roots.

B The equation has no real roots.

C The equation has unequal roots.

D The equation has rational roots.

$$\begin{aligned} \Delta &= b^2 - 4ac \\ &= (-1)^2 - 4 \times 2 \times 5 \\ &= 1 - 40 \\ &= -39 \\ \therefore &\text{ No real roots} \end{aligned}$$

- 9 The probability distribution of a random variable X is shown below:

x	0	1	2	3	4
$P(X=x)$	k	$2k$	$3k$	$2k$	k

What is the value of k ?

- A $\frac{1}{10}$
 B $\frac{1}{9}$
 C $\frac{1}{8}$
 D $\frac{1}{5}$

$$k + 2k + 3k + 2k + k = 1$$

$$9k = 1$$

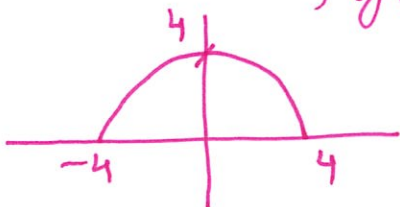
$$k = \frac{1}{9}$$

- 10 Which of the following is true if $f(x) = \sqrt{x}$ and $g(x) = 16 - x^2$

- A Domain of $f(g(x))$ is $[0, 4]$ and range of $f(g(x))$ is $[0, 4]$
 B Domain of $f(g(x))$ is $[-4, 4]$ and range of $f(g(x))$ is $(0, 4)$
 C Domain of $f(g(x))$ is $(-4, 4)$ and range of $f(g(x))$ is $[0, 4]$
 D Domain of $f(g(x))$ is $[-4, 4]$ and range of $f(g(x))$ is $[0, 4]$

$$f(g(x)) = f(16 - x^2)$$

$$f(g(x)) = \sqrt{16 - x^2}$$



$$\text{Domain: } [-4, 4]$$

$$\text{Range: } [0, 4]$$

Mathematics Advanced
Section II Answer Booklet

70 marks

Attempt Questions 11–27

Allow about 1 hour and 45 minutes for this section

Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
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Question 11 (5 marks)

If $f(x) = x^2 + 2x - 8$

- (a) Find the x and y intercepts.

2

y-intercept when $x = 0$
 $f(0) = 0^2 + 2 \times 0 - 8$
 $= -8$

x-intercept when $y = 0$

$$x^2 + 2x - 8 = 0$$
$$(x+4)(x-2) = 0$$
$$x = 2 \text{ or } x = -4$$

- (b) Find the axis of symmetry.

1

$$x = -\frac{b}{2a}$$

$$x = \frac{-2}{2 \times 1}$$

$$x = -1$$

- (c) Find the turning point. State whether it is a maximum or a minimum.

2

Turning point on the axis of symmetry

$$f(-1) = (-1)^2 + 2(-1) - 8$$
$$= 1 - 2 - 8$$
$$= -9$$

Since $a > 0 \therefore$ parabola is concave up
Turning pt. is a minimum at $(-1, -9)$

Question 12 (6 marks)

The volume V of a gas varies inversely with the pressure P it is under.

- (a) If the volume of the gas is 120 cm^3 when it is under a pressure of 40 kg/cm^3 , find an equation that relates the variables V and P . 2

