Name: ____

Section I (6 marks)

Teacher	Fentoullis	Hussein	Razzaghi	Kaur	Moon	Wilkinson
	▢	▢	▢	▢	▢	▢

Attempt Questions 1 to 6
Allow about 12 minutes for this section

Use the multiple-choice answer sheet for Questions 1 to 6

1. What is the gradient of any line which is perpendicular to $5x + 2y = 3$?

(A) $\frac{2}{5}$ **(B)** $\frac{-2}{5}$ **(C)** $\frac{5}{2}$ **(D)** $\frac{-5}{2}$

2. Expand and simplify $\left(3-\sqrt{7}\right)^2$

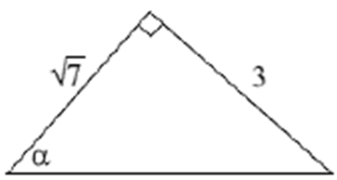
(A) $\left(16-2\sqrt{21}\right)$ **(B)** 16 **(C)** $\left(16-6\sqrt{7}\right)$ **(D)** 2

3. Simplify the expression $\sec x \sin x$

(A) $\operatorname{cosec} x$ **(B)** $\cos x$ **(C)** $\tan x$ **(D)** 1

4.

Calculate the size of angle α in the triangle, correct to the nearest second

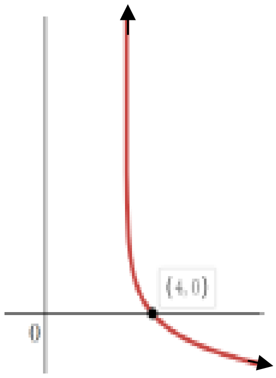


(A) $48°35'25\cdot36''$ **(B)** $48°35'$ **(C)** $48°35'25''$ **(D)** $48°35'26''$

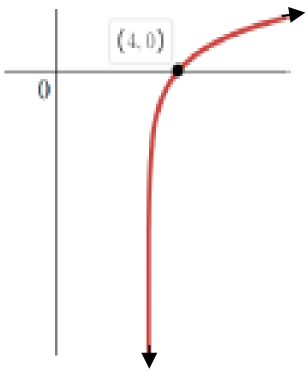
5.

Which of the following graphs refer to $y = -\log_e(x-3)$.

(A)



(B)



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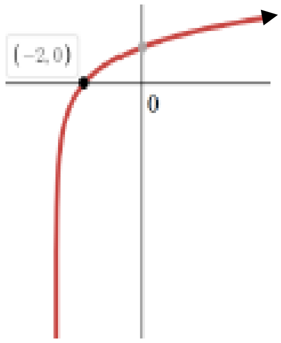
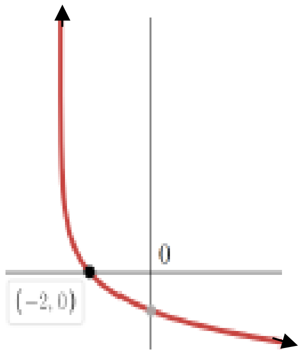


(D)



6.

Given $y = \dfrac{-1}{2x^2}$ find y'

(A) $\dfrac{4}{x^3}$ **(B)** $\dfrac{1}{x^3}$ **(C)** $\dfrac{1}{x}$ **(D)** $\dfrac{-1}{x^3}$

End of Section I

Name: ____

Teacher	Fentoullis	Hussein	Razzaghi	Kaur	Moon	Wilkinson
	▢	▢	▢	▢	▢	▢

Section II - Answer in the space provided

Question 7 (10 marks)

Marks

(a) **1**

Solve: $-\dfrac{2x+1}{3}=5$

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(b) Use the quadratic formula to solve $x^2+12x-9=0$, giving your answer in simplest surd form. **2**

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Question 7 continues on the following page

(c) Find the centre and radius of the circle with equation: **2**

$$x^2 - 6x + y^2 + 2y - 6 = 0$$

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Question 7 continues on the following page

(d) **(i)** Solve the equation: $|x-2|=4$ **1**

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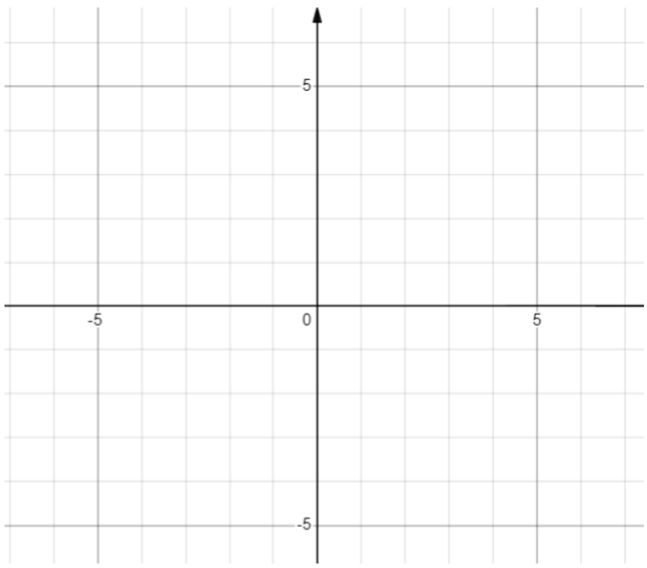
(ii) Draw a sketch of $y=|x-2|$ and $y=4$ on the same set of axes, clearly indicating the coordinates of any point of intersection and intercepts. **2**



(iii) Hence or otherwise solve the inequality $|x-2|\geq 4$ **2**

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End of Question 7

Extra writing space

If you use this space, clearly indicate which question you are answering.

Name: ____

Teacher	Fentoullis	Hussein	Razzaghi	Kaur	Moon	Wilkinson
	▢	▢	▢	▢	▢	▢

Section II - Answer in the space provided

Question 8 (15 marks)

Marks

(a) Consider the function given below: **2**

$$f(x) = \begin{cases} x^2 - 9 & when \;\; x \le 0 \\ -9 + x & when \;\; x > 0 \end{cases}$$

Evaluate $f(2) + f(0) - f(-2)$

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(b) Given $f(x) = \dfrac{3}{x}$ and $g(x) = x - 2$ **2**

State the domain and range for $f\left(g(x)\right)$

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Question 8 continues on the following page

(c) Calculate the exact value of: **2**

$$\left(\cos 315° + \tan 135°\right)\left(\sin 45° - \tan 135°\right)$$

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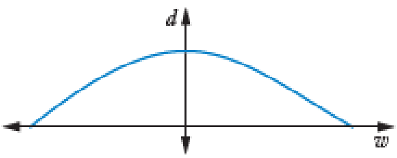
(d) A satellite dish has the shape of a parabola shown below, with equation

$d = \dfrac{-w^2}{800} + 200$ where d is the depth of the dish in metres.



