Section I

5 marks
Attempt Questions 1–5
Allow about 10 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 5

1. What is the solution to the equation $3x^2+7x-1=0$?

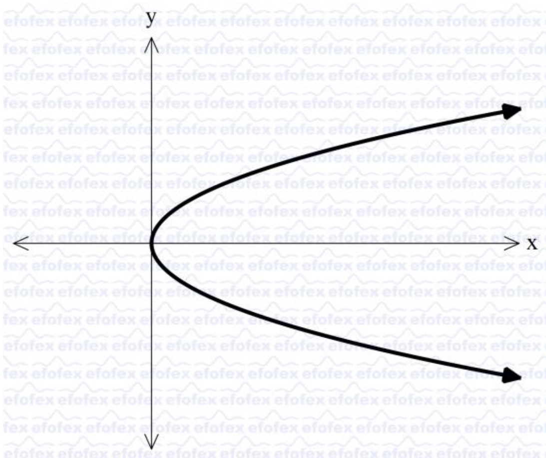
A. $x=\dfrac{-7\pm\sqrt{61}}{6}$

B. $x=\dfrac{-7\pm\sqrt{37}}{6}$

C. $x=\dfrac{7\pm\sqrt{37}}{6}$

D. $x=\dfrac{7\pm\sqrt{61}}{6}$

2. Classify the type of relation below.



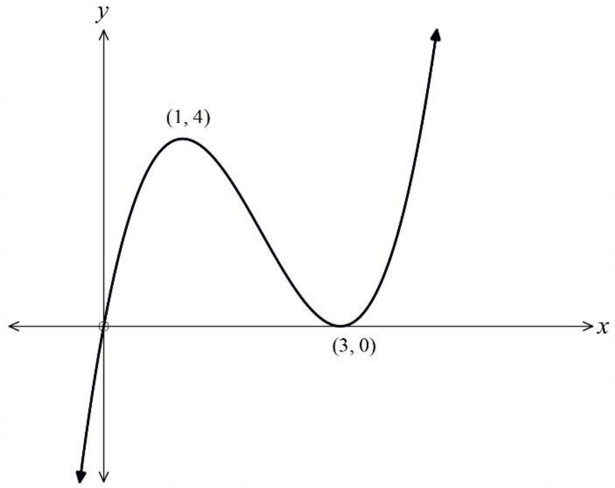
A. One-to-One

B. Many-to-One

C. One-to-Many

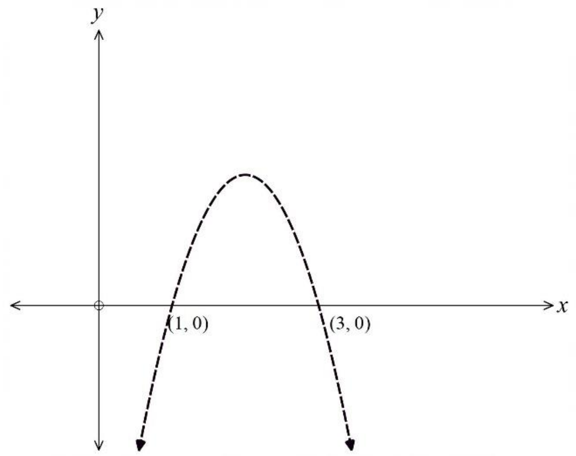
D. Many-to-Many

3. The graph of $y = f(x)$ is shown below.

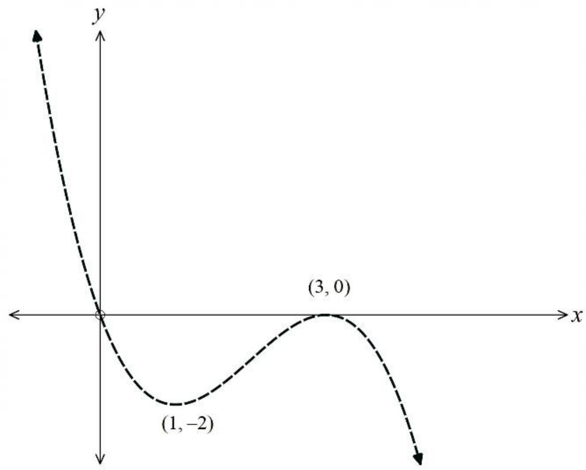


Which graph could represent $y = f'(x)$

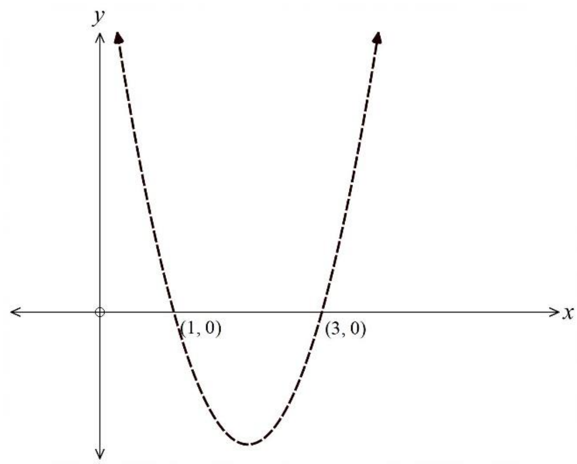
A.



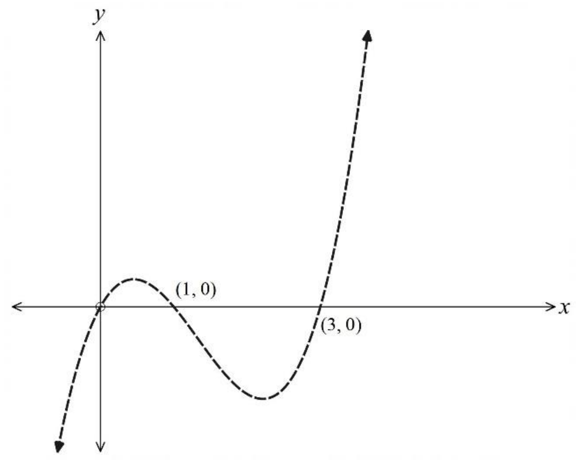
B.



C.



D.



4. Find the exact value of $\tan\dfrac{5\pi}{3}$.

A. $-\sqrt{3}$

B. $-\dfrac{1}{\sqrt{3}}$

C. $\dfrac{1}{\sqrt{3}}$

D. $\sqrt{3}$

5. Find the derivative of $y=\sqrt{\left(x^3+4\right)^3}$.

A. $3\left(x^3+4\right)^2$

B. $18\sqrt{x^3+4}$

C. $\dfrac{\sqrt{x^3+4}}{18x^2}$

D. $\dfrac{9x^2\sqrt{x^3+4}}{2}$

Section II

Student Number:

75 marks

Attempt Questions 6–28

Allow about 1 hour and 50 minutes for this section

Section II – Part A

Question 6 (2 marks) **2**

Solve $x^3 = 4x$.

……………………………………………………………………………………………………….

……………………………………………………………………………………………………….

……………………………………………………………………………………………………….

……………………………………………………………………………………………………….

……………………………………………………………………………………………………….

……………………………………………………………………………………………………….

Question 7 (2 marks)

ABC is a triangle such that $\angle ABC = 60^\circ$, $\angle ACB = 45^\circ$ and $AC = 9$ m.

Find the exact length of AB. Leave your answer with a rational denominator. **2**

……………………………………………………………………………………………………….

……………………………………………………………………………………………………….

……………………………………………………………………………………………………….

……………………………………………………………………………………………………….

……………………………………………………………………………………………………….

Question 8 (3 marks)

Let $y = \dfrac{9x - x^2 - x^3}{2}$.

(a) Find the gradient of the tangent at the point $x = \sqrt{3}$. **2**

…………………………………………………………………………………………………

…………………………………………………………………………………………………

…………………………………………………………………………………………………

…………………………………………………………………………………………………

…………………………………………………………………………………………………

…………………………………………………………………………………………………

…………………………………………………………………………………………………

(b) Find the angle of inclination of the tangent to the positive *x*-axis. **1**

…………………………………………………………………………………………………

…………………………………………………………………………………………………

…………………………………………………………………………………………………

…………………………………………………………………………………………………

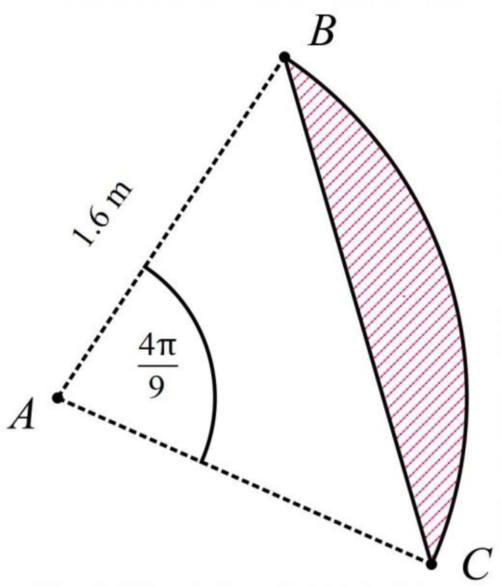
…………………………………………………………………………………………………

…………………………………………………………………………………………………

…………………………………………………………………………………………………

Question 9 (3 marks)

The shaded segment shown is formed by an arc BC of a circle radius 1.6m, which subtends an angle of $\frac{4\pi}{9}$ radians at its centre A. **3**



Find the area of the shaded segment, correct to 3 significant figures.

