Name: ....

Teacher: ....

Fort Street High School

2019

Preliminary HSC Assessment Task 3

Mathematics Advanced

Outcomes Assessed							
MA11-1	MA11-2	MA11-3	MA11-4	MA11-5	MA11-6	MA11-7	MA11-9

General Instructions:

- Marks may be deducted for careless or badly arranged work
- NESA-approved calculators may be used
- A Reference Sheet is provided with this paper
- Use a separate booklet for each question

Time allowed: 2 hours (+ 5 minutes reading time)

Question	Mark
1	/12
2	/12
3	/12
4	/12
5	/12
6	/12
7	/12
8	/12
Total	/96
	%

Question 1 (12 marks) *Start a new booklet*

(a) Express $\dfrac{2}{\sqrt{6}}$ with a rational denominator in simplest form. **1**

(b) Simplify $\dfrac{p^2q^{-3}}{p^{-1}q^2}$ **1**

(c) Two lines ℓ_1 and ℓ_2 have equations $y = 2x + 1$ and $2y + x + 8 = 0$ respectively.

i) Show that ℓ_1 is perpendicular to ℓ_2. **2**

ii) Find the point at which ℓ_1 and ℓ_2 intersect. **2**

(d) Solve $|3x - 1| = 5$ **2**

(e) Simplify fully $\dfrac{6x+18}{9(x^2+4x+4)} \times \dfrac{x^2-4}{x+3}$ **2**

(f) Find the equation of the tangent to the curve $y = 5x^3 - 4$ at the point $(1,1)$. **2**

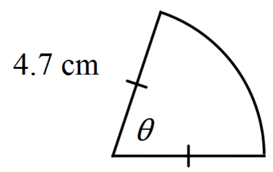
Question 2 (12 marks) *Start a new booklet*

(a) Given $g(x) = 2e^{3x}$ and $h(x) = x^2$, find:

i) $g(x) \times h(x)$ **1**

ii) $h\left(g(x)\right)$ in simplest form **1**

(b) A sector has perimeter 15.2 cm and radius 4.7 cm. Find the size of the angle subtended by the arc. Give your answer in radians, correct to 2 decimal places. **2**



NOT TO SCALE

(c) Given that $\cos\theta = \dfrac{4}{5}$ and $\dfrac{3\pi}{2} \le \theta \le 2\pi$, find the exact value of $\sin\theta \sec\theta$ **2**

(d) One cube has two red faces, two blue faces and two green faces. Another cube has three red faces and three blue faces. If both cubes are rolled simultaneously, find the probability that the upper faces have:

i) two red faces **1**

ii) no red faces **2**

(e) Solve for x: $4^x - 9 \times 2^x + 8 = 0$ **3**

Question 3 (12 marks) *Start a new booklet*

(a) Find the exact value of:

i) $\cos\frac{5\pi}{4}$ **1**

ii) $\cot\frac{17\pi}{6}$ **2**

(b) Consider the function $f(x) = 3e^{x-2}$

i) Sketch $y = f(x)$, labelling any intercepts, asymptotes and other key features. **2**

ii) State the domain and range of $f(x)$ **2**

(c) Two identical biased coins are tossed at the same time. The probability that both coins land on heads is 0.16. Find the probability that:

i) at least one coin lands on tails **1**

ii) both coins land on tails **2**

(d) Prove that $\frac{\sec^4 x - \tan^4 x}{\sec^2 x + \tan^2 x} = 1$ **2**

Question 4 (12 marks) *Start a new booklet*

(a) Differentiate the following with respect to x:

i) $y=\left(x\sqrt{x}+1\right)^3$ **2**

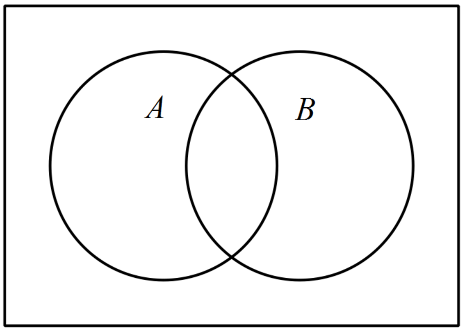
ii) $y=\dfrac{x^2-3}{2x}$ **2**

iii) $y=5xe^{2x}$ **2**

(b) Show that $f(x)=2xe^{x^2}$ is an odd function. **2**

(c) Let A be the set of letters in the word SENIOR and B be the set of letters in the word FOCUS. Sharon randomly selects a letter from the English alphabet.

i) Copy and complete the Venn diagram to represent the *number* of elements. **2**



ii) Find the probability that Sharon chooses a letter belonging to the set:

α) $A\cup B$ **1**

β) $A\cap B'$ **1**

Question 5 (12 marks) *Start a new booklet*

(a) The point $P(-1,-3)$ lies on the curve $y = 2x^2 + 3x - 2$. The normal to the curve at P intersects the curve again at point Q.

i) Show that the equation of the normal at P is $x - y - 2 = 0$. **2**

ii) Find the coordinates of Q. **2**

(b) Solve for $0 \le x \le 2\pi$:

