FINAL MARK

GIRRAWEEN HIGH SCHOOL
YEAR 11
MATHEMATICS (ADVANCED)
2019 Yearly Examination

Name: _____

QUESTION	MARK	MA11-1	MA11-2	MA11-3	MA11-4	MA11-5	MA11-6	MA11-9
1-5	/5	✓	✓					✓
6	/18	✓						✓
7	/26	✓				✓		✓
8	/19	✓	✓				✓	✓
9	/20	✓	✓				✓	✓
10	/12	✓	✓					✓
11	/13	✓	✓	✓				✓
12	a,d/8	✓	✓	✓	✓			✓
12	b,c/7	✓	✓	✓				✓
TOTAL								
	/128	/128	/84	/28	/8	/26	/39	/128

Year 11 Mathematics Advanced outcomes

A student:

MA11-1 uses algebraic and graphical techniques to solve, and where appropriate, compare alternative solutions to problems

MA11-2 uses the concepts of functions and relations to model, analyse and solve practical problems

MA11-3 uses the concepts and techniques of trigonometry in the solution of equations and problems involving geometric shapes

MA11-4 uses the concepts and techniques of periodic functions in the solutions of trigonometric equations or proof of trigonometric identities

MA11-5 interprets the meaning of the derivative, determines the derivative of functions and applies these to solve simple practical problems

MA11-6 manipulates and solves expressions using the logarithmic and index laws, and uses logarithms and exponential functions to solve practical problems

MA11-7 uses concepts and techniques from probability to present and interpret data and solve problems in a variety of contexts, including the use of probability distributions

MA11-8 uses appropriate technology to investigate, organise, model and interpret information in a range of contexts

MA11-9 provides reasoning to support conclusions which are appropriate to the context

GIRRAWEEN HIGH SCHOOL

MATHEMATICS (ADVANCED)

YEAR 11

Yearly Examination 2019

Year 11 Mathematics

Time allowed: 120 minutes (plus10 minutes reading time)

Instructions:

- Attempt ALL questions.
- For Questions 1 to 5, colour in the bubble next to the letter corresponding to the correct response on your paper.
- For Questions 6 to 12:
    - All necessary working should be shown in the space provided.
    - Marks will be deducted for careless or badly arranged work.
    - Board-approved calculators may be used.
    - Reference sheet is provided.
    - Diagrams are NOT to scale.

Section 1 (5 marks)
Use the multiple-choice answer sheet for questions 1-5.
Multiple Choice (5 marks).

1. The simplest form of the expression $\frac{1}{\sqrt{3}-\sqrt{2}}-\sqrt{3}$ is:

A) $\sqrt{2}$ B) $\sqrt{3}$ C) $\sqrt{3}+\sqrt{2}$ D) $\sqrt{3}-\sqrt{2}$

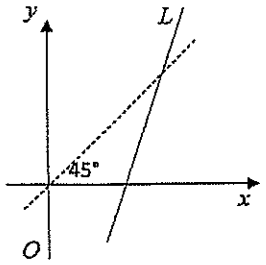
2. The simplest form of the algebraic fraction $\frac{n}{n-1}+\frac{n}{1+n}$ is :

A) 1 B) -1 C) 0 D) $\frac{2n^2}{n^2-1}$

3. The degree of the polynomial $\left(x^5+1\right)^2$, is

A) 2 B) 5 C) 7 D)10

4) Given the line L in the diagram below, the gradient of the line is m.
Which one of the following is correct about m.



A) $m=1$ B) $m=0$ C) $m<1$ D) $m>1$

5.What is the derivative of e^{x^2}.

A) $x^2e^{x^2-1}$ B) $2xe^{2x}$ C) $2xe^{x^2}$ D) $2e^{x^2}$

Answers to Multiple Choice Questions

Select the alternative A, B, C or D that best answers the question.

1.	A	○	B	○	C	○	D	○
2.	A	○	B	○	C	○	D	○
3.	A	○	B	○	C	○	D	○
4.	A	○	B	○	C	○	D	○
5.	A	○	B	○	C	○	D	○

Section11

- **Answer questions in the spaces provided.**
- **Your responses should include relevant mathematical reasoning and/or calculations.**
- **Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.**

Question 6 (18 marks)

a) Find the value of $4\pi\sqrt{\frac{a}{g}}$ if a=4.1 and g=9.8. Give your answer to 2 significant figures.