$$V = \frac{K}{P} \quad \therefore K = 4800$$

$$120 = \frac{K}{40} \quad V = \frac{4800}{P} \text{ or } V \times P = 4800$$

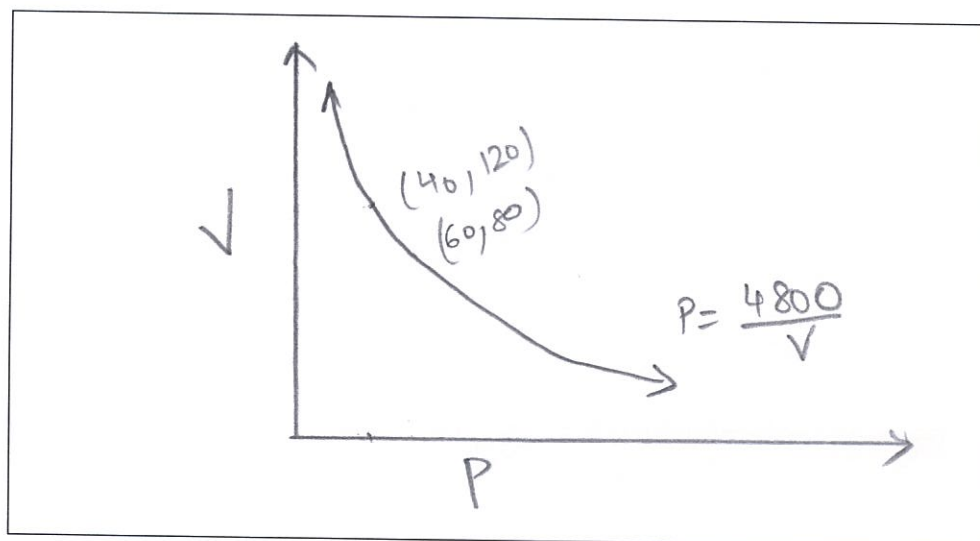
$$\text{or } P = \frac{4800}{V}$$

- (b) What amount of pressure is needed so that the volume of the gas is 80 cm^3 ? 1

$$P = \frac{4800}{80}$$

$$= 60 \text{ kg/cm}^3$$

- (c) Draw a graph of the relationship between V and P , using P as the independent variable and V as the dependent variable, where $P > 0$. Label the axes. 2

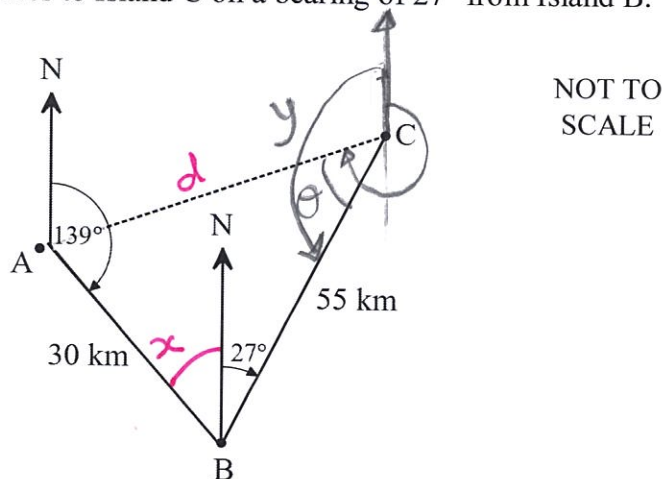


- (d) Describe what would happen to the volume of the gas if the pressure approaches zero. 1

As $P \rightarrow 0$, $V \rightarrow \infty$

Question 13 (4 marks)

A boat sails 30 kilometres from Island A to Island B on a bearing of 139° . It then sails 55 kilometres to Island C on a bearing of 27° from Island B.



Calculate the distance and bearing the boat must sail on to return to Island A from Island C. 4

$$x = 41^\circ \text{ (Co-int } \angle \text{ s on parallel lines are supplementary)}$$

$$\therefore \angle ABC = 41 + 27 = 68^\circ$$

$$d = \sqrt{30^2 + 55^2 - 2 \times 30 \times 55 \times \cos 68^\circ}$$

$$d = 51.8536 \approx 51.85$$

$$\frac{\sin \theta}{30} = \frac{\sin 68}{55}$$

$$\sin \theta = \frac{30 \times \sin 68}{55}$$

$$\theta \approx 32^\circ$$

$$y = 180 - 27^\circ \text{ (Co-int } \angle \text{ s on } \parallel \text{ lines are supplementary)}$$

