GIRRAWEEN HIGH SCHOOL

MATHEMATICS (ADVANCED)

YEAR 11

Yearly Examination 2020

Year 11 Mathematics **September 2020**

Reading time: 10 minutes

Working time: 120 minutes

Section I	**5** marks
Section II	**100** marks
TOTAL	**105** marks

Instructions:

- Attempt ALL questions.
- For Questions 1 to 5, colour in the bubble next to the letter corresponding to the correct response on your paper.
- For Questions 6 to 12:
  - All necessary working should be shown in the space provided.
  - Marks will be deducted for careless or badly arranged work.
  - Board-approved calculators may be used.
  - Reference sheet with formulae is attached at the end of the paper.
  - Diagrams are NOT to scale.

Section I (5 marks)

- **Use the multiple-choice answer sheet for questions 1-5.**

---

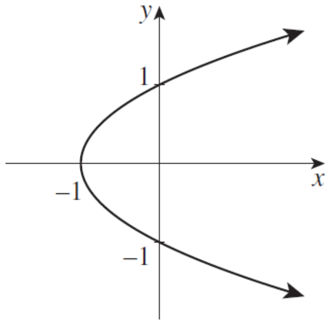
1. What type of relation is shown by the graph on the right?

(A) One-to-one

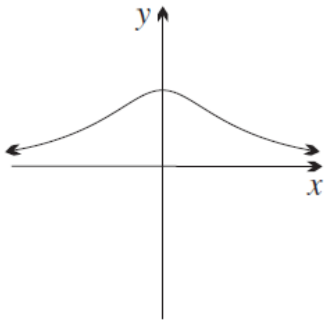
(B) Many-to-one

(C) One-to-many

(D) Many-to-many

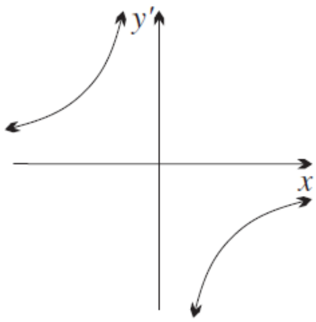


2. The diagram below shows the graph of $y = f(x)$.

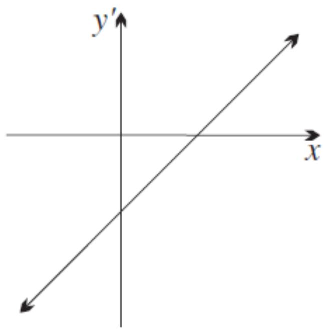


Which of the following graphs show the graph of y'?

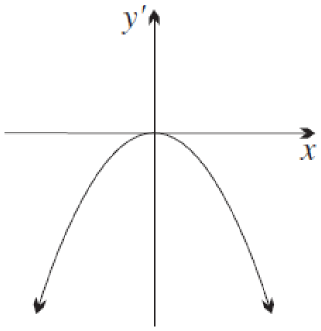
(A)



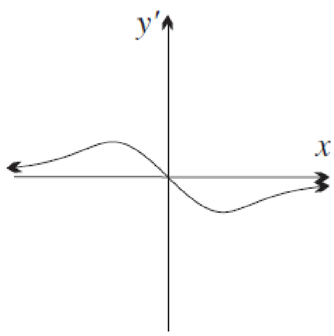
(B)



(C)

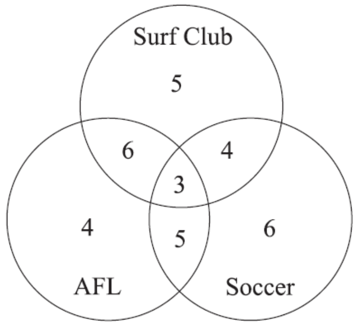


(D)



3. In a classroom, students are asked what sports club they are members of and the results are shown in the Venn diagram on the right.

A student who is a member of a AFL club is chosen at random. What is the probability that he/she is also a member of a soccer club?



(A) $\frac{4}{9}$ (B) $\frac{5}{18}$

(C) $\frac{6}{11}$ (D) $\frac{8}{33}$

4. What is the derivative of $(x^2 - 6)^3$?

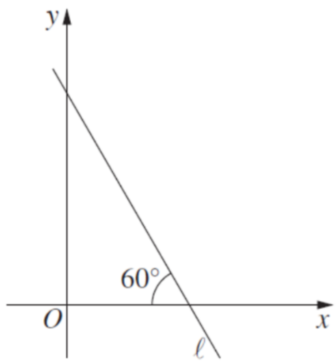
(A) $3(x^2 - 6)^2$

(B) $3(2x - 6)^2$

(C) $6x(x^2 - 6)^2$

(D) $6x(2x - 6)^2$

5. What is the slope of the line l shown on the right?



(A) $\frac{1}{\sqrt{3}}$

(B) $-\frac{1}{\sqrt{3}}$

(C) $\sqrt{3}$

(D) $-\sqrt{3}$

Answers to Multiple Choice Questions

Select the alternative A, B, C or D that best answers the question.

1.	A	○	B	○	C	○	D	○
2.	A	○	B	○	C	○	D	○
3.	A	○	B	○	C	○	D	○
4.	A	○	B	○	C	○	D	○
5.	A	○	B	○	C	○	D	○

Section II (105 marks)

- **Answer questions in the spaces provided.**
- **Your responses should include relevant mathematical reasoning and/or calculations.**
- **Extra writing space is provided at the back of this booklet. If you use this space, indicate in the writing space under the question and clearly write the question number on the extra writing space.**

Question 6 (14 marks)

(a) Evaluate, correct to 3 significant figures: $\dfrac{e^2+\sqrt[4]{1.6\times2.6}}{\pi log_{10}5}$ **1**

………………………………………………………………………………………………………

(b) Factorise fully: $3x^2-7x-6$ **2**

………………………………………………………………………………………………………

………………………………………………………………………………………………………

(c) Express $475°$ in radians. Leave your answers in terms of π. **1**

………………………………………………………………………………………………………

………………………………………………………………………………………………………

(d) Expand and simplify: $(\sqrt{5}-3)(2\sqrt{5}+4)$ **2**

………………………………………………………………………………………………………

………………………………………………………………………………………………………

(e) Simplify:

(i) $\dfrac{x^2+5x+6}{x^2-16}\div\dfrac{x+3}{x-4}$ **2**

………………………………………………………………………………………………………

………………………………………………………………………………………………………

………………………………………………………………………………………………………

………………………………………………………………………………………………………

………………………………………………………………………………………………………

(ii) $\frac{1}{x-y}+\frac{2x-y}{x^2-y^2}$ **3**

....