..... 1

b) Write down the value of $|-9|-|-10|$

..... .1

c) Factorise fully:

i) x^2+5x-6

.... 1

ii) $2x^2-7x-15$

..... 2

.....

d) Simplify:

i) $\frac{2x^2-xy}{4x^2-y^2}$ 2

.....

.....

.....

ii) $\frac{2}{3}-\frac{x-1}{4}$ 2

.....

.....

.....

.....

iii) $\frac{1}{x^2-4}-\frac{2}{x-2}-\frac{3}{x+2}$ 3

.....

.....

.....

.....

.....

.....

.....

.....

e) Express $\frac{21}{\sqrt{63}}-\frac{3}{\sqrt{7}+2}$ as an integer. (show necessary working) 3

.....

.....

.....

.....

f) If $a=\left(\frac{1}{5}\right)^{\frac{1}{3}}$ and $b=\left(\frac{1}{5}\right)^{\frac{1}{2}}$, evaluate the expression $\frac{a^3+b^2}{(ab)^6}$. 3

.....

.....

.....

.....

.....

.....

Question 7 (26 marks)

(a) Solve :

i) $3(2x+1)-2(3-x)=53$ 2

.....

.....

.....

.....

ii) $3x^2-4x-4=0$ 2

.....

.....

.....

.....

.....

iii) $16^x=64$ 2

.....

.....

.....

iv) $|4x-3|=7$ 3

.....

.....

.....

.....

.....

.....

v) $\frac{x-5}{3}-\frac{x+1}{4}=5$ 3

.....

.....

.....

.....

.....

b) Differentiate :

i) $y=\frac{7}{2x}$ 2

.....

.....

.....

.....

ii) $y=\frac{5-x^2+5x^4}{x^2}$ 3

.....

.....

.....

.....

.....

.....

iii) $y=5(4-x)^9$ 3

.....

.....

.....

.....

.....

iv) $y = x^2(3x+2)^3$ 3

.....

.....

.....

.....

.....

.....

v) $y = \dfrac{x^2-4}{x-1}$ 3

.....

.....

.....

.....

.....

.....

Question 8 (19 marks).

a) Differentiate $y = \dfrac{2x}{e^x+1}$ 3

.....

.....

.....

.....

.....

b) i) Differentiate $y = e^x - x$ 1

.....

.....

.....

ii) For what value of x is the curve stationary? 2

.....

.....

c) i) Find the gradient of the tangent to $y = e^{-x}$ at the point $P(-1, e)$. 2

.....

.....

.....

.....

.....

ii) Hence write down the gradient of the normal at this point. 1

.....

.....

.....

iii) Hence determine the equation of this normal. 2

.....

.....

.....

.....

iv) Find the x and y intercepts of the normal. 2

.....

.....

.....

v) Find the area of the triangle whose vertices lie at the intercepts and the origin. 3

.....

.....

.....

.....

d) Find the equation of the normal to the curve $y=\sqrt{x+2}$ at the point (7,3). 3

.....

.....

.....

.....

.....

Question 9 (20 marks).

a) Evaluate

i) $\log_{10} 875$ correct to 2 decimal places. 1

.....

ii) Evaluate $\log_e 80$ to 3 significant figures. 1

.....

iii) $\log_5 2$ correct to 2 decimal places 2

.....

b) Evaluate:

i) $\log_2 16$ 2

.....

.....

ii) $\log_5 25+1$ 2

.....

.....

.....

.....

iii) $\log_3 \frac{1}{\sqrt[3]{3}}$ 2

.....

.....

.....

.....

.....

c) Given $\log_5 3 = 0.68$ and $\log_5 4 = 0.86$, find

i) $\log_5 12$ 1

.....

.....

.....

ii) $\log_5 0.75$ 1

.....

.....

.....

.....

iii) $\log_5 9$ 2

.....

.....

.....

.....

.....

iv) $\log_5 20$ 2

.....

.....

.....

.....

d) The amount of money A in a bank account after n years grows with compound interest according to the formula $A = 850(1.025)^n$.

.....

.....

.....

i) Find the initial amount in the bank. 1

.....

ii) Find the amount after 7 years. 1

.....

.....

iii) Find how many years it will take for the amount in the bank to be \$1000. 2

.....

.....

.....

Question 10 (12 marks)

a) Show that the lines $5x - y - 10 = 0$, $x + y - 8 = 0$ and $2x - 3y + 9 = 0$ are concurrent? 3

.....

.....

.....

