



**2021**

**Year 11 Assessment Task 3 - Online examination**

## Mathematics Advanced

Total Marks: 55

Working time: 65 minutes

### General Instructions

- This is an open book task
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet will be provided
- Show relevant mathematical reasoning and/or calculations

### Specific Instructions:

- You will not be able to leave your desk for the duration of the task.
- Mobile phones must be turned off and out of sight.
- Your microphone and cameras must be on, but you can turn the volume on your devices down so that any noise from other students does not disturb you.
- You are not permitted to have headphones/ear buds and if you have long hair, please tie it back.
- You can ask for assistance through the direct chat function of Zoom/Teams or ask as your microphone is on.
- You are to manage your time and make sure you have a timer at hand to keep within the assessment time limit.
- At the end of the assessment, you will have 15 minutes to scan and submit your task on Google Classroom. During this time, if you have any difficulty with submitting your task, please communicate this with your teacher immediately.
- Please note that submission of the task is your responsibility.

**Question 1. (6 marks)**

(a) Rationalise the denominator of  $\frac{4+\sqrt{2}}{4-\sqrt{2}}$ , simplifying your answer. **2**

(b) Simplify  $\frac{2}{x+1} - \frac{1}{x^2-1}$ . **2**

(c) Solve by completing the square  $2x^2 - 14x + 5 = 0$  leaving you answer in exact form **2**

**Question 2. (7 marks)***Start a separate Booklet*

- (a) Consider the function  $f(x) = (x-1)(x+4)^2$ .
- (i) Find  $f(0)$  **1**
- (ii) Determine whether  $f(x)$  is considered odd, even or neither.  
Justify your answer. **1**
- (iii) Describe how the graph intersects with the  $x$  axis? **2**
- (b) Consider  $f(x) = \sqrt{18-x^2}$ .
- (i) State the domain and range of  $f(x)$  **2**
- (ii) Identify the type of relation as one-to-one or one-to-many or many-to-one or many-to-many **1**

**Question 3. (8 marks)**

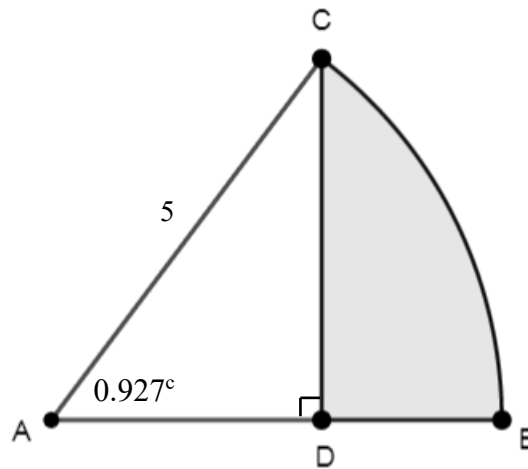
*Start a separate Booklet*

- (a) Starting from point  $A$ , a hiker walks 800 metres on a bearing of  $053^\circ T$ , reaching point  $B$ . From point  $B$  the hiker walks a further 1.35 kilometres on a bearing of  $165^\circ T$  until point  $C$ .
- (i) Draw a one-third page diagram of the hiker's path. **1**
- (ii) What is the distance from  $A$  to  $C$  correct to the nearest metre? **2**
- (iii) What bearing should the hiker take to get back to  $A$  from  $C$ ? **3**
- (b) Show that  $(\operatorname{cosec} x + \cot x)(\operatorname{cosec} x - \cot x) = 1$  **2**

**Question 4. (7 marks)**

*Start a separate Booklet*

- (a) Sector  $ABC$  has an angle of  $0.927$  radians and a radius of  $5$  cm.



Find the area of the shaded region  $BCD$  correct to 2 decimal places.

**3**

- (b) Solve  $6\sin^2 \theta - \sin \theta - 1 = 0$  for  $0^\circ \leq \theta \leq 360^\circ$ , correct to the nearest degree.

**4**

**Question 5. (7 marks)***Start a separate Booklet*

- (a) A dragster is a car that races in a straight line.

Its displacement  $d$  over 400 m is modelled by the equation

$$d(t) = t^2 + 3t - 2 \text{ where time } t \text{ in seconds is } 0 \leq t \leq 20.$$

- (i) The difference quotient is given by  $\frac{d(t+h) - d(t)}{h}$

What is the meaning of the difference quotient in this context?

**1**

- (ii) Use the difference quotient to find the average speed of the car over the first 10 seconds.

**2**

- (b) (i) Use the product rule to differentiate  $x^2 e^x$  with respect to  $x$ .