....

....

....

....

....

(f) Show that $\frac{2}{6-3\sqrt{3}}-\frac{1}{2\sqrt{3}+3}$ is rational. **3**

....

....

....

....

....

....

Question 7 (10 marks)

(a) Solve:

(i) $\frac{a}{7}-4=\frac{a}{2}+11$ **2**

....

....

....

....

(ii) $3^{-4x} = 9^{6-x}$ **2**

………

………

………

(iii) $|3x - 8| = 4$ **2**

………

………

………

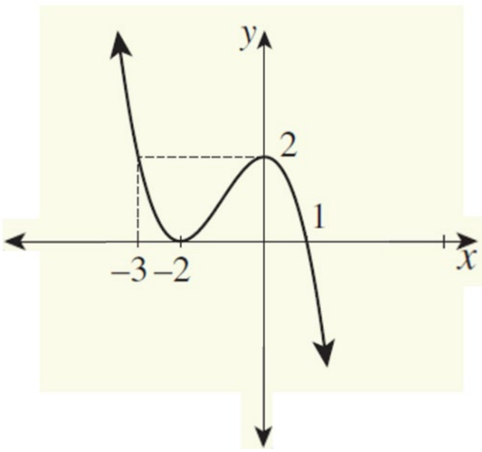
………

(b) Write the function for when $y = 2^x - x$ is rotated $180°$ about the origin. **2**

………

………

(c) The graph below shows the curve $y = f(x)$. On the same set of axes, sketch $y = f(x + 2)$ showing all important features. **2**



Question 8 (17 marks)

(a) (i) Expand $\left(x - \frac{1}{x}\right)^2$. **1**

………

………

(ii) Show that $x - \frac{1}{x} = 2\sqrt{11}$, given that $x = \sqrt{11} + 2\sqrt{3}$. **2**

………

………

………

………

(iii) Use the answer in (i) and (ii) to find the value of $x^2 + \frac{1}{x^2}$. **2**

………

………

………

(b) Consider the function $y = -x^2 + 2x + 8$.

(i) Find the value of the discriminant. **1**

………

………

(ii) Find the coordinates of the vertex. **1**

………

………

(iii) Sketch the graph, showing all the intercepts. **2**

(iv) State the domain and the range. **2**

………

………

(c) Differentiate:

(i) $y = \frac{x+2}{3x-4}$ **2**

………

………

………

………

(ii) $y = (2x+1)^4$. **2**

………

………

………

(iii) $y = \sqrt[3]{x^2 - 5}$ **2**

………

………

………

………

Question 9 (17 marks)

(a) Express $cosec^2\theta - 1$ in terms of $cot^2\theta.$ **1**

……………………………………………………………………………………………………

……………………………………………………………………………………………………

(b) Prove that: $4sec^2\theta - 3 = 1 + 4tan^2\theta$ **2**

……………………………………………………………………………………………………

……………………………………………………………………………………………………

……………………………………………………………………………………………………

……………………………………………………………………………………………………

(c) Solve $2\cos x - \sqrt{3} = 0$ for $0 \leq x \leq 2\pi$. **2**

……………………………………………………………………………………………………

……………………………………………………………………………………………………

……………………………………………………………………………………………………

……………………………………………………………………………………………………

(d) You may use the space below to draw a diagram for parts (i) to (v) on the next page.
There is no mark allocated for the sketch.

(i) Find the gradient, length and midpoint M of the interval joining A(10, 2) and B(2, 8). **3**

………

………

………

………

………

(ii) Show that the perpendicular bisector of the interval AB has equation $4x - 3y - 9 = 0$. **2**

………

………

………

………

(iii) Find the point C where the perpendicular bisector meets the line $x - y + 2 = 0$. **2**

………

………

………

………

(iv) Find the length of CM and hence find the area of ΔABC. **3**

………

………

………

………

(v) Find $\angle ACB$, correct to the nearest minute. **2**

………

………

………

………

Question 10 (16 marks)

(a) Use the log laws and identities to simplify and evaluate:

$log_5 2 - log_5 250$ **2**

……………………………………………………………………………………………………

……………………………………………………………………………………………………

……………………………………………………………………………………………………

(b) A bag contains 6 red balls and 4 white balls. A sample of 3 balls were randomly drawn from the bag. If the ball is white it is replaced, and if it is red it is not replaced.

(i) Draw a probability tree to show the all possible outcomes of the sample. **2**

(ii) Find the probability of drawing at least one red ball. **1**

……………………………………………………………………………………………………

……………………………………………………………………………………………………

(iii) Find the probability of drawing a sample with two balls of the same colour. **1**

……………………………………………………………………………………………………

……………………………………………………………………………………………………

(c) Solve $log_2(6-x) - log_2(4-x) = 2$ for x, where $x < 4$. **3**

....

....

....

....

(d) Differentiate $y = (2x^3 - 1)(x^4 - 2)$. **2**

....

....

....

....

(e) Let $f(x) = x^2 + kx + 3$ where k is a constant.
Find the value of k given that $f(x)$ has a tangent with the gradient of zero at $x = -1$. **2**

....

....

....

....

....

....

(f) Consider the function $y = 2x^2 - x + 1$.
Find the equation of the normal to the curve at the point $x = 1$. **3**

....

....