.....

.....

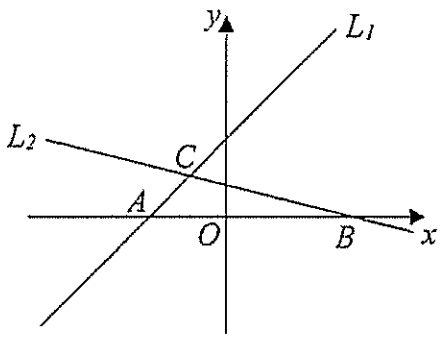
.....

.....

.....

.....

b) In the diagram below, the line L_1: $x-y+2=0$ cuts the x axis at A and the line L_2: $x+4y-3=0$ cuts the x axis at B.



i) Show that the co-ordinates of A and B are (-2,0) and (3,0) respectively. 2

.....

.....

.....

ii) If C is the point of intersection of the lines L_1 and L_2, use simultaneous equations to show that the co-ordinates of C are (-1,1). 2

.....

.....

.....

.....

.....

.....

iii) Find the length of AC. 1

.....

.....

.....

.....

.....

.....

iv) Calculate the gradient of the line AC. 1

.....

.....

.....

v) Show that the line L_1 makes an angle of 45^0 with the positive direction of the x-axis. 1

.....

.....

.....

vi) Find the area of ΔABC. 2

.....

.....

.....

.....

.....

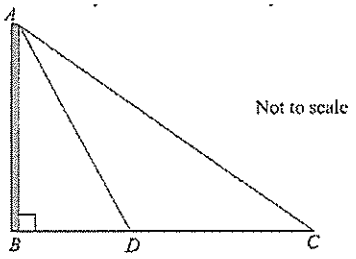
.....

.....

.....

Question 11 (13 marks).

a) A vertical tower AB with points B, C and D in a straight line on the ground is shown below. The distance from CD is 100 metres. The angle of elevation to the top of the tower from point C is 35^0 and from point D is 60^0.



i) Show that $AD = \dfrac{100\sin 35^0}{\sin 25^0}$ 3

.....

.....

.....

.....

.....

.....

ii) Calculate the height of the tower. Answer to the nearest metre. 2

.....

.....

.....

.....

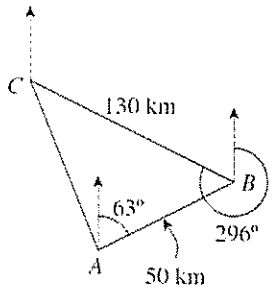
.....

.....

.....

.....

b) A ship sails $50km$ from port A to port B on a bearing of 063^0T, then sails $130km$ From port B to port C on a bearing of 296^0T.



i) Show that $\angle ABC = 53^0$ 2

.....

.....

.....

.....

.....

.....

ii) Find, correct to the nearest km, the distance of port A from port C. 3

.....

.....

.....

.....

.....

iii) Use the cosine rule to find $\angle ACB$, and hence find the bearing of port A from port C, correct to the nearest degree. 3

.....

.....

.....

.....

.....

Question 12.(15 marks).

a) Prove that $\tan\theta - \tan\theta\sin^2\theta = \sin\theta\cos\theta$. 2

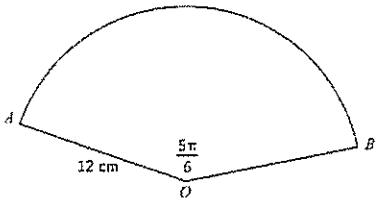
.....

.....

.....

.....

b) In the diagram below,



AB is an arc of a circle with centre O. The radius $OA = 12cm$ and $\angle AOB = \frac{5\pi}{6}$ radians.

i) Find the length of the arc AB. 2

.....

.....

.....

.....

ii) Find the area of the sector AOB. 2

.....

.....

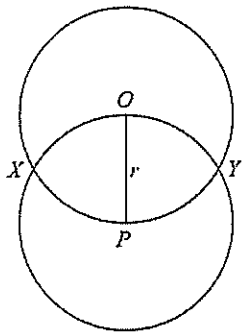
.....

.....

.....

.....

c) Two equal circles with centres O and P intersect at X and Y as shown below. The centres of each circle lie on the circumference of the other circle.



Find the exact area of the region $XOYP$. (Answer in terms of r) 3

.....

.....

.....

.....

.....

.....