(i) Show, algebraically, that the quadratic function is even. **1**

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Question 8 continues on the following page

(ii) Find the maximum depth of the dish from the top of the span. **1**

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(iii) What is the total width of the satellite dish? **2**

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(iv) Calculate the depth of the dish at a point 100 metres from its endpoint. **2**

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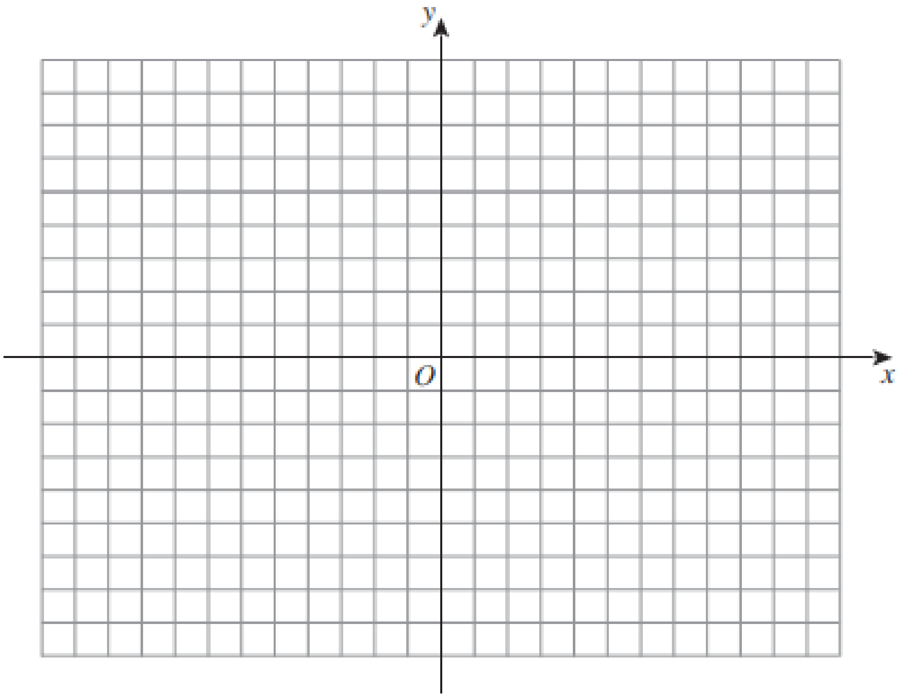
Question 8 continues on the following page

(e) Sketch the graph of $y = e^x + 1$ **3**

Label the features of your graph clearly.



Extra writing space

If you use this space, clearly indicate which question you are answering.

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End of Question 8

Name: ____________________________________

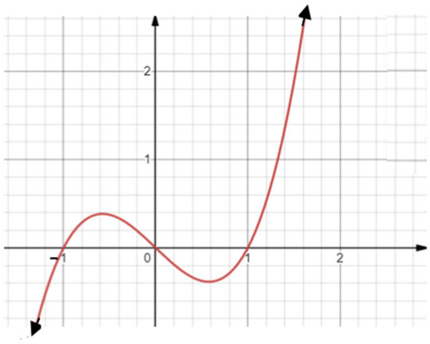
Teacher	Fentoullis	Hussein	Razzaghi	Kaur	Moon	Wilkinson
	▢	▢	▢	▢	▢	▢

Section II - Answer in the space provided

Question 9 (15 marks)

Marks

(a) **(i)** Write the equation of the monic polynomial shown below, in expanded form. **2**



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(ii) The graph in part (i) is shifted 1 unit to the left. Show that this new function is: **2**

$$g(x)=x^3+3x^2+2x$$

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Question 9 continues on the following page

(b) **(i)** Find the equation of the tangents to the curve $y=x^2-4x$, at the points where the curve cuts the x-axis. **2**

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(ii) Hence, what is the point of intersection of these tangents? **1**

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(c) Prove $\dfrac{\tan\theta}{\sec\theta-1}-\dfrac{\tan\theta}{\sec\theta+1}=2\cot\theta$ **2**

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Question 9 continues on the following page

(d) Find the derivative of the function $f(x) = 2x^2 - 3x$ from first principles. **3**

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(e) Solve $2\cos 2x = 1$ for $0 \le x \le \pi$. **3**

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End of Question 9

Extra writing space

If you use this space, clearly indicate which question you are answering.

Name: ______________________________

Teacher	Fentoullis	Hussein	Razzaghi	Kaur	Moon	Wilkinson
	▢	▢	▢	▢	▢	▢

Section II - Answer in the space provided

Question 10 (15 marks)

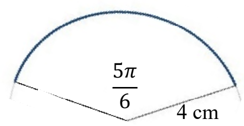
Marks

(a) Give the exact value of $195°$ in radians **1**

(b) **(i)** XY is an arc of a circle with centre O. **2**

The radius OX = 4 cm and angle XOY = $\frac{5\pi}{6}$ radians.

Find the length of the arc XY correct to 1 decimal place.



(ii) The arc *XY* is extended to point *Z* forming the new sector *XOZ*. **2**

The sector *XOZ* has an area of $\frac{26\pi}{3}\ cm^2$

Find angle *XOZ* subtended at the centre, in radians.

Question 10 continues on the following page

(c) Solve $\cos^2 x - \cos x = 0$ for $[0, 2\pi]$ **3**

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(d) Differentiate:

(i) $f(x) = 5x(x^3 + 1)$ **1**

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(ii) $y = \dfrac{4x - 3}{3x + 2}$ **2**

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(iii) $y = 6x^2 e^{-x}$ **2**

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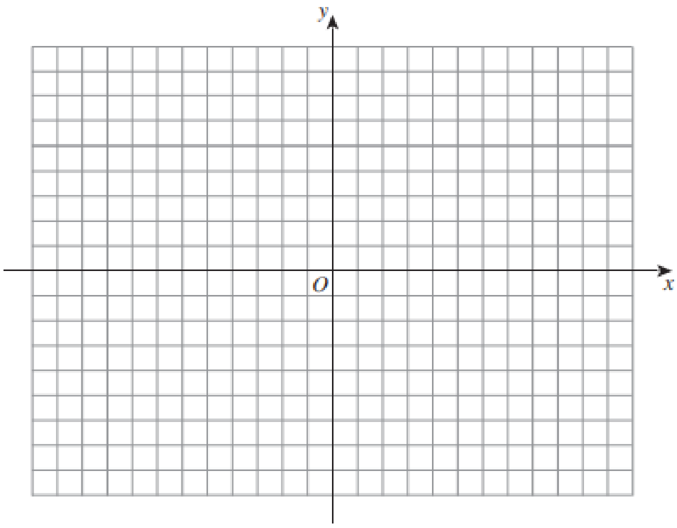
Question 10 continues on the following page

(e) Use the axes below to show graphically and give the number of solutions of the equation $\log_e x - x = -2$ **2**



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End of Question 10

Extra writing space

If you use this space, clearly indicate which question you are answering.