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

Question 10 (3 marks)

Given the points $A(4,2)$, $B(1,-4)$ and $C(7,8)$

(a) Find the equation of the line AB **2**

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

(b) Determine if the points A, B and C are collinear. **1**

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

Question 11 (3 marks)

Let $f(x) = x^2 - 2x$ and $g(x) = \dfrac{1}{2x}$.

(a) Show that $g(f(x)) = \dfrac{1}{2x(x-2)}$. **1**

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

(b) Find the value(s) of x for which $g(f(x))$ is discontinuous. **2**

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

Extra Writing Paper

Section II – Part B

Student Number:

Question 12 (2 marks) **2**

Differentiate $y = \dfrac{6}{(x^4+3)^3}$.

…………………………………………………………………………………………

…………………………………………………………………………………………

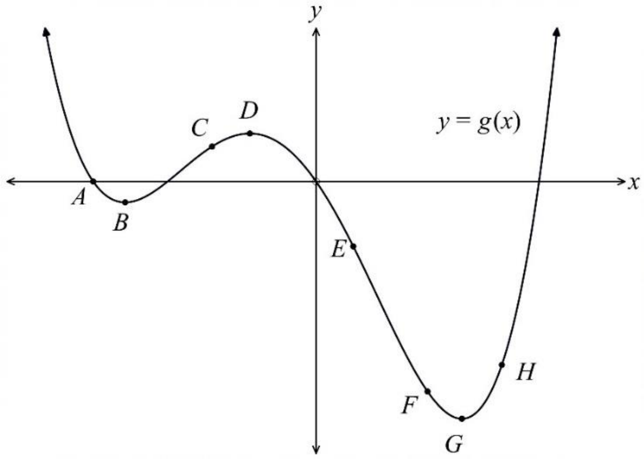
…………………………………………………………………………………………

…………………………………………………………………………………………

Question 13 (3 marks)

The curve $y = g(x)$ is shown below.

Eight points on the curve are labelled A – H.



(a) Name the point(s) where the tangent to the curve is horizontal. **1**

…………………………………………………………………………………………

…………………………………………………………………………………………

(b) Name the point(s) where the curve is increasing at a decreasing rate. **1**

…………………………………………………………………………………………

…………………………………………………………………………………………

(c) Explain why the average rate of change between A and H is negative. **1**

…………………………………………………………………………………………

…………………………………………………………………………………………

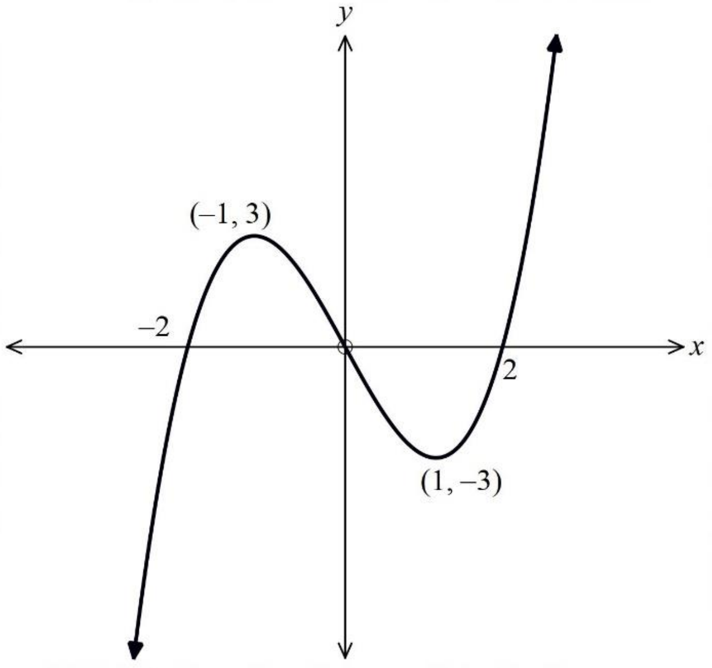
…………………………………………………………………………………………

Question 14 (3 marks)

Classify the two functions below, as odd, even or neither.

Justify your answer with reasons or calculations.

(a) The function whose graph is shown below. **1**



…………………………………………………………………………………………………

…………………………………………………………………………………………………

…………………………………………………………………………………………………

(b) The function whose equation is $y = 2x^2 + 3x - 1$ **2**

…………………………………………………………………………………………………

…………………………………………………………………………………………………

…………………………………………………………………………………………………

…………………………………………………………………………………………………

…………………………………………………………………………………………………

…………………………………………………………………………………………………

Question 15 (4 marks)

(a) Given that $y = x^3(x+4)^5$, find $\frac{dy}{dx}$. Fully factorise your answer. **2**

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

(b) Find $\frac{d}{dx}\left(\frac{ax^2}{ax+b}\right)$. Fully factorise your answer. **2**

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

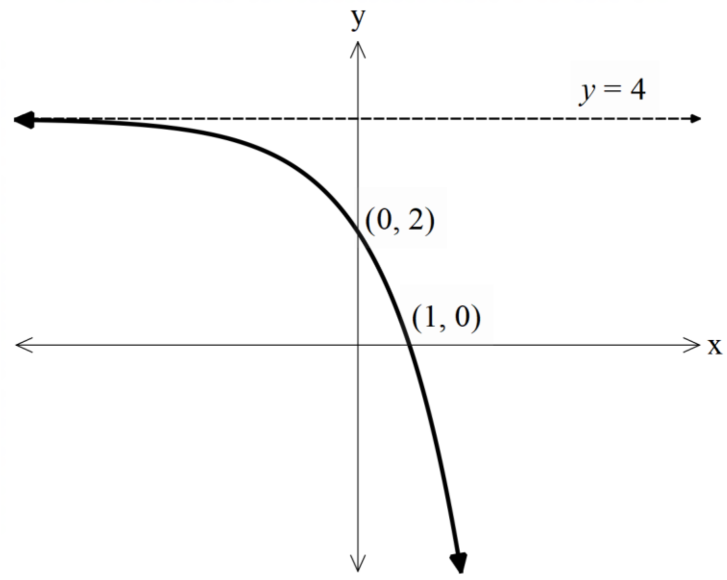
Question 16 (3 marks)

Given the sketch of $y = f(x)$ below, sketch $y = -f(-x)$ on the same set of axes. Ensure you label any asymptotes, x-intercepts, and y-intercepts. **3**

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………



Extra Writing Paper

Section II – Part C

Student Number:

Question 17 (4 marks)

(a) Simplify the expression $\frac{a^2-3a}{2ax}\times\frac{4a^2x}{2ax-6x}$, giving your answer as a simple fraction. **2**

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

(b) Solve the equation $\frac{3x+5}{2}=4x-1$. **2**

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

Question 18 (3 marks)

Solve the following trigonometric equations for $0\le x\le 2\pi$.

(a) $\tan x=\sqrt{3}$ **1**

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

Question 21 (b) is on the next page.

(b) $\sin 2x = \frac{1}{2}$ **2**

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

Question 19 (2 marks)

Solve $8^x = 2^{2x+1}$

2

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

Question 20 (2 marks)

Find the value(s) of m for which the quadratic equation $x^2 + (m-1)x + 4 = 0$ has equal roots **2**

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

Question 21 (3 marks)

(a) Find the centre and radius of the circle represented by the equation $x^2 - 4x + y^2 + 6y + 4 = 0$ **2**