.....

d) Solve these trigonometric equations in radians, for $0 \leq x \leq 2\pi$.

i) $\sin^2 x = \frac{3}{4}$ 3

.....

.....

.....

.....

ii) $2\cos^2 x - \cos x - 1 = 0$ 3

.....

.....

.....

.....

.....

.....

End of Examination

GIRRAWEEN HIGH SCHOOL

MATHEMATICS (ADVANCED)

YEAR 11

Yearly Examination 2019

Year 11 Mathematics

Time allowed: 120 minutes (plus10 minutes reading time)

Instructions:

- Attempt ALL questions.
- For Questions 1 to 5, colour in the bubble next to the letter corresponding to the correct response on your paper.
- For Questions 6 to 12:
    - All necessary working should be shown in the space provided.
    - Marks will be deducted for careless or badly arranged work.
    - Board-approved calculators may be used.
    - Reference sheet is provided.
    - Diagrams are NOT to scale.

Section 1 (5 marks)
Use the multiple-choice answer sheet for questions 1-5.
Multiple Choice (5 marks).

1. The simplest form of the expression $\frac{1}{\sqrt{3}-\sqrt{2}}-\sqrt{3}$ is:

A) $\sqrt{2}$ B) $\sqrt{3}$ C) $\sqrt{3}+\sqrt{2}$ D) $\sqrt{3}-\sqrt{2}$

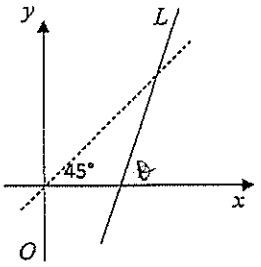
2. The simplest form of the algebraic fraction $\frac{n}{n-1}+\frac{n}{1+n}$ is :

A) 1 B) -1 C) 0 D) $\frac{2n^2}{n^2-1}$

3. The degree of the polynomial $\left(x^5+1\right)^2$, is

A) 2 B) 5 C) 7 D)10

4) Given the line L in the diagram below, the gradient of the line is m.
Which one of the following is correct about m.



A) $m=1$ B) $m=0$ C) $m<1$ D) $m>1$

5.What is the derivative of e^{x^2}.

A) $x^2e^{x^2-1}$ B) $2xe^{2x}$ C) $2xe^{x^2}$ D) $2e^{x^2}$

Answers to Multiple Choice Questions

Select the alternative A, B, C or D that best answers the question.

1.	A	●	B	○	C	○	D	○
2.	A	○	B	○	C	○	D	●
3.	A	○	B	○	C	○	D	●
4.	A	○	B	○	C	○	D	●
5.	A	○	B	○	C	●	D	○

Section11

- **Answer questions in the spaces provided.**
- **Your responses should include relevant mathematical reasoning and/or calculations.**
- **Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.**

Question 6 (18 marks)

a) Find the value of $4\pi\sqrt{\frac{a}{g}}$ if a =4.1 and g =9.8. Give your answer to 2 significant figures.

$= 4\pi\sqrt{\frac{4.1}{9.8}} = 8.1$ (1)

b) Write down the value of $|-9|-|-10|$

$= 9 - 10 = -1$ (1)

c) Factorise fully:

i) x^2+5x-6

$(x+6)(x-1)$ (1)

ii) $2x^2-7x-15$ $= (2x^2-10x)+(3x-15)$

$= 2x(x-5)+3(x-5)$ (2)

$= (x-5)(2x+3)$

d) Simplify:

i) $\frac{2x^2-xy}{4x^2-y^2}$

$= \frac{x(2x-y)}{(2x-y)(2x+y)} = \frac{x}{2x+y}$ (2)

ii) $\frac{2}{3}-\frac{x-1}{4}$

$= \frac{8-3(x-1)}{12} = \frac{8-3x+3}{12}$

$= \frac{11-3x}{12}$ (2)

iii) $\frac{1}{x^2-4}-\frac{2}{x-2}-\frac{3}{x+2}$ $= \frac{1}{(x-2)(x+2)} - \frac{2}{x-2} - \frac{3}{x+2}$

$= \frac{1-2(x+2)-3(x-2)}{(x-2)(x+2)} = \frac{1-2x-4-3x+6}{(x-2)(x+2)}$

$= \frac{-5x+3}{(x-2)(x+2)}$

e) Express $\frac{21}{\sqrt{63}}-\frac{3}{\sqrt{7}+2}$ as an integer. 3

$= \frac{21\times\sqrt{63}}{63} - \frac{3(\sqrt{7}-2)}{(\sqrt{7}+2)(\sqrt{7}-2)} = \frac{\sqrt{63}}{3} - \frac{3(\sqrt{7}-2)}{3}$