....

....

....

....

Question 11 (12 marks)

(a) Solve for x. $e^{2x} + 3e^x - 10 = 0.$ **2**

....

....

....

....

....

....

(b) Find the exact value of $\sin\theta$, for $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, given $\cot\theta = -\frac{24}{7}$. **2**

....

....

(c) For events A and B from a sample space, $P(A|B) = \frac{3}{4}$ and $P(B) = \frac{1}{3}$.

(i) Calculate $P(A \cap B)$. **1**

....

....

(ii) Draw a Venn diagram, hence calculate $P(\bar{A} \cap B)$. **2**

....

....

(iii) If events A and B are independent, calculate $P(A \cup B)$. **2**

....

....

....

(d) Differentiate $f(x) = x^2 - 3x$ from first principles. **3**

....

....

....

....

....

....

....

....

Question 12 (14 marks)

(a) The population, D, of Tasmanian Devils in a sanctuary is given by $D(t)$, where t is the time in years after the sanctuary was established.
The devil population is modelled by the function $D = 28e^{0.35t}$.

(i) What was the population of Tasmanian Devils at the time when the sanctuary was established? **1**

....

(ii) Find the expression to show the rate of change of population of Tasmanian Devils. **1**

....

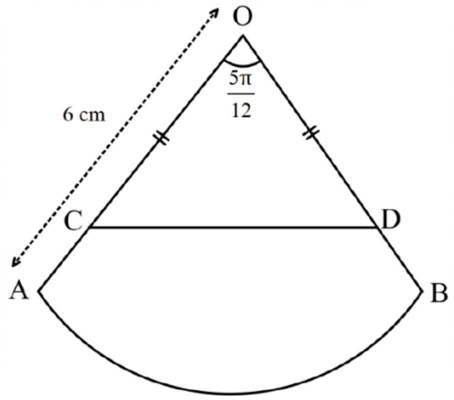
(iii) Calculate the increase in the number of Tasmanian Devils at the end of the first 8 years. Give your answer correct to two significant figures. **1**

....

....

(iv) Sketch the function of the population of Tasmanian Devils over time. **2**

(b) The diagram shows a sector AOB. The length OA is 6 cm and $\angle AOB = \frac{5\pi}{12}$. The points C and D lie on OA and OB respectively, and $OC = OD$.



(i) Find the exact area of sector AOB. **1**

………

………

(ii) If the area of ΔCOD is half the area of sector AOB, find the length of OC correct to two decimal places. **3**

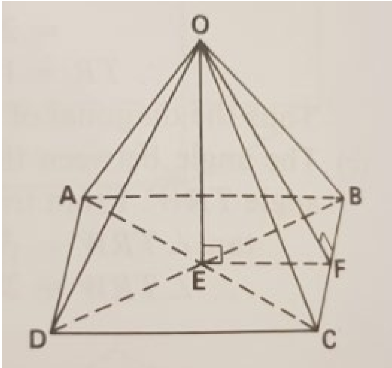
………

………

………

………

(c) The square pyramid shown below has a height of 16 cm and a base edge of 24 cm. Find:



(i) the length of OF. **1**

………………………………………………………………………………………………………

………………………………………………………………………………………………………

(ii) the size of the angle between the plane OBC and the base $ABCD$. **1**

………………………………………………………………………………………………………

………………………………………………………………………………………………………

………………………………………………………………………………………………………

………………………………………………………………………………………………………

(iii) the length DE and hence find the size of the angle between the edge OD and the base $ABCD$. **3**

………………………………………………………………………………………………………

………………………………………………………………………………………………………

………………………………………………………………………………………………………

………………………………………………………………………………………………………

………………………………………………………………………………………………………

………………………………………………………………………………………………………

………………………………………………………………………………………………………

………………………………………………………………………………………………………

End of Examination

Extra Writing Space. Clearly write the question number

GIRRAWEEN HIGH SCHOOL

MATHEMATICS (ADVANCED)

YEAR 11

Yearly Examination 2020

Year 11 Mathematics **September 2020**

Reading time: 10 minutes

Working time: 120 minutes

Section I	**5** marks
Section II	**100** marks
TOTAL	**105** marks

Instructions:

- Attempt ALL questions.
- For Questions 1 to 5, colour in the bubble next to the letter corresponding to the correct response on your paper.
- For Questions 6 to 12:
  - o All necessary working should be shown in the space provided.
  - o Marks will be deducted for careless or badly arranged work.
  - o Board-approved calculators may be used.
  - o Reference sheet with formulae is attached at the end of the paper.
  - o Diagrams are NOT to scale.

Section I (5 marks)

- **Use the multiple-choice answer sheet for questions 1-5.**

---

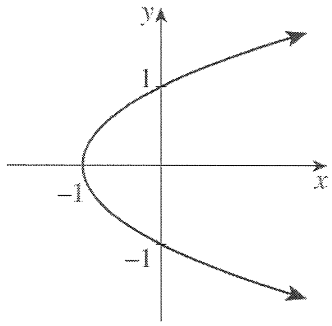
1. What type of relation is shown by the graph on the right?

(A) One-to-one

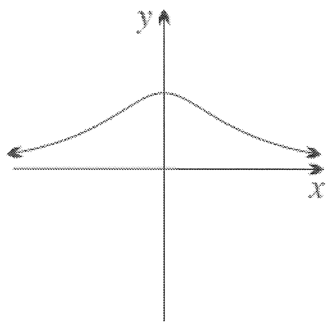
(B) Many-to-one

(C) One-to-many

(D) Many-to-many

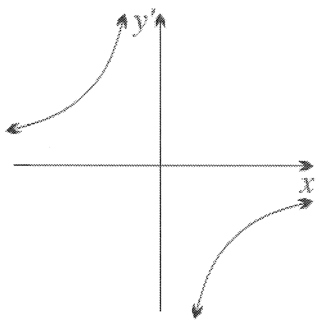


2. The diagram below shows the graph of $y = f(x)$.

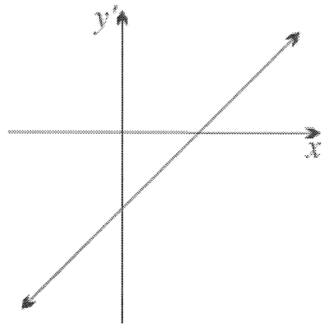


Which of the following graphs show the graph of y'?