……………………………………………………………………………………………

……………………………………………………………………………………………

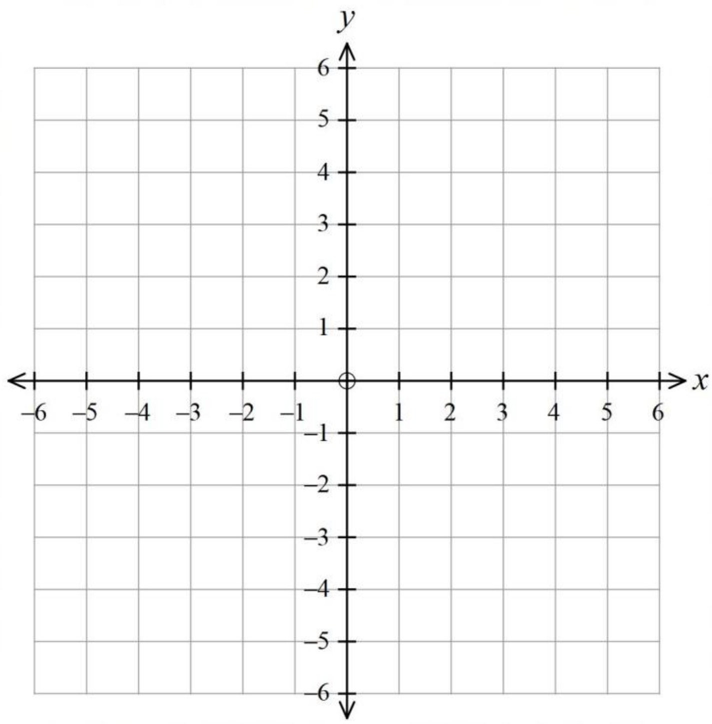
……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

(b) Draw the circle on the axes below. **1**



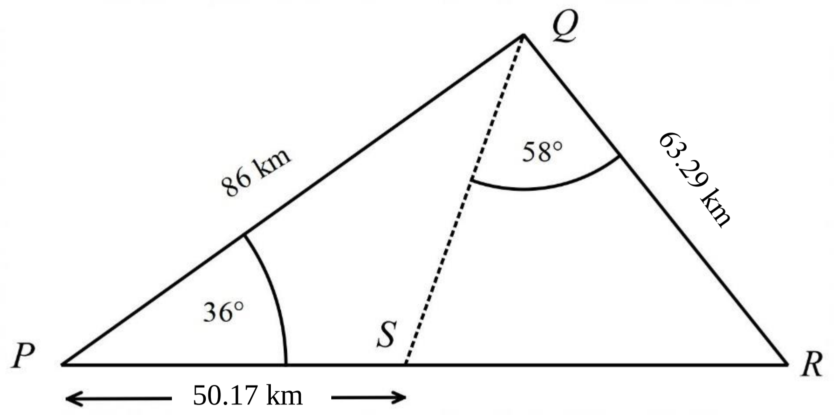
Extra Writing Paper

Section II – Part D

Student Number:

Question 22 (4 marks)

Triangles PQS and QRS have a common side QS, as shown.



(a) Calculate the length of QS, correct to 2 decimal places. **2**

....

....

....

....

....

....

(b) Find the area of ΔPQR, correct to the nearest square kilometre. **2**

....

....

....

....

....

....

Question 23 (5 marks)

Bobby is on a game show where the objective is to run through a shutter door before it closes. The shutter door falls from the roof, and the distance from the floor to the bottom of the shutter door is given by $y = 6 - t$ where y is in metres and t is in seconds. Bobby starts a fixed distance away from the shutter door and can start running when the door begins to drop. His distance from the start line is given by $x = \frac{t^2}{4} + 4t$ where x is in metres and t is in seconds.

(a) Bobby knows he can make it under the shutter door if it is at least 1.5m from the ground. How much time does Bobby have to make it past the door? **1**

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

(b) If Bobby starts 20m away, show that he makes it past the shutter door with 0.5s to spare. **2**

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

(c) Find Bobby's velocity when he passes the shutter door. **2**

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

Question 24 (3 marks)

Show that $\sin\theta\cot\theta - \cos\theta\sin^2\theta = \cos^3\theta$. **3**

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

Question 25 (3 marks)

Given that $f(x) = \dfrac{4}{x^2}$

(i) Show that $f(x)$ is an even function. **1**

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

(ii) State the domain and range of $f(x)$. **2**

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

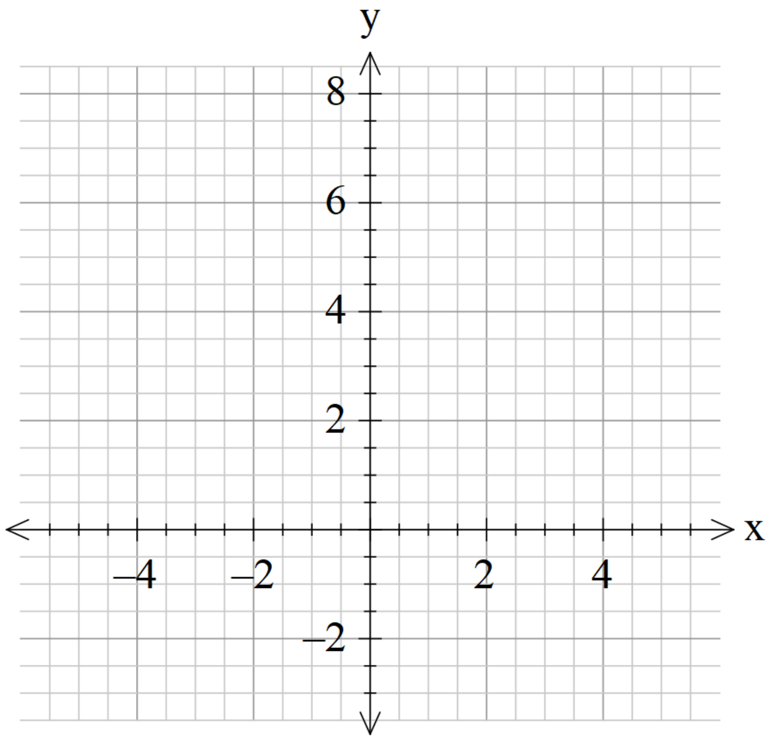
Extra Writing Paper

Section II – Part E

Student Number:

Question 26 (5 marks)

(a) On the axes below, draw the graphs of $y = |x+1|$ and $y = \frac{2}{x} + 2$. Label any intercepts, asymptotes, or other key information. **4**



……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

(b) Use the graph to find solutions to $|x+1| = \frac{2}{x} + 2$ **1**

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

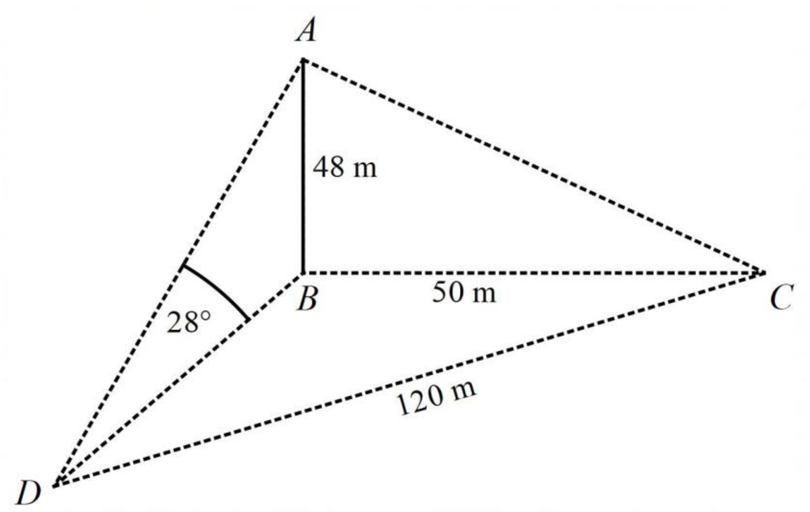
Question 27 (4 marks)

A 48-metre high, vertical lighting pole *AB* stands in the middle of a flat sporting ground.

Diana starts from the base of the pole and walks 50 m due east to a point *C* where she places a peg.

She attaches a 120 m long string to the peg and walks in a roughly south-westerly direction until the string is taut, at point *D*.

She then measures the angle of elevation of the pole to be 28°.



Use the information provided to find the *actual* bearing on which she was walking from *C* to *D*, correct to the nearest degree. **4**

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

Question 28 (6 marks)

Given the parabola $y = x^2 - 4x + 4$

(a) Show that the equation of the normal to the tangent of the parabola at the point $P(4,4)$ is $y = -\frac{x}{4} + 5$ **3**

…………………………………………………………………………………………………

…………………………………………………………………………………………………

…………………………………………………………………………………………………

…………………………………………………………………………………………………

…………………………………………………………………………………………………

…………………………………………………………………………………………………

…………………………………………………………………………………………………

…………………………………………………………………………………………………

(b) Find the other point of intersection $Q(x_1, y_1)$ between the parabola and the normal to the tangent at $P(4,4)$. **3**

…………………………………………………………………………………………………

…………………………………………………………………………………………………

…………………………………………………………………………………………………

…………………………………………………………………………………………………

…………………………………………………………………………………………………

…………………………………………………………………………………………………

…………………………………………………………………………………………………

…………………………………………………………………………………………………

…………………………………………………………………………………………………

…………………………………………………………………………………………………

End of Paper

Extra Writing Paper

Section I

5 marks
Attempt Questions 1–5
Allow about 10 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 5