i) $\sin x = \dfrac{1}{\sqrt{2}}$ **2**

ii) $\tan x = 2\sin x$ **3**

(c) i) Neatly sketch $y = |x-2|$ and $y = |x+1|$ on the same set of axes. **2**

ii) Hence, or otherwise, solve $|x-2| = |x+1|$ **1**

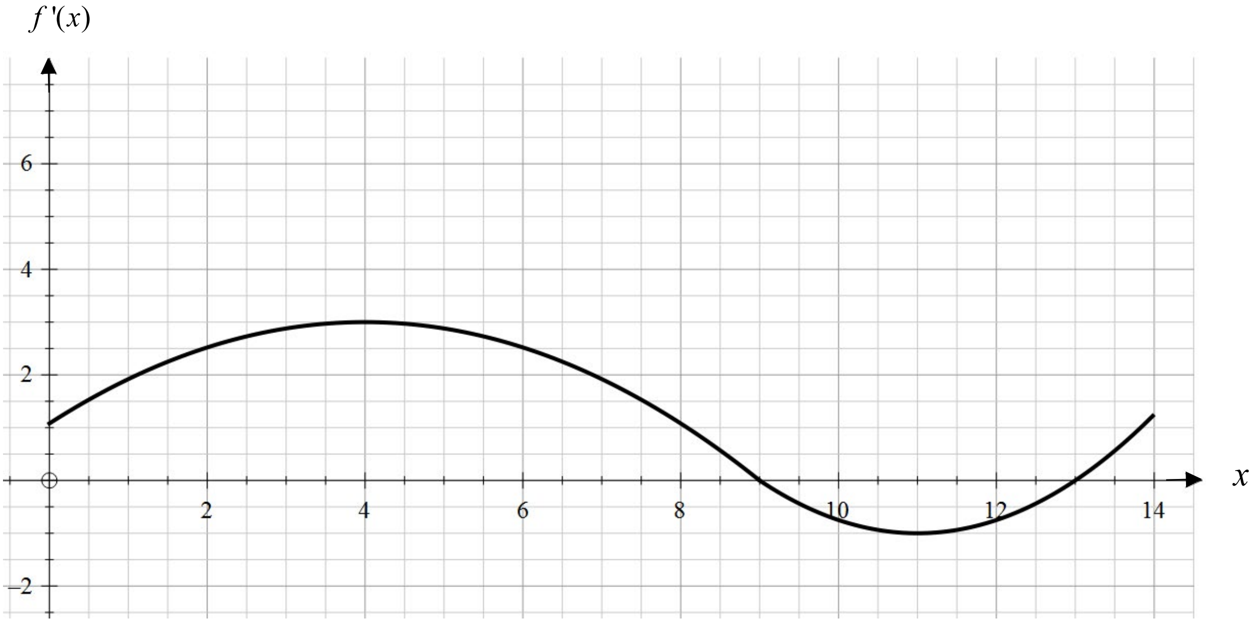
Question 6 (12 marks) *Start a new booklet*

(a) Given the gradient function $f'(x)$ below, state the values of x in the domain $[0,14]$ for which $f(x)$ is:

i) increasing **1**

ii) decreasing **1**

iii) stationary **1**

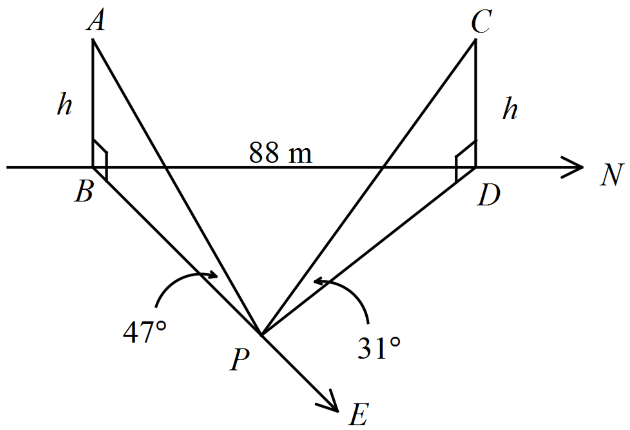


(b) Using the graph of $f'(x)$ above and your responses in part (a), draw a possible sketch of $y = f(x)$. **2**

Question 6 continues on the next page

(Question 6 continued)

(c) AB and CD are towers of equal height, h metres. CD is due north of AB. From a point P on the same horizontal plane as the feet B and D of the towers, and bearing due east of tower AB, the angles of elevation of A and C, the tops of the towers, are 47° and 31° respectively. The distance between the towers is 88 metres.



i) Show that $BP = h\tan 43°$ **1**

ii) Find the value of h to the nearest metre. **2**

(d) Jerry's distance from home, s km, after time t minutes is given by $s = t^3 - 4t^2 + 5t + 8$.

i) How far does Jerry travel in the first minute? **2**

ii) At what speed is Jerry travelling after 5 minutes? **2**

End of Question 6

Question 7 (12 marks) *Start a new booklet*

(a) Differentiate $f(x) = x^2 - 3x$ from first principles. **2**

(b) Consider the function $f(x) = \begin{cases} (x+1)^3 & x > -2 \\ -1 & x \leq -2 \end{cases}$

i) Sketch $y = f(x)$ **2**

ii) Find the values of x for which $f(x)$ is:

α) continuous **1**

β) differentiable **1**

(c) The population of salmon, P, at time t days is given by $P = -800e^{-0.005t} + 1200$.

i) Determine the initial population. **1**

ii) Find $\dfrac{dP}{dt}$ **1**

iii) Explain why the population is always increasing. **1**

iv) What number is the population approaching? **1**

v) Sketch the graph of the population of salmon over time. **2**

Question 8 (12 marks) *Start a new booklet*

(a) i) Show that $\dfrac{2x^2+x-1}{x^2-1}=2+\dfrac{1}{x-1}$ **2**

ii) Sketch a neat graph of $y=\dfrac{2x^2+x-1}{x^2-1}$ labelling any intercepts, discontinuities and other key features. **3**

(b) The probability of a cure with Drug A is 0.8 and with Drug B is 0.6.

One randomly selected patient is treated with Drug A and another with Drug B.

Find the probability that:

i) both patients are cured **1**

ii) neither patient is cured **1**

iii) the patient treated with Drug B is cured, given that only one of the selected patients is cured **2**

(c) The line $y=mx$ is tangent to the curve $y=e^{2x}$ at a point *T*.

Find the coordinates of *T*. **3**

End of Examination

Name: ....

Teacher: ....

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Preliminary HSC Assessment Task 3

Mathematics Advanced

Outcomes Assessed							
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SOLUTIONS

Question	Mark
1	/12
2	/12
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4	/12
5	/12
6	/12
7	/12
8	/12
Total	/96
	%

Question 1 (12 marks) *Start a new booklet*

(a) Express $\dfrac{2}{\sqrt{6}}$ with a rational denominator in simplest form.

Solution

$$=\frac{2}{\sqrt{6}}\times\frac{\sqrt{6}}{\sqrt{6}}$$

$$=\frac{2\sqrt{6}}{6}$$

$$=\frac{\sqrt{6}}{3}$$

Marking guideline:

1 Correct response

Marker's comments:

Mostly done well.

(b) Simplify $\dfrac{p^2q^{-3}}{p^{-1}q^2}$

Solution

$$=\frac{p^3}{q^5}$$

Marking guideline:

1 Correct response

Marker's comments:

Mostly done well.

(c) Two lines ℓ_1 and ℓ_2 have equations $y=2x+1$ and $2y+x+8=0$ respectively.

i) Show that ℓ_1 is perpendicular to ℓ_2.

$\ell_1 : m_1 = 2$ and $\ell_2 : y=-\dfrac{1}{2}x-4$

$$m_2=-\frac{1}{2}$$

$$m_1\times m_2 = 2\times -\frac{1}{2}=-1$$

Therefore m_1 and m_2 are perpendicular.

Marking guideline:

2 Correct response

1 Partially correct response

Marker's comments:

Mostly done well.

ii) Find the point at which ℓ_1 and ℓ_2 intersect.