$$= 153^\circ$$

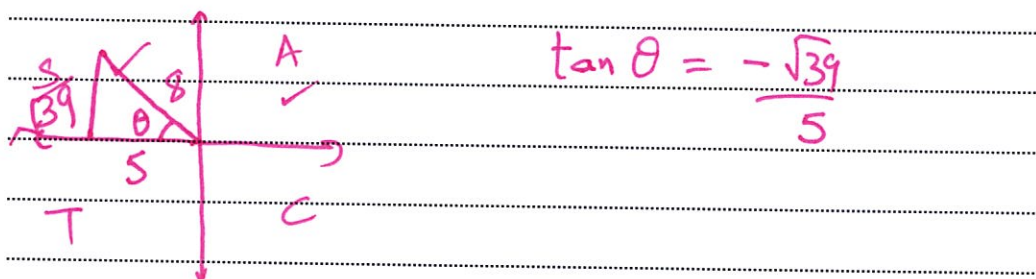
$$\text{The bearing of A from C is } 360^\circ - 153 + 32$$

$$= 239^\circ \text{ T}$$

Question 14 (2 marks)

Given that $\cos \theta = -\frac{5}{8}$ and $0 < \theta < \pi$, find the value of $\tan \theta$.

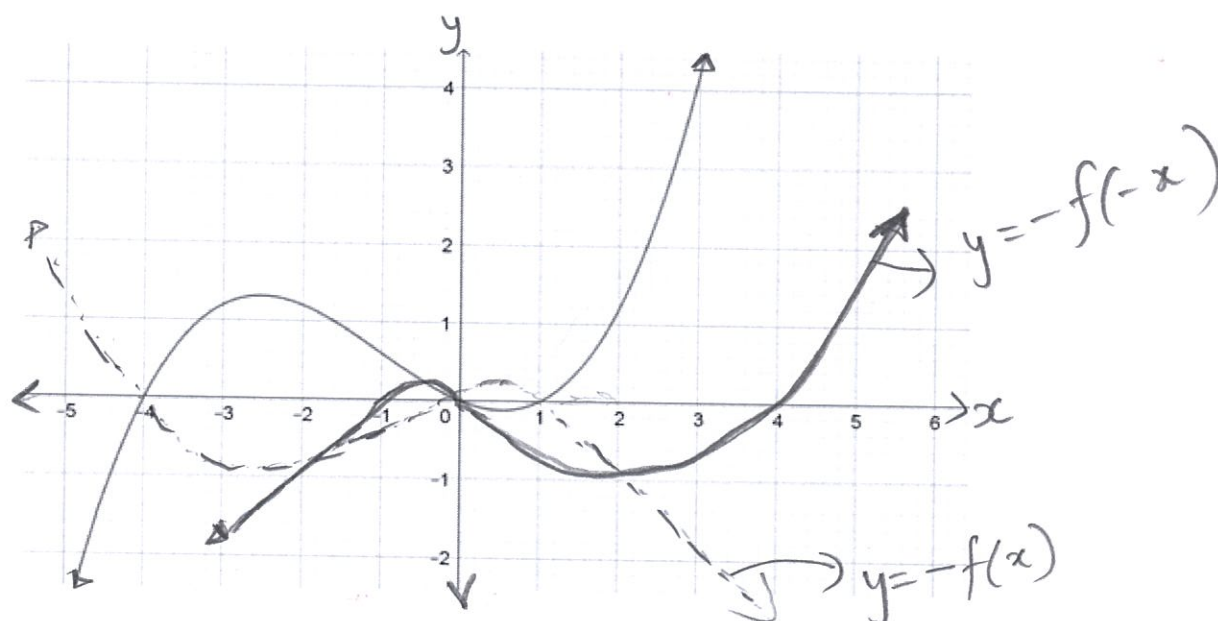
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Question 15 (2 marks)