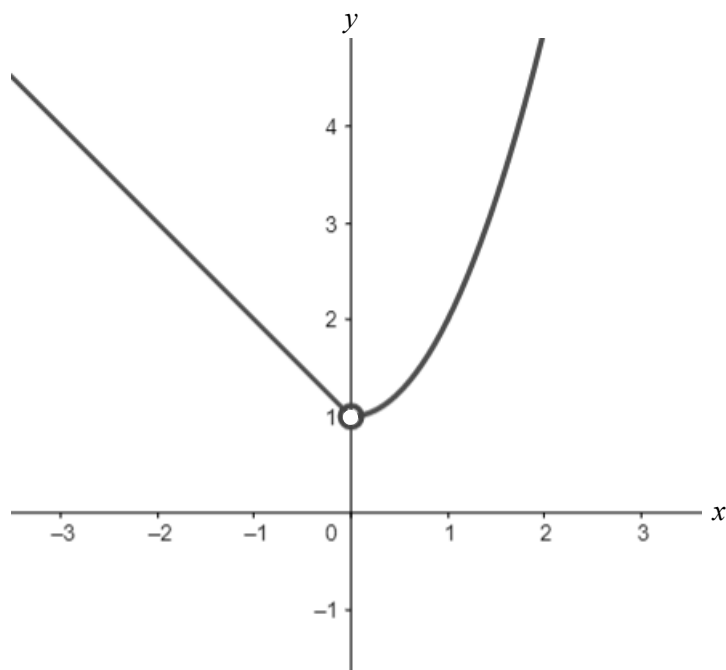
**2**

- (ii) Use the quotient rule to differentiate  $\frac{x^3}{(x-1)^2}$  with respect to  $x$ , simplify your answer.

**2**

**Question 6. (7 marks)***Start a separate Booklet*

- (a) The graph of  $y = f(x)$  where  $x \neq 0$  is shown below.



- |       |                                                        |          |
|-------|--------------------------------------------------------|----------|
| (i)   | Explain why $y = f(x)$ is continuous or discontinuous? | <b>1</b> |
| (ii)  | For what values of $x$ is $f(x)$ decreasing?           | <b>1</b> |
| (iii) | Sketch a one-third of the page graph of $y = f'(x)$    | <b>2</b> |
- (b) Find the equation of a normal to the curve  $y = x^3 - 2x$  at the point  $P$  where  $x = 2$  **3**

**Question 7. (6 marks)***Start a separate Booklet*

- (a) Loudness,  $L$  in decibels dB, is given by the formula

$$L = 10 \log_{10} \left( \frac{I}{I_0} \right),$$

where  $I$  is the intensity level and  $I_0$  is the threshold sound (or sound that can barely be heard).

During a particular day, city traffic noise level was measured to be 67 decibels.

- (i) How many more times the threshold of sound is city traffic? **2**
- (ii) Late at night, the traffic noise's intensity was measured to be half of what it is during the day.
- How many decibels is this, correct to one decimal place? **1**

- (b) Solve for  $x$ ,  $3 \ln x = \ln x - \ln 3$ . **3**



**Question 8. (7 marks)***Start a separate Booklet*

- (a) Gertrude heard that the number of branches on a tree after it was planted could be modelled using the formula

$$N(t) = A(1.04)^t$$

where  $N(t)$  is the number of branches after time  $t$  in years.

- (i) What does the value of  $A$  represent in this equation? **1**

- (ii) Gertrude bought a tree from a nursery and noted it had 9 branches when she planted it.

To the nearest month, how long does it take for the tree to have 200 branches? **2**

- (b) A large stone is sinking through a layer of mud. At time  $t$  seconds, the displacement of the stone in metres below the surface is given by

$$x = 20 \left( 1 - e^{-\frac{1}{2}t} \right)$$

where downwards is taken to be in the positive direction.

- (i) Find an equation for the velocity,  $v$ , of the stone in terms of  $t$  where  $v = \frac{dx}{dt}$ . **1**

- (ii) Hence find the acceleration,  $a$ , of the stone after 2 seconds, where  $a = \frac{dv}{dt}$ . **2**

- (iii) What happens to displacement as  $t \rightarrow \infty$ ? **1**

**End of examination**

# Fort St High School



**2021**

Year 11 Assessment Task 3 - Online examination

# Mathematics Advanced SOLUTIONS

**Question 1. (6 marks)**

- (a) Rationalise the denominator of  $\frac{4+\sqrt{2}}{4-\sqrt{2}}$ , simplifying your answer.

**Solution**

$$\begin{aligned}\frac{4+\sqrt{2}}{4-\sqrt{2}} &= \frac{4+\sqrt{2}}{4-\sqrt{2}} \times \frac{4+\sqrt{2}}{4+\sqrt{2}} \\ &= \frac{16+8\sqrt{2}+2}{16-2} \\ &= \frac{18+8\sqrt{2}}{14} \\ &= \frac{9+4\sqrt{2}}{7}\end{aligned}$$

| Marking Guidelines                                                                                                                                                                                                                                                                                               |                                                     |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------|
| 2                                                                                                                                                                                                                                                                                                                | Correct response                                    |
| 1                                                                                                                                                                                                                                                                                                                | Un-simplified answer or multiplied by the conjugate |
| Marker's comments                                                                                                                                                                                                                                                                                                |                                                     |
| <p>Too many careless errors. Take your time with the first few questions to avoid unnecessary errors.</p> <p>Extremely poor communication, involving working across the page and lack of equivalence (= signs).</p> <p>Basic arithmetic skills, such as knowing how to cancel factors, are severely lacking.</p> |                                                     |

- (b) Simplify  $\frac{2}{x+1} - \frac{1}{x^2-1}$ .

