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Name: \_\_\_\_\_

Class: 11MTA\_\_\_\_\_

## CHERRYBROOK TECHNOLOGY HIGH SCHOOL



2020

YEAR 11

AP2

## MATHEMATICS ADVANCED

*Time allowed – 2 hours plus 10 minutes reading time*

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### General Instructions

- Attempt all questions
- Write your name and student number on the question paper
- Write using black pen
- NESA approved calculators may be used
- The NESA reference sheet has been provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations

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**Total marks:**  
**65**

### Section I – 10 marks (pages 2 – 4)

- Attempt Questions 1-10
- Allow about 15 minutes for this section

### Section II – 55 marks (pages 5 – 18)

- Attempt Questions 11-26
- Allow about 1 hour and 45 minutes for this section

## Section I

10 marks

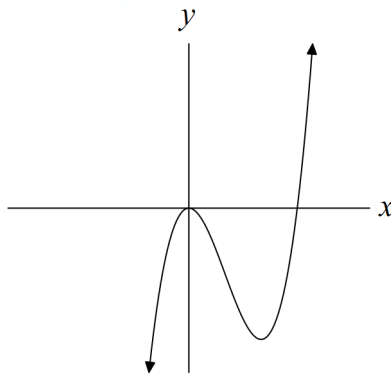
Attempt Questions 1–10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

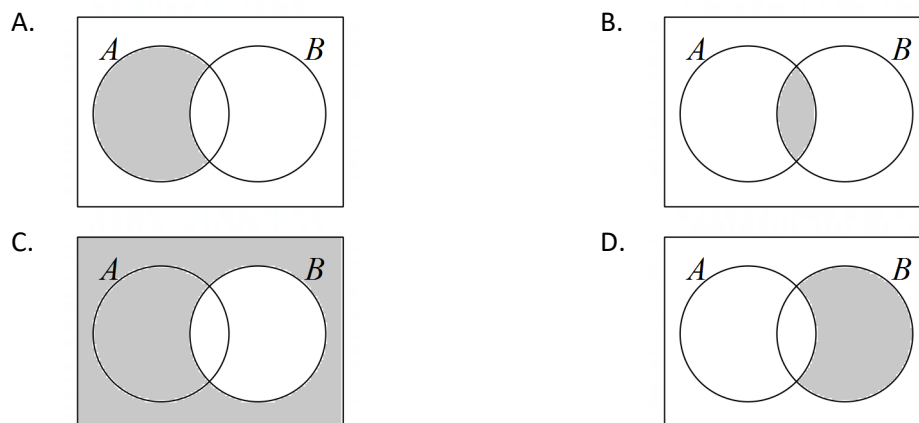
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1. What is the value of  $f(2a + 1)$  if  $f(x) = 3x - 2$ 
  - A.  $6a - 2$
  - B.  $6a - 1$
  - C.  $6a + 1$
  - D.  $6a + 2$
  
2. The line  $5x - y - 4 = 0$  passes through the point  $(k, 26)$ . What is the value of  $k$ ?
  - A. 3
  - B. 4
  - C. 5
  - D. 6
  
3. What type of relation is shown by the graph below?



- A. One - to - One
- B. One - to - Many
- C. Many - to - One
- D. Many - to - Many

4. Which of the Venn diagrams shows  $A \cap B'$  shaded?



5. The probability distribution for the random variable  $X$  is shown below.

| $x$        | 2              | 3             | 4              | 5             | 6              | 7             | 8              |
|------------|----------------|---------------|----------------|---------------|----------------|---------------|----------------|
| $P(X = x)$ | $\frac{1}{16}$ | $\frac{1}{8}$ | $\frac{3}{16}$ | $\frac{1}{4}$ | $\frac{3}{16}$ | $\frac{1}{8}$ | $\frac{1}{16}$ |

What is  $P(3 < X \leq 7)$ ?

- A.  $\frac{1}{2}$
- B.  $\frac{5}{8}$
- C.  $\frac{3}{4}$
- D.  $\frac{7}{8}$

6. Solve  $|x - 3| - 8 = 0$

- A.  $x = 11$
- B.  $x = 5$  or  $x = -11$
- C.  $x = -5$  or  $x = 11$
- D.  $x = 5$  or  $x = 11$

7. Which of the following is true for the quadratic equation  $3x^2 + x + 4 = 0$ ?
- A. The roots are rational
  - B. There are no real roots
  - C. The roots are equal
  - D. The roots are irrational
8. A sector has an angle of  $\frac{\pi}{3}$  subtended at the centre of a circle. The radius is  $6\text{ cm}$ .  
The exact area of the sector is?
- A.  $\pi\text{ cm}^2$
  - B.  $3\pi\text{ cm}^2$
  - C.  $6\pi\text{ cm}^2$
  - D.  $12\pi\text{ cm}^2$
9. A function is defined as  $g(x) = 12x^3 - 8x + 14$   
Evaluate  $g'(-2)$ .
- A.  $-152$
  - B.  $-66$
  - C.  $136$
  - D.  $150$
10. The variance of a discrete random variable  $X$  is  $\text{Var}(X) = 5.01$   
Calculate  $\text{Var}(5X - 2)$
- A.  $25.05$
  - B.  $26.05$
  - C.  $125.25$
  - D.  $126.25$

**End of Section I**

## Section II

55 marks

### Attempt Questions 11 – 26

Allow about 1 hour and 45 minutes for this section

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
  - Your responses should include relevant mathematical reasoning and/or calculations.
  - Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.
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#### Question 11

- (a) Express  $(2\sqrt{3} + 1)(2 - \sqrt{3})$  in the form of  $a + b\sqrt{3}$  where  $a$  and  $b$  are integers. 2

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- (b) Show algebraically that  $f(x) = \frac{2x}{x^2+3}$  is an odd function. 2

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(c) Rewrite the following expression without any negative or fractional indices.

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$$\left(\frac{12x^{-3}y^2}{xyz^{\frac{1}{2}}}\right)^2$$

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Question 12

If  $f(x) = x^2$  and  $g(x) = 2x + 1$ :

(a) Find the composite function  $f(g(x))$

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(b) What is the domain and range of the composite function  $f(g(x))$ ?

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### Question 13

Differentiate  $f(x) = 2x^2 - x + 3$  from first principles.

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Question 14

Differentiate with respect to  $x$ :

(a)  $4x^{-3}$

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(b)  $y = (5 + 2x^2)^{\frac{1}{2}}$

2

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Question 15

A circle is given by the equation  $x^2 + y^2 + 6x - 12y = 19$

3

What is the centre and radius of this circle?

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## Question 16

(a) If  $f(x) = \frac{x^2-4}{2x+5}$  show that  $f'(x) = \frac{2(x+4)(x+1)}{(2x+5)^2}$

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(b) Hence, or otherwise, find the equation of the tangent to  $f(x) = \frac{x^2-4}{2x+5}$  at the point  $\left(1, \frac{-3}{7}\right)$ .  
Give your answer in general form.