The graph of $y = f(x)$ is given below. Sketch the graph of $y = -f(-x)$

2

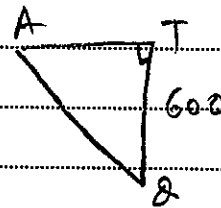
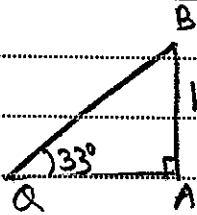
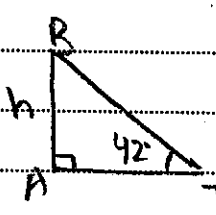
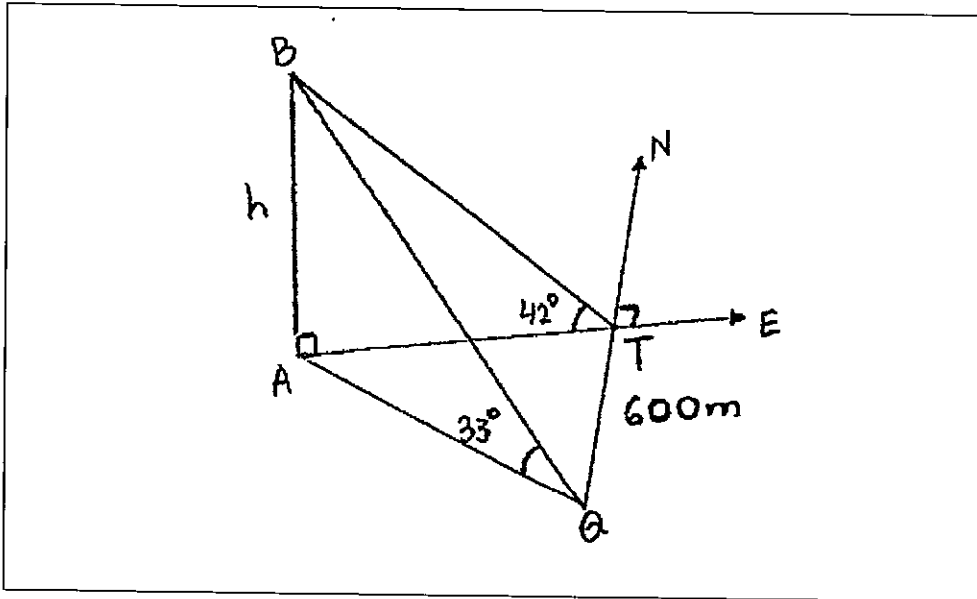


Question 16 (4 marks)

4

The elevation of a building from T due east of it is 42° ; at a place Q due south of T, the elevation of the building is 33° . The distance from T to Q is 600 metres.

Let the height (h) of the building be AB where (A) is the foot of the building. Determine the height of the building to 2 decimal places



$$\tan 42^\circ = \frac{h}{AT}$$

$$\tan 33^\circ = \frac{h}{AQ}$$

$$AT = \frac{h}{\tan 42^\circ}$$

$$AQ = \frac{h}{\tan 33^\circ}$$

$$AQ^2 - AT^2 = 600^2$$

$$\frac{h^2}{\tan^2 33^\circ} - \frac{h^2}{\tan^2 42^\circ} = 600^2$$

$$h^2 \left[\frac{1}{\tan^2 33^\circ} - \frac{1}{\tan^2 42^\circ} \right] = 600^2$$

$$h^2 \times \left[\frac{\tan^2 42^\circ - \tan^2 33^\circ}{\tan^2 33^\circ \times \tan^2 42^\circ} \right] = 600^2$$

$$h^2 = \frac{600^2 \times \tan^2 33^\circ \times \tan^2 42^\circ}{\tan^2 42^\circ - \tan^2 33^\circ}$$

$$h = \sqrt{\frac{600^2 \times \tan^2 33^\circ \times \tan^2 42^\circ}{\tan^2 42^\circ - \tan^2 33^\circ}} = 562.51 \text{ m}$$

Question 17 (4 marks)

For the given discrete probability distribution

X	1	2	3	4
$P(X = x)$	0.3	0.2	0.4	0.1

(a) Find the expected value.