**Solution**

$$\begin{aligned}\frac{2}{x+1} - \frac{1}{x^2-1} &= \frac{2}{x+1} - \frac{1}{(x-1)(x+1)} \\ &= \frac{2(x-1)}{(x+1)(x-1)} - \frac{1}{(x-1)(x+1)} \\ &= \frac{2x-3}{(x+1)(x-1)}\end{aligned}$$

| Marking Guidelines                                                                           |                                                                                              |
|----------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------|
| 2                                                                                            | Correct response                                                                             |
| 1                                                                                            | Un-simplified answer or one arithmetic error (ok for the denominator to be in factored form) |
| Marker's comments                                                                            |                                                                                              |
| <p>Basic arithmetic skills, such as knowing how to cancel factors, are severely lacking.</p> |                                                                                              |

(c) Solve by completing the square  $2x^2 - 14x + 5 = 0$  leaving you answer in exact form

**Solution**

$$2x^2 - 14x + 5 = 0$$

$$2x^2 - 14x = -5$$

$$x^2 - 7x = -\frac{5}{2}$$

$$x^2 - 7x + \left(-\frac{7}{2}\right)^2 = \left(-\frac{7}{2}\right)^2 - \frac{5}{2}$$

$$\left(x - \frac{7}{2}\right)^2 = \frac{49}{4} - \frac{10}{4}$$

$$\left(x - \frac{7}{2}\right)^2 = \frac{39}{4}$$

$$x - \frac{7}{2} = \pm \frac{\sqrt{39}}{2}$$

$$x = \frac{7 \pm \sqrt{39}}{2}$$

| Marking Guidelines                                                                                                                                                                                                                                                                                                                                                                                                          |                                                                                                  |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------|
| 2                                                                                                                                                                                                                                                                                                                                                                                                                           | Correct response (requires steps that demonstrate understanding of completing the square method) |
| 1                                                                                                                                                                                                                                                                                                                                                                                                                           | Evidence of correct procedure with incorrect answer.                                             |
| Marker's comments                                                                                                                                                                                                                                                                                                                                                                                                           |                                                                                                  |
| <p>Extremely poor communication, involving working across the page and lack of equivalence (= signs).<br/> Many students didn't divide through by 2 to make the equation monic before completing the square.<br/> Stating answers without working or using the quadratics formula <b>did not</b> attract any marks as you are required to demonstrate the completing the square method.<br/> Avoid the use of decimals.</p> |                                                                                                  |

**Question 2. (7 marks)**

(a) Consider the function  $f(x) = (x-1)(x+4)^2$ .

(i) Find  $f(0)$

**Solution**

$$\begin{aligned} f(0) &= (0-1)(0+4)^2 \\ &= -16 \end{aligned}$$

| Marking Guidelines |                  |
|--------------------|------------------|
| 1                  | Correct response |
| Marker's comments  |                  |
| Mostly Done Well.  |                  |

(ii) Determine whether  $f(x)$  is considered odd, even or neither. Justify your answer.

**Solution**

The test for odd or even is to find the result of  $f(-x)$

$$\begin{aligned} f(-x) &= (-x-1)(-x+4)^2 \\ &= -(x+1)(4-x)^2 \\ &\neq f(x) \text{ or } -f(x) \end{aligned}$$

| Marking Guidelines                                                                                                      |                                                                                    |
|-------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------|
| 1                                                                                                                       | Correct response (requires the result of $f(-x)$ or refer to symmetry of a sketch) |
| Marker's comments                                                                                                       |                                                                                    |
| Mostly done well. Make sure you show all steps. It is not enough to state the rule, you need to show it mathematically. |                                                                                    |

By the test it is neither an odd or even function

(iii) Describe how the graph intersects with the  $x$  axis?

**Solution**

From the factored form of the equation

$$f(x) = (x-1)(x+4)^2$$

We can deduce there is a single root at  $x = 1$  and

A double root at  $x = -4$

A single root has a graph cut through the  $x$ -axis.

A double root is a tangent to the  $x$ -axis.

| Marking Guidelines                                                                                                                                                                                                                                      |                                                                    |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------|
| 2                                                                                                                                                                                                                                                       | Correct response (requires description i.e. cuts and is a tangent) |
| 1                                                                                                                                                                                                                                                       | States single and double root.                                     |
| Marker's comments                                                                                                                                                                                                                                       |                                                                    |
| Mostly done well. To get full marks you needed to state that there is a double root/turning point/tangent to the $x$ -axis when $x = -4$ . It was not enough to simply state the $x$ value of both intercepts without mentioning the type of intercept. |                                                                    |

(b) Consider  $f(x) = \sqrt{18 - x^2}$ .