$= \frac{(\sqrt{63} - 3\sqrt{7} + 6)}{3}$

$= \frac{3\sqrt{7} - 3\sqrt{7} + 6}{3} = \frac{6}{3} = 2$

f) If $a=\left(\frac{1}{5}\right)^{\frac{1}{3}}$ and $b=\left(\frac{1}{5}\right)^{\frac{1}{2}}$, evaluate the expression $\frac{a^3+b^2}{(ab)^6}$. 3

$= \frac{\frac{1}{5} + \frac{1}{5}}{\left(\left(\frac{1}{5}\right)^{\frac{5}{6}}\right)^6} = \frac{\frac{2}{5}}{\left(\frac{1}{5}\right)^5} = \frac{2}{5} \times \frac{5^5}{1}$

$= 2\times 5^4 = 1250$

Question 7 (26 marks)

(a) Solve :

i) $3(2x+1)-2(3-x)=53$ 2

$6x+3-6+2x=53$

$8x-3=53$

$8x=56$

$x=7$

ii) $3x^2-4x-4=0$ 2

$(3x+2)(x-2)=0$

$x=-\frac{2}{3}$ or 2

iii) $16^x=64$ 2

$16^x=64$ equating powers

$4^{2x}=4^3 \therefore 2x=3 \therefore x=\frac{3}{2}$

OR $2^{4x}=2^6 \therefore 4x=6 \therefore x=\frac{3}{2}$

iv) $|4x-3|=7$ 3

$4x-3=7$ or $-(4x-3)=7$

$4x=10$

$x=\frac{5}{2}$ or $-4x+3=7$

$-4x=4$

$x=-1$

v) $\frac{x-5}{3}-\frac{x+1}{4}=5$ 3

(×12) $4(x-5)-3(x+1)=60$

$4x-20-3x-3=60$

$x-23=60$

$x=83$

b) Differentiate :

i) $y=\frac{7}{2x}$ $=\frac{7}{2}x^{-1}$ 2

$\frac{dy}{dx}=-\frac{7}{2}x^{-2}=\frac{-7}{2x^2}$

ii) $y=\frac{5-x^2+5x^4}{x^2}$ $=\frac{5}{x^2}-1+5x^2=5x^{-2}-1+5x^2$ 3

$\frac{dy}{dx}=-10x^{-3}+10x=\frac{-10}{x^3}+10x$

iii) $y=5(4-x)^9$ 3

$\frac{dy}{dx}=5\times 9\cdot(4-x)^8\times(-1)$

$=-45(4-x)^8$

iv) $y=x^2(3x+2)^3$ 3

$u=x^2$ | $v=(3x+2)^3$

$u'=2x$ | $v'=3(3x+2)^2\times 3$

$=9(3x+2)^2$

$\frac{dy}{dx}=vu'+uv'=(3x+2)^3(2x)+x^2\times 9(3x+2)^2$

$=3x(3x+2)^2[6x+4+9x]$

$=x[3x+2]^2[15x+4]$

v) $y = \dfrac{x^2-4}{x-1}$

$u = x^2 - 4 \quad | \quad v = x - 1$

$u' = 2x \quad | \quad v' = 1$

$\dfrac{dy}{dx} = \dfrac{vu' - uv'}{v^2} = \dfrac{(x-1)\,2x - (x^2-4)\,1}{(x-1)^2}$

$= \dfrac{2x^2 - 2x - x^2 + 4}{(x-1)^2} = \dfrac{x^2 - 2x + 4}{(x-1)^2}$

Question 8 (19 marks)

a) Differentiate $y = \dfrac{2x}{e^x + 1}$ 3

$u = 2x \quad | \quad v = e^x + 1$

$u' = 2 \quad | \quad v' = e^x$

$\dfrac{dy}{dx} = \dfrac{vu' - uv'}{v^2} = \dfrac{(e^x+1)2 - 2x(e^x)}{(e^x+1)^2}$

$= \dfrac{2e^x + 2 - 2xe^x}{(e^x+1)^2}$

b) i) Differentiate $y = e^x - x$ 1

$\dfrac{dy}{dx} = e^x - 1$

ii) For what value of x is the curve stationary? 2

stationary $\Rightarrow \dfrac{dy}{dx} = 0 \quad \therefore e^x - 1 = 0$

$e^x = 1 \quad \therefore x = 0.$

b) Find the equation of the normal to the curve $y = \sqrt{x+2}$ at the point (7,3). 3