Name: ____

Teacher	Fentoullis	Hussein	Razzaghi	Kaur	Moon	Wilkinson
	▢	▢	▢	▢	▢	▢

Section II - Answer in the space provided

Question 11 (15 marks)

Marks

(a) Simplify, using the log laws, **2**

$$2\log_2 25 + \log_2 16 - \log_2 20$$

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(b) Solve for x, correct to 1 decimal place **2**

$$e^{2x} = 200$$

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(c) Find $\frac{d}{dx}\left[\left(4e^{3x}-1\right)^8\right]$ **2**

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Question 11 continues on the following page

(d) If $R = t^2 - 12t + 2$

(i) Find the average rate of change of R between $t = 1 \quad and \quad t = 4$ **1**

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(ii) What is the instantaneous rate of change of R at $t = 4$ **1**

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(e) Angie leaves island A in a boat and sails 142 km on a bearing of $078°$, to island B. She then sails on a bearing of $191°$ for 220 km to island C, as shown in the diagram.

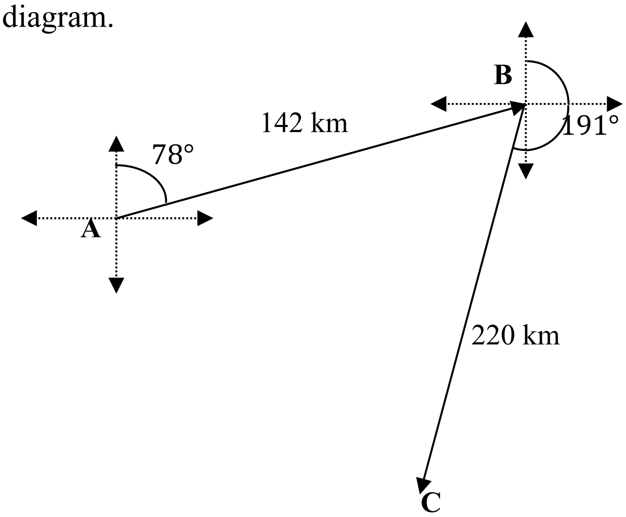

Diagram not to scale

(i) Show that $\angle ABC = 67°$ **2**

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(ii) Calculate the distance from island C to island A, to the nearest km. **2**

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(iii) If Angie wants to sail from island C directly back to island A on what bearing should Angie sail?
(Answer to the nearest degree) **3**

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End of Question 11

Extra writing space

If you use this space, clearly indicate which question you are answering.

Name: ____

Teacher	Fentoullis	Hussein	Razzaghi	Kaur	Moon	Wilkinson
	▢	▢	▢	▢	▢	▢

Section II - Answer in the space provided
Question 12 (14 marks)

Marks

(a) Use the Chain Rule to differentiate: **2**

$$y = \sqrt{5 - x}$$

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(b) The displacement, x metres, of an object moving along a straight line over time, t seconds, is given by

$$x = \left(t^3 + 1\right)^4$$

(i) Find the initial displacement and velocity. **2**

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(ii) Find its acceleration after 2 seconds (answer in scientific notation, correct to 3 significant figures). **2**

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Question 12 continues on the following page

(iii) Show that the particle is never at the origin. **2**

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(c) The amount A grams of a given carbon isotope in a dead tree trunk is given by

$$A = A_0 e^{-kt}$$

where A_0 is the original amount of isotope in the dead tree, k is a positive constant, and where the time t is measured in years from the death of the tree.

(i) Show that A satisfies the equation $\frac{dA}{dt} = -kA$ **1**

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(ii) The amount of isotope is halved every 5500 years. **3**

Show that the value of k is 1.26027×10^{-4}

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Question 12 continues on the following page

(c)

(iii) For a particular dead tree trunk, the amount of isotope is only15% of the original amount in the living tree. How long ago did the tree die? Answer to the nearest 1000 years. **2**

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End of paper

Extra writing space

If you use this space, clearly indicate which question you are answering.

Name: (Solutions)

Teacher: ______

Fort Street High School 2022

Preliminary HSC Assessment Task 3

Mathematics Advanced

Time allowed: 2 hours (plus 10 minutes reading time)

General Instructions

- Marks may be deducted for careless or untidy work
- Write using blue or black pen
- NESA approved calculators may be used
- A formulae sheet is provided with this paper
- In Questions in Section II, show relevant mathematical reasoning and/or calculations

Total Marks – 90

Section I: Pages 1 to 2 **6 marks**

- Attempt questions 1-6, using the answer sheet, page 26, provided
- Allow about 10 minutes for this section

Section II: Pages 3 to 26 **84 marks**

- Attempt questions 7-12, using the space provided in the booklet
- Allow about 1 hour and 50 minutes for this section

Outcome	Assessment Area Description and Marking Guidelines
MA11-1	Uses algebraic and graphical techniques to solve, and where appropriate, compare alternative solutions to problems
MA11-2	Uses the concepts of functions and relations to model, analyse and solve practical problems
MA11-3	Uses the concepts and techniques of Trigonometry in the solution of equations and problem solving involving geometric shapes
MA11-4	Uses the concepts and techniques of periodic functions in the solutions of trigonometric equations or proof of trigonometric identities
MA11-5	Interprets the meaning of the derivative, determines the derivative of functions and applies these to solve simple practical problems
MA11-6	Manipulates and solves expressions using the logarithmic and index laws, and uses logarithms and exponential functions to solve practical problems
MA11-9	Provides reasoning to support conclusions which are appropriate to the context

Final Marks

Question	Mark
Section I **Multiple Choice** **Questions 1-6**	/6
Section II **Question 7**	/10
Question 8	/15
Question 9	/15
Question 10	/15
Question 11	/15
Question 12	/14
Total	/90
	%

2022 Year 11 Advanced Final Examination
Mathematics Advanced
Section I – Multiple Choice Answer Sheet

(Answers)