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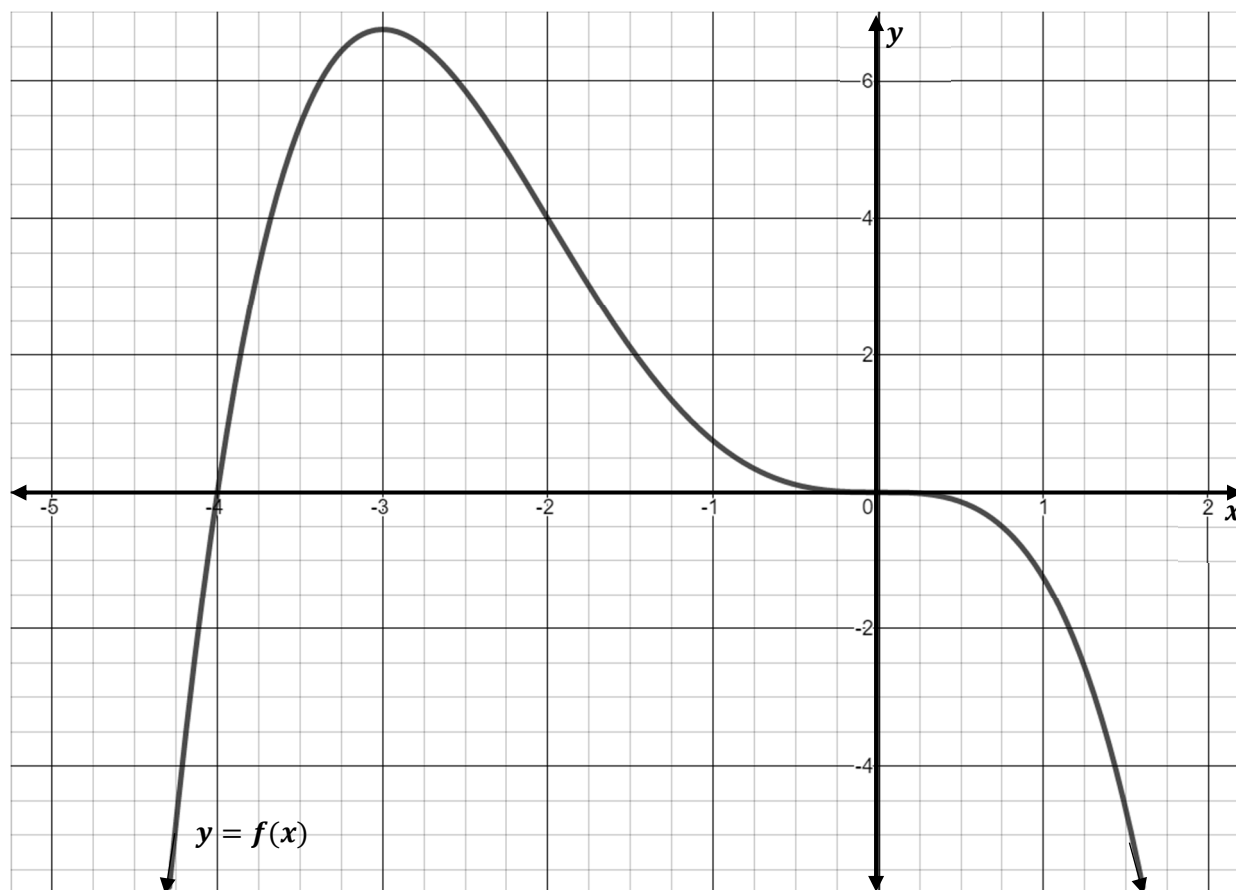
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(c) The graph below shows the graph of  $y = f(x)$ .

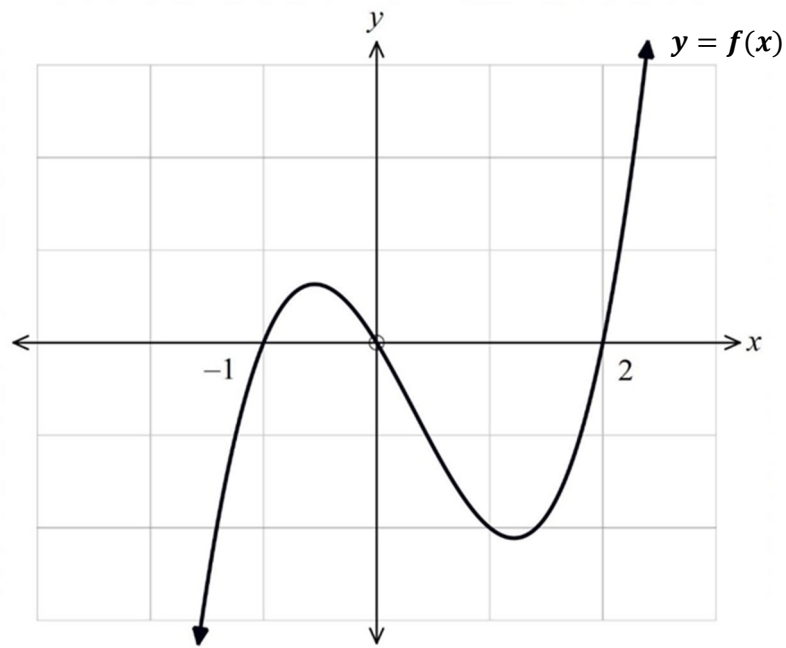
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On the same set of axes sketch the gradient function.



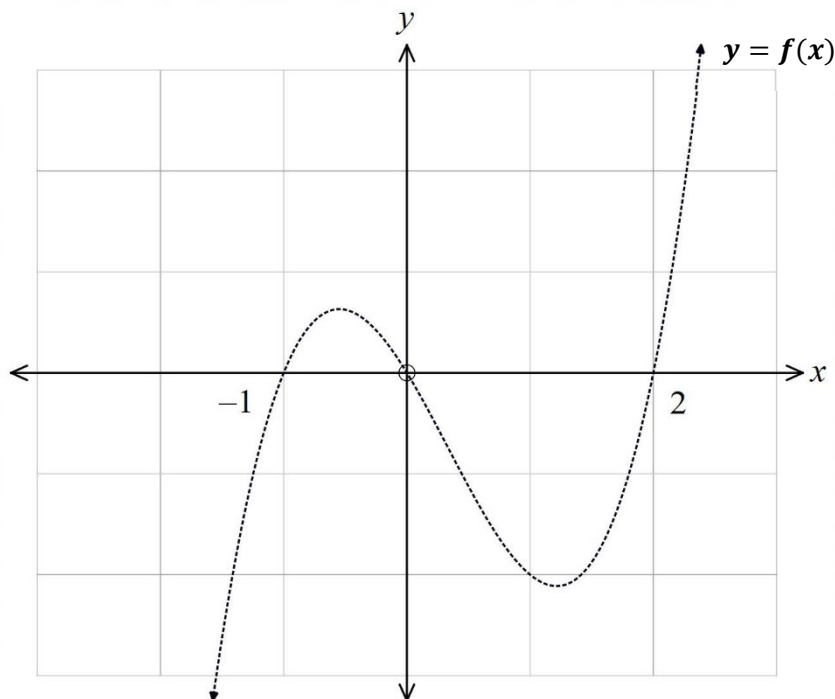
### Question 17

The sketch of  $y = f(x)$  is shown below.