2

$$E(X) = \sum x p(x)$$

$$= 1 \times 0.3 + 2 \times 0.2 + 3 \times 0.4 + 4 \times 0.1$$

$$= 0.3 + 0.4 + 1.2 + 0.4$$

$$= 2.3$$

(b) Find the variance.

2

$$\text{Var}(X) = 1^2 \times 0.3 + 2^2 \times 0.2 + 3^2 \times 0.4 + 4^2 \times 0.1 - (2.3)^2$$

$$= 1.01$$

or

X	1	2	3	4
$(x - \mu)$	-1.3	-0.3	0.7	1.7
$P(X = x)$	0.3	0.2	0.4	0.1

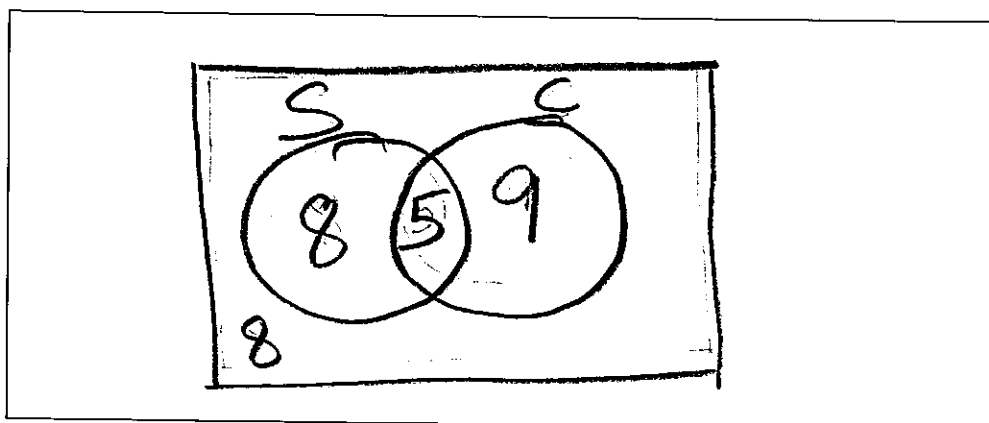
$$\text{Var}(X) = (-1.3)^2 \times 0.3 + (-0.3)^2 \times 0.2 + (0.7)^2 \times 0.4 + 1.7^2 \times 0.1$$
$$= 1.01$$

Question 18 (3 marks)

In a class of 30 students, 13 students are in the soccer club, 14 students are in the chess club, and 8 students don't belong to either club.

- (a) Construct a Venn diagram to display this information.

2



- (b) If a student is chosen at random, what is the probability that the student belongs to both the clubs?

1

$$\frac{5}{30} = \frac{1}{6}$$

Question 19 (2 marks)

John enters a biathlon which includes cross-country skiing and rifle shooting. He has a 39% chance of winning the cross-country skiing and a 32% chance of winning both races. Find the probability that he wins the rifle shooting, if he has won the cross-country skiing race?

2

(The **biathlon** is a winter sport that combines cross-country skiing and rifle shooting. It is treated as a race where the contestant with the shortest total time wins.)

$$P(\text{win x-ski}) = 0.39$$

$$P(\text{win both}) = 0.32$$

$$P(\text{win rifle} / \text{x-ski}) = \frac{P(\text{win both})}{P(\text{win x-ski})} = \frac{0.32}{0.39} = 0.82 \text{ or } 82\%$$

Question 20 (6 marks)

- (a) Using the method of first principles, find $f'(x)$ if $f(x) = 3x^2 - 5$

3

$$\begin{aligned}f(x+h) &= 3(x+h)^2 - 5 \\&= 3(x^2 + 2xh + h^2) - 5 \\&= 3x^2 + 6xh + 3h^2 - 5\end{aligned}$$

$$\begin{aligned}f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\&= \lim_{h \rightarrow 0} \frac{(3x^2 + 6xh + 3h^2 - 5) - (3x^2 - 5)}{h} \\&= \lim_{h \rightarrow 0} \frac{6xh + 3h^2}{h} = \lim_{h \rightarrow 0} \cancel{h}(6x + 3h) \\&= 6x\end{aligned}$$

- (b) Hence, find the equation of the line that is tangent to $f(x)$ at $x = 2$.