Name ____________________________________

Teacher	Fentoullis	Hussein	Razzaghi	Kaur	Moon	Wilkinson
	▢	▢	▢	▢	▢	▢

Allow about 10 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ~~●~~ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

correct

A ~~●~~ B ~~●~~ C ○ D ○

1. A ● B ○ C ○ D ○
2. A ○ B ○ C ● D ○
3. A ○ B ○ C ● D ○
4. A ○ B ○ C ● D ○
5. A ● B ○ C ○ D ○
6. A ○ B ● C ○ D ○

Name: (Answers)

Teacher	Fentoullis	Hussein	Razzaghi	Kaur	Moon	Wilkinson
	▢	▢	▢	▢	▢	▢

Section I (6 marks)

Attempt Questions 1 to 6
Allow about 12 minutes for this section

Use the multiple-choice answer sheet for Questions 1 to 6

1. What is the gradient of any line which is perpendicular to $5x+2y=3$?

$2y = -5x + 3$
$y = -\frac{5}{2}x + \frac{3}{2}$

(A) $\frac{2}{5}$ (B) $\frac{-2}{5}$ (C) $\frac{5}{2}$ (D) $\frac{-5}{2}$

2. Expand and simplify $\left(3-\sqrt{7}\right)^2 = (3-\sqrt{7})(3-\sqrt{7}) = 16-6\sqrt{7}$

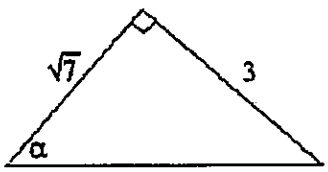
(A) $\left(16-2\sqrt{21}\right)$ (B) 16 (C) $\left(16-6\sqrt{7}\right)$ (D) 2

3. Simplify the expression $\sec x \sin x = \frac{1}{\cos x} \cdot \sin x = \frac{\sin x}{\cos x}$

(A) $\operatorname{cosec} x$ (B) $\cos x$ (C) $\tan x$ (D) 1

4.

Calculate the size of angle α in the triangle, correct to the nearest second



$\tan \alpha = \frac{3}{\sqrt{7}}$

$\alpha = \tan^{-1}(\quad)$
$= 48° 35' 25.36''$
$\doteq 48° 35' 25''$

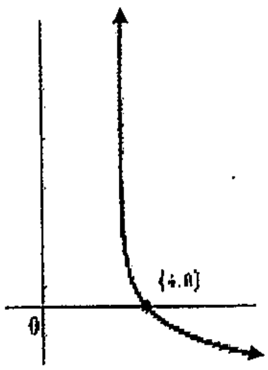
(A) 48°35'25·36" (B) 48°35' (C) 48°35'25" (D) 48°35'26"

5.

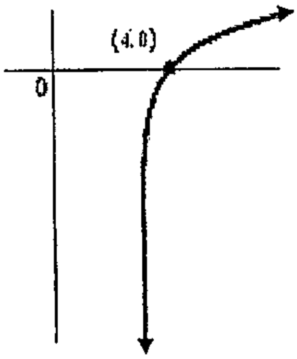
Which of the following graphs refer to $y=-\log_e(x-3)$. translated 3 units to the right

reflection in x-axis

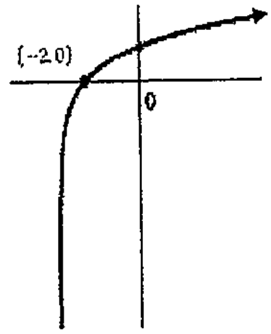
(A)



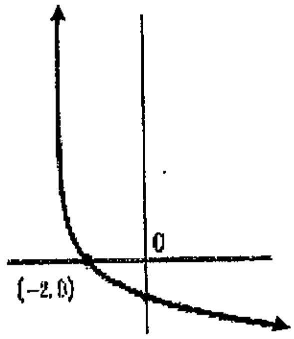
(B)



(C)



(D)



6.

Given $y=\dfrac{-1}{2x^2}$ find y' $y=\frac{-1}{2}\cdot x^{-2}$, $y'=x^{-3}=\frac{1}{x^3}$

(A) $\dfrac{4}{x^3}$ (B) $\dfrac{1}{x^3}$ (C) $\dfrac{1}{x}$ (D) $\dfrac{-1}{x^3}$

End of Section I

Name: ______________________________

Teacher	Fentoullis	Hussein	Razzaghi	Kaur	Moon	Wilkinson
	▢	▢	▢	▢	▢	▢

Section II - Answer in the space provided

Question 7 (10 marks)

Marks

(a) Solve : $-\frac{2x+1}{3}=5$ 1

$2x+1=-15$

$2x=-16$

$\boxed{x=-8}$ ✓

(b) Use the quadratic formula to solve $x^2+12x-9=0$, giving your answer in simplest surd form. 2

$x=\frac{-12\pm\sqrt{12^2-4\times1\times-9}}{2(1)}$

$=\frac{-12\pm\sqrt{180}}{2}$ ✓

$\sqrt{180}$

$=\sqrt{36\times5}$

$=6\sqrt{5}$

$=\frac{-12\pm6\sqrt{5}}{2}$

$=\frac{2(-6\pm3\sqrt{5})}{2}$

$=\boxed{-6\pm3\sqrt{5}}$ ✓

Question 7 continues on the following page

(c) Find the centre and radius of the circle with equation: **2**

$$x^2 - 6x + y^2 + 2y - 6 = 0$$

$x^2 - 6x + 9 + y^2 + 2y + 1 = 6 + 10$

$(x-3)^2 + (y+1)^2 = 16$

$\therefore$ radius $= 4$, centre $(3, -1)$

1 working

1 answer including (both radius + centre)

Question 7 continues on the following page

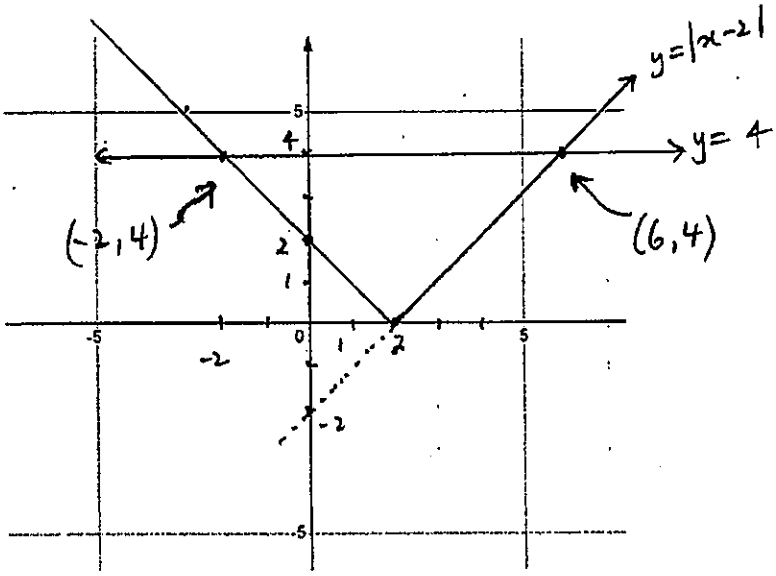
(d) **(i)** Solve the equation: $|x-2| = 4$ **1**