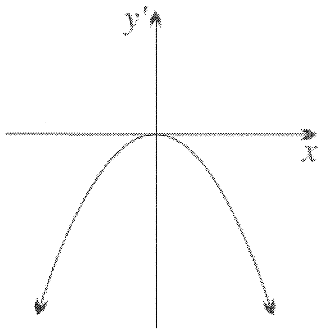
(A)



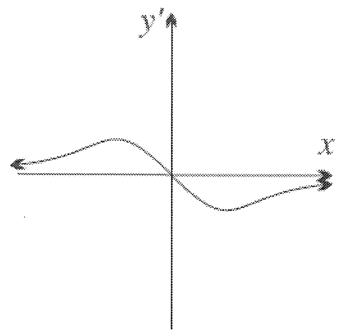
(B)



(C)

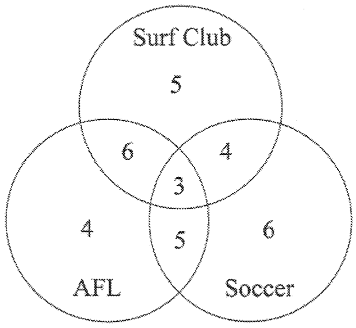


(D)



3. In a classroom, students are asked what sports club they are members of and the results are shown in the Venn diagram on the right.

A student who is a member of a AFL club is chosen at random. What is the probability that he/she is also a member of a soccer club?



(A) $\frac{4}{9}$ (B) $\frac{5}{18}$

(C) $\frac{6}{11}$ (D) $\frac{8}{33}$

4. What is the derivative of $(x^2 - 6)^3$?

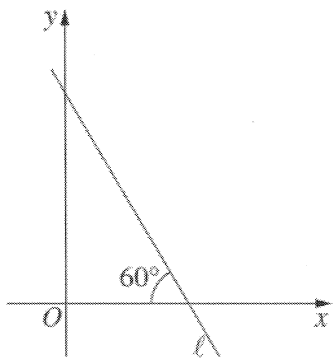
(A) $3(x^2 - 6)^2$

(B) $3(2x - 6)^2$

(C) $6x(x^2 - 6)^2$

(D) $6x(2x - 6)^2$

5. What is the slope of the line l shown on the right?



(A) $\frac{1}{\sqrt{3}}$

(B) $-\frac{1}{\sqrt{3}}$

(C) $\sqrt{3}$

(D) $-\sqrt{3}$

Answers to Multiple Choice Questions

Select the alternative A, B, C or D that best answers the question.

1.	A	○	B	○	C	●	D	○
2.	A	○	B	○	C	○	D	●
3.	A	●	B	○	C	○	D	○
4.	A	○	B	○	C	●	D	○
5.	A	○	B	○	C	○	D	●

Section II (105 marks)

- **Answer questions in the spaces provided.**
- **Your responses should include relevant mathematical reasoning and/or calculations.**
- **Extra writing space is provided at the back of this booklet. If you use this space, indicate in the writing space under the question and clearly write the question number on the extra writing space.**

Question 6 (14 marks)

(a) Evaluate, correct to 3 significant figures: $\dfrac{e^2+\sqrt[4]{1.6\times 2.6}}{\pi \log_{10}5}$ **1**

4.02

(b) Factorise fully: $3x^2-7x-6$ **2**

$= 3x^2-9x+2x-6 = 3x(x-2)+2(x-3)$

$=(3x+2)(x-3)$

(c) Express 475° in radians. Leave your answers in terms of π. **1**

$475 \times \dfrac{\pi}{180} = \dfrac{95\pi}{36}$

(d) Expand and simplify: $(\sqrt{5}-3)(2\sqrt{5}+4)$ **2**

$= 10+4\sqrt{5}-6\sqrt{5}-12$

$= -2-2\sqrt{5}$ or $-2(1+\sqrt{5})$

(e) Simplify:

(i) $\dfrac{x^2+5x+6}{x^2-16} \div \dfrac{x+3}{x-4}$ **2**

$= \dfrac{(x+2)\cancel{(x+3)}}{\cancel{(x-4)}(x+4)} \times \dfrac{\cancel{(x-4)}}{\cancel{(x+3)}}$

$= \dfrac{x+2}{x+4}$

(ii) $\frac{1}{x-y}+\frac{2x-y}{x^2-y^2}$ **3**

$= \frac{1}{(x-y)} + \frac{2x-y}{(x+y)(x-y)}$

$= \frac{(x+y)+2x-y}{(x+y)(x-y)} = \frac{3x}{(x+y)(x-y)}$ OR $\frac{3x}{x^2-y^2}$

(f) Show that $\frac{2}{6-3\sqrt{3}}-\frac{1}{2\sqrt{3}+3}$ is rational. **3**

$= \frac{2(2\sqrt{3}+3)-(6-3\sqrt{3})}{(6-3\sqrt{3})(2\sqrt{3}+3)} = \frac{4\sqrt{3}+6-6+3\sqrt{3}}{12\sqrt{3}+18-18-9\sqrt{3}}$

$= \frac{7\sqrt{3}}{3\sqrt{3}} = \frac{7}{3}$ Hence it's rational

OR

$\frac{2\ (6+3\sqrt{3})}{(6-3\sqrt{3})(6+3\sqrt{3})} - \frac{(2\sqrt{3}-3)}{(2\sqrt{3}+3)(2\sqrt{3}-3)} = \frac{12+6\sqrt{3}}{9} - \frac{(2\sqrt{3}-3)}{3} = \frac{12+6\sqrt{3}-6\sqrt{3}+9}{9}$