(i) State the domain and range of  $f(x)$

**Solution**

Domain:  $[-3\sqrt{2}, 3\sqrt{2}]$

Range:  $[0, 3\sqrt{2}]$

| Marking Guidelines |                                                 |
|--------------------|-------------------------------------------------|
| 2                  | Correct response (accept inequalities)          |
| 1                  | One correct answer or confused domain and range |
| Marker's comments  |                                                 |
| Mostly done well.  |                                                 |

(ii) Identify the type of relation as one-to-one or one-to-many or many-to-one or many-to-many

**Solution**

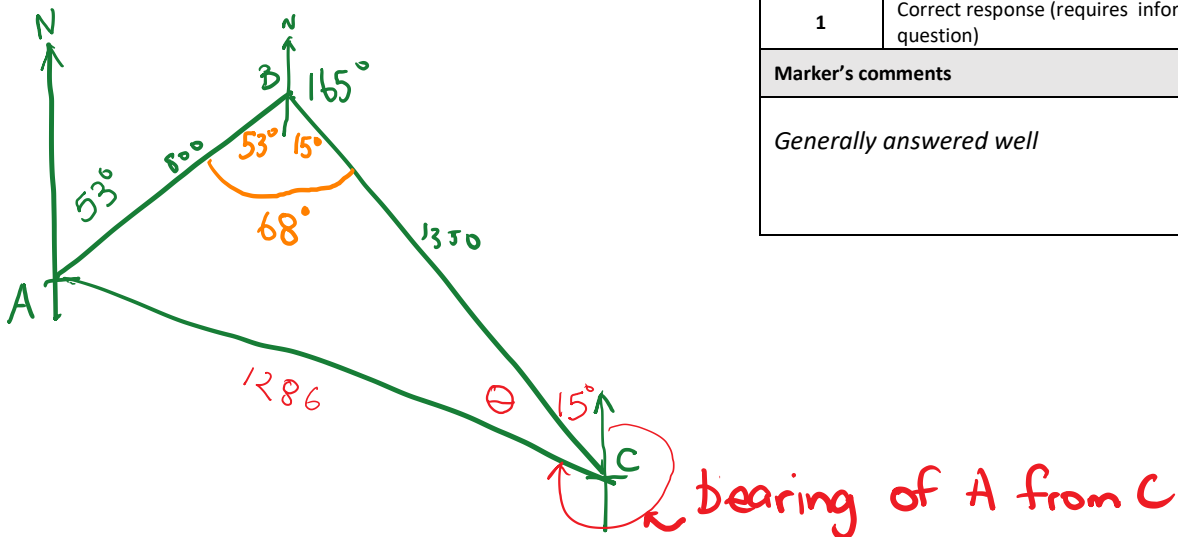
Many-to-one

| Marking Guidelines |                                                                                    |
|--------------------|------------------------------------------------------------------------------------|
| 1                  | Correct response (requires the result of $f(-x)$ or refer to symmetry of a sketch) |
| Marker's comments  |                                                                                    |
| Mostly done well.  |                                                                                    |

**Question 3. (8 marks)**

- (a) Starting from point  $A$ , a hiker walks 800 metres on a bearing of  $053^\circ T$ , reaching point  $B$ . From point  $B$  the hiker walks a further 1.35 kilometres on a bearing of  $165^\circ T$  until point  $C$ .
- (i) Draw a one-third page diagram of the hiker's path.

**Solution**



| Marking Guidelines      |                                                           |
|-------------------------|-----------------------------------------------------------|
| 1                       | Correct response (requires information from the question) |
| Marker's comments       |                                                           |
| Generally answered well |                                                           |

- (ii) What is the distance from  $A$  to  $C$  correct to the nearest metre?

**Solution**

$$AC^2 = AB^2 + BC^2 - 2(AB)(BC)\cos B$$

$$AC^2 = 800^2 + 1350^2 - 2 \times 800 \times 1350 \times \cos 68$$

$$AC = 1286 \text{ m correct to the nearest metre}$$

| Marking Guidelines                                                                                                                        |                                               |
|-------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------|
| 2                                                                                                                                         | Correct response                              |
| 1                                                                                                                                         | One correct substitution into the cosine rule |
| Marker's comments                                                                                                                         |                                               |
| <p>Generally answered well</p> <p>Although the cosine rule is provided in the formula sheet, a few students used an incorrect formula</p> |                                               |

(iii) What bearing should the hiker take to get back to  $A$  from  $C$ ?

**Solution**

To find the bearing, first find  $\theta$  in the above diagram

$$\begin{aligned}\frac{\sin \theta}{800} &= \frac{\sin 68^\circ}{1286} \\ \sin \theta &= \frac{800 \sin 68^\circ}{1286} \\ &= 35.22^\circ\end{aligned}$$

Therefore the bearing is  $= 360^\circ - 15^\circ - 35.22^\circ$   
 $= 309^\circ 47' T$  to the nearest minute

| Marking Guidelines                                                                                                                                                                                                                                        |                                    |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------|
| 3                                                                                                                                                                                                                                                         | Correct response (ignore rounding) |
| 2                                                                                                                                                                                                                                                         | Found $\theta$                     |
| 1                                                                                                                                                                                                                                                         | Progress towards answer            |
| Marker's comments                                                                                                                                                                                                                                         |                                    |
| <p><i>Generally answered well</i><br/> <i>A mark was deducted where answers were given as compass bearings instead of true bearing.</i><br/> <i>Although the sine rule is provided in the formula sheet, a few students used an incorrect formula</i></p> |                                    |