$y = (x+2)^{1/2} \quad \therefore \dfrac{dy}{dx} = \dfrac{1}{2}(x+2)^{-1/2} = \dfrac{1}{2\sqrt{x+2}}$

at (7,3), $\dfrac{dy}{dx} = \dfrac{1}{2\sqrt{9}} = \dfrac{1}{6} \quad \therefore m_{normal} = -6$

eq^n : $y - 3 = -6(x - 7) \quad \therefore$ ~~$6y + 18 = x + 7$~~

$y - 3 = -6x + 42$

$6x + y - 45 = 0$

c) i) Find the gradient of the tangent to $y = e^{-x}$ at the point $P(-1, e)$. 2

$\frac{dy}{dx} = -e^{-x}$, at $(-1, e)$ $\frac{dy}{dx} = -e^{1} = -e$

ii) Thus write down the gradient of thr normal at this point. 1

$\therefore$ gradient of normal $= \frac{1}{e}$

iii) Hence determine the equation of this normal. 2

$y - e = \frac{1}{e}(x + 1)$

$ey - e^2 = x + 1$

$\therefore x - ey + e^2 + 1 = 0$

iv) Find the x and y intercepts of the normal. $x - ey + e^2 + 1 = 0$ 2

x-intercept $\Rightarrow y = 0$ $\therefore x = -(1 + e^2)$

y-intercept $\Rightarrow x = 0$ $\therefore y = \frac{e^2 + 1}{e}$

v) Find the area of the triangle whose vertices lie at the intercepts and the origin. 3

Area $= \frac{1}{2}bh = \frac{1}{2}(1 + e^2)\left(\frac{e^2 + 1}{e}\right)$

$= \frac{1}{2e}(e^4 + 2e^2 + 1)$

$= \frac{1}{2}\left(e^3 + 2e + e^{-1}\right) u^2$

Question 9 (20 marks).

a) Evaluate

i) $\log_{10} 875$ correct to 2 decimal places. 1

$= 2.94$

ii) Evaluate $\log_e 80$ to 3 significant figures. 1

$= 4.38$

iii) $\log_5 2$ correct to 2 decimal places 2

$= \frac{\log_{10} 2}{\log_{10} 5} = 0.43$

b) Evaluate:

i) $\log_2 16$ 2

$= \log_2 2^4 = 4\log_2 2 = 4$

ii) $\log_5 25+1$ 2

$= \log_5 5^2 + 1 = 2+1 = 3$

iii) $\log_3 \frac{1}{\sqrt[3]{3}}$ 2

$= \log_3 3^{-\frac{1}{3}} = -\frac{1}{3}\log_3 3 = -\frac{1}{3}$

c) Given $\log_5 3 = 0.68$ and $\log_5 4 = 0.86$, find

i) $\log_5 12$ 1

$= \log_5 (3 \times 4) = \log_5 3 + \log_5 4$

$= 0.68 + 0.86 = 1.54$

ii) $\log_5 0.75$ 1

$= \log_5 \frac{3}{4} = \log_5 3 - \log_5 4$

$= 0.68 - 0.86$

$= -0.18$

iii) $\log_5 9$ 2

$= \log_5 3^2 = 2\log_5 3$

$= 2 \times 0.68$

$= 1.36$

iv) $\log_5 20$ 2

$= \log_5 (5 \times 4) = \log_5 5 + \log_5 4$

$= 1 + 0.86 = 1.86$

d) The amount of money A in a bank account after n years grows with compound interest according to the formula $A = 850(1.025)^n$.

i) Find the initial amount in the bank. $A = 850(1.025)^n$ 1

$n = 0 \Rightarrow A = \$850$

ii) Find the amount after 7 years. $n = 7$ 1

$A = 850(1.025)^7 = \$1010.38$

iii) Find how many years it will take for the amount in the bank to be \$1000. 2

$1000 = 850(1.025)^n$

$\frac{1000}{850} = 1.025^n = \frac{20}{17}$

$n \log(1.025) = \log\left(\frac{20}{17}\right)$

$\therefore n = \frac{\log \frac{20}{17}}{\log 1.025} \doteq 6.6$

Question 10 (12 marks)