Solution

$y = 2x + 1$ (1) and $2y + x + 8 = 0$ (2)

(1) $\rightarrow$ (2)

$$\begin{aligned} 2(2x+1) + x + 8 &= 0 \\ 5x + 10 &= 0 \\ 5x &= -10 \\ x &= -2 \end{aligned}$$

$$\begin{aligned} &\textit{Sub}\; x = -2 \text{ into } (1) \\ &y = 2(-2) + 1 \\ &y = -3 \end{aligned}$$

Marking guideline:

2 Correct response

1 Partially correct response

Marker's comments:

Some students incorrectly found x, which made their value for y incorrect. One carry-on mark still given in this case.

(d) Solve $|3x - 1| = 5$

Solution

$$\begin{aligned} 3x - 1 &= 5 \\ 3x &= 6 \\ x &= 2 \end{aligned} \quad \textit{or} \quad \begin{aligned} -(3x-1) &= 5 \\ -3x + 1 &= 5 \\ -3x &= 4 \\ x &= -\frac{4}{3} \end{aligned}$$

Marking guideline:

2 Correct response

1 Partially correct response

Marker's comments:

Some students neglected find both possible values for x.

(e) Simplify fully $\dfrac{6x+18}{9(x^2+4x+4)} \times \dfrac{x^2-4}{x+3}$

Solution

$$\begin{aligned} &= \frac{6(x+3)}{9(x+2)^2} \times \frac{(x-2)(x+2)}{x+3} \\ &= \frac{6}{9(x+2)} \times \frac{x-2}{1} \\ &= \frac{2(x-2)}{3(x+2)} \end{aligned}$$

Marking guideline:

2 Correct response

1 Partially correct response

Marker's comments:

Some students expanded the quadratic term first instead of factorising. Doing this meant you were unable to find the correct terms to cancel. The other most common mistake was leaving the final expression in expanded form.

(f) Find the equation of the tangent to the curve $y = 5x^3 - 4$ at the point $(1,1)$.

Solution

$$y' = 15x^2$$
$$At\ x = 1,$$
$$y' = 15(1)^2 = 15$$

The equation of the line is therefore

$$y - y_1 = m(x - x_1)$$
$$y - 1 = 15(x - 1)$$
$$y = 15x - 14$$

Marking guideline:

2	Correct response
1	Correct gradient

Marker's comments:

Mostly done well, some students used $m = 5$ as their gradient of the line. The coefficient of x^3 is not the gradient of the line. This method only works for lines of the form $y = mx + b$.

Question 2 (12 marks) *Start a new booklet*

(a) Given $g(x)=2e^{3x}$ and $h(x)=x^2$, find:

i) $g(x)\times h(x)$

Solution

$g(x)\times h(x)=2x^2e^{3x}$

Marking guideline:

1 Correct response

Marker's comments:

Mostly well done. Some students tried to incorrectly use index laws to simplify the expression, attaining $2e^{3x+2}$

ii) $h\left(g(x)\right)$ in simplest form

$=(2e^{3x})^2$
$=4e^{6x}$

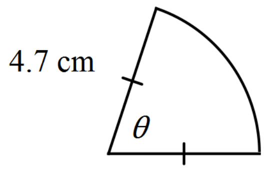
Marking guideline:

1 Correct response

Marker's comments:

Common errors included $2e^{6x}$ and $4e^{9x^2}$

(b) A sector has perimeter 15.2 cm and radius 4.7 cm. Find the size of the angle subtended by the arc. Give your answer in radians, correct to 2 decimal places. **2**



NOT TO SCALE

$$P=15.2=2r+\ell$$
$$15.2=2(4.7)+\ell$$
$$l=5.8$$

$$l=r\theta$$
$$\theta=\frac{l}{r}=\frac{5.8}{4.7}$$
$$\theta=1.23$$

Marking guideline:

2 Correct response

1 Correct arc length

Marker's comments:

Some students did not read the question properly, assuming the arc length was 15.2 cm. Many students did not realise the formula $\ell=r\theta$ on their reference sheet already has θ in radians (not degrees).

(c) Given that $\cos\theta = \frac{4}{5}$ and $\frac{3\pi}{2} \le \theta \le 2\pi$, find the exact value of $\sin\theta\sec\theta$

Solution

$$\sin\theta = -\frac{3}{5}$$

$$\sec\theta = \frac{5}{4}$$

$$\sin\theta\sec\theta = \left(-\frac{3}{5}\right)\left(\frac{5}{4}\right)$$

$$= -\frac{3}{4}$$

Marking guideline:

2 Correct response

1 Partially correct response

Marker's comments:

Mostly well done. Many students did not have the negative sign. Some students evaluated $\sin\theta$ and $\sec\theta$ separately but did not find $\sin\theta\sec\theta$ – very unfortunate.

(d) One cube has two red faces, two blue faces and two green faces. Another cube has three red faces and three blue faces. If both cubes are rolled simultaneously, find the probability that the upper faces have:

i) two red faces

Solution

$$P(RR) = \frac{1}{3} \times \frac{1}{2}$$

$$P(RR) = \frac{1}{6}$$

Marking guideline:

1 Correct response

Marker's comments:

Well done.

ii) no red faces

Solution

$$P\left(\overline{R}\overline{R}\right) = \left(1 - \frac{1}{3}\right)\left(\frac{1}{2}\right)$$

$$P\left(\overline{R}\overline{R}\right) = \frac{1}{3}$$

Marking guideline:

2 Correct response, fully simplified

1 Partially correct response

Marker's comments:

Poorly done. Most common error was $\frac{5}{6}$.

(e) Solve for x: $4^x - 9\times 2^x + 8 = 0$

Solution

$$\left(2^2\right)^x - 9\left(2^x\right) + 8 = 0$$
$$\left(2^x\right)^2 - 9\left(2^x\right) + 8 = 0$$

Let $u = 2^x$

$$(u-8)(u-1) = 0$$

$u = 8$ or $u = 1$

so $2^x = 8$ or $2^x = 1$

$x = 3$ or $x = 0$

Marking guideline:

3 Correct solution for x

2 Correct solutions for u

1 Correct substitution

Marker's comments:

Poorly done. Students clearly lacked basic understanding of index laws.