$x - 2 = 4$, $x - 2 = -4$

$\underline{x = 6}$, $\underline{x = -2}$

✓

(ii) Draw a sketch of $y = |x-2|$ and $y = 4$ on the same set of axes, clearly indicating the coordinates of any point of intersection and intercepts. **2**



✓ pts of int. and intercepts

✓ graphs / sketch

(iii) Hence or otherwise solve the inequality $|x-2| \geq 4$ **2**

$x - 2 \geq 4$, $x - 2 \leq -4$

$\underline{x \geq 6}$, $\underline{x \leq -2}$

✓ ✓

End of Question 7

Name: ______________________________

Teacher	Fentoullis	Hussein	Razzaghi	Kaur	Moon	Wilkinson
	☐	☐	☐	☐	☐	☐

Section II - Answer in the space provided

Question 8 (15 marks)

Marks

(a) Consider the function given below: **2**

$$f(x) = \begin{cases} x^2 - 9 & when \ x \le 0 \\ -9 + x & when \ x > 0 \end{cases}$$

Evaluate $f(2) + f(0) - f(-2)$

$= (-9+2) + (0^2 - 9) - [(-2)^2 - 9]$ ✓

$= -7 - 9 - (-5)$

$= -11$ ✓

(b) Given $f(x) = \frac{3}{x}$ and $g(x) = x - 2$ **2**

State the domain and range for $f(g(x))$

$f(g(x)) = \frac{3}{x-2}$

Domain: $(-\infty, 2) \cup (2, \infty)$ ✓

Range: $(-\infty, 0) \cup (0, \infty)$ ✓

Question 8 continues on the following page

(c) Calculate the exact value of: 2

$$(\cos 315° + \tan 135°)(\sin 45° - \tan 135°)$$

$= \left[\frac{1}{\sqrt{2}} + (-1)\right]\left[\frac{1}{\sqrt{2}} - (-1)\right]$

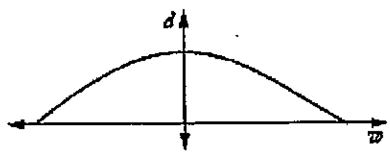
$= \left(\frac{1}{\sqrt{2}} - 1\right)\left(\frac{1}{\sqrt{2}} + 1\right)$ ✓

$= \frac{1}{2} - 1$

$= -\frac{1}{2}$ ✓

(d) A satellite dish has the shape of a parabola shown below, with equation

$d = \frac{-w^2}{800} + 200$ where d is the depth of the dish in metres.



(i) Show, algebraically, that the quadratic function is even. 1

$f(w) = \frac{-w^2}{800} + 200$

$f(-w) = -\frac{(-w)^2}{800} + 200$ ✓

$= \frac{-w^2}{800} + 200$

$= f(w)$

$\therefore$ since $f(w) = f(-w)$ the function is even

Question 8 continues on the following page

(ii) Find the maximum depth of the dish from the top of the span. **1**

When $w = 0$, $d = \frac{-(0)^2}{800} + 200$

$= 200$

$\therefore$ the maximum depth is 200m

(iii) What is the total width of the satellite dish? **2**

When $d = 0$, $0 = \frac{-w^2}{800} + 200$

$\frac{w^2}{800} = 200$

$w^2 = 160\,000$

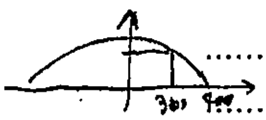
$\therefore w = \pm\sqrt{160\,000}$

$= \pm 400$

$\therefore$ the total width $= 400 + 400$

$= 800$ m

(iv) Calculate the depth of the dish at a point 100 metres from its endpoint. **2**



when $w = 300$

$d = \frac{-(300)^2}{800} + 200$

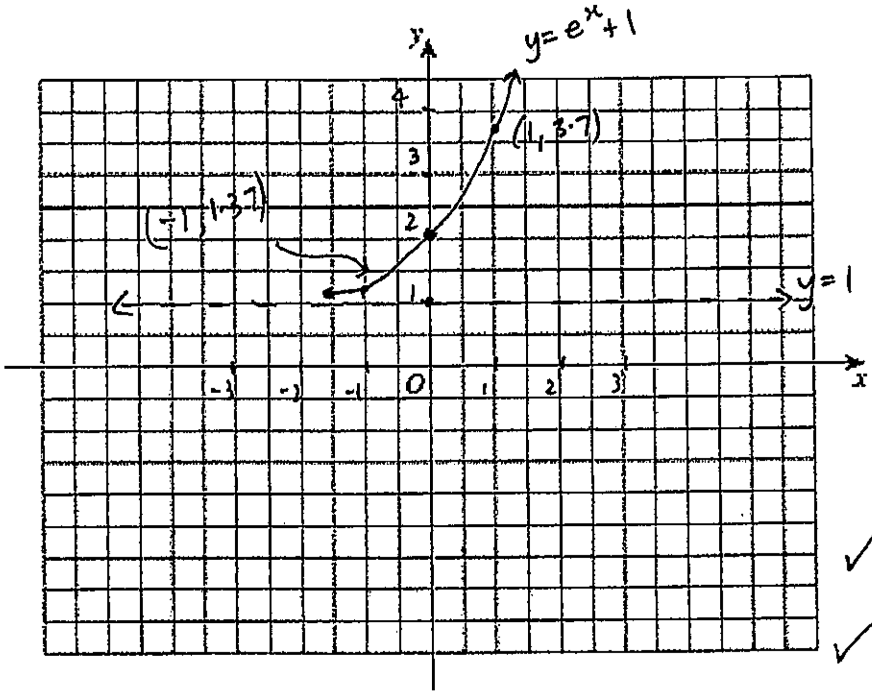
$= -\frac{90000}{800} + 200$

$= 87.5$ m

Question 8 continues on the following page

(e) Sketch the graph of $y = e^x + 1$ 3

Label the features of your graph clearly.



✓ y intercept / shape

✓ asymptote

✓ additional point/s on the curve.