(b) Show that  $(\operatorname{cosec} x + \cot x)(\operatorname{cosec} x - \cot x) = 1$

**Solution**

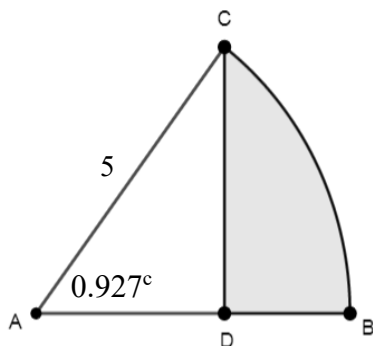
$$\begin{aligned}LHS &= (\operatorname{cosec} x + \cot x)(\operatorname{cosec} x - \cot x) \\ &= \operatorname{cosec}^2 x - \cot^2 x \\ &= 1 + \cot^2 x - \cot^2 x \\ &= 1 \\ &= RHS\end{aligned}$$

| Marking Guidelines                                                                                        |                              |
|-----------------------------------------------------------------------------------------------------------|------------------------------|
| 2                                                                                                         | Correct response             |
| 1                                                                                                         | Correct expansion of factors |
| Marker's comments                                                                                         |                              |
| <p><i>Generally answered well but some students did not include significant steps in the process.</i></p> |                              |



**Question 4. (7 marks)**

- (a) Sector  $ABC$  has an angle of  $0.927$  radians and a radius of  $5$  cm.



Find the area of the shaded region  $BCD$  correct to 2 decimal places.

**Solution**

Area shaded = Area of the sector  $ACB$  – Area of the triangle  $ACD$

$$\begin{aligned}\text{Area of the sector} &= \frac{1}{2}(5)^2(0.927) \\ &= 11.5875\end{aligned}$$

To find the area of the triangle, first find  $AD$

$$\begin{aligned}AD &= 5\cos(0.927) \\ &\approx 3.001\ldots\end{aligned}$$

$$\begin{aligned}\text{Area of the triangle} &= \frac{1}{2} \times 5 \times (AD) \times \sin(0.927) \\ &\approx 6.001\end{aligned}$$

$$\begin{aligned}\text{Area shaded} &= 11.5875 - 6.001 \\ &\approx 5.59 \text{ cm}^2\end{aligned}$$

| Marking Guidelines                                                                                                                                                                                                                               |                                                                  |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------|
| 3                                                                                                                                                                                                                                                | Correct response (ignore rounding and units)                     |
| 2                                                                                                                                                                                                                                                | Found $AD$ and area of the sector <b>or</b> area of the triangle |
| 1                                                                                                                                                                                                                                                | Found area of the sector                                         |
| Marker's comments                                                                                                                                                                                                                                |                                                                  |
| <i>Poorly done. Common errors were using the incorrect formula for the area of a sector, rounding answers early and saying triangle <math>ADC</math> is a 3,4,5 right triangle. Many students lost 1 mark for rounding early more than once.</i> |                                                                  |

(b) Solve  $6\sin^2 \theta - \sin \theta - 1 = 0$  for  $0^\circ \leq \theta \leq 360^\circ$ , correct to the nearest degree.

**Solution**

Let  $m = \sin \theta$

$$6m^2 - m - 1 = 0$$

$$(3m + 1)(2m - 1) = 0$$

$$m = \frac{1}{2}, \text{ or } m = -\frac{1}{3}$$

$$\sin \theta = \frac{1}{2} \text{ or } \sin \theta = -\frac{1}{3}$$

$$\sin \theta = \frac{1}{2}$$

$$\theta = 30^\circ, 150^\circ$$

$$\sin \theta = -\frac{1}{3}$$

$$\theta \approx 199^\circ \text{ or } 341^\circ$$

| Marking Guidelines                                                                                                                                                   |                                                          |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------|
| 4                                                                                                                                                                    | Correct response (ignore rounding)                       |
| 3                                                                                                                                                                    | Solutions with one error                                 |
| 2                                                                                                                                                                    | Finding the ratios (i.e. solving the quadratic)          |
| 1                                                                                                                                                                    | For recognising the equation is reducible to a quadratic |
| Marker's comments                                                                                                                                                    |                                                          |
| Generally, well done. Please note that answers had to be given to the nearest degree, However, this time it was ignored if answers were given to the nearest minute. |                                                          |

**Question 5. (7 marks)**

- (a) A dragster is a car that races in a straight line. Its displacement  $d$  over 400 m is modelled by the equation

$$d(t) = t^2 + 3t - 2 \text{ where time } t \text{ in seconds is } 0 \leq t \leq 20.$$

- (i) The difference quotient is given by  $\frac{d(t+h) - d(t)}{h}$

What is the meaning of the difference quotient in this context?

**Solution**

The difference quotient is the average rate of change.

In this context the wording of part (ii) gives it away.