1. What is the solution to the equation $3x^2+7x-1=0$?

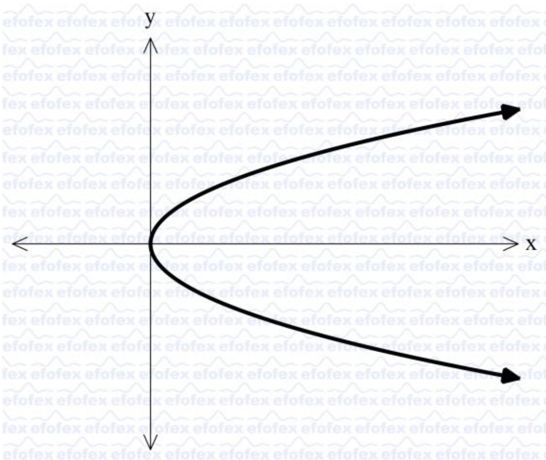
A. $x=\dfrac{-7\pm\sqrt{61}}{6}$

B. $x=\dfrac{-7\pm\sqrt{37}}{6}$

C. $x=\dfrac{7\pm\sqrt{37}}{6}$

D. $x=\dfrac{7\pm\sqrt{61}}{6}$

2. Classify the type of relation below.



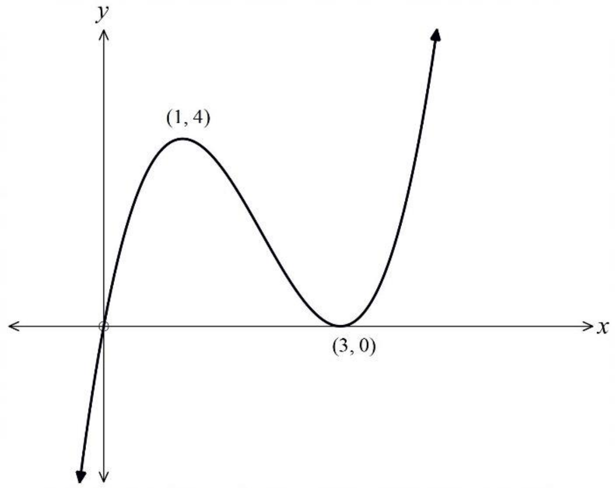
A. One-to-One

B. Many-to-One

C. One-to-Many

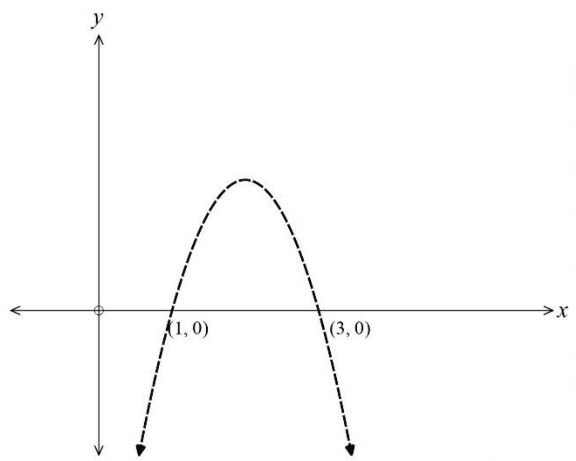
D. Many-to-Many

3. The graph of $y = f(x)$ is shown below.

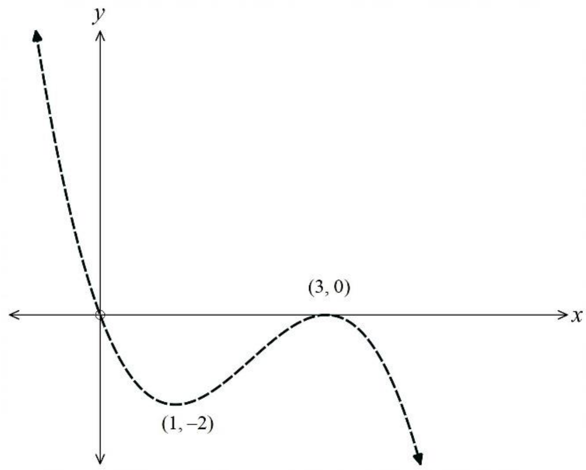


Which graph could represent $y = f'(x)$

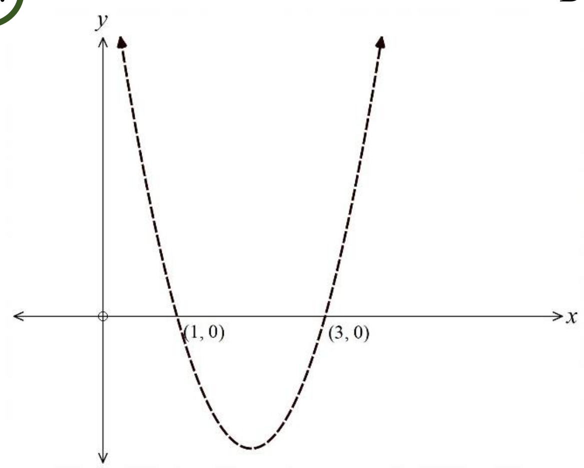
A.



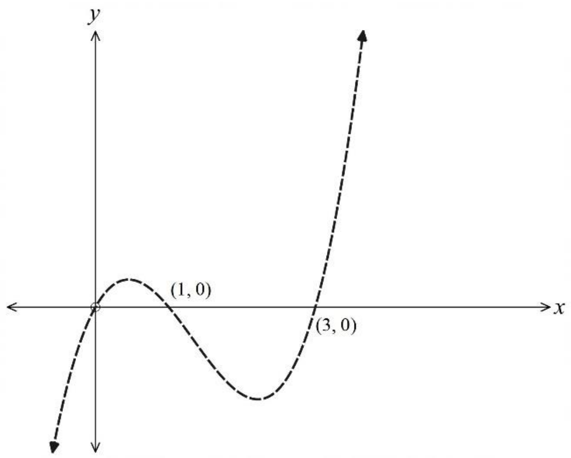
B.



C.



D.



4. Find the exact value of $\tan\frac{5\pi}{3}$.

A. $-\sqrt{3}$

B. $-\frac{1}{\sqrt{3}}$

C. $\frac{1}{\sqrt{3}}$

D. $\sqrt{3}$

5. Find the derivative of $y=\sqrt{\left(x^3+4\right)^3}$.

A. $3\left(x^3+4\right)^2$

B. $18\sqrt{x^3+4}$

C. $\frac{\sqrt{x^3+4}}{18x^2}$

D. $\frac{9x^2\sqrt{x^3+4}}{2}$

Section II

Student Number:

75 marks
Attempt Questions 6–28
Allow about 1 hour and 50 minutes for this section

Section II – Part A

Question 6 (2 marks)

Solve $x^3 = 4x$.

$x^3 - 4 = 0x$

$x(x^2 - 4) = 0$

$x(x-2)(x+2) = 0$

$x = 0,\ x = 2,\ x = -2$

Marking guideline:

2 Correct response

1 Partially correct response

Marker's comments:

There were two common and significant errors in this question. Students either/or

1) Divided both sides by x as their first step. Doing this neglects the fact that x can equal 0.
2) Students neglected the fact that taking the square root of 4 yields 2 answers.

Question 7 (2 marks)

ABC is a triangle such that $\angle ABC = 60^\circ$, $\angle ACB = 45^\circ$ and $AC = 9$ m.

Find the exact length of AB. Leave your answer with a rational denominator. **2**

$$\frac{x}{\sin(45)} = \frac{9}{\sin(60)}$$

$$\frac{x}{(\frac{1}{\sqrt{2}})} = \frac{9}{(\frac{\sqrt{3}}{2})}$$

$$x = \frac{9\left(\frac{1}{\sqrt{2}}\right)}{\left(\frac{\sqrt{3}}{2}\right)}$$

$$x = 9\left(\frac{1}{\sqrt{2}}\right)\times\left(\frac{2}{\sqrt{3}}\right)$$

$$x = \frac{18}{\sqrt{6}}$$

$$x = \frac{18}{\sqrt{6}}\times\frac{\sqrt{6}}{\sqrt{6}}$$

$$x = \frac{18\sqrt{6}}{6}$$

$$x = 3\sqrt{6}$$

Marking guideline:

2 Correct response

1 Correct use of Sine Rule

Marker's comments:

Mostly done well; however, some students used the wrong angles and sides for sine rule or neglected to rationalise the denominator.

Question 8 (3 marks)

Let $y = \dfrac{9x - x^2 - x^3}{2}$.

(a) Find the gradient of the tangent at the point $x = \sqrt{3}$. **2**

$$y = \frac{1}{2}\left(9x - x^2 - x^3\right)$$
$$\frac{dy}{dx} = \frac{1}{2}(9 - 2x - 3x^2)$$

At $x = \sqrt{3}$

$$= \frac{1}{2}(9 - 2\sqrt{3} - 3(\sqrt{3})^2)$$
$$= \frac{1}{2}\left(9 - 2\sqrt{3} - 9\right)$$
$$= \frac{1}{2}(-2\sqrt{3})$$
$$= -\sqrt{3}$$

Marking guideline:

2 Correct response

1 Correct Differentiation

Marker's comments:

Mostly done well.