$= \frac{21}{9} = \frac{7}{3}$

Question 7 (10 marks)

(a) Solve:

(i) $\frac{a}{7}-4=\frac{a}{2}+11$ **2**

$\frac{a}{7} - \frac{a}{2} = 15$

$\frac{2a-7a}{14} = 15$ $\therefore a = -42$

$-5a = 210$

(ii) $3^{-4x} = 9^{6-x}$ **2**

$3^{-4x} = (3^2)^{6-x}$

$-4x = 2(6-x)$

$-4x = 12 - 2x$

$-2x = 12$

$\therefore x = -6$

(iii) $|3x - 8| = 4$ **2**

$3x - 8 = 4$ OR $3x - 8 = -4$

$3x = 12$ $\qquad$ $3x = 4$

$\therefore x = 4$ $\qquad$ $\therefore x = \frac{4}{3}$

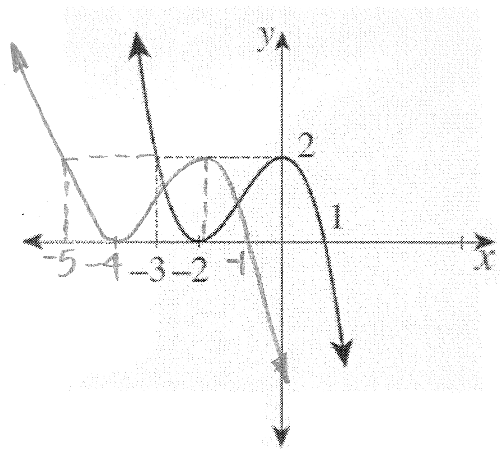
(b) Write the function for when $y = 2^x - x$ is rotated $180°$ about the origin. **2**

order is not important

replace x by $-x$: $y = 2^{-x} + x$

replace y by $-y$: $-y = 2^{-x} + x$ $\quad$ $\therefore y = -2^{-x} - x$

(c) The graph below shows the curve $y = f(x)$. On the same set of axes, sketch $y = f(x+2)$ showing all important features. **2**



Question 8 (17 marks)

(a) (i) Expand $\left(x - \frac{1}{x}\right)^2$. **1**

$= x^2 - 2(x)\left(\frac{1}{x}\right) + \left(\frac{1}{x}\right)^2 = x^2 - 2 + \frac{1}{x^2}$

(ii) Show that $x - \frac{1}{x} = 2\sqrt{11}$, given that $x = \sqrt{11} + 2\sqrt{3}$. **2**

$LHS = x - \frac{1}{x} = (\sqrt{11} + 2\sqrt{3}) - \frac{1}{(\sqrt{11}+2\sqrt{3})}\frac{(\sqrt{11}-2\sqrt{3})}{(\sqrt{11}-2\sqrt{3})} = (\sqrt{11}+2\sqrt{3}) - \frac{(\sqrt{11}-2\sqrt{3})}{(11-12)}$

$= (\sqrt{11}+2\sqrt{3}) + (\sqrt{11}-2\sqrt{3}) = 2\sqrt{11}$

(iii) Use the answer in (i) and (ii) to find the value of $x^2 + \frac{1}{x^2}$. **2**

$x^2 + \frac{1}{x^2} = \left(x - \frac{1}{x}\right)^2 + 2 = (2\sqrt{11})^2 + 2 = 46$

(b) Consider the function $y = -x^2 + 2x + 8$.

(i) Find the value of the discriminant. **1**

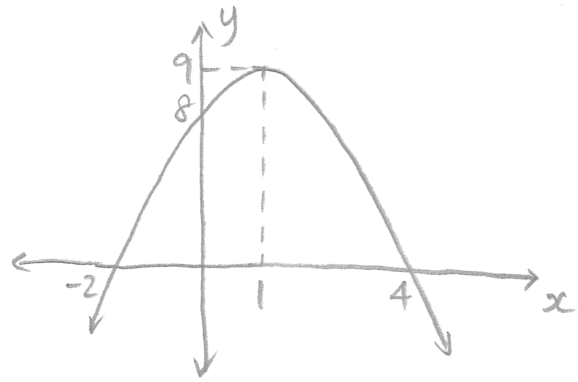
$\Delta = b^2 - 4ac = 4 + 32 = 36$

(ii) Find the coordinates of the vertex. **1**

$x_{vertex} = -\frac{b}{2a} = 1 \qquad y_{vertex} = -(1)^2 + 2(1) + 8 = 9$

$\therefore$ vertex is at $(1, 9)$

(iii) Sketch the graph, showing all the intercepts. **2**



(iv) State the domain and the range. **2**

Domain : all real values of x

Range : $y \leq 9$

(c) Differentiate:

(i) $y = \frac{x+2}{3x-4}$ **2**

$\frac{dy}{dx} = \frac{(3x-4)-3(x+2)}{(3x-4)^2} = -\frac{10}{(3x-4)^2}$

(ii) $y = (2x+1)^4$. **2**

$\frac{dy}{dx} = 4 \times 2 \times (2x+1)^3$

$= 8(2x+1)^3$

(iii) $y = \sqrt[3]{x^2 - 5}$ $= (x^2-5)^{1/3}$ **2**

$\frac{dy}{dx} = \frac{1}{3}(x^2-5)^{-2/3} \times 2x$

$= \frac{2x}{3\sqrt[3]{(x^2-5)^2}}$

Question 9 (17 marks)

(a) Express $cosec^2\theta - 1$ in terms of $cot^2\theta$. **1**

$\cos^2\theta + \sin^2\theta = 1$ → $\cot^2\theta + 1 = cosec^2\theta$

$\frac{\cos^2\theta}{\sin^2\theta} + 1 = \frac{1}{\sin^2\theta}$ $\therefore cosec^2\theta - 1 = \cot^2\theta$