It is the average velocity over a period of time or

The average change in displacement over change in time

Note: The derivative  $d'(t)$  is the instantaneous rate of change

| Marking Guidelines                                                                                                                                                                                                                                                                                                                        |                                                                                              |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------|
| 1                                                                                                                                                                                                                                                                                                                                         | Correct response (requires key concept of average rate of change in velocity – accept speed) |
| Marker's comments                                                                                                                                                                                                                                                                                                                         |                                                                                              |
| Not well done. The vast majority of students confused the difference quotient with first principles differentiation and consequently scored 0 marks for answers like velocity or speed as these are instantaneous and are not the average rate of change represented by the gradient of a secant between $t=0$ and $t=10$ (where $h=10$ ) |                                                                                              |

- (ii) Use the difference quotient to find the average speed of the car over the first 10 seconds.

**Solution**

Here  $t = 0, h = 10$  Velocity and speed are both positive over 10 seconds.

Therefore you can use them interchangeably.

$$\begin{aligned}
 &= \frac{d(t+h) - d(t)}{h} \\
 &= \frac{d(0+10) - d(0)}{10} \\
 &= \frac{(10^2 + 3 \times 10 - 2) - (0^2 - 3 \times 0 - 2)}{10} \\
 &= \frac{130}{10}
 \end{aligned}$$

Average speed =  $13 \text{ ms}^{-1}$

| Marking Guidelines                                                                                                                                                                                                                                      |                                                                       |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------|
| 2                                                                                                                                                                                                                                                       | Correct response (requires substitution into the difference quotient) |
| 1                                                                                                                                                                                                                                                       | Partially correct solution                                            |
| Marker's comments                                                                                                                                                                                                                                       |                                                                       |
| Many students worked out the derivative using the first principles of differentiation formula which was not required by the question. Many students that did understand what the difference quotient is, substituted incorrect values for $t$ and $h$ . |                                                                       |

- (b) (i) Use the product rule to differentiate  $x^2e^x$  with respect to  $x$ .

**Solution**

$$\begin{aligned}\frac{d}{dx} x^2 e^x &= x^2 \times (e^x) + e^x (2x) \\ &= x e^x (x + 2)\end{aligned}$$

| Marking Guidelines                                                    |                                                                               |
|-----------------------------------------------------------------------|-------------------------------------------------------------------------------|
| 2                                                                     | Correct response (requires evidence that the product rule was used correctly) |
| 1                                                                     | Partially correct solution                                                    |
| Marker's comments                                                     |                                                                               |
| Well done. Some students did not find the correct derivative of $e^x$ |                                                                               |

- (ii) Use the quotient rule to differentiate  $\frac{x^3}{(x-1)^2}$  with respect to  $x$ , simplify your answer.

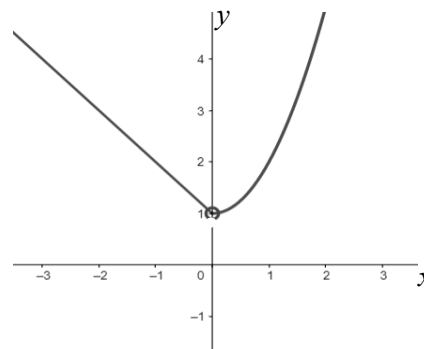
**Solution**

$$\begin{aligned}\frac{d}{dx} \frac{x^3}{(x-1)^2} &= \frac{(x-1)^2 \times (3x^2) - x^3 \times 2(x-1)(1)}{(x-1)^4} \\ &= \frac{x^2(x-1)(3(x-1) - 2x)}{(x-1)^4} \\ &= \frac{x^2(x-3)}{(x-1)^3}\end{aligned}$$

| Marking Guidelines                                                                                                              |                                                                                |
|---------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------|
| 2                                                                                                                               | Correct response (requires evidence that the quotient rule was used correctly) |
| 1                                                                                                                               | Partially correct solution                                                     |
| Marker's comments                                                                                                               |                                                                                |
| Most students correctly substituted into the quotient rule. Some students were unable to simplify and consequently lost a mark. |                                                                                |

**Question 6. (7 marks)**

(a) The graph of  $y = f(x)$  where  $x \neq 0$  is shown below.



(i) Explain why  $y = f(x)$  is continuous or discontinuous?

**Solution**

$y = f(x)$  is discontinuous because  $f(0)$  does not exist.

| Marking Guidelines                                                                                                                  |                                                         |
|-------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------|
| 1                                                                                                                                   | Correct response (requires understanding of continuity) |
| Marker's comments                                                                                                                   |                                                         |
| Mostly well done. Accepted most informal definitions of discontinuity as well, as outlined in syllabus i.e. 'break'/'gap' in curve. |                                                         |

(ii) For what values of  $x$  is  $f(x)$  decreasing?