(b) Find the angle of inclination of the tangent to the positive *x*-axis. **1**

The gradient is negative so the angle of inclination is in the second quadrant.

$$\tan\theta = -\sqrt{3}$$
$$\theta = \pi - \frac{\pi}{3}$$
$$\theta = \frac{2\pi}{3} \ or\ \theta = 120^\circ$$

Marking guideline:

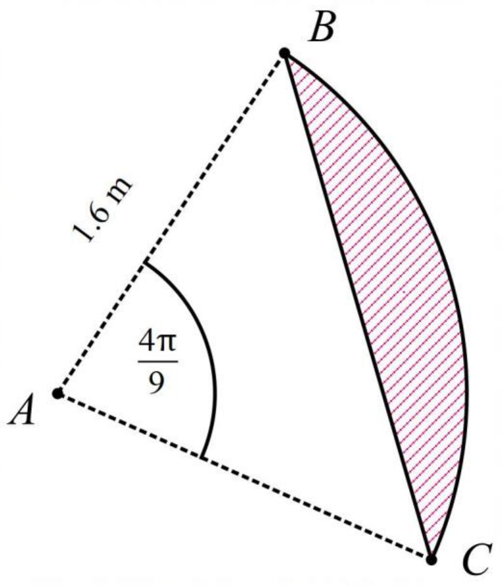
1 Correct response

Marker's comments:

Generally not answered well. Many students did not know what the angle of inclination is or gave an answer of 60 degrees. The angle of inclination is the angle between the positive x-axis and the line.

Question 9 (3 marks)

The shaded segment shown is formed by an arc BC of a circle radius 1.6m, which subtends an angle of $\frac{4\pi}{9}$ radians at its centre A. **3**



Find the area of the shaded segment, correct to 3 significant figures.

Converting radians to degrees $\frac{4\pi}{9} \times \frac{180}{\pi} = 80^{\circ}$

$$A_{sector} = \frac{80}{360} \times \pi (1.6)^2$$
$$= 1.787m^2$$

$$A_{triangle} = \frac{1}{2}(1.6)^2 \sin(80)$$
$$= 1.261m^2$$

$$A_{shaded} = A_{sector} - A_{triangle}$$
$$= 1.787m^2 - 1.261m^2$$
$$= 0.527m^2$$

Marking guideline:

3 Correct Response

2 One error

1 Makes some progress towards finding the answer

Marker's comments:

Most common errors were students having their calculator in the wrong mode or failing to convert between radians and degrees correctly. Carry on marks were awarded for certain errors as long as the students understood how to answer the question conceptually.

Question 10 (3 marks)

Given the points $A(4,2)$, $B(1,-4)$ and $C(7,8)$

(a) Find the equation of the line AB **2**

$$m_{AB} = \frac{-4-2}{1-4} = \frac{-6}{-3} = 2$$

$$y-2 = 2(x-4)$$
$$y-2 = 2x-8$$
$$y = 2x-6$$

Marking guideline:

2 Correct response

1 Finds Gradient correctly

Marker's comments:

Motly done well

(b) Determine if the points A, B and C are collinear. **1**

Test point C

$$C(7,8)$$
$$y = 2x-6$$
$$8 = 2(7)-6$$
$$8 = 14-6$$
$$8 = 8$$

Therefore A, B and C are collinear

Marking guideline:

1 Correct response

Marker's comments:

Mostly done well, but students who did not justify their response did not get awarded a mark.

Question 11 (3 marks)

Let $f(x) = x^2 - 2x$ and $g(x) = \dfrac{1}{2x}$.

(a) Show that $g(f(x)) = \dfrac{1}{2x(x-2)}$. **1**

$$g(f(x)) = \frac{1}{2(x^2-2x)}$$
$$= \frac{1}{2x(x^2-2)}$$
$$as\ req'd$$

Marking guideline:

1 Correct response

Marker's comments:

Mostly done well.

(b) Find the value(s) of x for which $g(f(x))$ is discontinuous. **2**

$x = 0$ *and* $x = 2$ since the denominator cannot equal 0.

Marking guideline:

2 Finds both discontinuities

1 Finds one discontinuity

Marker's comments:

Mostly done well.

Section II – Part B

Student Number:

Question 12 (2 marks) **2**

Differentiate $y = \dfrac{6}{(x^4+3)^3}$.

$y = 6(x^4+3)^{-3}$

$\dfrac{dy}{dx} = -18(x^4+3)^{-4} \times 4x^3$

$= \dfrac{-72x^3}{(x^4+3)^4}$

Marking guideline:

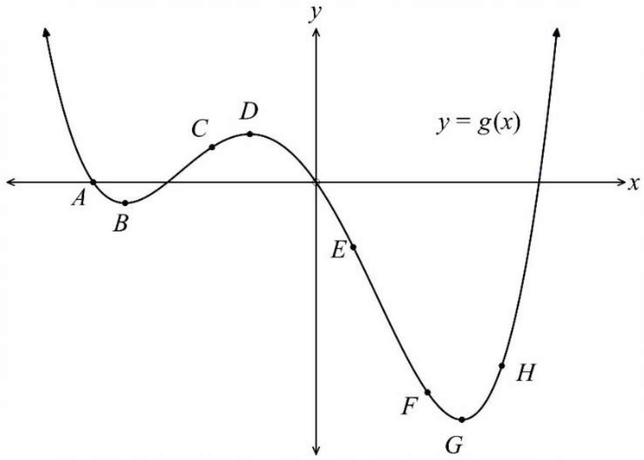
2 Correct Response

1 Makes progress towards finding the answer

Marker's comments:
Many students used the quotient rule which complicated the process and led to careless errors

Question 13 (3 marks)

The curve $y = g(x)$ is shown below.
Eight points on the curve are labelled A – H.



Marking guideline:

3 3 Correct Responses

2 2 correct responses

1 1 correct response

Marker's comments:
Part (a) was generally answered well. In part (b) some students also included point H. In part (c) students confused negative y values with a negative rate of

(a) Name the point(s) where the tangent to the curve is horizontal. **1**

B, D, G………………………………………………………………………………….

…………………………………………………………………………………………….

(b) Name the point(s) where the curve is increasing at a decreasing rate. **1**

…C……………………………………………………………………………………….

…………………… ……………………………………………………………………….

(c) Explain why the average rate of change between A and H is negative. **1**

…………………………………………………………………………………………

…………Because the gradient of AH is negative……………………………………….

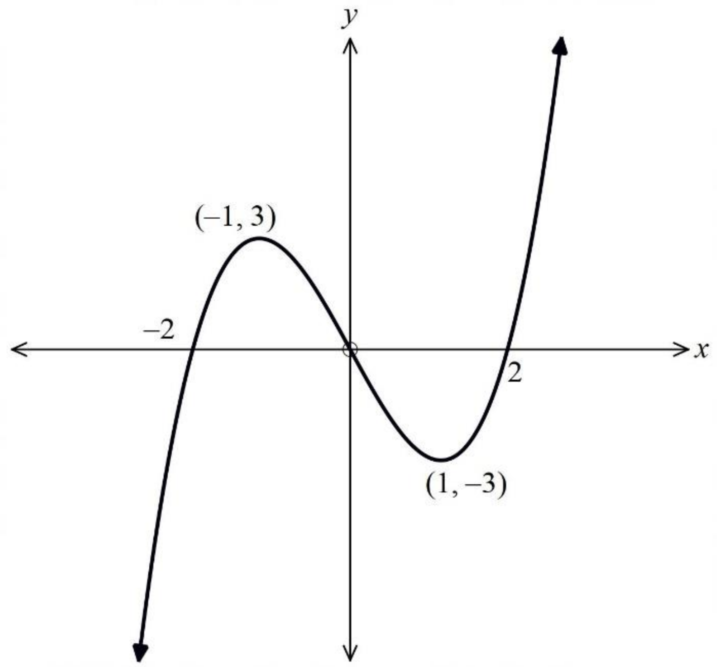
…………………………………………………………………………………………

Question 14 (3 marks)

Classify the two functions below, as odd, even or neither.

Justify your answer with reasons or calculations.

(a) The function whose graph is shown below. **1**



Marking guideline:

1 Correct response

Marker's comments:
Students were not stating that rotational/ point symmetry was about the origin. Mark not awarded if reasons/ calculations were not also given.

…Odd since there is rotational symmetry 180 degrees about the origin

(b) The function whose equation is $y = 2x^2 + 3x - 1$ **2**

Test

$$f(-x) = 2(-x)^2 + 3(-x) - 1$$
$$= 2x^2 - 3x - 1$$
$$\therefore f(-x) \neq -f(x) \text{ and } f(-x) \neq f(x)$$

So the function is neither odd nor even.