Question 3 (12 marks) *Start a new booklet*

(a) Find the exact value of:

i) $\cos\frac{5\pi}{4}$

Solution

$$\cos\frac{5\pi}{4} = -\cos\frac{\pi}{4}$$
$$= -\frac{1}{\sqrt{2}}$$

Marking guideline:

1 Correct response

Marker's comments:

Generally well done

ii) $\cot\frac{17\pi}{6}$

Solution

$$\cot\frac{17\pi}{6} = \cot\frac{5\pi}{6}$$
$$= -\cot\frac{\pi}{6}$$
$$= -\sqrt{3}$$

Marking guideline:

2 Correct response

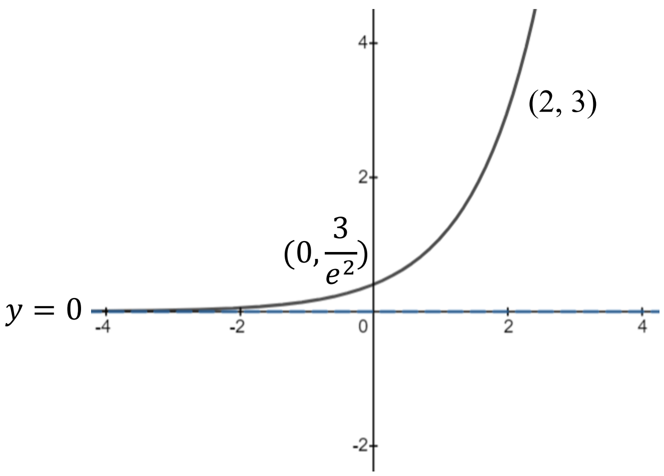
1 Correct reference angle

Marker's comments:

Poorly done. Many students forgot the negative sign, some had the incorrect magnitude for the exact value and some gave answers as angles in terms of pi.

(b) Consider the function $f(x) = 3e^{x-2}$

i) Sketch $y = f(x)$, labelling any intercepts, asymptotes and other key features. **2**



Marking guideline:

2 Correct response

1 Partially correct response

Marker's comments:

Terrible! Students did not indicate or draw the asymptote. Others only plotted one point on the curve which is bewildering because a line requires 2 points so why would only 1 point be sufficient for a curve? Many students labelled co-ordinates of points with decimal approximations rather than exact values.

ii) State the domain and range of $f(x)$ **2**

$$Domain: All\,real\,x$$
$$Range: y > 0$$

Marking guideline:

2 Correct response

1 Partially correct response

Marker's comments:

Well done

(c) Two identical biased coins are tossed at the same time. The probability that both coins land on heads is 0.16. Find the probability that:

i) at least one coin lands on tails

$$P(At\,least\,1\,tail) = 1 - P(HH)$$
$$= 0.84$$

Marking guideline:

1 Correct response

Marker's comments:

Generally well done. Many students who attempted to add the probabilities of TT, TH and HT forgot either HT or TH.

ii) both coins land on tails

$$p^2 = 0.16$$
$$p = 0.4 = P(H)$$
$$P(T) = 0.6$$
$$P(TT) = P(T) \times P(T)$$
$$= 0.6 \times 0.6$$
$$= 0.36$$

Marking guideline:

2 Correct response

1 Partially correct response

Marker's comments:

Well done.

(d) Prove that $\dfrac{\sec^4 x - \tan^4 x}{\sec^2 x + \tan^2 x} = 1$

$$LHS = \frac{\sec^4 x - \tan^4 x}{\sec^2 x + \tan^2 x}$$
$$= \frac{(\sec^2 x + \tan^2 x)(\sec^2 x - \tan^2 x)}{(\sec^2 x + \tan^2 x)}$$
$$= \sec^2 x - \tan^2 x$$
$$= \tan^2 x + 1 - \tan^2 x$$
$$= 1$$
$$= RHS$$

Marking guideline:

2 Correct response

1 Partially correct response

Marker's comments: Many students did not set out their proof correctly i.e.

$$LHS =$$
$$=$$
$$\vdots$$
$$= RHS$$

In proofs, students are reminded to never start from what they are asked to prove and attempt to manipulate both sides.

Others omitted steps and used incorrect version of the pythagorean identities like for example $1 - \sin^4 x = \cos^4 x$

Question 4 (12 marks) *Start a new booklet*

(a) Differentiate the following with respect to x:

i) $y=\left(x\sqrt{x}+1\right)^3$ **2**

Solution

$$y' = 3\left(x^{\frac{3}{2}}+1\right)^2 \times \frac{3}{2}x^{\frac{1}{2}}$$

$$= \frac{9\sqrt{x}}{2}\left(x\sqrt{x}+1\right)^2$$

Marking guideline:

2 Correct response

1 Partially correct response

Marker's comments:

A number of students didn't use the chain rule correctly and lost one mark.

ii) $y=\dfrac{x^2-3}{2x}$

Solution

$$y' = \frac{2x(2x)-2(x^2-3)}{4x^2}$$

$$= \frac{4x^2-2x^2+6}{4x^2}$$

$$= \frac{2x^2+6}{4x^2}$$

$$= \frac{x^2+3}{2x^2}$$

Marking guideline:

2 Correct response

1 Partially correct response

Marker's comments:

Poorly done. A number of students didn't use quotient rule properly and some didn't even use quotient rule.

iii) $y=5xe^{2x}$

Solution

$$y' = 5(e^{2x}) + 2(5x)e^{2x}$$

$$= 5e^{2x} + 10xe^{2x}$$

$$= 5e^{2x}(1+2x)$$

Marking guideline:

2 Correct response

1 Partially correct response

Marker's comments:

Not well done. Most students didn't know the differentiation of exponential functions.

(b) Show that $f(x) = 2xe^{x^2}$ is an odd function.

Solution

$f(x)$ is odd if $f(-x) = -f(x)$

$$f(-x) = 2(-x)e^{(-x)^2}$$

$$= -2xe^{x^2}$$

$$= -f(x)$$

Therefore $f(x)$ is an odd function.

Marking guideline:

2 Correct response

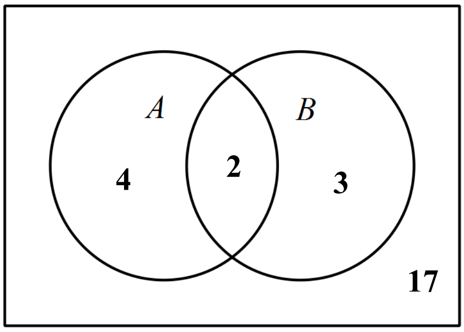
1 Partially correct response

Marker's comments:

Generally well done.

(c) Let A be the set of letters in the word SENIOR and B be the set of letters in the word FOCUS. Sharon randomly selects a letter from the English alphabet.

i) Copy and complete the Venn diagram to represent the *number* of elements. **2**



Marking guideline:

2 Correct response

1 Partially correct response

Marker's comments:

Not well done. The students showed the lack of knowledge in drawing Venn diagrams.

ii) Find the probability that Sharon chooses a letter belonging to the set:

α) $A \cup B$

Solution

$$P(A \cup B) = \frac{6}{26} + \frac{5}{26} - \frac{2}{26} = \frac{9}{26}$$

Marking guideline:

1 Correct response

Marker's comments:

Poorly done. Most students showed confusion between $A \cup B$ and $A \cap B$

β) $A \cap B'$

Solution

$$P(A \cap B') = \frac{4}{26} = \frac{2}{13}$$

Marking guideline:

1 Correct response

Marker's comments:

Not well done.