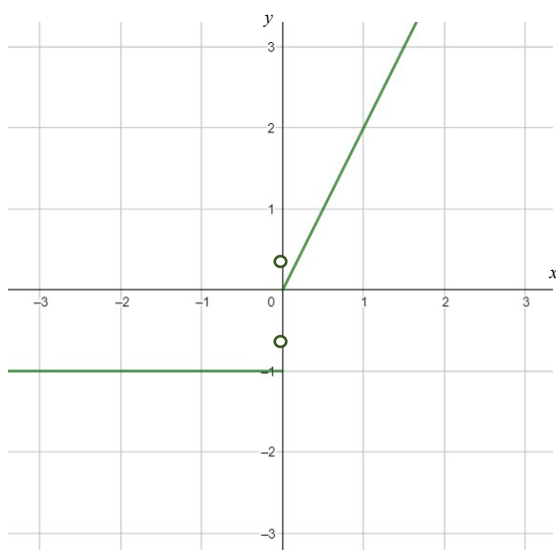
**Solution**

$y = f(x)$  is decreasing for  $x < 0$

| Marking Guidelines                                                                             |                  |
|------------------------------------------------------------------------------------------------|------------------|
| 1                                                                                              | Correct response |
| Marker's comments                                                                              |                  |
| Students need to write <i>entire</i> domain. Make sure to use inequality or interval notation. |                  |

(iii) Sketch a one-third of the page graph of  $y = f'(x)$

**Solution**



| Marking Guidelines                                                                                                                                                                                                                          |                                                                                                                                                           |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------|
| 2                                                                                                                                                                                                                                           | Correct response (requires axes labelled)<br>RHS of the graph should illustrate the gradient is increasing, and LHS should have horizontal line at $x=-1$ |
| 1                                                                                                                                                                                                                                           | Partially correct solution                                                                                                                                |
| Marker's comments                                                                                                                                                                                                                           |                                                                                                                                                           |
| Most students obtained partial marks.<br>Students are encouraged to make graph as correct as possible – having correct features (intercepts, points), if the question permits. i.e. LHS having a gradient of 1 can be found via inspection. |                                                                                                                                                           |

(b) Find the equation of a normal to the curve  $y = x^3 - 2x$  at the point  $P$  where  $x = 2$

**Solution**

The gradient of the tangent is found by

$$m = \frac{dy}{dx} = 3x^2 - 2$$

$$x = 2, \quad m = 3 \times (2)^2 - 2$$

$$\text{At} \quad = 10$$

The gradient of the normal at  $P$

$$10 \times m_2 = -1$$

$$m_2 = -\frac{1}{10}$$

$$x = 2, \quad y = (2)^3 - 2 \times 2$$

$$\text{When} \quad = 4$$

The equation of the normal at  $P$  is

$$y - 4 = -\frac{1}{10}(x - 2)$$

$$10y - 40 = -x + 2$$

$$x + 10y - 42 = 0$$

| Marking Guidelines                                                                                                               |                                                                          |
|----------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------|
| 3                                                                                                                                | Correct response (general form or $y=mx+b$ form)                         |
| 2                                                                                                                                | Found equation of the tangent <b>or</b> one error with correct procedure |
| 1                                                                                                                                | Found the gradient of the tangent at $P$                                 |
| Marker's comments                                                                                                                |                                                                          |
| Be careful when differentiating.<br>Make sure to find the equation specified in the question. i.e. normal equation, not tangent. |                                                                          |

**Question 7. (6 marks)**

- (a) Loudness,  $L$  in decibels dB, is given by the formula  $L = 10\log_{10}\left(\frac{I}{I_0}\right)$ , where  $I$  is the intensity level and  $I_0$  is the threshold sound (or sound that can barely be heard).

During a particular day, city traffic noise level was measured to be 67 decibels.

- (i) How many more times the threshold of sound is city traffic?

**Solution**

Let  $I$  be the intensity of city traffic

$$L = 10\log_{10}\left(\frac{I}{I_0}\right)$$

$$67 = 10\log_{10}\left(\frac{I}{I_0}\right)$$

$$6.7 = \log_{10}\left(\frac{I}{I_0}\right)$$

$$10^{6.7} = \frac{I}{I_0}$$

$$I = I_0 \times 10^{6.7}$$

| Marking Guidelines                                                                                                                                                                                                                                        |                                         |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------|
| 2                                                                                                                                                                                                                                                         | Correct response (accept approximation) |
| 1                                                                                                                                                                                                                                                         | Partially correct solution              |
| Marker's comments                                                                                                                                                                                                                                         |                                         |
| Poorly done . Most students got confused in the wording of the question . They should have read the question carefully . The practice on applications of logarithmic and exponential functions is recommended to improve in the future assessment tasks . |                                         |

Therefore city traffic is  $10^{6.7}$  or  $\approx 5011872$  more intense than city traffic

- (ii) Late at night, the traffic noise's intensity was measured to be half of what it is during the day.

How many decibels is this, correct to one decimal place?

**Solution**

Late at night intensity  $I$  is  $= \frac{1}{2} \times 10^{6.7} I_0$

$$L = 10\log_{10}\left(\frac{\frac{1}{2} \times 10^{6.7} I_0}{I_0}\right)$$

$$6L = 10\log_{10}\left(\frac{10^{6.7}}{2}\right)$$

$$L \approx 64.0 \text{ dB}$$

| Marking Guidelines                                                                                                                              |                                    |
|-------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------|
| 1                                                                                                                                               | Correct response (ignore rounding) |
| Marker's comments                                                                                                                               |                                    |
| Not Well done. Again the same problem , students had trouble understanding the question . More practice on working mathematically is required . |                                    |



(b) Solve for  $x$ ,  $3\ln x = \ln x - \ln 3$ .