a) Are the lines $5x-y-10=0$, $x+y-8=0$ and $2x-3y+9=0$ concurrent? 3

$5x-y-10=10$ —①

$x+y-8=0$ —②

① + ② $6x-18=0$

$x=3$

sub $x=3$ in ① $15-y-10=0$ $\therefore y=5$

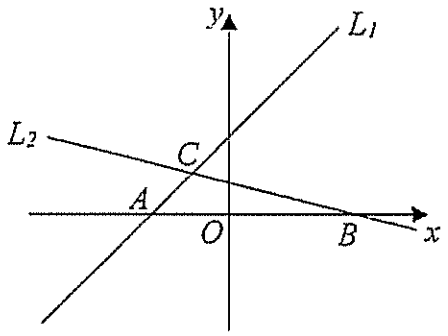
These lines intersect at $(3,5)$.

sub in $(3,5)$ in the line $2x-3y+9=0$

LHS. $6-15+9=0=RH$

$\therefore$ Three lines are concurrent.

) In the diagram below, the line L_1: $x-y+2=0$ cuts the x axis at A and the line L_2: $x+4y-3=0$ cuts the x axis at B.



i) Show that the co-ordinates of A and B are (-2,0) and (3,0) respectively. 2

co-ordinates of A $\Rightarrow y=0$ $x-y+2=0$ $\therefore x=-2$ $\therefore A(-2,0)$

co-ordinates of B $\Rightarrow y=0$ in $x+4y-3=0$ $\therefore x=3$ $\therefore B(3,0)$.

ii) If C is the point of intersection of the lines L_1 and L_2, use simultaneous equations to show that the co-ordinates of C are (-1,1). 2

$x-y+2=0$ —①

$x+4y-3=0$ —②

① − ② $-5y+5=0$

$\therefore y=1$

sub in $y=1$ in ①

$x-1+2=0$

$\therefore x=-1$

$\therefore C(-1,1)$

iii) Find the length of AC. 1

$A(-2,0)$; $C(-1,1)$

$AC = \sqrt{(-2+1)^2+(0-1)^2} = \sqrt{1+1} = \sqrt{2}$

iv) Calculate the gradient of the line AC. 1

$m_{AC} = \frac{1-0}{-1+2} = \frac{1}{1} = 1$

v) Show that the line L_1 makes an angle of 45^0 with the positive direction of the x-axis. 1

$\tan\theta = m = \tan^{-1} 1 = 45^0$

vi) Find the area of ΔABC. 2

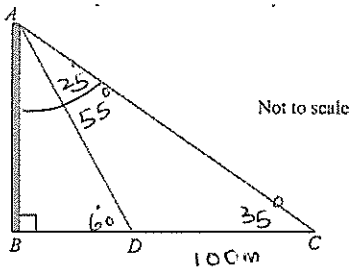
$\text{Area} = \frac{1}{2} \cdot AB \cdot AC \sin 45^0$

$= \frac{1}{2} (3+2) \cdot \sqrt{2} \times \frac{1}{\sqrt{2}}$

$= \frac{1}{2} \times 5 = 2.5u^2$

Question 11 (13 marks).

a) A vertical tower AB with points B, C and D in a straight line on the ground is shown below. The distance from CD is 100 metres. The angle of elevation to the top of the tower from point C is 35^0 and from point D is 60^0.



i) Show that $AD = \dfrac{100\sin 35^0}{\sin 25^0}$ 3

$\angle CAD = 25^\circ$ (Ext. $\angle ADB = \angle CAD + 35^\circ$)

Using sine rule in $\triangle ADC$, $\dfrac{AD}{\sin 35^\circ} = \dfrac{100}{\sin 25^\circ}$

$AD = \dfrac{100\sin 35^\circ}{\sin 25^\circ}$

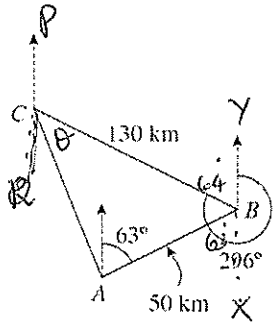
ii) Calculate the height of the tower. Answer to the nearest metre. 2

$\sin 60^\circ = \dfrac{AB}{AD}$

$\therefore AB = \dfrac{100\sin 35^\circ}{\sin 25^\circ} \times \sin 60^\circ = 117.5367\ldots$

$= 118\text{m}$

b) A ship sails $50km$ from port A to port B on a bearing of 063^0T, then sails $130km$ From port B to port C on a bearing of 296^0T.