(b) Prove that: $4sec^2\theta - 3 = 1 + 4tan^2\theta$ **2**

$LHS = 4\sec^2\theta - 3 = 4(1+\tan^2\theta) - 3$

$= 1 + 4\tan^2\theta$

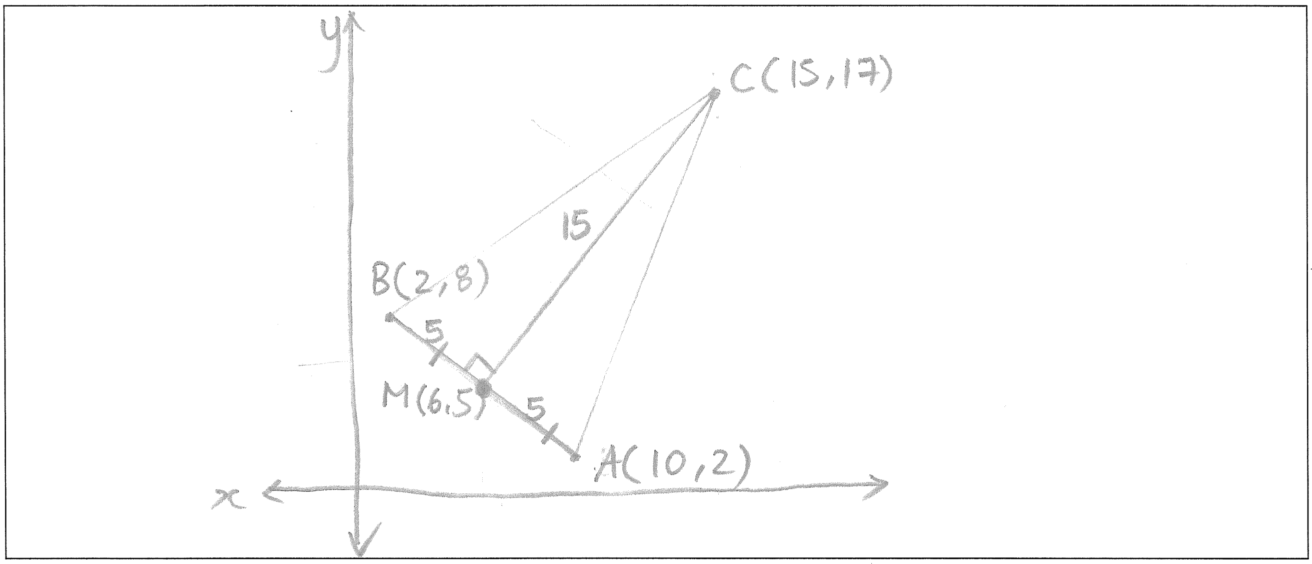
$= RHS.$

(c) Solve $2\cos x - \sqrt{3} = 0$ for $0 \le x \le 2\pi$. **2**

$\cos x = \frac{\sqrt{3}}{2}$ (Quad 1 & 4)

$\therefore x = \frac{\pi}{6}, \frac{11\pi}{6}$

(d) You may use the space below to draw a diagram for parts (i) to (v) on the next page.
There is no mark allocated for the sketch.



(i) Find the gradient, length and midpoint M of the interval joining A(10, 2) and B(2, 8). **3**

$m_{AB} = \frac{8-2}{2-10} = -\frac{3}{4}$

$M_{AB} = \left(\frac{10+2}{2}, \frac{2+8}{2}\right)$

$AB = \sqrt{(10-2)^2 + (2-8)^2}$

$= 10$ units

$= (6, 5)$

(ii) Show that the perpendicular bisector of the interval AB has equation $4x - 3y - 9 = 0$. **2**

$m_{\perp} = \frac{4}{3}$

$y - y_1 = m(x - x_1)$

$y - 5 = \frac{4}{3}(x - 6)$

$3y - 15 = 4x - 24$

$\therefore 4x - 3y - 9 = 0$

(iii) Find the point C where the perpendicular bisector meets the line $x - y + 2 = 0$. **2**

$4x - 3y - 9 = 0$

$-4\times[x - y + 2 = 0]$

$y - 17 = 0$

$\therefore y = 17$

$x - 17 + 2 = 0$

$\therefore x = 15$

$\therefore C$ is at $(15, 17)$

(iv) Find the length of CM and hence find the area of ΔABC. **3**

$CM = \sqrt{(15-6)^2 + (17-5)^2}$

$= 15$ units

$A_{\Delta ABC} = \frac{1}{2} \times AB \times CM$

$= \frac{1}{2} \times 10 \times 15$

$= 75$ units^2

(v) Find $\angle ACB$, correct to the nearest minute. **2**

$\angle ACB = 2 \times \angle ACM$

$= 2 \times \tan^{-1}\left(\frac{5}{15}\right)$

$= 36^\circ 52'$

Question 10 (16 marks)