3

when $x = 2$

$$\begin{aligned}f(x) &= 3(2)^2 - 5 \\&= 7\end{aligned}$$

$$f'(x) = 6x$$

$$f'(2) = 12$$

$$y - 7 = 12(x - 2)$$

$$y = 12x - 17$$

Question 21 (6 marks)

Differentiate the following expressions with respect to x .

(a) $2x^2 + \frac{5}{x}$

2

$$\frac{d}{dx} \left(2x^2 + \frac{5}{x} \right)$$

$$= 4x - \frac{5}{x^2}$$

$$= 4x - \frac{5}{x^2}$$

(b) $(3x^2 - 5x)^5$

2

$$\frac{d}{dx} (3x^2 - 5x)^5$$

$$5(3x^2 - 5x)^4 \times (6x - 5)$$

(c) $(3x - 4)(4x + 5)$

2

$$\frac{d}{dx} (3x - 4)(4x + 5)$$

$$4(3x - 4) + (4x + 5) \times 3$$

$$12x - 16 + 12x + 15$$

$$24x - 1$$

Question 22 (4 marks)

If $y = \frac{2x+4}{\sqrt{x+1}}$. Show that $\frac{dy}{dx} = \frac{x}{\sqrt{(1+x)^3}}$

4

$$u = 2x+4$$

$$v = \sqrt{x+1}$$

$$u' = 2$$

$$v' = \frac{1}{2\sqrt{x+1}}$$

$$y' = \frac{vu' - uv'}{v^2}$$

$$= \frac{2\sqrt{x+1} - (2x+4)}{(x+1)^{3/2}}$$

$$= \frac{2\sqrt{x+1} - 2(x+2)}{(x+1)^{3/2}}$$

$$= \frac{2(\sqrt{x+1})^2 - (x+2)}{(x+1)\sqrt{x+1}} = \frac{2(x+1) - (x+2)}{\sqrt{(x+1)^3}}$$

$$= \frac{x}{\sqrt{(x+1)^3}}$$

Question 23 (3 marks)

Velocity, V metres per second, of a particle at time t seconds is given by

3

$$V = \frac{8t^3}{3} - 8t^2 + 12t + 8$$

Show that the acceleration is never less than 4 metres per second per second.

$$V = \frac{8t^3}{3} - 8t^2 + 12t + 8$$

$$a = \frac{dV}{dt} = \frac{8}{3} \times 3t^2 - 16t + 12$$

$$a = 8t^2 - 16t + 12$$

$$\text{Axis of symmetry } t = \frac{-b}{2a} = \frac{-(-16)}{2 \times 8}$$

$$t = 1$$

Vertex. $\rightarrow (1, 4)$ Since minimum turning point value of

$$a = 8(1)^2 - 16 \times 1 + 12$$
$$\underline{a = 4.}$$

Question 24 (2 marks)

Differentiate $y = -2e^{-3x} + 5x^3$

2

$$\frac{dy}{dx} = -2e^{-3x} \times (-3) + 15x^2$$

$$= 6e^{-3x} + 15x^2$$

Question 25 (3 marks)

The amount A of money in a bank account after n years grows with compound interest according to the formula $A = 820(1.035)^n$. 3

Find how many years it will take for the amount in the bank to be \$2000?

$$A = 820(1.035)^n$$

$$2000 = 820(1.035)^n$$

$$1.035^n = \frac{2000}{820}$$

$$n = \frac{\log 2000}{\log 1.035 \cdot 820}$$

$$= \frac{\log_{10} 2000}{\log_{10} 820}$$

$$\log_{10} 1.035$$

$$= 25.9 \approx 26 \text{ year s.}$$

Question 26 (4 marks)

The pH value of a solution is given by the formula $\text{pH} = -\log_{10}[\text{H}^+]$, where $[\text{H}^+]$ is the concentration of hydrogen ions in moles per litre.