Extra writing space

If you use this space, clearly indicate which question you are answering.

when $x = -1$

$y = e^{-1} + 1$

$= \frac{1}{e} + 1$

$= 1\frac{1}{e}$

$\doteqdot 1.368$

when $x = 1$

$y = e + 1$

$\doteqdot 3.7$

End of Question 8

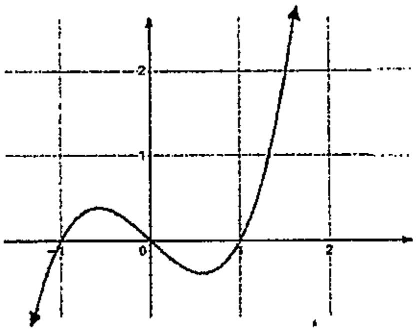
Name: ____________________________

Teacher	Fentoullis	Hussein	Razzaghi	Kaur	Moon	Wilkinson
	▢	▢	▢	▢	▢	▢

Section II - Answer in the space provided

Question 9 (15 marks) **Marks**

(a) (i) Write the equation of the monic polynomial shown below, in expanded form. **2**



Using zeros: $y = (x+1)(x)(x-1)$

$= x(x^2-1)$

$\underline{y = x^3 - x}$

(ii) The graph in part (i) is shifted 1 unit to the left. Show that this new function is: **2**

$$g(x)=x^3+3x^2+2x$$

Using zeros: $-2, -1, 0$

$(x+2)$

$(x+1)$

x

$g(x) = x(x+1)(x+2)$

$= x(x^2+3x+2)$

$= x^3+3x^2+2x$

Question 9 continues on the following page

(b) (i) $\boxed{\text{Find the equation of the tangents}}$ to the curve $y = x^2 - 4x$, at the points where the curve cuts the x-axis. **2**

Curve cuts the x-axis when $y = 0$

$0 = x(x-4)$

at $x = 0, 4$

$\therefore$ $(0,0)$ and $(4,0)$

$m_T = \frac{dy}{dx} = 2x - 4$

at $x = 0$, $m_T = 2(0) - 4 = -4$

at $x = 4$, $m_T = 2(4) - 4 = 4$

Eqn of tangent$_1$ $y - 0 = -4(x - 0)$

$y = -4x$

Eqn of tangent$_2$ $y - 0 = 4(x - 4)$

$y = 4x - 16$

(ii) Hence, what is the point of intersection of these tangents? **1**

~~points~~ pt of int: $y = -4x$ —①

$y = 4x - 16$ —②

①+②

$2y = -16$

$y = -8$

when $y = -8$, $-8 = -4(x)$ $x = 2$

pt. of intersection is $(2, -8)$

(c) Prove $\frac{\tan\theta}{\sec\theta - 1} - \frac{\tan\theta}{\sec\theta + 1} = 2\cot\theta$ **2**

$$LHS = \frac{\tan\theta(\sec\theta + 1) - \tan\theta(\sec\theta - 1)}{\sec^2\theta - 1}$$

$$= \frac{\tan\theta\sec\theta + \tan\theta - \tan\theta\sec\theta + \tan\theta}{\tan^2\theta}$$

$$= \frac{2\tan\theta}{\tan^2\theta} = \frac{2}{\tan\theta} = 2\cot\theta = RHS$$

Question 9 continues on the following page

(d) Find the derivative of the function $f(x) = 2x^2 - 3x$ from first principles. 3

$f(x) = 2x^2 - 3x$

$f(x+h)$

$= 2(x+h)^2 - 3(x+h)$

$= 2(x^2 + 2xh + h^2) - 3x - 3h$

$= 2x^2 + 4xh + 2h^2 - 3x - 3h$

$f'(x) = \lim_{h \to 0} \frac{2x^2 + 4xh + 2h^2 - 3x - 3h - (2x^2 - 3x)}{h}$

$= \lim_{h \to 0} \frac{4xh + 2h^2 - 3h}{h}$

$= \lim_{h \to 0} \frac{\cancel{h}(4x + 2h - 3)}{\cancel{h}}$

$= \lim_{h \to 0} 4x + 2h - 3$

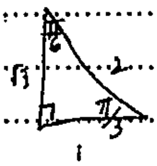
$= 4x - 3$

(e) Solve $2\cos 2x = 1$ for $0 \le x \le \pi$ 3

$\cos 2x = \frac{1}{2}$ for $0 \le x \le \frac{\pi}{2}$ then

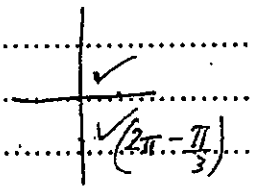
$0 \le 2x \le \pi$

$2x$ acute/reference $\frac{\pi}{3}$



$\cos \frac{\pi}{3} = \frac{1}{2}$

$2x = \frac{\pi}{3}, \frac{5\pi}{3}$



$\therefore x = \frac{\pi}{6}, \frac{5\pi}{6}$

(answers)

End of Question 9

Name: ______________________

Teacher	Fentoullis	Hussein	Razzaghi	Kaur	Moon	Wilkinson
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Section II - Answer in the space provided

Question 10 (15 marks)

Marks

(a) Give the exact value of 195° in radians 1

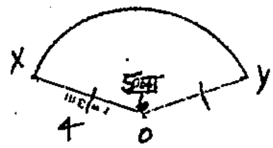
$195 \times \frac{\pi}{180} = \frac{13\pi}{12}$ ✓

(b)

(i) XY is an arc of a circle with centre O. 2

The radius OX = 4 cm and angle XOY = $\frac{5\pi}{6}$ radians.

Find the length of the arc XY correct to 1 decimal place.



$l = r\theta$

$= 4 \times \frac{5\pi}{6}$

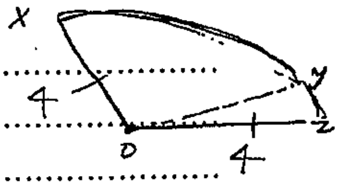
$= \frac{10\pi}{3}$ ✓

$= 10.4719\ldots \doteq \boxed{10.5\text{ cm}}$ ✓

(ii) The arc XY is extended to point Z forming the new sector XOZ. 2

The sector XOZ has an area of $\frac{26\pi}{3}$ cm^2

Find angle XOZ subtended at the centre, in radians.