**Solution**

$$3\ln x = \ln x - \ln 3$$

$$\ln x^3 = \ln\left(\frac{x}{3}\right)$$

$$x^3 = \frac{x}{3}$$

$$3x^3 - x = 0$$

$$x(3x^2 - 1) = 0$$

$$x = \frac{1}{\sqrt{3}} \quad \text{as } x > 0$$

| Marking Guidelines                                                                                                                                                                                      |                                         |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------|
| 3                                                                                                                                                                                                       | Correct response                        |
| 2                                                                                                                                                                                                       | Provided additional incorrect solutions |
| 1                                                                                                                                                                                                       | Demonstrates a log law                  |
| Marker's comments                                                                                                                                                                                       |                                         |
| Generally answered well . Some students couldn't recognise that $x$ should be greater than zero. Due to this inability , they gave three or two answers instead of giving one final answer with reason. |                                         |

**Question 8. (7 marks)**

- (a) Gertrude heard that the number of branches on a tree after it was planted could be modelled using the formula

$$N(t) = A(1.04)^t \quad \text{where } N(t) \text{ is the number of branches after time } t \text{ in years.}$$

- (i) What does the value of  $A$  represent in this equation?

**Solution**

$A$  represents the initial number of branches on the tree

| Marking Guidelines |                  |
|--------------------|------------------|
| 1                  | Correct response |
| Marker's comments  |                  |
| Well done          |                  |

- (ii) Gertrude bought a tree from a nursery and noted it had 9 branches when she planted it.

To the nearest month, how long does it take for the tree to have 200 branches?

**Solution**

$$\begin{aligned}
 N(t) &= A(1.04)^t \\
 200 &= 9(1.04)^t \\
 1.04^t &= \frac{200}{9} \\
 \ln(1.04^t) &= \ln\left(\frac{200}{9}\right) \\
 t \ln(1.04) &= \ln\left(\frac{200}{9}\right) \\
 t &= \frac{\ln\left(\frac{200}{9}\right)}{\ln(1.04)} \\
 t &\approx 79 \text{ years} \\
 &\approx 949 \text{ months}
 \end{aligned}$$

| Marking Guidelines                                                                                                          |                                    |
|-----------------------------------------------------------------------------------------------------------------------------|------------------------------------|
| 2                                                                                                                           | Correct response (ignore rounding) |
| 1                                                                                                                           | Using log laws with error          |
| Marker's comments                                                                                                           |                                    |
| Many students could not apply the log laws to simplify their answer. Lots of students gave an incorrect answer of 79 years. |                                    |

- (b) A large stone is sinking through a layer of mud. At time  $t$  seconds, the displacement of the stone in metres below the surface is given by  $x = 20\left(1 - e^{-\frac{1}{2}t}\right)$  where downwards is taken to be in the positive direction.

- (i) Find an equation for the velocity,  $v$ , of the stone in terms of  $t$  where  $v = \frac{dx}{dt}$ .

**Solution**

$$\begin{aligned} v &= \frac{dx}{dt} \\ &= 20\left(\frac{1}{2}e^{-\frac{1}{2}t}\right) \\ &= 10e^{-\frac{1}{2}t} \end{aligned}$$

| Marking Guidelines |                  |
|--------------------|------------------|
| 1                  | Correct response |
| Marker's comments  |                  |
| Well done.         |                  |

- (ii) Hence find the acceleration,  $a$ , of the stone after 2 seconds, where  $a = \frac{dv}{dt}$ .

**Solution**

$$\begin{aligned} a &= \frac{dv}{dt} \\ &= -5e^{-\frac{1}{2}t} \end{aligned}$$

at  $t = 2$

$$\begin{aligned} a &= -5e^{-\frac{1}{2}(2)} \\ &= -5e^{-1} \\ &= -\frac{5}{e} \\ &\approx -1.84 \text{ ms}^{-2} \end{aligned}$$

| Marking Guidelines                                                                         |                                         |
|--------------------------------------------------------------------------------------------|-----------------------------------------|
| 2                                                                                          | Correct response (accept approximation) |
| 1                                                                                          | Found expression for acceleration only  |
| Marker's comments                                                                          |                                         |
| Most students differentiated to get the acceleration but many forgot to substitute $t=2$ . |                                         |

(iii) What happens to displacement as  $t \rightarrow \infty$ ?

**Solution**

$$\text{as } t \rightarrow \infty \quad e^{-\frac{1}{2}t} \rightarrow 0$$

$$\begin{aligned}\text{Therefore } x &\rightarrow 20(1-0) \\ &\rightarrow 20 \text{ m}\end{aligned}$$

| Marking Guidelines                                                                                                |                  |
|-------------------------------------------------------------------------------------------------------------------|------------------|
| 1                                                                                                                 | Correct response |
| Marker's comments                                                                                                 |                  |
| Not well done. Students need further revision of taking limits as t approaches infinity in exponential functions. |                  |