Marking guideline:

2 correct answer with supporting calculations or

1 correct answer without support or equivalent merit

Marker's comments:
Marks were deducted if some form of a conclusion was not given. Students incorrectly substituted values for x when providing supporting calculations.

Question 15 (4 marks)

(a) Given that $y = x^3(x+4)^5$, find $\frac{dy}{dx}$. Fully factorise your answer. **2**

$u = x^3 \qquad v = (x+4)^5$

$\frac{du}{dx} = 3x^2 \qquad \frac{dv}{dx} = 5(x+4)^2$

$$\begin{aligned}\frac{dy}{dx} &= 5x^3(x+4)^4 + 3x^2(x+4)^5 \\ &= x^2(x+4)^4[3(x+4)+5x] \\ &= x^2(x+4)^4[3x+12+5x] \\ &= x^2(x+4)^4[8x+12] \\ &= 4x^2(x+4)^4[2x+3]\end{aligned}$$

Marking guideline:

2 correct answer

1 correct use of product rule

Marker's comments:
Students were not factorising fully. Algebra skills were not very good and a few students did not attempt the question.

(b) Find $\frac{d}{dx}\left(\frac{ax^2}{ax+b}\right)$. Fully factorise your answer. **2**

$u = ax^2 \qquad v = ax+b$

$\frac{du}{dx} = 2ax \qquad \frac{dv}{dx} = a$

$$\begin{aligned}\frac{dy}{dx} &= \frac{(ax+b)(2ax) - a^2x^2}{(ax+b)^2} \\ &= \frac{2a^2x^2 + 2axb - a^2x^2}{(ax+b)^2} \\ &= \frac{a^2x^2 + 2axb}{(ax+b)^2} \\ &= \frac{ax(ax+2b)}{(ax+b)^2}\end{aligned}$$

Marking guideline:

2 correct answer

1 correct use of quotient rule

Marker's comments:
Some students recalled the quotient rule incorrectly and a few did not attempt the question.

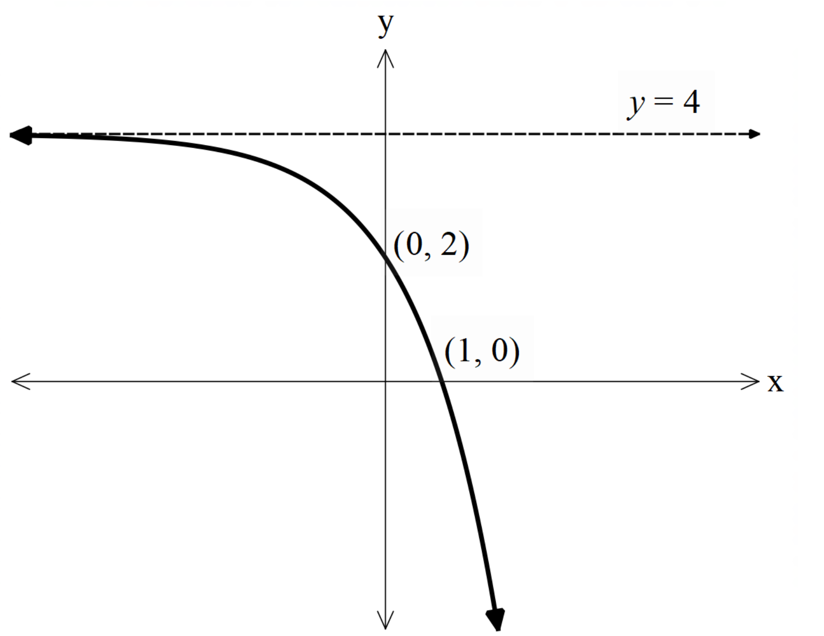
Question 16 (3 marks)

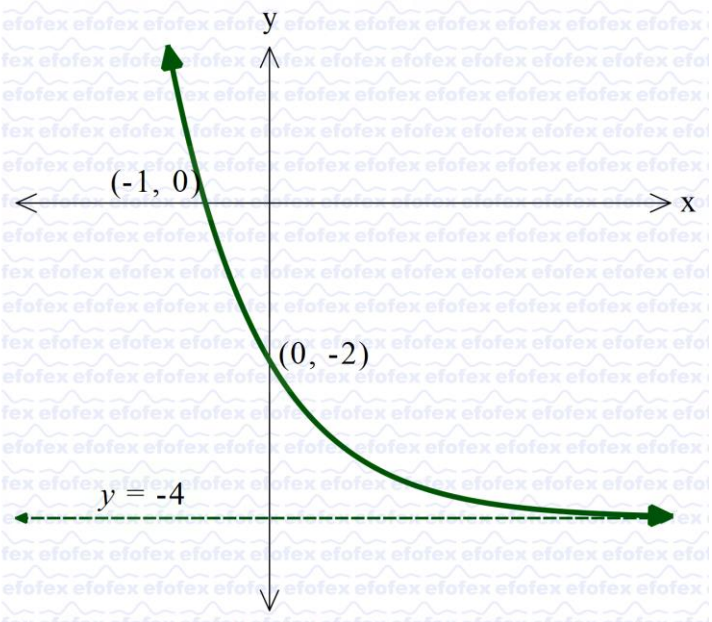
Given the sketch of $y = f(x)$ below, sketch $y = -f(-x)$ on the same set of axes. Ensure you label any asymptotes, x-intercepts, and y-intercepts. **3**

………………………………………………………………………………………………………

………………………………………………………………………………………………………

………………………………………………………………………………………………………





Marking guideline:

3	Correct Answer
2	Any 2 of correct shape, asymptote and intercepts
1	Any 1 of correct shape, asymptote and intercepts

Marker's comments:
Generally answered well.

Section II – Part C **Student Number:**

Question 17 (4 marks)

(a) Simplify the expression $\frac{a^2-3a}{2ax}\times\frac{4a^2x}{2ax-6x}$, giving your answer as a simple fraction. **2**

$$\frac{a(a-3)}{2ax}\times\frac{4a^2x}{2x(a-3)}=\frac{a^2}{x}$$

Marking guideline:

2 correct answer

1 some correct factorisation and or manipulation

Marker's comments:

(b) Solve the equation $\frac{3x+5}{2}=4x-1$. **2**

$$3x+5=2(4x-1)$$
$$3x+5=8x-2$$
$$5x=7$$
$$x=\frac{7}{5}$$

Marking guideline:

2 correct answer

1 some correct factorisation and or manipulation

Marker's comments:

Question 18 (3 marks)

Solve the following trigonometric equations for $0\le x\le 2\pi$.

(a) $\tan x = \sqrt{3}$ **1**

1^{st} and 3^{rd} quadrants

$$x = \frac{\pi}{3},\ \pi + \frac{\pi}{3}$$
$$x = \frac{\pi}{3},\ \frac{4\pi}{3}$$

Marking guideline:

1 Correct response

Marker's comments:

Question 21 (b) is on the next page.

(b) $\sin 2x = \frac{1}{2}$ **2**

1^{st} and 2^{nd} quadrants

$Domain$:

$$0 \le x \le 2\pi$$
$$0 \le 2x \le 4\pi$$

$$2x = \sin^{-1}(\frac{1}{2})$$
$$2x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$$
$$x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$$

Marking guideline:

2 correct answer

1 Finds partially correct answer. E.g. failing to consider the change in domain or equivalent

Marker's comments:

Question 19 (2 marks)

Solve $8^x = 2^{2x+1}$ **2**

$$(2^3)^x = 2^{2x+1}$$
$$2^{3x} = 2^{2x+1}$$
$$3x = 2x+1$$
$$x = 1$$

Marking guideline:

2 correct answer

1 correct equates indices

Marker's comments:

Question 20 (2 marks)

Find the value(s) of m for which the quadratic equation $x^2+(m-1)x+4=0$ has equal roots **2**

Equal roots when

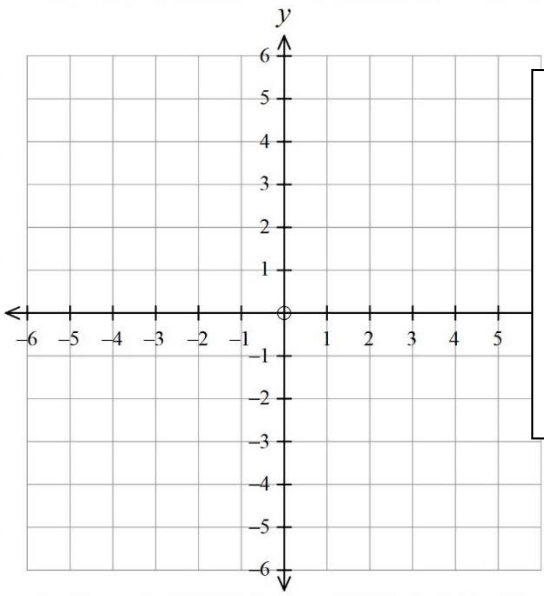
$$\Delta = b^2-4ac=0$$
$$\Delta = (m-1)^2-4(1)(4)=0$$
$$\Delta = (m-1)^2-16=0$$
$$\Delta = m^2-2m+1-16=0$$
$$\Delta = m^2-2m-15=0$$
$$\Delta = (m-5)(m+3)=0$$
$$m=5 \;\; or \;\; m=-3$$