Question 5 (12 marks) *Start a new booklet*

(a) The point $P(-1,-3)$ lies on the curve $y = 2x^2 + 3x - 2$. The normal to the curve at P intersects the curve again at point Q.

i) Show that the equation of the normal at P is $x - y - 2 = 0$.

Solution

$y' = 4x + 3$

When $x = -1$,

$y' = 4(-1) + 3 = -1$

So the gradient of the normal is $m = 1$

Using the equation of a line $y - y_1 = m(x - x_1)$

$y + 3 = 1(x + 1)$

$x - y - 2 = 0$

Marking guideline:

2 Correct response

1 Partially correct response

Marker's comments:

Some students incorrectly stated that 4 in $y' = 4x + 3$ is the gradient of the normal.

ii) Find the coordinates of Q.

Solution

(1) $y = x - 2$
(2) $y = 2x^2 + 3x - 2$

$$(1) = (2)$$
$$x - 2 = 2x^2 + 3x - 2$$
$$0 = 2x^2 + 2x$$
$$0 = x(x + 1)$$
$$x = 0 \;\; or \;\; x = -1$$

So P intersects at $x = -1$ and Q intersects at $x = 0$

When $x = 0$,
$y = 0 - 2$
$y = -2$

Therefore Q intersects at (0, -2)

Marking guideline:

2 Correct response

1 Partially correct response

Marker's comments:

Mostly well done, some algebraic errors.

(b) Solve for $0 \le x \le 2\pi$:

i) $\sin x = \frac{1}{\sqrt{2}}$

Solution

$$x = \sin^{-1}\frac{1}{\sqrt{2}}$$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}$$

Marking guideline:

2 Correct response

1 Partially correct response

Marker's comments:

Answer must be in radian form to match the domain given. 1 mark lost for this.

ii) $\tan x = 2\sin x$

Solution

$$\frac{\sin x}{\cos x} = 2\sin x$$

$$\sin x \sec x = 2\sin x$$

$$\sin x \sec x - 2\sin x = 0$$

$$\sin x(\sec x - 2) = 0$$

$$\sin x = 0 \quad or \quad \sec x = 2$$

$$\cos x = \frac{1}{2}$$

$$x = 0,\ \pi,\ 2\pi \quad or \quad x = \frac{\pi}{3}, \frac{5\pi}{3}$$

Marking guideline:

3 Correct response

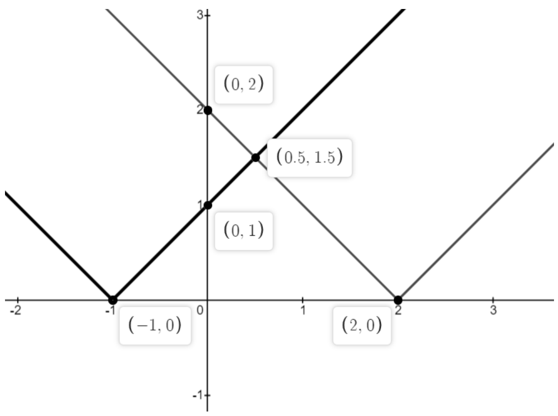
2 Partially correct response

1 Attempt to solve equation correctly

Marker's comments:

After writing $\sin x \sec x = 2\sin x$, some students divided by $\sin x$. This is an incorrect step as $\sin x$ may be equal to 0, additionally some answers are lost.

(c) i) Neatly sketch $y = |x-2|$ and $y = |x+1|$ on the same set of axes. **2**



Marking guideline:

2 Correct response

1 Gets shapes correct without labelling key features, or gets one graph correct.

Marker's comments:

Mostly well done, some incorrect graphs

ii) Hence, or otherwise, solve $|x-2|=|x+1|$

Solution

Graphically $x=\frac{1}{2}$

Marking guideline:

1 Correct response

Marker's comments:

Some students did not answer the question, which was to find the value of x. Stating only the point of intersection of the two graphs was not accepted.

Question 6 (12 marks) *Start a new booklet*

(a) Given the gradient function $f\,'(x)$ below, state the values of x in the domain $[0,14]$

for which $f(x)$ is:

i) Increasing

$0 \le x < 9,\ 13 < x \le 14$
or $[0,9) \cap (13,14]$

Marking guideline:

1 Correct response

Marker's comments:

Very poorly done. Students need to learn proper notation. i.e. $0 \le x < 9 \ \Rightarrow\ [0,9)$

ii) Decreasing

$9 < x < 13$
or $(9,13)$

Marking guideline:

1 Correct response

Marker's comments:

Error carried awarded from (i) for some students

iii) Stationary

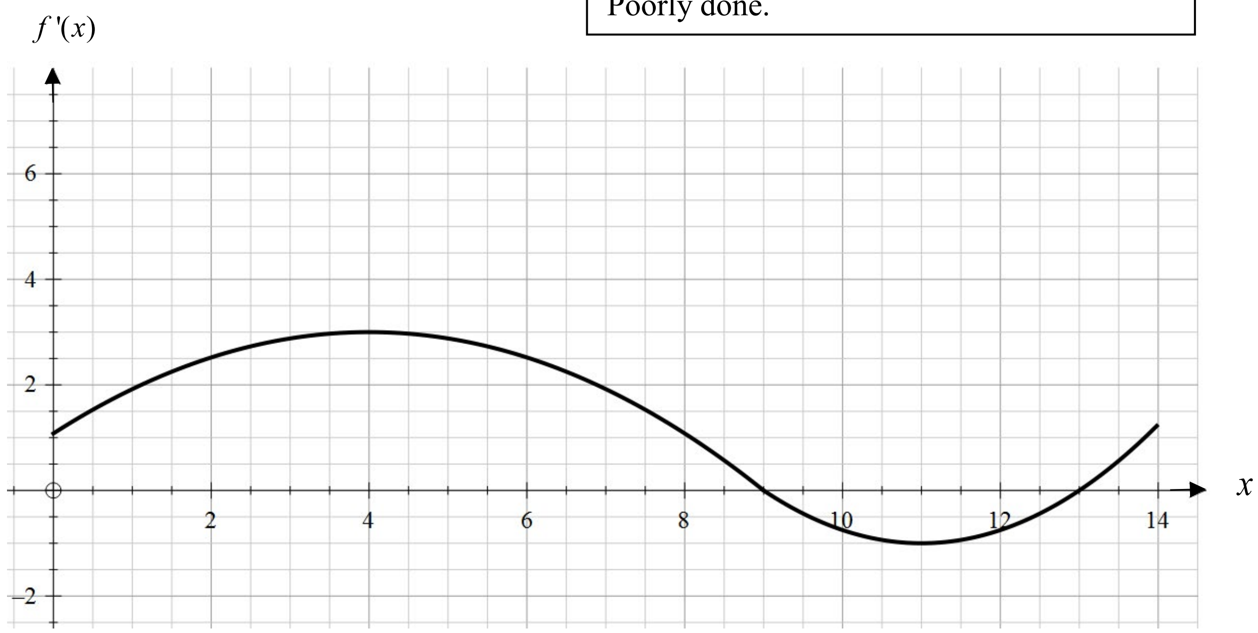
$x = 9$ and $x = 13$

Marking guideline:

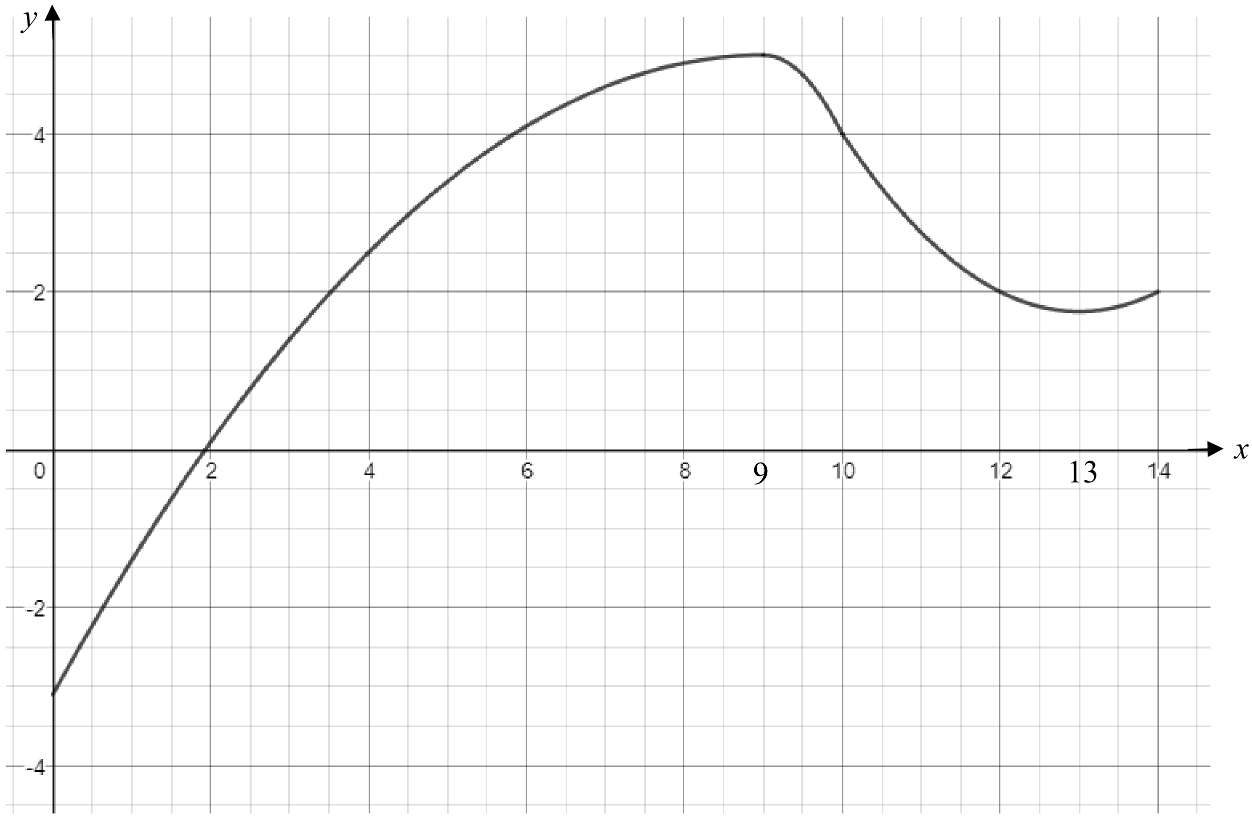
1 Correct response

Marker's comments:

Poorly done.



(b) Using the graph of $f'(x)$ above and your responses in part (a), draw a possible sketch of $y = f(x)$.



Marking guideline:

2 Correct response

1 Partially correct response

Marker's comments:

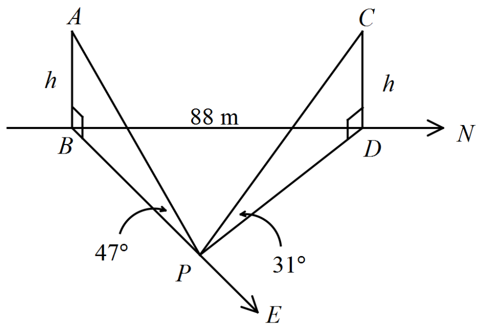
Many students did not have an understanding of drawing of $y = f(x)$.

Graphs were often poorly done. Must use a rule, proper relative scale and label axes/

Question 6 continues on the next page

(Question 6 continued)

(c) AB and CD are towers of equal height, h metres. CD is due north of AB. From a point P on the same horizontal plane as the feet B and D of the towers, and bearing due east of tower AB, the angles of elevation of A and C, the tops of the towers, are 47° and 31° respectively. The distance between the towers is 88 metres.



i) Show that $BP = h\tan 43°$

$$\angle BAP = 43^\circ$$
$$\tan(\angle BAP) = \frac{BP}{h}$$
$$\tan 43^\circ = \frac{BP}{h}$$
$$BP = h\tan 43^\circ$$

Marking guideline:

1 Correct response

Marker's comments:

Well done

ii) Find the value of h to the nearest metre.

$$BP = h\tan 43^\circ$$
$$PD = h\tan 59^\circ$$
$$BD = 88m$$
$$\Delta BPD \text{ is right triangled with } BP^2 + BD^2 = PD^2$$
$$(h\tan 43^\circ)^2 + 88^2 = (h\tan 59^\circ)^2$$
$$h^2\tan^2 59^\circ - h^2\tan^2 43^\circ = 88^2$$
$$h^2(\tan^2 59^\circ - \tan^2 43^\circ) = 88^2$$
$$h^2 = \frac{88^2}{\tan^2 59^\circ - \tan^2 43^\circ}$$
$$h = \frac{88}{\sqrt{\tan^2 59^\circ - \tan^2 43^\circ}}$$
$$h = 63.84...$$
$$h = 64 \text{ m}$$

Marking guideline:

2 Correct response

1 Partially correct response

Marker's comments:

Many students incorrectly used $\angle BPD$ as the right angle rather than $\angle DBP$. This gave an answer of 46 m and was awarded 1 mark for correct method.