- (a) Find the pH value of a solution in a chemistry lab whose label indicates its concentration of hydrogen ions is 2.5×10^{-4} moles per litre. 1
Give your answer to one decimal place.

$$\text{pH} = -\log_{10} 2.5 \times 10^{-4}$$

$$= 3.6$$

- (b) Find the concentration of hydrogen ions in moles per litre for vinegar if a test strip indicates it has a pH of 2.4. 3
Give your answer as $a \times 10^b$ to two significant figures.

$$2.4 = -\log_{10}[\text{H}^+]$$

$$-2.4 = \log_{10}[\text{H}^+]$$

$$[\text{H}^+] = 10^{-2.4}$$

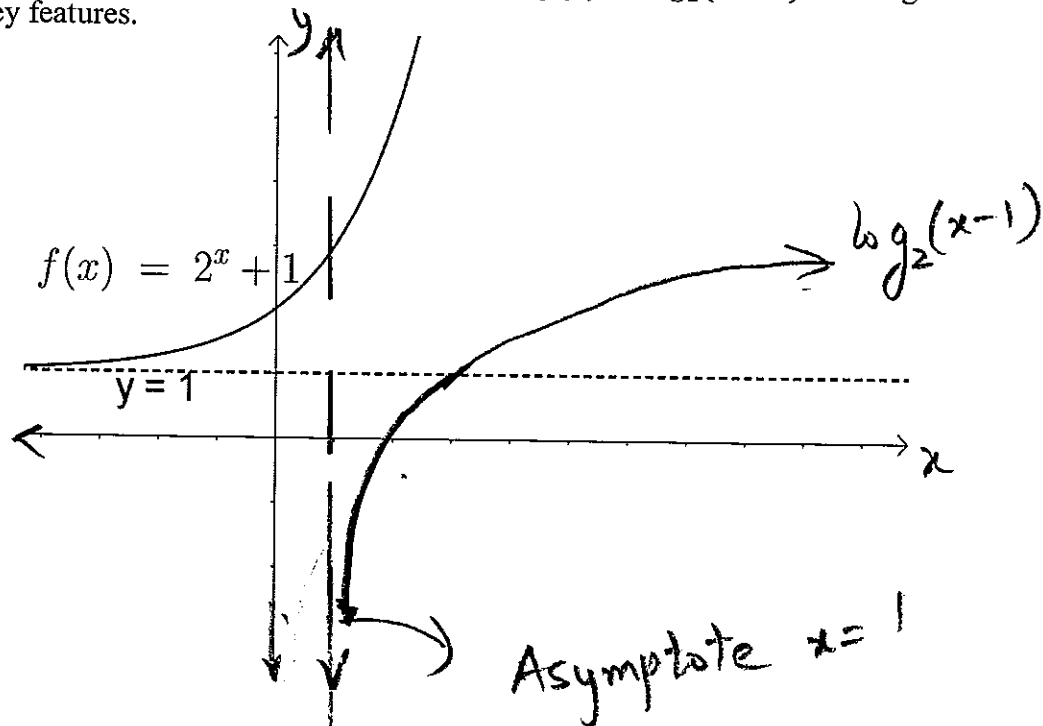
$$= 4.0 \times 10^{-3}$$

Question 27 (5 marks)

- (a) The graph of $f(x) = 2^x + 1$ is given below.

3

On the same set of axis sketch the graph of $g(x) = \log_2(x - 1)$ showing key features.



- (b) Hence, state the domain and range of $g(x)$.

2

Domain $\rightarrow (1, \infty)$
Range $\rightarrow (-\infty, \infty)$

End of examination