Marking guideline:	
2	correct answer
1	Correct application of discriminant
Marker's comments:	

Question 21 (3 marks)

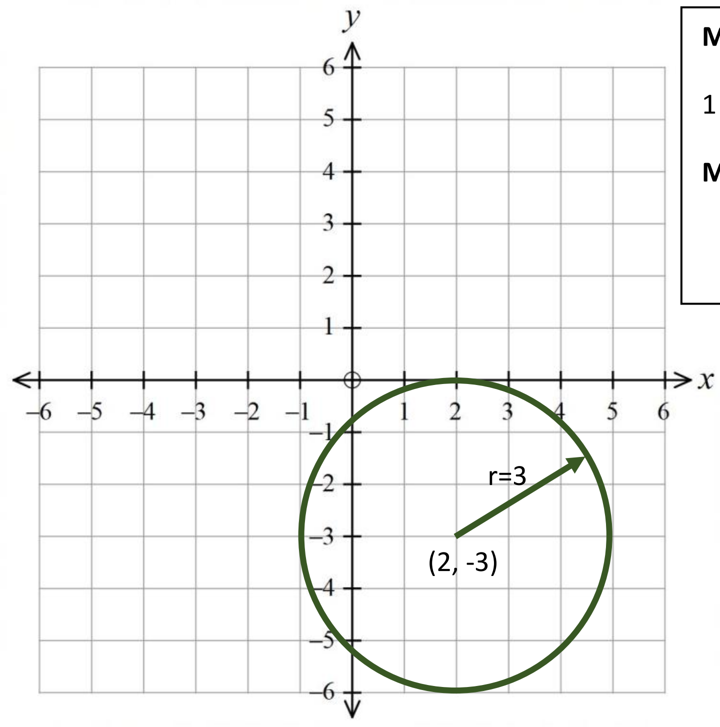
(a) Find the centre and radius **2**

$$x^2-4x+y^2+6y+4=0$$
$$(x-2)^2+(y+3)^2+4-4-$$
$$(x-2)^2+(y+3)^2=9$$
$$so\ centre: (2,-3)$$
$$radius: r=3$$



Marking guideline:	
2	correct centre and radius
1	either correct centre or radius
Marker's comments:	

(b) Draw the circle on the axes below. **1**



Marking guideline:

1 Correct response

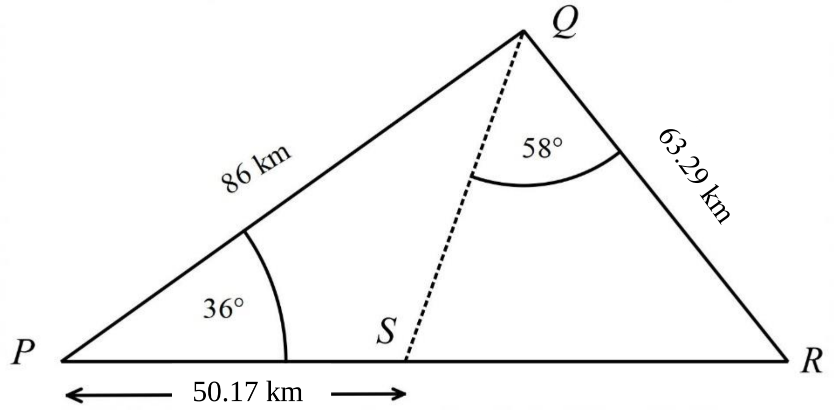
Marker's comments:

Student Number:

Section II – Part D

Question 22 (4 marks)

Triangles PQS and QRS have a common side QS, as shown.



(a) Calculate the length of QS, correct to 2 decimal places. **2**

$$QS^2 = 86^2 - 50.17^2 - 2(86)(50.17)\times\cos(36)$$
$$= 2931.827$$
$$QS = \sqrt{2931.827}$$
$$QS = 54.15km$$

Marking guideline:

2 correct answer rounded to 2 d.p.

1 correct use of cosine rule

Marker's comments:

A few students had their calculators on RADIANS and therefore ended up with the wrong answer.
Many different ways of finding *QS*, although cosine rule was the easiest.

(b) Find the area of $\Delta\,PQR$, correct to the nearest square kilometre. **2**

$$Area = A_{PQS} + A_{QRS}$$
$$= \frac{1}{2}(86)(50.17)\times\sin(36) + \frac{1}{2}(63.29)(54.15)\sin(58)$$
$$= 2721km^2$$

Marking guideline:

2 correct answer to the nearest km^2

1 makes progress towards finding the correct answer

Marker's comments:

Again, many different possible solutions.

Question 23 (5 marks)

Bobby is on a game show where the objective is to run through a shutter door before it closes. The shutter door falls from the roof, and the distance from the floor to the bottom of the shutter door is given by $y = 6 - t$ where y is in metres and t is in seconds. Bobby starts a fixed distance away from the shutter door and can start running when the door begins to drop. His distance from the start line is given by $x = \frac{t^2}{4} + 4t$ where x is in metres and t is in seconds.

(a) Bobby knows he can make it under the shutter door if it is at least 1.5m from the ground. How much time does Bobby have to make it past the door? **1**

$$1.5 = 6 - t$$
$$t = 4.5s$$

Marking guideline:

1 Correct response

Marker's comments:

Done well.

(b) If Bobby starts 20m away, show that he makes it past the shutter door with 0.5s to spare. **2**

$$20 = \frac{t^4}{4} + 4t$$
$$80 = t^2 + 16t$$
$$0 = t^2 + 16t - 80$$
$$0 = (t + 20)(t - 4)$$
$$\therefore t \neq -20 \text{ } so \text{ } t = 4$$

Therefore Bobby makes it with 0.5s to spare

since 4.5 – 4 = 0.5s

Marking guideline:

2 correct answer and shows that time left is 0.5 *s*

1 correct answer

Marker's comments:

Mostly done well. Students needed to show that there was 0.5 seconds left to spare to receive two marks.

(c) Find Bobby's velocity when he passes the shutter door. **2**

$$\frac{dx}{dt} = \frac{2t}{4} + 4 = \frac{t}{2} + 4$$

$$When \text{ } t = 4, \text{ } \frac{2(2)}{2} + 4 = 6m/s$$

Marking guideline:

2 correct answer

1 differentiates correctly

Marker's comments:

Mostly done well.

Question 24 (3 marks)

Show that $\sin\theta\cot\theta - \cos\theta\sin^2\theta = \cos^3\theta$. **3**

$$\begin{aligned}
LHS &= \sin\theta\cot\theta - \cos\theta\sin^2\theta \\
&= \sin\theta \times \frac{\cos\theta}{\sin\theta} - \cos\theta \times \left(1-\cos^2\theta\right) \\
&= \cos\theta - \cos\theta + \cos^3\theta \\
&= \cos^3\theta \\
&= RHS
\end{aligned}$$

Marking guideline:

3 complete and correct proof

2 significant progress on proof

1 some correct use of trig identities

Marker's comments:

Mostly done well.

Students need to practise properly setting out their trigonometric proofs.

Question 25 (3 marks)

Given that $f(x) = \dfrac{4}{x^2}$

(i) Show that $f(x)$ is an even function. **1**

$$\begin{aligned} f(-x) &= \frac{4}{(-x)^2} \\ &= \frac{4}{x^2} \\ &= f(x) \\ &\therefore f(x) \text{ is an even function} \end{aligned}$$

Marking guideline:

1 correct response

Marker's comments:

It is not enough to prove that $f(-x) = f(x)$ for one particular point (e.g. $x = 1$). You need to show that this condition holds for all x.

(ii) State the domain and range of $f(x)$. **2**

Domain: All reals except $x = 0$

Range: $y > 0$

Marking guideline:

2 correct domain and range

1 only one correct domain or range

Marker's comments:

Many students had trouble finding the range.