(d) Jerry's distance from home, s km, after time t minutes is given by $s = t^3 - 4t^2 + 5t + 8$.

i) How far does Jerry travel in the first minute?

$At\ t = 0,\ s = 8$

$At\ t = 1,\ s = (1)^3 - 4(1)^2 + 5(1) + 8 = 10$

$10 - 8 = 2\,\text{km}$

Marking guideline:

2 Correct response

1 Partially correct response

Marker's comments:

Most students received 1 mark only for $s = 10$ when $t = 1$. They failed to realise that they needed to find the difference in s for $t = 0$ and $t = 1$.

ii) At what speed is Jerry travelling after 5 minutes?

$\frac{ds}{dt} = 3t^2 - 8t + 5$

$At\ t = 5,$

$\frac{ds}{dt} = 3(5)^2 - 8(5) + 5$

$= 40$

$= 40$ km/min

Marking guideline:

2 Correct response

1 Partially correct response

Marker's comments:

Mostly well done.

Many students need to watch units (no mark deducted)

Common error was to find the average speed.

End of Question 6

Question 7 (12 marks) *Start a new booklet*

(a) Differentiate $f(x) = x^2 - 3x$ from first principles.

Solution

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \\
&= \lim_{h\to 0}\frac{[(x+h)^2-3(x+h)]-(x^2-3x)}{h} \\
&= \lim_{h\to 0}\frac{x^2+2xh+h^2-3x-3h-x^2+3x}{h} \\
&= \lim_{h\to 0}\frac{2xh+h^2-3h}{h} \\
&= \lim_{h\to 0}\frac{h(2x+h-3)}{h} \\
&= \lim_{h\to 0}(2x+h-3) \\
&= 2x-3
\end{aligned}
$$

Marking guideline:

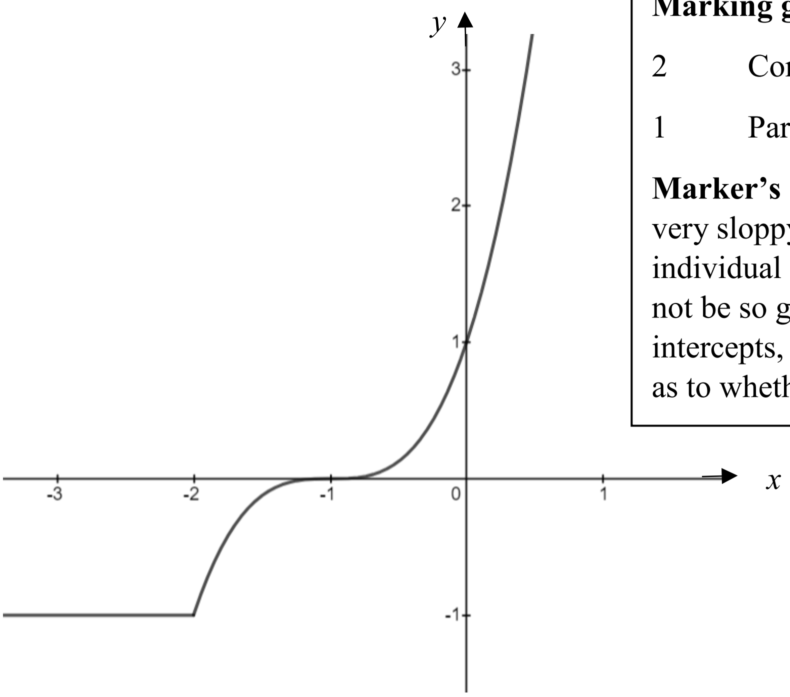
2 Correct response

1 Partially correct response

Marker's comments:
Zero marks were given if 'first principles' was not used. There were a lot of sign errors. Most students need to learn to set out such questions correctly. The limit notation was in the incorrect spot or left out. Students do not understand that f '(x) is a limit.

(b) Consider the function $f(x) = \begin{cases} (x+1)^3 & x > -2 \\ -1 & x \le -2 \end{cases}$

i) Sketch $y = f(x)$



Marking guideline:

2 Correct response

1 Partially correct response

Marker's comments: Most students' graphs were very sloppy, poorly drawn!! Take note of individual feedback as next time the marker will not be so generous. Marks were lost for leaving out intercepts, discontinuous graph and for uncertainty as to whether the point (-2, -1) was included**.**

ii) Find the values of x for which $f(x)$ is:

α) continuous

All real x

Marking guideline:

1 Correct response

Marker's comments:
Common error was to say $x \neq -2$. The new notation for domain and range needs to be revised.

β) differentiable

All real x, $x \neq -2$

Marking guideline:

1 Correct response

Marker's comments:
Common error was to just write $x \neq -2$. This is not the complete answer. What happens at all other x values?

(c) The population of salmon, P, at time t days is given by $P = -800e^{-0.005t} + 1200$.

i) Determine the initial population.

$$At\ t = 0,\ P = -800e^{0} + 1200$$
$$P = 400$$

Marking guideline:

1 Correct response

Marker's comments:

Well done

ii) Find $\dfrac{dP}{dt}$

$P'(t) = 4e^{-0.005t}$

Marking guideline:

1 Correct response

Marker's comments:

Well done

iii) Explain why the population is always increasing.

As $e^{-0.005t} > 0$ for all t, $P'(t) > 0$

Therefore P is always increasing.

Marking guideline:

1 Correct response

Marker's comments: The more complete answer is to say 'As $e^{-0.005t} > 0$ for all t, $P'(t) > 0$ always $\therefore$ P is always increasing .

iv) What number is the population approaching?

$As\ t \to \infty$

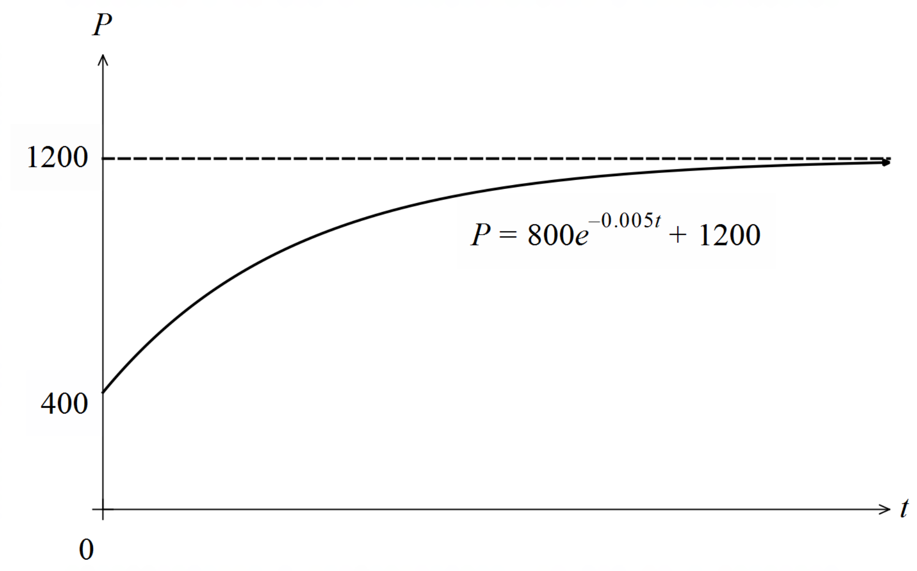
$P \to 1200$

Marking guideline:

1 Correct response

Marker's comments:
Well done

v) Sketch the graph of the population of salmon over time.