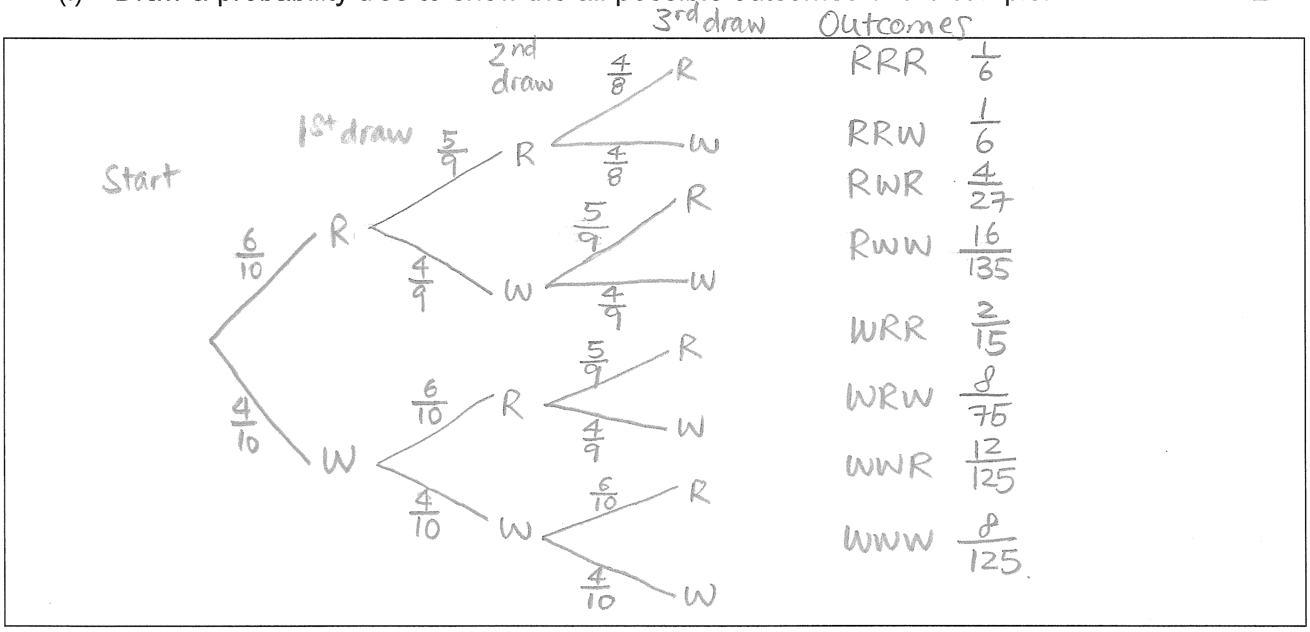
(a) Use the log laws and identities to simplify and evaluate:

$log_5 2 - log_5 250$ **2**

$= \log_5 \frac{2}{250} = \log_5 (125)^{-1} = -\log_5 125 = -\log_5 (5)^3 = -3$

(b) A bag contains 6 red balls and 4 white balls. A sample of 3 balls were randomly drawn from the bag. If the ball is white it is replaced, and if it is red it is not replaced.

(i) Draw a probability tree to show the all possible outcomes of the sample. **2**



(ii) Find the probability of drawing at least one red ball. **1**

$1 - P(WWW) = 1 - \frac{8}{125} = \frac{117}{125}$

(iii) Find the probability of drawing ~~at least one of red or white ball.~~ a sample with two balls of the same colour. **1**

$P(\text{two R or two W}) = 1 - P(RRR) - P(WWW)$

$= \frac{577}{750}$

(c) Solve $log_2(6-x) - log_2(4-x) = 2$ for x, where $x < 4$. **3**

$\log_2\left(\frac{6-x}{4-x}\right) = 2$

$\frac{6-x}{4-x} = 4$

$6-x = 16-4x$

$3x = 10$

$\therefore x = \frac{10}{3}$

(d) Differentiate $y = (2x^3 - 1)(x^4 - 2)$. **2**

$\frac{dy}{dx} = (2x^3-1)(4x^3) + (x^4-2)(6x^2)$

$= 4x^3(2x^3-1) + 6x^2(x^4-2)$

$= 2x^2\left[2x(2x^3-1) + 3(x^4-2)\right]$

$= 2x^2(7x^4 - 2x - 6)$

(e) Let $f(x) = x^2 + kx + 3$ where k is a constant.
Find the value of k given that $f(x)$ has a tangent with the gradient of zero at $x = -1$. **2**

$f'(x) = 2x + k$

when $x = -1$, $f'(x) = 0$

$2(-1) + k = 0$

$\therefore k = 2$

(f) Consider the function $y = 2x^2 - x + 1$.
Find the equation of the normal to the curve at the point ~~A(1, 1)~~. $x = 1$ at $A(1, 1)$ **3**

$\frac{dy}{dx} = 4x - 1$

When $x = 1$, $\frac{dy}{dx} = 3$

$\therefore m_{Normal} = -\frac{1}{3}$

$y - y_1 = m(x - x_1)$

$y - 2 = -\frac{1}{3}(x-1)$

$3y - 6 = -x + 1$

$\therefore x + 3y - 7 = 0$

$y - 1 = -\frac{1}{3}(x-1)$

$3y - 3 = -x + 1$

$\therefore x + 3y - 4 = 0$

Question 11 (12 marks)

(a) Solve for x. $\qquad e^{2x} + 3e^{x} - 10 = 0.$ **2**

let $u = e^{x}$, $\quad u^2 + 3u - 10 = 0$

$(u+5)(u-2) = 0$

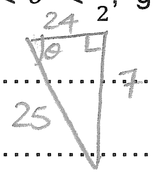
$u = -5 \quad$ or $\quad u = 2$

$e^{x} = -5 \qquad e^{x} = 2$

no soln $\qquad \therefore x = \ln 2$

(b) Find the exact value of $\sin\theta$, for $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, given $\cot\theta = -\frac{24}{7}$. **2**

$\tan\theta = -\frac{7}{24}$ (Quad 4)



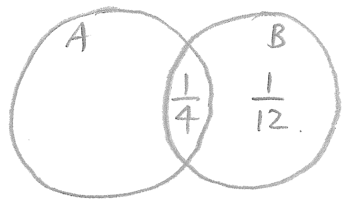
$\therefore \sin\theta = -\frac{7}{25}$.

(c) For events A and B from a sample space, $P(A|B) = \frac{3}{4}$ and $P(B) = \frac{1}{3}$.

(i) Calculate $P(A \cap B)$. **1**

$P(A|B) = \frac{P(A \cap B)}{P(B)} \qquad \therefore P(A \cap B) = \frac{3}{4} \times \frac{1}{3} = \frac{1}{4}$.