$A = \frac{1}{2} r^2 \theta$

$\frac{26\pi}{3} = \frac{1}{2}(16)\theta$ ✓

$\frac{52\pi}{3} = 16\theta$

$\theta = \frac{52\pi}{3} \times \frac{1}{16}$

$\boxed{\theta = \frac{13\pi}{12}}$ ✓

Question 10 continues on the following page

(c) Solve $\cos^2 x - \cos x = 0$ for $[0, 2\pi]$ **3**

$\cos x(\cos x - 1) = 0$

$\cos x = 0$, $\cos x - 1 = 0$

$\cos x = 1$

$\therefore x = \frac{\pi}{2}, \frac{3\pi}{2}, x = 0, 2\pi$

(d) Differentiate:

(i) $f(x) = 5x(x^3 + 1)$ **1**

$f(x) = 5x^4 + 5x$

$f'(x) = 20x^3 + 5$

(OR)

$f'(x) = 5x \cdot 3x^2 + (x^3+1) \cdot 5$

$= 15x^3 + 5x^3 + 5$

$= 20x^3 + 5$

(ii) $y = \frac{4x-3}{3x+2}$ **2**

Let $u = 4x - 3$, $v = 3x + 2$

$u' = 4$, $v' = 3$

$y' = \frac{u'v + v'u}{v^2} = \frac{12x + 8 - 12x + 9}{(3x+2)^2}$

$= \frac{17}{(3x+2)^2}$

(iii) $y = 6x^2 e^{-x}$ **2**

Let $u = 6x^2$, $v = e^{-x}$

$u' = 12x$, $v' = -e^{-x}$

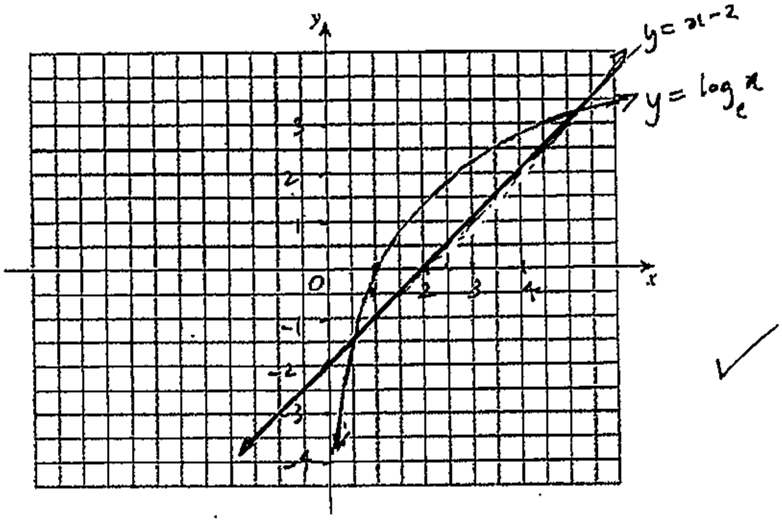
$y' = uv' + vu'$

$= (6x^2)(-e^{-x}) + (12x)(e^{-x})$

$= 6x^2 e^{-2}(2 - x)$ (OR) $-6xe^{-x}(x-2)$

Question 10 continues on the following page

(e) Use the axes below to show graphically, the number of solutions of the equation $\log_e x - x = -2$ **2**



$\log_e x = x - 2$

$y = \log_e x$ $\therefore$ 2 solutions

$y = x - 2$

End of Question 10

	▢	▢	▢	▢	▢	▢

Section II - Answer in the space provided

Question 11 (15 marks) **Marks**

(a) Simplify, using the log laws, **2**

$$2\log_2 25 + \log_2 16 - \log_2 20$$

$= 2\log_2 5^2 + \log_2 2^4 - \log_2(4\times5)$ ✓

$= 4\log_2 5 + 4\log_2 2 - [2\log_2 2 + \log_2 5]$

$= 3\log_2 5 + 2$ ✓

(b) Solve for x, correct to 1 decimal place **2**

$$e^{2x} = 200$$

$\ln e^{2x} = \ln 200$ ✓

$2x \ln e = \ln 200$

$2x = \ln 200$

$x = \frac{\ln 200}{2}$ $= 2.649\ldots$

$\doteq 2.6$ ✓

(c) Find $\frac{d}{dx}\left[\left(4e^{3x}-1\right)^8\right]$ **2**

$= 8(4e^{3x}-1)^7 \cdot 4e^{3x} \cdot 3$ ✓

$= 96(4e^{3x}-1)^7 e^{3x}$ ✓

Question 11 continues on the following page

(d) If $R = t^2 - 12t + 2$

(i) Find the average rate of change of R between $t = 1$ *and* $t = 4$ 1

$R = t^2 - 12t + 2$

$f(1) = 1^2 - 12(1) + 2 = -9$

$f(4) = 4^2 - 12(4) + 2 = -30$

$\therefore$ Rate of Change $= \dfrac{-30 - -9}{4-1} = \dfrac{-21}{3} = \underline{-7}$

(ii) What is the instantaneous rate of change of R at $t = 4$ 1

$f'(4) = 2t - 12$

$= 2(4) - 12$

$= 8 - 12$

$= \underline{-4}$

(e) Angie leaves island A in a boat and sails 142 km on a bearing of $078°$, to island B. She then sails on a bearing of $191°$ for 220 km to island C, as shown in the diagram.

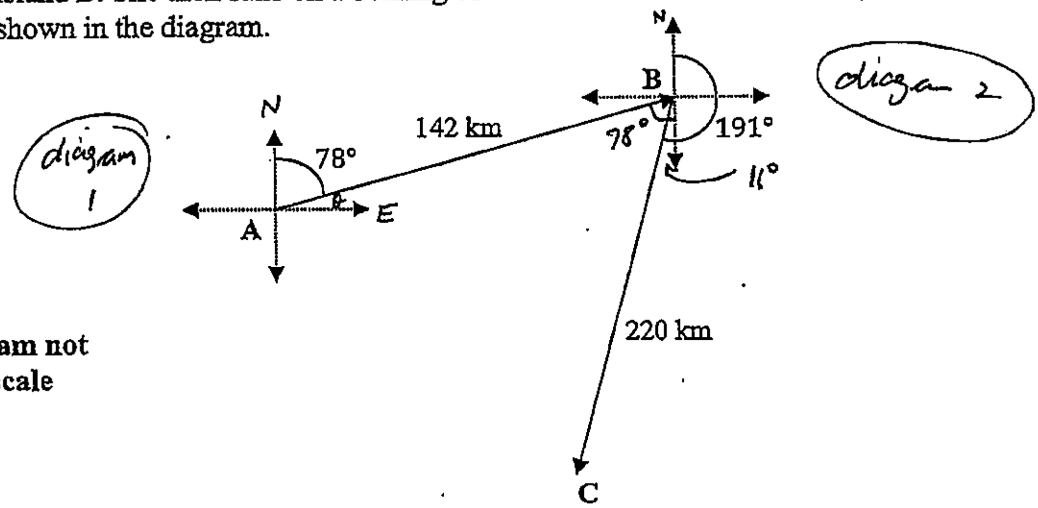


Diagram not to scale

(i) Show that $\angle ABC = 67°$ 2

In diagram 1: $\theta = 90 - 78 = 12°$

In diagram 2: alternate $\angle$'s $= 78°$ ($\angle AB$ South)