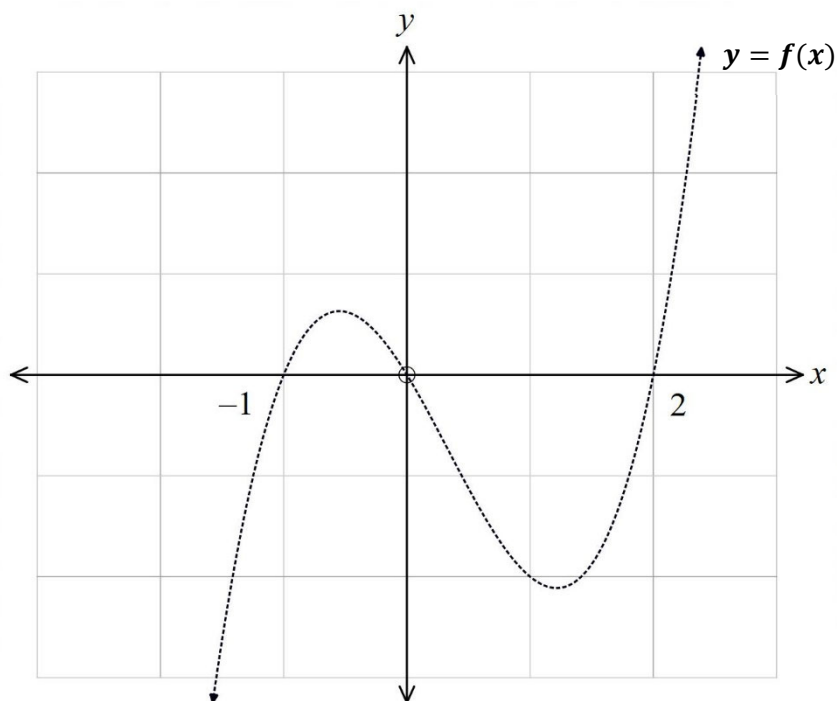
(a) Use the graph of  $y = f(x)$  to draw a neat sketch of  $y = -f(x)$  on the diagram below.

1



(b) Use the graph of  $y = f(x)$  to draw a neat sketch of  $y = -f(-x)$  on the diagram below.

1



### Question 18

(a) Convert  $\frac{11\pi}{12}$  to degrees.

1

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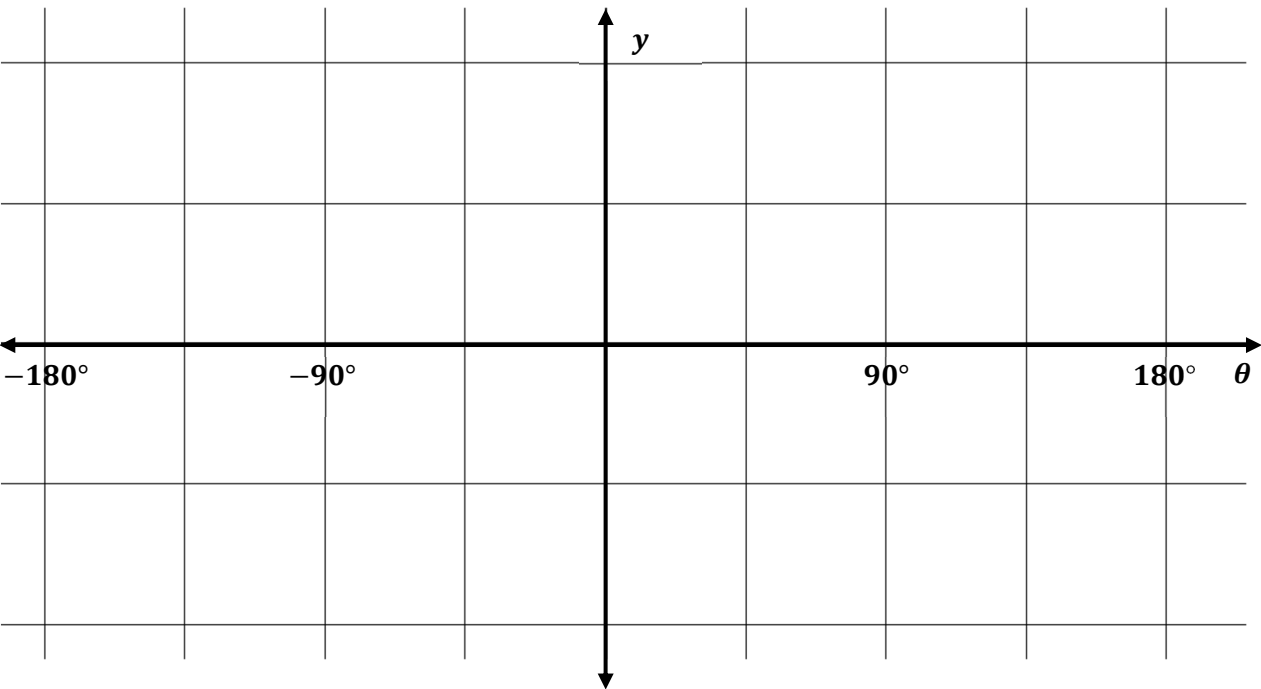
(b) Find the exact value of  $\tan 330^\circ$

2

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Question 19

Sketch the graph of  $y = 2 \sin \theta$  over the domain  $\theta \in [-180^\circ, 180^\circ]$  2



Question 20

Determine the value(s) of  $\theta$  for  $2 \cos \theta + 1 = 0$  over the domain  $\theta \in [0, 4\pi]$  3

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Question 21

The probability distribution of a random variable  $X$  is shown below

2

|            |     |     |     |     |     |
|------------|-----|-----|-----|-----|-----|
| $x$        | -2  | -1  | 0   | 1   | 2   |
| $P(X = x)$ | 0.1 | $a$ | 0.2 | 0.2 | $a$ |

Calculate the value of  $a$ .

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Question 22

Jasmine is planting flowers in her garden. The probability that an individual flower survives is 0.35. Jasmine plants 5 flowers.

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What is the probability that at least one flower survives? Answer correct to two decimal places.

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Question 23

In a game of chance, Mr Mathface tosses 2 coins. He wins \$10 for 2 heads, \$5 for 2 tails and nothing for a head and a tail.

- (a) Use a tree diagram to complete the probability distribution table below for the game of chance.
- 2

|            |     |     |      |
|------------|-----|-----|------|
| x          | \$0 | \$5 | \$10 |
| $P(X = x)$ |     |     |      |

- (b) Determine the expected value of this game
- 1

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- (c) If the game costs \$4 to play, would Mr Mathface expect to win or lose money in the long term? Justify your answer with suitable calculations.
- 2

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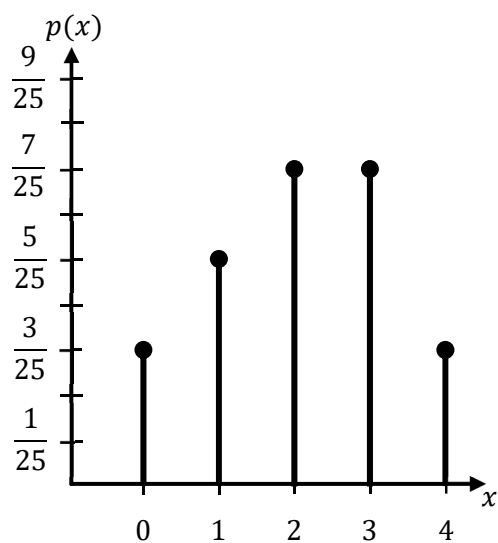
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## Question 24

The probability distribution of a discrete random variable  $X$  is shown below.

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Given that  $E(X) = 2.08$  calculate the variance,  $Var(X)$ .

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Question 25

Four identical balls numbered 1, 2, 3 and 4 are placed into a box.  
A ball is randomly drawn from the box and not returned to the box.  
A second ball is then randomly drawn from the box.

- (a) What is the probability that the first ball drawn is numbered 4 and the second ball drawn is numbered 1? **1**

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- (b) What is the probability that the sum of the two numbers on the two balls is 5? **1**

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- (c) Given that the sum on the two balls is 5, what is the probability that the second ball drawn is a number 1? **1**

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Question 26

5

The displacement of a particle moving in a straight line is given by:

$$x = 9t^2 - t^4$$

where  $x$  is the displacement from the origin in metres and  $t$  is the time in seconds, for  $t > 0$ .