(ii) Draw a Venn diagram, hence calculate $P(\bar{A} \cap B)$. **2**



$\therefore P(\bar{A} \cap B) = \frac{1}{12}$

(iii) If events A and B are independent, calculate $P(A \cup B)$. **2**

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{3}{4} + \frac{1}{3} - \frac{1}{4}$

$= \frac{5}{6}$

$P(A \cap B) = P(A) \times P(B)$

$\frac{1}{4} = P(A) \times \frac{1}{3}$

$\therefore P(A) = \frac{3}{4}$

(d) Differentiate $f(x) = x^2 - 3x$ from first principles. **3**

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{(x+h)^2 - 3(x+h) - (x^2 - 3x)}{h}$

$= \lim_{h \to 0} \frac{x^2 + 2hx + h^2 - 3x - 3h - x^2 + 3x}{h}$

$= \lim_{h \to 0} \frac{h^2 + 2hx - 3h}{h}$

$= \lim_{h \to 0} \frac{h(h + 2x - 3)}{h} = \lim_{h \to 0} h + 2x - 3 = 2x - 3$

Question 12 (14 marks)

(a) The population, D, of Tasmanian Devils in a sanctuary is given by $D(t)$, where t is the time in years after the sanctuary was established.
The devil population is modelled by the function $D = 28e^{0.35t}$.

(i) What was the population of Tasmanian Devils at the time when the sanctuary was established? **1**

When $t=0$, $D=28$

(ii) Find the expression to show the rate of change of population of Tasmanian Devils. **1**

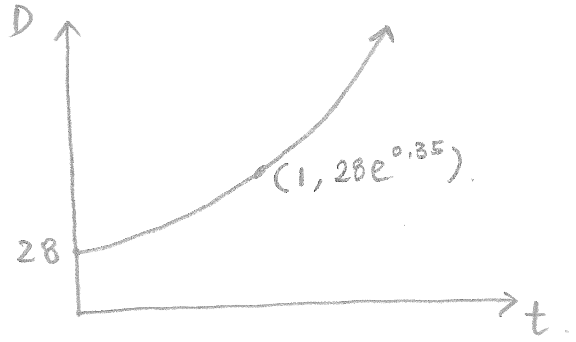
$\frac{dD}{dt} = (28 \times 0.35)e^{0.35t} = 9.8e^{0.35t}$

(iii) Calculate the increase in the number of Tasmanian Devils at the end of the first 8 years. Give your answer correct to two significant figures. **1**

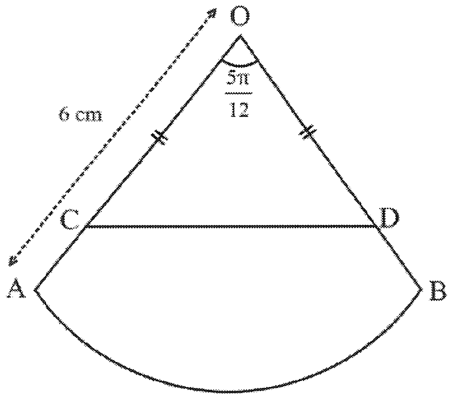
When $t=8$, $D = 28e^{0.35(8)} = 460.45$

$\therefore$ Increase in the number $= 430$ (2 sig. fig.)

(iv) Sketch the function of the population of Tasmanian Devils over time. **2**



(b) The diagram shows a sector AOB. The length OA is 6 cm and $\angle AOB = \frac{5\pi}{12}$. The points C and D lie on OA and OB respectively, and $OC = OD$.



(i) Find the exact area of sector AOB. **1**

$A_{AOB} = \frac{1}{2} \times 36 \times \left(\frac{5\pi}{12}\right) = \frac{15\pi}{2}\ cm^2$

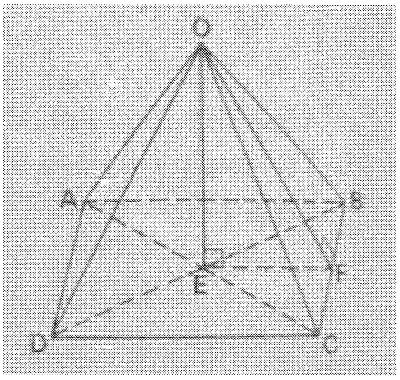
(ii) If the area of ΔCOD is half the area of sector AOB, find the length of OC correct to two decimal places. **3**

$A_{\Delta COD} = \frac{15\pi}{4}$ $\quad A_{\Delta COD} = \frac{1}{2}(OC)^2\left(\sin\frac{5\pi}{12}\right)$

$\therefore OC = \sqrt{2 \times \frac{15\pi}{4} \times \frac{1}{\sin\frac{5\pi}{12}}}$

$= 4.94cm$

(c) The square pyramid shown below has a height of 16 cm and a base edge of 24 cm. Find:

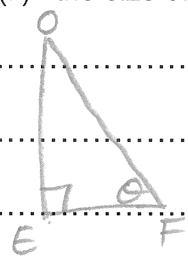


(i) the length of OF. **1**

$OF^2 = OE^2 + EF^2$

$OF = \sqrt{16^2+12^2} = 20cm$

(ii) the size of the angle between the plane OBC and the base $ABCD$. **1**

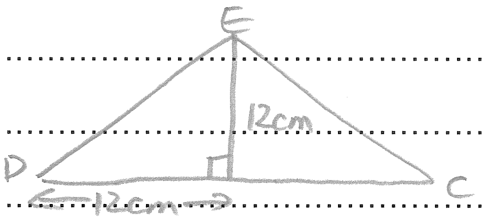


$\tan\theta = \frac{OE}{EF}$

$\tan\theta = \frac{16}{12}$

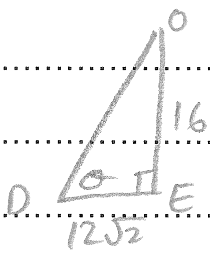
$\therefore \theta = 51°20'$

(iii) the length DE and hence find the size of the angle between the edge OD and the base $ABCD$. **3**



$DE = \sqrt{12^2+12^2}$

$= 12\sqrt{2}$ cm



$\tan\theta = \frac{16}{12\sqrt{2}}$

$\therefore \theta = 43°19'$

End ~~of~~ Examination