Also accepted set notation:

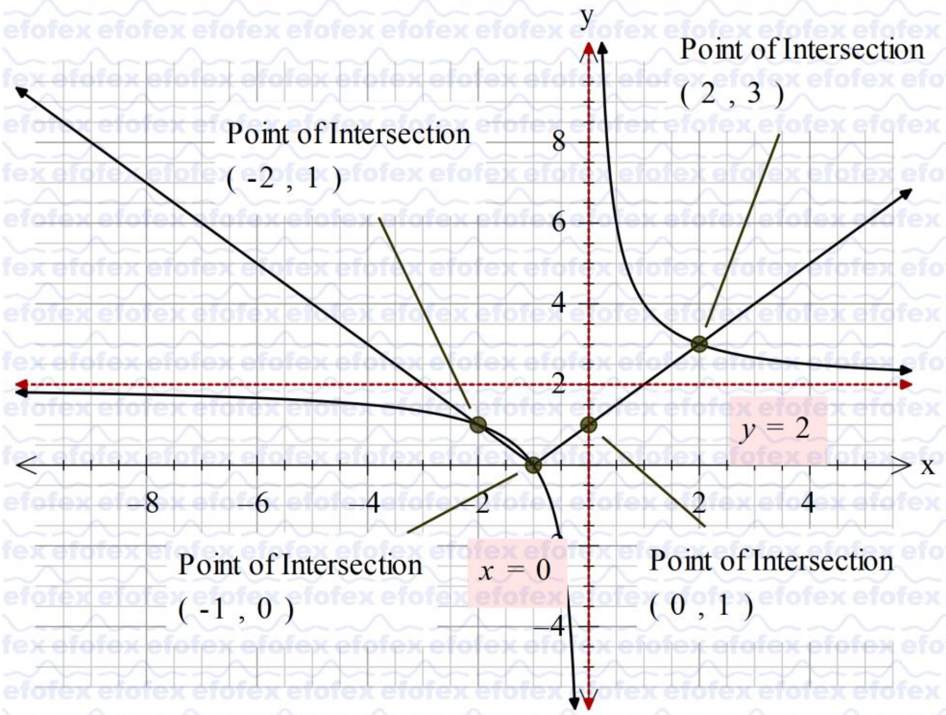
$D: (-\infty, 0) \cup (0, \infty)$ and $R: (0, \infty)$

Section II – Part E

Student Number:

Question 26 (5 marks)

(a) On the axes below, draw the graphs of $y=|x+1|$ and $y=\dfrac{2}{x}+2$. Label any intercepts, asymptotes, or other key information. **4**



Marking guideline:

4 Correct answer with intercepts and asymptotes clearly labelled

3 One curve is correct and one error on the other curve

2 Either both curves are partially correct or one curve is correct and the other is incorrect

1 One curve is partially correct, while the other curve is incorrect

Marker's comments:

Poorly graphed; points of intersection & intercepts and asymptotes (or their equations) NOT labelled. Students need to use the method in the solution where they use an arrow to label instead of writing the point so small it cannot be seen. Some left out one branch of the hyperbola. Graphs need to be neat/ clearly drawn with one continuous line/ showing main features (intercepts) on clearly labelled axes.

(b) Use the graph to find solutions to $|x+1|=\dfrac{2}{x}+2$

The solutions are the points of intersection so $x=-2,-1,2$

Marking guideline:

1 Correct response

Marker's comments: Mostly answered well. Some included the points of intersection instead of the solutions; some did not transfer these points to their graph in part (a).

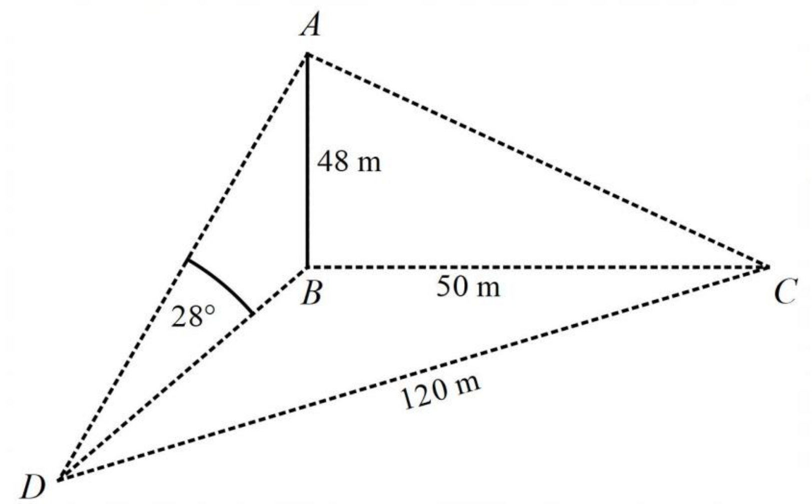
Question 27 (4 marks)

A 48-metre high, vertical lighting pole *AB* stands in the middle of a flat sporting ground.

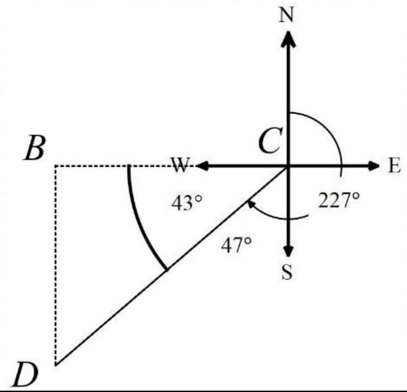
Diana starts from the base of the pole and walks 50 m due east to a point *C* where she places a peg.

She attaches a 120 m long string to the peg and walks in a roughly south-westerly direction until the string is taut, at point *D*.

She then measures the angle of elevation of the pole to be 28°.



Use the information provided to find the *actual* bearing on which she was walking from *C* to *D*, correct to the nearest degree. **4**



Find the length of BD using Δ ABD.

$\tan 28° = \dfrac{48}{BD}$

$BD = \dfrac{48}{\tan 28°}$

$= 90.27487033$

Find the size of $\angle BCD$ using $\Delta\, BCD$

$\cos \angle BCD = \dfrac{120^2 + 50^2 - 90.27^2}{2 \times 120 \times 50}$

$= \dfrac{8750.447787}{12000}$

$= 0.72920398$

$\angle BCD = \cos^{-1}(0.72920398)$

$= 43.1803$

$= 43°$

The bearing is 227° or $S47^{\circ}W$

Marking guideline:

4	Correct answer
3	mainly correct working with a minor error or angle only, not bearing
2	working with more than one error in calculation or reasoning or only partially completed
1	working with some correct and relevant calculations

Marker's comments: Mostly answered very well. Some students could not interpret the information or diagram correctly (3D problem). Some incorrectly used angle B as a right angle. Some students wasted time by finding unnecessary information e.g. length of AC or < BCA.

Question 28 (6 marks)

Given the parabola $y = x^2 - 4x + 4$

(a) Show that the equation of the normal to the tangent of the parabola at the point $P(4,4)$ is $y = -\frac{x}{4} + 5$ **3**

$y = x^2 - 4x + 4$

$\frac{dy}{dx} = 2x - 4$

$at\ x = 4,\ \frac{dy}{dx} = 8 - 4$

$so\ m = 4$

$\therefore\ perpendicular\ \ gradient\ \ is\ m = -\frac{1}{4}$

$m = -\frac{1}{4}$

$(x_1, y_1) = (4,\ 4)$

$y - 4 = -\frac{1}{4}(x - 4)$

$y - 4 = \frac{-x}{4} + 1$

$y = -\frac{x}{4} + 5$

Marking guideline:

3	correct answer
2	Finds correct gradient of the normal, but incorrectly finds equation of the line, or equivalent
1	Correctly finds the gradient of the tangent

Marker's comments: Students need to practise answering "Show" questions.

Students did not "show" instead going from $y - 4 = -\frac{1}{4}(x - 4)$ to $y = -\frac{x}{4} + 5$

(b) Find the other point of intersection $Q(x_1, y_1)$ between the parabola and the normal to the tangent at $P(4,4)$. **3**

(1) $y = x^2 - 4x + 4$

(2) $y = -\frac{x}{4} + 5$

$(1) = (2)$

$x^2 - 4x + 4 = -\frac{x}{4} + 5$

$4x^2 - 16x + 16 = -x + 20$

$4x^2 - 15x - 4 = 0$

$$x = \frac{15 \pm \sqrt{15^2 - 4(4)(-4)}}{2(4)}$$

$x = 4, -\frac{1}{4}$

But since point P is at $x = 4$, then Q is at $x = \frac{-1}{4}$

So the y value of Q is

$$y = \frac{-x}{4} + 5$$

$$y = \frac{-(-\frac{1}{4})}{4} + 5$$

$$y = \frac{81}{6}$$

$$\therefore Q : (-\frac{1}{4}, \frac{81}{16})$$

Marking guideline:

3 complete answer

2 correctly equates (1) and (2) and finds the possible solutions for x

1 attempts to equate (1) and (2) and starts to solve for x

Marker's comments:

(b) Mostly very well done.

BUT:

(i)Careless errors made when substituting negative numbers, see below:

$$y = \frac{-(-\frac{1}{4})}{4} + 5$$

(ii)Students need to remember to answer the question writing at least

$$\therefore Q : (-\frac{1}{4}, \frac{81}{16})$$