Sketch the displacement time graph and then determine the total distance travelled in the first 3 seconds.

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End of Paper

## Section II Extra writing space

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**If you use this space, clearly indicate which question you are answering.**

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Student No.

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Name: \_\_\_\_\_

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2020

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## Section I

10 marks

Attempt Questions 1–10

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Use the multiple-choice answer sheet for Questions 1–10.

1. What is the value of  $f(2a + 1)$  if  $f(x) = 3x - 2$

A.  $6a - 2$

B.  $6a - 1$

C.  $6a + 1$

D.  $6a + 2$

$$\begin{aligned} f(2a+1) &= 3(2a+1) - 2 \\ &= 6a + 3 - 2 \\ &= 6a + 1 \end{aligned}$$

2. The line  $5x - y - 4 = 0$  passes through the point  $(k, 26)$ . What is the value of  $k$ ?

A. 3

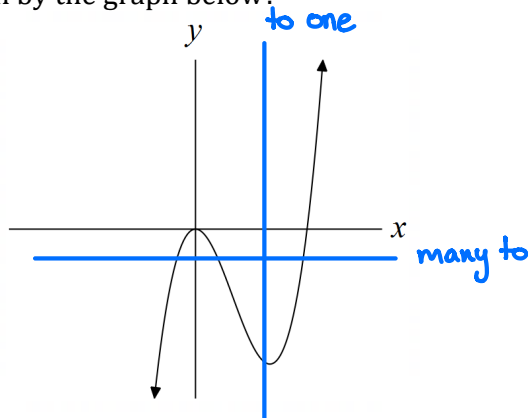
B. 4

C. 5

D. 6

$$\begin{aligned} 5(k) - 26 - 4 &= 0 \\ 5k &= 30 \\ k &= 6 \end{aligned}$$

3. What type of relation is shown by the graph below?



A. One - to - One

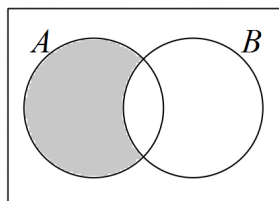
B. One - to - Many

C. Many - to - One

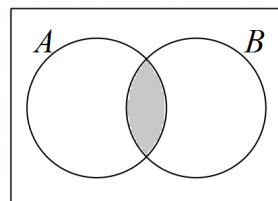
D. Many - to - Many

4. Which of the Venn diagrams shows  $A \cap B'$  shaded?

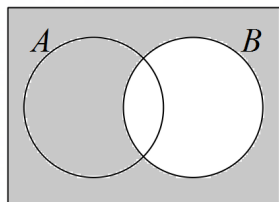
A.



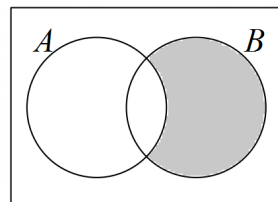
B.



C.



D.



5. The probability distribution for the random variable  $X$  is shown below.

|            |                |               |                |               |                |               |                |
|------------|----------------|---------------|----------------|---------------|----------------|---------------|----------------|
| x          | 2              | 3             | 4              | 5             | 6              | 7             | 8              |
| $P(X = x)$ | $\frac{1}{16}$ | $\frac{1}{8}$ | $\frac{3}{16}$ | $\frac{1}{4}$ | $\frac{3}{16}$ | $\frac{1}{8}$ | $\frac{1}{16}$ |

What is  $P(3 < X \leq 7)$ ?  $P(3 < X \leq 7) = \frac{3}{16} + \frac{1}{4} + \frac{3}{16} + \frac{1}{8}$

$$= \frac{3}{4}$$

A.  $\frac{1}{2}$

B.  $\frac{5}{8}$

C.  $\frac{3}{4}$

D.  $\frac{7}{8}$

6. Solve  $|x - 3| - 8 = 0$

A.  $x = 11$

B.  $x = 5$  or  $x = -11$

C.  $x = -5$  or  $x = 11$

D.  $x = 5$  or  $x = 11$

$$(x-3) - 8 = 0 \quad \text{or} \quad -(x-3) - 8 = 0$$

$$x - 11 = 0 \quad \text{or} \quad -x - 5 = 0$$

$$x = 11 \quad \text{or} \quad x = -5$$

7. Which of the following is true for the quadratic equation  $3x^2 + x + 4 = 0$ ?

- A. The roots are rational  
B. There are no real roots  
C. The roots are equal  
D. The roots are irrational

$$\begin{aligned}\Delta &= (1)^2 - 4(3)(4) \\ &= 1 - 48 \\ &= -47.\end{aligned}$$

Since  $\Delta < 0$ , no real roots

8. A sector has an angle of  $\frac{\pi}{3}$  subtended at the centre of a circle. The radius is 6 cm.

The exact area of the sector is?

- A.  $\pi \text{ cm}^2$   
B.  $3\pi \text{ cm}^2$   
C.  $6\pi \text{ cm}^2$   
D.  $12\pi \text{ cm}^2$

$$\begin{aligned}A &= \frac{1}{2}r^2\theta \\ &= \frac{1}{2} \times 6^2 \times \frac{\pi}{3} \\ &= 18 \times \frac{\pi}{3} \\ &= 6\pi \text{ cm}^2\end{aligned}$$

9. A function is defined as  $g(x) = 12x^3 - 8x + 14$

Evaluate  $g'(-2)$ .

- A. -152  
B. -66  
C. 136  
D. 150

$$\begin{aligned}g'(x) &= 36x^2 - 8 \\ g'(-2) &= 36(-2)^2 - 8 \\ &= 136\end{aligned}$$

10. The variance of a discrete random variable  $X$  is  $\text{Var}(X) = 5.01$

Calculate  $\text{Var}(5X - 2)$

- A. 25.05  
B. 26.05  
C. 125.25  
D. 126.25

$$\begin{aligned}\text{Change in scale by } 5^2 \\ \text{Var}(5x-2) &= 5^2 \text{Var}(X) \\ &= 5^2(5.01) \\ &= 125.25\end{aligned}$$

End of Section I



## Section II

55 marks

Attempt Questions 11 – 26

Allow about 1 hour and 45 minutes for this section

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
  - Your responses should include relevant mathematical reasoning and/or calculations.
  - Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.
- 

### Question 11

- (a) Express  $(2\sqrt{3} + 1)(2 - \sqrt{3})$  in the form of  $a + b\sqrt{3}$  where  $a$  and  $b$  are integers. 2

$$\begin{aligned}(2\sqrt{3} + 1)(2 - \sqrt{3}) &= 4\sqrt{3} - 2(3) + 2 - \sqrt{3} \\ &= 3\sqrt{3} - 4 \\ \therefore &-4 + 3\sqrt{3}\end{aligned}$$

- (b) Show algebraically that  $f(x) = \frac{2x}{x^2 + 3}$  is an odd function. 2

$$\begin{aligned}\text{Odd functions } f(-x) &= -f(x) \\ f(-x) &= \frac{2(-x)}{(-x)^2 + 3} \\ &= \frac{-2x}{x^2 + 3} \\ &= -\left(\frac{2x}{x^2 + 3}\right) \\ &= -f(x) \\ \therefore f(x) &\text{ is an odd function.}\end{aligned}$$

(c) Rewrite the following expression without any negative or fractional indices.