Marking guideline:

2 Correct response

1 Partially correct response

Marker's comments:

Poorly done. Many students forgot the asymptote. Axes should be labelled. Many were unable to pick a suitable scale for the t axis. This resulted in the wrong shape for the curve. A few more points should have been plotted to improve the accuracy of the graph.

Question 8 (12 marks) *Start a new booklet*

(a) i) Show that $\dfrac{2x^2+x-1}{x^2-1}=2+\dfrac{1}{x-1}$

$$\begin{aligned} LHS &= \frac{2x^2+x-1}{x^2-1} \\ &= \frac{(2x-1)(x+1)}{(x-1)(x+1)} \\ &= \frac{2x-1}{x-1} \\ &= \frac{2x-2+1}{x-1} \\ &= \frac{2(x-1)+1}{x-1} \\ &= \frac{2(x-1)}{x-1}+\frac{1}{x-1} \\ &= 2+\frac{1}{x-1} \end{aligned}$$

Marking guideline:

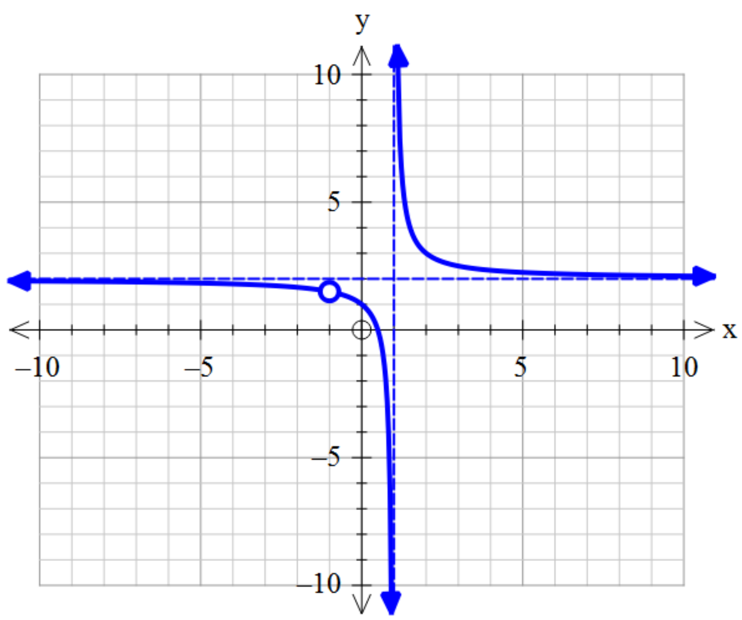
2 Correct response

1 Partially correct response

Marker's comments:

- Showing RHS = LHS was also accepted as a correct response as was polynomial division.
- Some students were unable to make the transition from $\dfrac{2x+1}{x-1}$ to $\dfrac{2x-2+1}{x-1}$ to $2+\dfrac{1}{x-1}$

vi) Sketch a neat graph of $y=\dfrac{2x^2+x-1}{x^2-1}$ labelling any intercepts, discontinuities and other key features.



Marking guideline:

3 Correct response

2 Partially correct response (Fails to show discontinuity)

1 Correct graph shape

Marker's comments:

- Most recognised the vertical asymptote at $x = 1$ and the horizontal asymptote at $y = 2$
- The majority of students omitted the discontinuity at $x = -1$
- A few were unable to sketch the correct shape

(b) The probability of a cure with Drug A is 0.8 and with Drug B is 0.6.

One randomly selected patient is treated with Drug A and another with Drug B.

Find the probability that:

i) both patients are cured

$$P(AB) = 0.6 \times 0.8 = 0.48$$

Marking guideline:

1 Correct response

Marker's comments:

- Generally answered well

ii) neither patient is cured

$$P(\overline{A}\overline{B}) = 0.2 \times 0.4 = 0.08$$

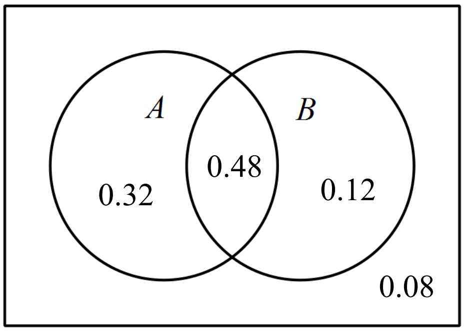
Marking guideline:

1 Correct response

Marker's comments:

- Generally answered well

iii) the patient treated with Drug B is cured, given that only one of the selected patients is cured



$$P(B \mid One\ Cured) = \frac{0.12}{0.32 + 0.12} = \frac{0.12}{0.44} = \frac{3}{11}$$

Marking guideline:

2 Correct response

1 Partially correct response

Marker's comments:

- Not answered very well.
- Many students did not attempt the questions
- Most were unable to determine the event probability and/ or the sample probability.

(c) The line $y = mx$ is tangent to the curve $y = e^{2x}$ at a point T.

Find the coordinates of T.

(1) $y = mx$

(2) $y = e^{2x}$

(3) $y' = 2e^{2x} = m$

$(3) \to (1) = (4)$

(4) $y = 2xe^{2x}$

$(2) = (4)$

$2xe^{2x} = e^{2x}$

$2x = 1$

$x = \frac{1}{2}$

$x = \frac{1}{2} \to (2)$

$y = e^{2(\frac{1}{2})}$

$y = e$

So $T = \left(\frac{1}{2}, e\right)$

Marking guideline:

3	Correct response
2	Partially correct response
1	Attempt to solve equation correctly

Marker's comments:

- Not answered very well
- Students did not recognise $\frac{dy}{dx}$ as the gradient of the line and continued to use an unknown value of m which complicated their calculations.
- A few were unable to differentiate $y = e^{2x}$
- For some reason students equated the x coordinate of T with zero.

End of Examination