i) Show that $\angle ABC = 53^0$ 2

$\angle ABX = 63^\circ$ (alternate $\angle$'s on $\parallel$ lines)

$\angle CBY = 360 - 296 = 64^\circ$ (angles at a point)

$\therefore \angle ABC = 180 - (63^\circ + 64^\circ) = 180 - 127 = 53^\circ$

ii) Find, correct to the nearest km, the distance of port A from port C. 3

Using cosine rule,

$AC^2 = 130^2 + 50^2 - 2 \times 130 \times 50 \cos 53^\circ$

$\therefore AC = 107.594\ldots = 108km$

iii) Use the cosine rule to find $\angle ACB$, and hence find the bearing of port A from port C, correct to the nearest degree. 3

$\cos\theta = \dfrac{130^2 + 108^2 - 50^2}{2 \times 130 \times 108} = 0.928$

$\theta = 22^\circ$

$\angle BCP = 180 - 64 = 116^\circ$

$\therefore$ bearing of A from port C $= 116 + 22 = 138^\circ$

Question 12.(15 marks).

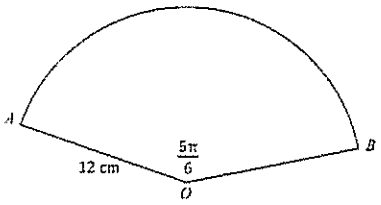
Prove that $\tan\theta - \tan\theta\sin^2\theta = \sin\theta\cos\theta$. 2

LHS $\tan\theta - \tan\theta\sin^2\theta$

$= \tan\theta(1-\sin^2\theta) = \tan\theta \times \cos^2\theta$

$= \frac{\sin\theta}{\cos\theta} \times \cos^2\theta = \sin\theta\cos\theta = RHS$

b) In the diagram below,



AB is an arc of a circle with centre O. The radius $OA = 12cm$ and $\angle AOB = \frac{5\pi}{6}$ radians.

i) Find the length of the arc AB. 2

$l = r\theta$

$= 12 \times \frac{5\pi}{6} = 10\pi$ cm

ii) Find the area of the sector AOB. 2

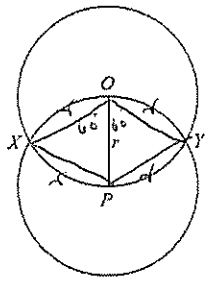
Area $= \frac{1}{2} r^2 \sin\theta$

$= \frac{1}{2} \times 12^2 \sin\frac{5\pi}{6}$

$= \frac{1}{2} \times 144 \times \frac{1}{2} = 36$cm^2

$= \frac{1}{2} \times 144 \times \frac{5\pi}{6} = 60\pi$ cm^2.

b) Two equal circles with centres O and P intersect at X and Y as shown below. The centres of each circle lie on the circumference of the other circle



Find the exact area of the region $XOYP$. 3

Area of segment XYP = A

$A = \frac{1}{2}r^2(\theta - \sin\theta)$

$= \frac{1}{2}r^2\left(\frac{2\pi}{3} - \sin\frac{2\pi}{3}\right)$

$= \frac{1}{2}r^2\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$

Area of the region XOYP is twice the area of segment XYP

$\therefore A = 2 \times \frac{1}{2}r^2\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right) = r^2\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$

c) Solve these trigonometric equations in radians, for $0 \le x \le 2\pi$.

i) $\sin^2 x = \frac{3}{4}$ ii) $2\cos^2 x - \cos x - 1 = 0$ (6)

$\sin x = \frac{\sqrt{3}}{2}$ or $\sin x = -\frac{\sqrt{3}}{2}$

related angle $= \frac{\pi}{3}$

$\therefore x = \frac{\pi}{3}$ or $\frac{2\pi}{3}$ or $\frac{4\pi}{3}$ or $\frac{5\pi}{3}$

Let $u = \cos x$

$2u^2 - u - 1 = 0$

$(2u+1)(u-1) = 0$

$u = -\frac{1}{2}$ or 1

$\therefore \cos x = -\frac{1}{2}$ or 1

$\cos x = -\frac{1}{2}$, related $\angle = \frac{\pi}{3}$

$\therefore x = \frac{2\pi}{3}$ or $\frac{4\pi}{3}$

$\cos x = 1$, $x = 0$ or 2π.

$\therefore x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$ or 2π

End of Examination