2

$$\left( \frac{12x^{-3}y^2}{xyz^{\frac{1}{2}}} \right)^2$$

$$\begin{aligned} \left( \frac{12x^{-3}y^2}{xyz^{\frac{1}{2}}} \right)^2 &= \frac{144x^{-6}y^4}{x^2y^2z} \\ &= \frac{144y^2}{x^8z} \end{aligned}$$

## Question 12

If  $f(x) = x^2$  and  $g(x) = 2x + 1$

(a) Find the composite function  $f(g(x))$

1

$$\begin{aligned} f(g(x)) &= (2x+1)^2 \\ &= 4x^2 + 4x + 1 \end{aligned}$$

(b) What is the domain and range of the composite function  $f(g(x))$ ?

2

domain:  $x \in (-\infty, \infty)$

domain: all real  $x$

range:  $y \in [0, \infty)$

range:  $y \geq 0$

Don't need to show this for full marks, but handy to help find the range.

axis of Symmetry =  $-\frac{b}{2a}$   
 $= \frac{-4}{2(4)}$   
 $= -0.5$

vertex:  $f(g(-0.5)) = (2(-0.5)+1)^2$   
 $= 0$   
 $\therefore \text{vertex} = (-0.5, 0)$

## Question 13

Differentiate  $f(x) = 2x^2 - x + 3$  by first principals.

3

$$\begin{aligned}f(x+h) &= 2(x+h)^2 - (x+h) + 3 \\&= 2x^2 + 4xh + 2h^2 - x - h + 3\end{aligned}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{2x^2} + 4xh + 2h^2 - \cancel{x} - h + \cancel{3} - (\cancel{2x^2} - \cancel{x} + \cancel{3})}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2h^2 + 4xh - h}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{h}(2h + 4x - 1)}{\cancel{h}}$$

$$= \lim_{h \rightarrow 0} 2h + 4x - 1$$

$$= 2(0) + 4x - 1$$

$$f'(x) = 4x - 1$$

### Question 14

Differentiate, with respect to  $x$ :

(a)  $4x^{-3}$

1

$$\frac{d}{dx}(4x^{-3}) = -12x^{-4}$$

(b)  $y = (5 + 2x^2)^{\frac{1}{2}}$

2

$$u = 5 + 2x^2 \quad y = u^{\frac{1}{2}}$$

$$u' = 4x \quad y' = \frac{1}{2}u^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = 4x \times \frac{1}{2} (5 + 2x^2)^{-\frac{1}{2}}$$

$$= \frac{2x}{\sqrt{5 + 2x^2}}$$

### Question 15

A circle is given by the equation  $x^2 + y^2 + 6x - 12y = 19$

3

What is the centre and radius of this circle?

$$x^2 + 6x + \underline{\quad} + y^2 - 12y + \underline{\quad} = 19$$

$$\quad \quad \quad +9 \quad \quad \quad +36 \quad \quad \quad +9 \quad +36$$

$$x^2 + 6x + 9 + y^2 - 12y + 36 = 64$$

$$(x+3)^2 + (y-6)^2 = 8^2$$

$\therefore$  Centre:  $(-3, 6)$

radius: 8 units.

Question 16

- (a) If  $f(x) = \frac{x^2-4}{2x+5}$  show that  $f'(x) = \frac{2(x+4)(x+1)}{(2x+5)^2}$   $u = x^2-4$   $v = 2x+5$   
 $u' = 2x$   $v' = 2$

3

$$f'(x) = \frac{u'v - v'u}{v^2}$$

$$= \frac{2x(2x+5) - 2(x^2-4)}{(2x+5)^2}$$

$$= \frac{4x^2 + 10x - 2x^2 + 8}{(2x+5)^2}$$

$$= \frac{2x^2 + 10x + 8}{(2x+5)^2}$$

$$= \frac{2(x^2 + 5x + 4)}{(2x+5)^2}$$

$$= \frac{2(x+1)(x+4)}{(2x+5)^2}$$

$$= f'(x) \text{ as required.}$$

- (b) Hence or otherwise find the equation of the tangent to  $f(x) = \frac{x^2-4}{2x+5}$  at the point  $(1, \frac{-3}{7})$ .  
 Give your answer in general form.

3

$$\text{Gradient of tangent: } f'(1) = \frac{2(1+1)(1+4)}{(2(1)+5)^2}$$

$$= \frac{2(2)(5)}{(7)^2}$$

$$= \frac{20}{49}$$

$$y - \left(\frac{-3}{7}\right) = \frac{20}{49}(x - 1)$$

$$y = \frac{20}{49}x - \frac{20}{49} - \frac{3}{7}$$

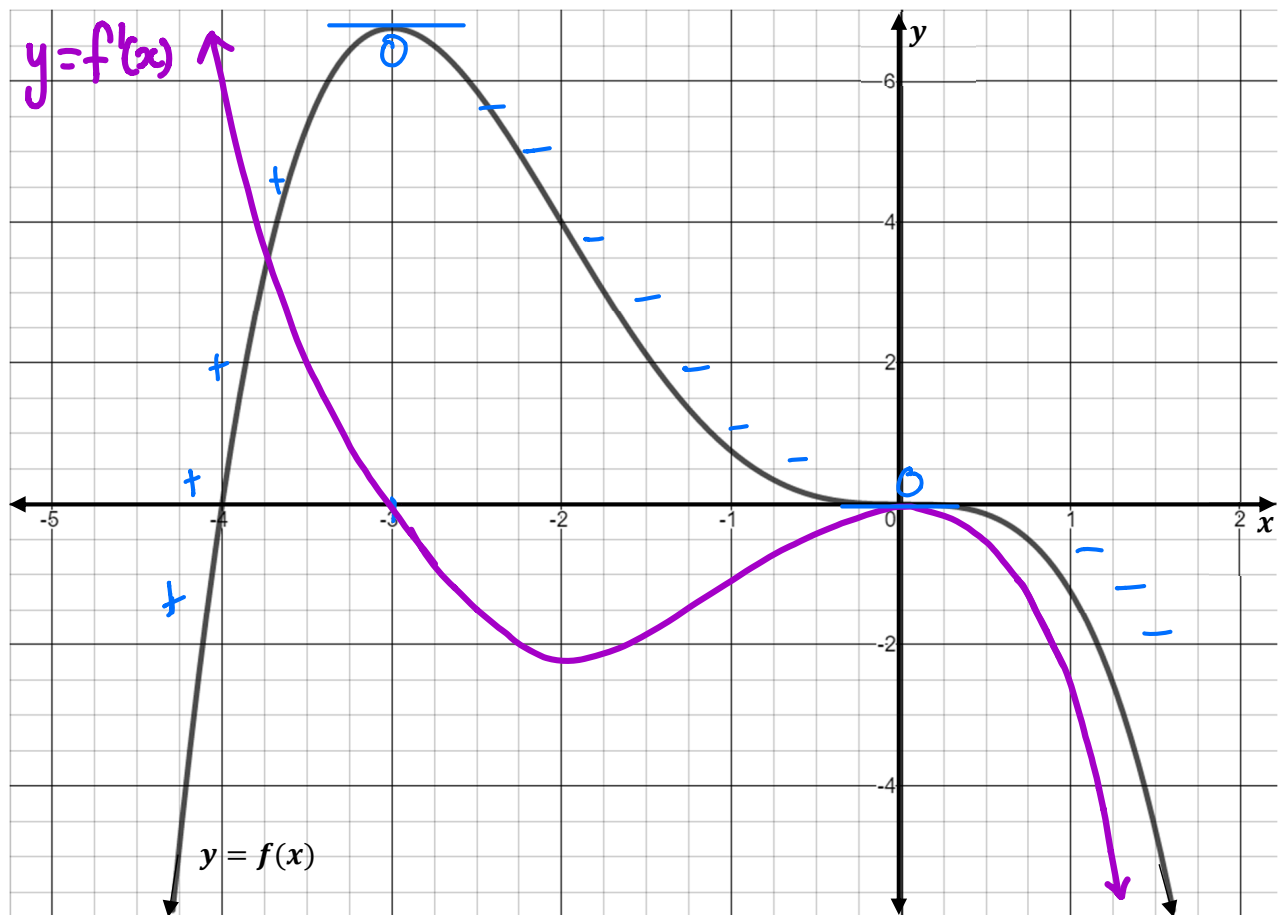
$$0 = \frac{20}{49}x - y - \frac{41}{49}$$

$$\therefore \text{equation of tangent is } 20x - 49y - 41 = 0$$

(c) The graph below shows the graph of  $y = f(x)$ .