$\angle ABC =$

$= 78 - (191 - 180)$

$= \underline{67°}$

(ii) Calculate the distance from island C to island A, to the nearest km. 2

$d_{AC} = \sqrt{142^2 + 220^2 - 2 \times 142 \times 220 \cos 67^\circ}$

$= \sqrt{44\,151 \cdot 119 \ldots}$

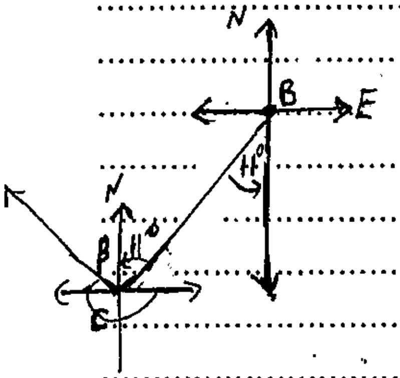
$= 210 \cdot 12$

$\doteqdot 210$ km

(iii) Angie wants to sail from island C directly back to island A. Given the bearing of B from C is 011°, on what bearing should Angie sail? (Answer to the nearest degree) 3

Since $191 - 180 = 11^\circ$

(The bearing of B from C)



$\angle ACB$:

$\cos \angle ACB = \dfrac{210^2 + 220^2 - 142^2}{2 \times 210 \times 220}$

$= 0.782956 \ldots$

$\angle ACB = \cos^{-1}(\;"\;)$

$= 38.467 \ldots$

$\doteqdot 38^\circ$

$\beta = 38 - 11$

$= 27^\circ$

$\therefore$ The bearing of A from C

$360 - 27$

$= 333^\circ$

End of Question 11

Name: ______________________________

Teacher	Fentoullis	Hussein	Razzaghi	Kaur	Moon	Wilkinson
	☐	☐	☐	☐	☐	☐

Section II - Answer in the space provided
Question 12 (14 marks)

Marks

(a) Use the Chain Rule to differentiate: **2**

$$y=\sqrt{5-x}$$

Let $u = 5-x$

$\therefore y = \sqrt{u} = u^{\frac{1}{2}}$

$\frac{du}{dx} = -1$

$\frac{dy}{du} = \frac{1}{2}u^{-\frac{1}{2}} = \frac{1}{2\sqrt{u}}$

$\therefore \frac{dy}{dx} = \frac{du}{dx} \cdot \frac{dy}{du}$

$= -1 \cdot \frac{1}{2\sqrt{u}}$

$= -1\left(\frac{1}{2\sqrt{5-x}}\right)$

$= \frac{-1}{2\sqrt{5-x}}$

OR

$y = (5-x)^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}(5-x)^{-\frac{1}{2}} \cdot -1$

$= -\frac{1}{2}\left(\frac{1}{\sqrt{5-x}}\right)$

(b) The displacement, x metres, of an object moving along a straight line over time, t seconds, is given by

$$x=\left(t^3+1\right)^4$$

(i) Find the initial displacement and velocity. **2**

$x(0) = (0^3+1)^4 = 1m$ ✓

$\dot{x} = 4(t^3+1)^3 \cdot 3t^2$

$= 12t^2(t^3+1)^3$

$\dot{x}(0) = 12(0)(0+1)^3$ ✓

$= 0 \ ms^{-1}$

(ii) Find its acceleration after 2 seconds (answer in scientific notation, correct to 3 significant figures). **2**

$\ddot{x} = 12t^2\left[3(t^3+1)^2 \cdot 3t^2\right] + (t^3+1)^3 \cdot 24t$

$= 108t^4\left[(t^3+1)^2\right] + 24t(t^3+1)^3$

$\therefore \ddot{x}(2) = 108(2)^4\left[(2^3+1)^2\right] + 24(2)(2^3+1)^3$

$= 174960$

$\doteqdot 175\,000 \ ms^{-2} = 1.75 \times 10^5$ (3 sig. fig.)

Question 12 continues on the following page

(iii) Show that the particle is never at the origin. **2**

$x = (t^3+1)^4$

for $x=0$, $t^3+1=0$

$t^3=-1$

as $t \geq 0$ $t^3 \geq 0$ $\therefore$ Never at the origin.

no-soln.

(c) The amount A grams of a given carbon isotope in a dead tree trunk is given by

$$A = A_0 e^{-kt}$$

where A_0 and k are positive constants, and where the time t is measured in years from the death of the tree.

(i) Show that A satisfies the equation $\frac{dA}{dt} = -kA$ **1**

$\frac{dA}{dt} = A_0 \times -k \times e^{-kt}$

$= -k A_0 e^{-kt}$

$= -k(A)$ (since $A_0 e^{-kt} = A$)

(ii) The amount of isotope is halved every 5500 years. **3**

Show that the value of k is $1 \cdot 26027 \times 10^{-4}$

When $t = 5500$, $A = \frac{1}{2} A_0$

$\therefore \frac{1}{2} A_0 = A_0 e^{-5500k}$

$\ln \frac{1}{2} A_0 = \ln A_0 e^{-5500k}$

$\ln \frac{1}{2} = -5500k$

$\therefore k = \frac{\ln \frac{1}{2}}{-5500} = 0.0001260276\ldots$

$= 1.260276 \times 10^{-4}$

Question 12 continues on the following page

(iii) For a particular dead tree trunk, the amount of isotope is only15% of the original amount in the living tree. How long ago did the tree die? Answer to the nearest 1000 years. **2**

$A = 0{\cdot}15 A_0$

$\therefore 0{\cdot}15 A_0 = A_0 e^{-kt}$

$0{\cdot}15 = e^{-kt}$ ✓

$\ln 0{\cdot}15 = \ln e^{-kt}$

$= -kt \ln e$

$\therefore t = \dfrac{\ln 0{\cdot}15}{-k}$ ✓

$= 15053{\cdot}3107\ldots$

$= \underline{15\,000 \text{ years}}$ (nearest 1000 yrs)

End of paper