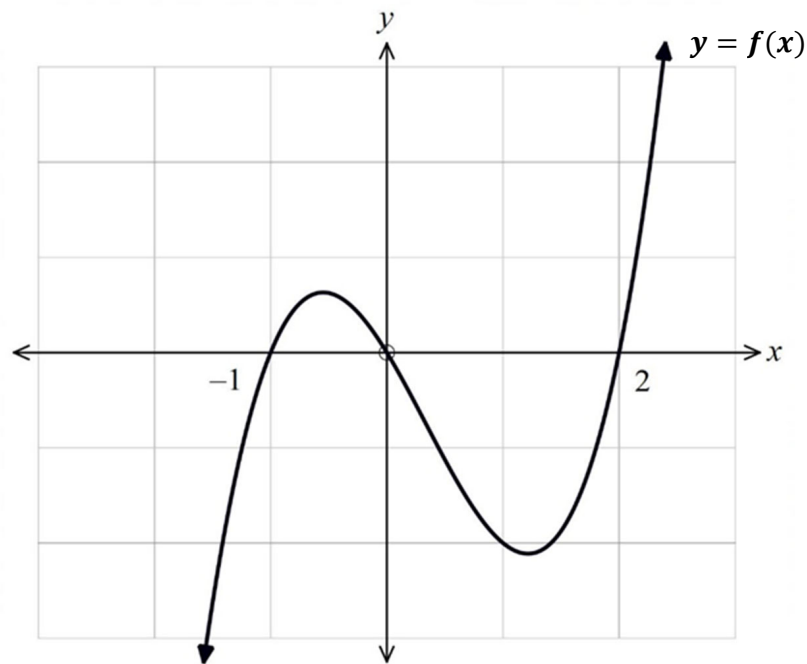
2

On the same set of axes sketch the gradient function.



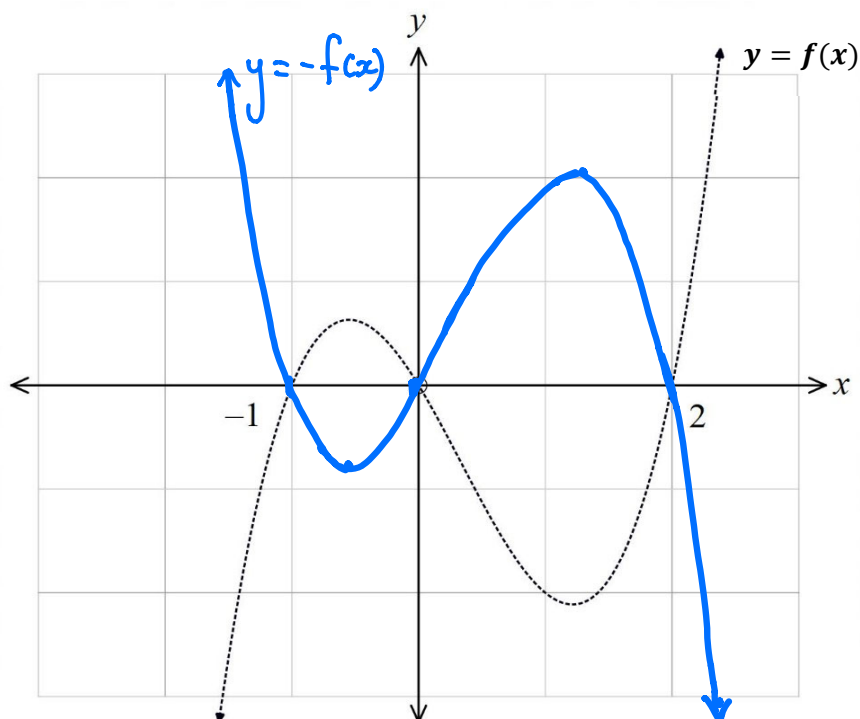
### Question 17

The sketch of  $y = f(x)$  is shown below.



(a) Use the graph of  $y = f(x)$  to draw a neat sketch of  $y = -f(x)$  on the diagram below.

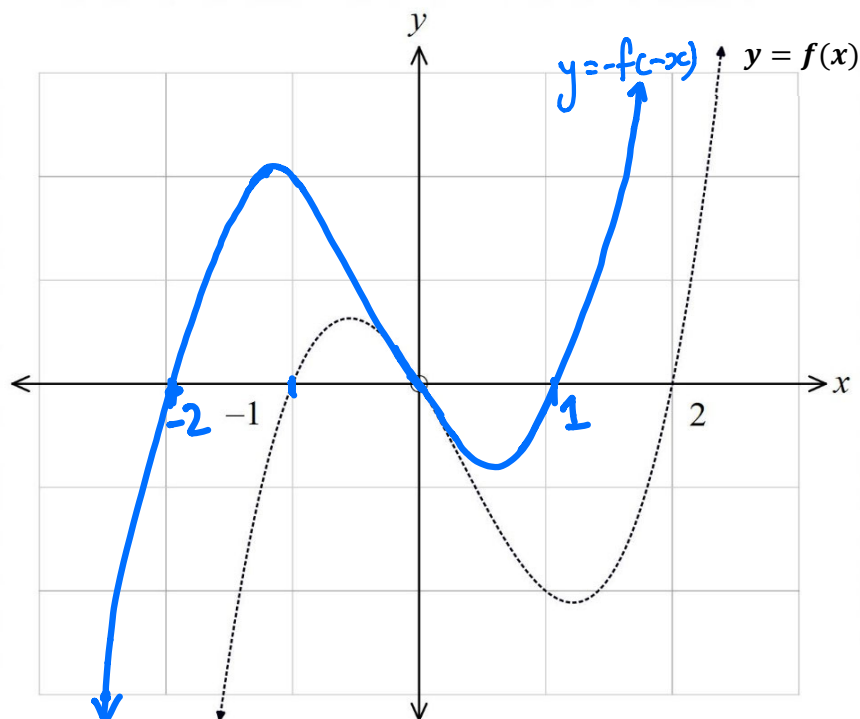
1



(b) Use the graph of  $y = f(x)$  to draw a neat sketch of  $y = -f(-x)$  on the diagram below.

1

Question amended to reflect correct use of notation



### Question 18

(a) Convert  $\frac{11\pi}{12}$  to degrees.

1

$$\frac{11\pi}{12} \times \frac{180}{\pi} = 165^\circ$$

(b) Find the exact value of  $\tan 330^\circ$

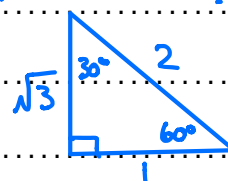
2

$$\begin{array}{c|c} s & A \\ \hline r & c \end{array}$$

$330^\circ = 4^{\text{th}}$  quad, Related Angle:  
 $360^\circ - \theta = 330^\circ$   
 $\therefore \theta = 30^\circ$

$$\tan 330^\circ = -\tan 30^\circ$$

$$= -\frac{1}{\sqrt{3}}$$

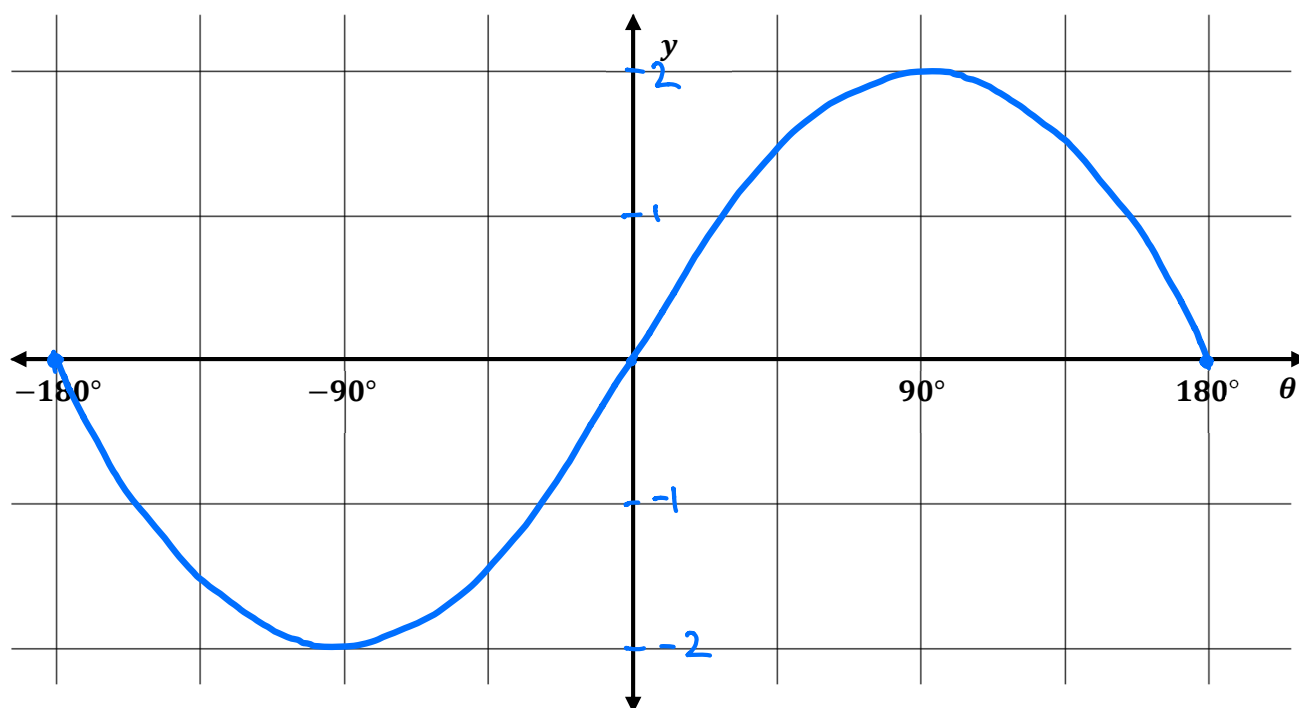




### Question 19

Sketch the graph of  $y = 2 \sin \theta$  over the domain  $\theta \in [-180^\circ, 180^\circ]$

2



### Question 20

Determine the value(s) of  $\theta$  for  $2 \cos \theta + 1 = 0$  over the domain  $\theta \in [0, 4\pi]$

3

$$2 \cos \theta + 1 = 0$$

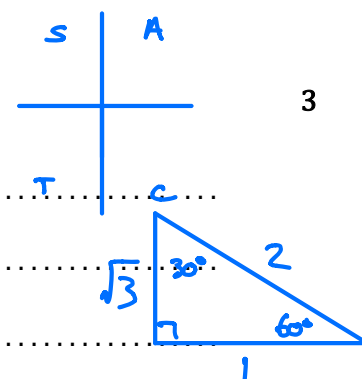
Related Angle:

$$\cos \theta = -\frac{1}{2}$$

$$\cos \theta = \frac{1}{2}, \text{ when } \theta = 60^\circ = \frac{\pi}{3}$$

$\theta$  is in Q2,  $\pi - \theta$

Q3,  $\pi + \theta$



$$\therefore \theta = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3} \text{ and } \frac{10\pi}{3}$$

$$\frac{2\pi}{3} + \frac{6\pi}{3} = \frac{8\pi}{3}$$

$$\frac{4\pi}{3} + \frac{6\pi}{3} = \frac{10\pi}{3}$$

### Question 21

The probability distribution of a random variable  $X$  is shown below

2

|          |     |     |     |     |     |
|----------|-----|-----|-----|-----|-----|
| $x$      | -2  | -1  | 0   | 1   | 2   |
| $P(X=x)$ | 0.1 | $a$ | 0.2 | 0.2 | $a$ |

Calculate the value of  $a$ .

$$0.1 + a + 0.2 + 0.2 + a = 1$$

$$0.5 + 2a = 1$$

$$2a = 0.5$$

$$a = 0.25$$

### Question 22

Jasmine is planting flowers in her garden. The probability that an individual flower survives is 0.35. Jasmine plants 5 flowers.

2

What is the probability that at least one flower survives? Answer correct to two decimal places.

$$P(\text{at least 1 survives}) = 1 - P(\text{No survive})$$

$$= 1 - 0.65^5$$

$$= 0.883...$$

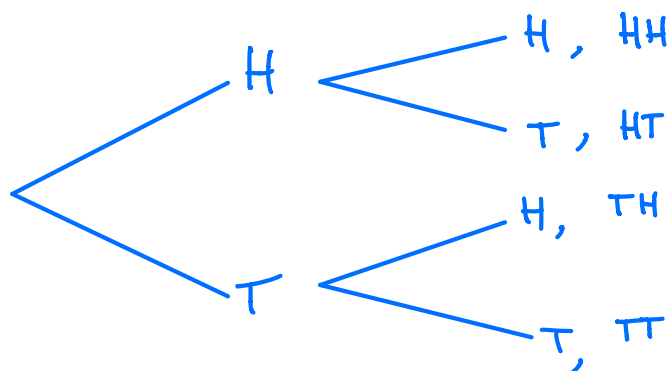
$$= 0.88$$

### Question 23

In a game of chance, Mr Mathface tosses 2 coins. He wins \$10 for 2 heads, \$5 for 2 tails and nothing for a head and a tail.

- (a) Use a tree diagram to complete the probability distribution table below for the game of chance.

2



|          |               |               |               |
|----------|---------------|---------------|---------------|
| x        | \$0           | \$5           | \$10          |
| P(X = x) | $\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{1}{4}$ |

- (b) Determine the expected value of this game

1

$$E(x) = \left(\frac{1}{2} \times 0\right) + \left(\frac{1}{4} \times 5\right) + \left(\frac{1}{4} \times 10\right)$$

$$= \$3.75$$

- (c) If the game costs \$4 to play, would Mr Mathface expect to win or lose money in the long term? Justify your answer with suitable calculations.

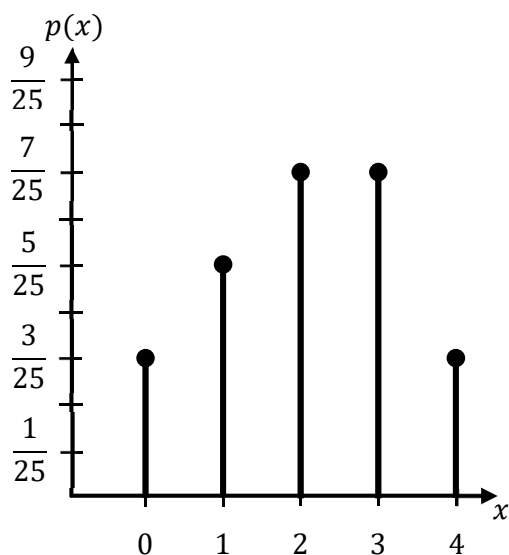
2

Since  $E(x)$ ,  $3.75 < 4$ , Mr Mathface will expect to lose money in the long term.

## Question 24

The probability distribution of a discrete random variable  $X$  is shown.

2



Given that  $E(X) = 2.08$  calculate the  $\text{Var}(X)$

$$\text{Var}(X) = \frac{3}{25}(0 - 2.08)^2 + \frac{5}{25}(1 - 2.08)^2 + \frac{7}{25}(2 - 2.08)^2 + \frac{7}{25}(3 - 2.08)^2 + \frac{3}{25}(4 - 2.08)^2$$

$$= 1.4336$$

OR

$$\text{Var}(X) = E(X^2) - E(X)^2$$

$$= \left[ (0^2 \times \frac{3}{25}) + (1^2 \times \frac{5}{25}) + (2^2 \times \frac{7}{25}) + (3^2 \times \frac{7}{25}) + (4^2 \times \frac{3}{25}) \right] - 2.08^2$$

$$= 1.4336$$

## Question 25

Four identical balls numbered 1, 2, 3 and 4 are placed into a box.

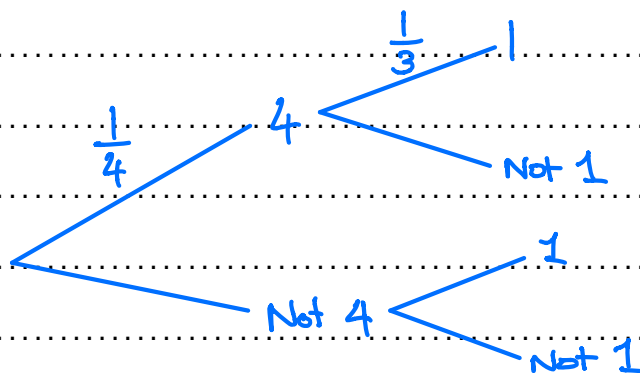
A ball is randomly drawn from the box and not returned to the box.

A second ball is then randomly drawn from the box.

- (a) What is the probability that the first ball drawn is numbered 4 and the second ball drawn is numbered 1? 1

$$P(4 \text{ then } 1) = \frac{1}{4} \times \frac{1}{3}$$

$$= \frac{1}{12}$$



- (b) What is the probability that the sum of the two numbers is 5? 1

$$P(\text{sum of } 5) = \frac{4}{12}$$

$$= \frac{1}{3}$$

1st

|   |              |              |              |              |
|---|--------------|--------------|--------------|--------------|
|   |              |              | 2nd          |              |
|   | 1            | 2            | 3            | 4            |
| 1 | <del>2</del> | 3            | 4            | 5            |
| 2 | 3            | <del>4</del> | 5            | 6            |
| 3 | 4            | 5            | <del>6</del> | 7            |
| 4 | 5            | 6            | 7            | <del>8</del> |

- (c) Given that the sum on the two balls is 5, what is the probability that the second ball drawn is numbered 1? 1

$$P(2^{\text{nd}} \text{ ball is } 1 \mid \text{sum } 5) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{\frac{1}{12}}{\frac{1}{3}}$$

$$= \frac{1}{4}$$

The displacement of a particle moving in a straight line is given by

$$x = 9t^2 - t^4$$

where  $x$  is the displacement from the origin in metres and  $t$  is the time in seconds, for  $t > 0$ .

Sketch the displacement time graph and then determine the total distance travelled in the first 3 seconds.

Leading Coefficient = -1, polynomial degree 4.

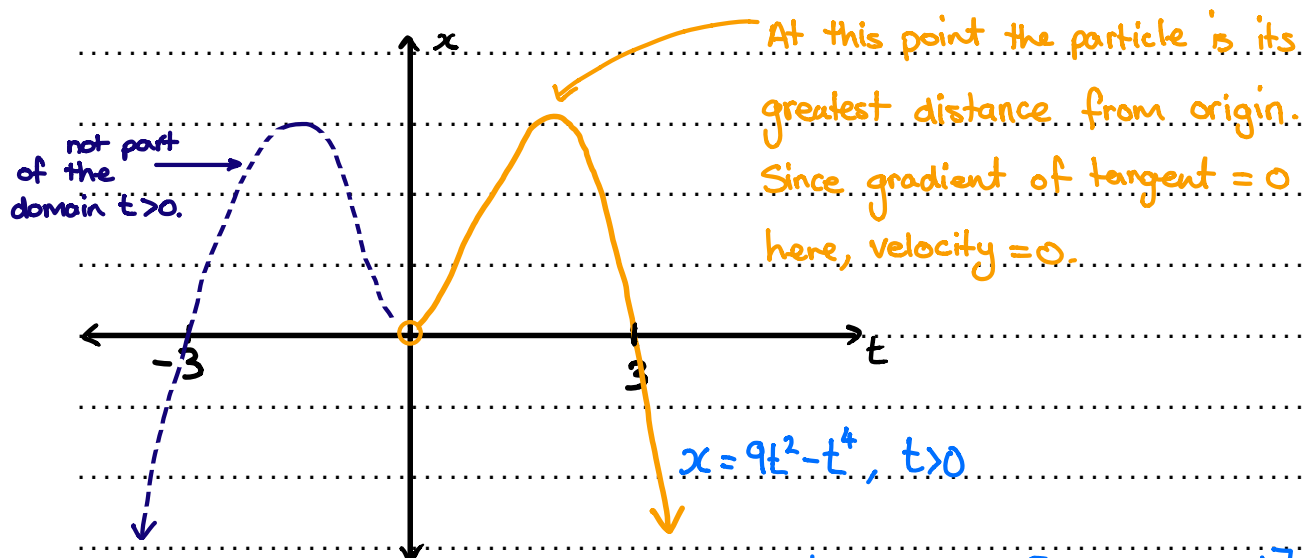
$$x = t^2(9 - t^2)$$

$$= t^2(3+t)(3-t)$$

double = bounce on x-axis

$\therefore x$  intercepts are  $t = 0, t = 0, t = -3, t = 3$ , domain:  $t > 0$

Vertical axis intercept = 0.



$$\frac{dx}{dt} = 18t - 4t^3$$

$$\frac{dx}{dt} = 0, \quad 0 = 2t(9 - 2t^2)$$

$$t = 0 \text{ or } 9 - 2t^2 = 0$$

$$t = \pm \sqrt{\frac{9}{2}}$$

$$= \pm \frac{3}{\sqrt{2}}$$

$$\text{Since } t > 0, \quad t = \frac{3}{\sqrt{2}}$$

$$\text{Distance} = 2 \left[ 9 \left( \frac{3}{\sqrt{2}} \right)^2 - \left( \frac{3}{\sqrt{2}} \right)^4 \right]$$

$$= 2 \left( \frac{81}{2} - \frac{81}{4} \right)$$

$$= \frac{81}{2} \text{ m}$$

$$= 40.5 \text{ m}$$

End of Paper

## Section II Extra writing space

**If you use this space, clearly indicate which question you are answering.**

[illegible]

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**If you use this space, clearly indicate which question you are answering.**

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