GIRRAWEEN HIGH SCHOOL

MATHEMATICS (ADVANCED)

YEAR 11

TASK 3

Year 11 Mathematics (Advanced) **September 2021**

TIME ALLOWED – 1 hour **TOTAL MARK – 45 marks**

Instructions:

- Write using black pen.
- Calculators approved by NESA may be used.
- This task is open book but no technology (ICT).
- For all questions, show relevant mathematical reasoning and/or calculations.
- Marks may be deducted for careless or badly arranged work.

You must have your camera (no virtual background) and speaker on, but have microphone off during examination.

No other technologies other than the computer displaying the exam, and your phone which must be on silent and displayed on the desk facing upwards, should be visible.

Question 1 (14 marks)

(a) Simplify:

(i) $\left(\frac{64x^{-3}}{y^6}\right)^{\frac{1}{3}}$ **2**

(ii) $\log_3 6 + \log_3 \frac{1}{486}$ **2**

(b) Solve for x:

(i) $\left(\frac{2}{3}\right)^x = \frac{27}{8}$ **1**

(ii) $\frac{1}{3} = \log_{216} x$ **2**

(c) Sketch $y = 2^x$ and $y = -2^{-x}$, on the same set of coordinate axes. **2**

(d) The number of bacteria growing in a Petri dish triples every two hours.

Initially there were 50 bacteria.

(i) Find the values of $\boldsymbol{x}$ so that P is the bacteria population after n hours.

$$P = 50 \times 3^{\frac{n}{x}}$$ **1**

(ii) How many bacteria are there after 13 hours? **1**

(iii) How long, to the nearest hour, will it take for the bacteria population to reach 1 000 000? **3**

Question 2 (18 marks)

(a) Differentiate: $y = e^{5x}(2x + 1)$ **2**

(b) (i) Find the gradient of the tangent to the curve $y = e^{-2x}$ at $x = 1$. **2**

(ii) Find the equation of the tangent at $x = 1$. **2**

(iii) Find the equation of the normal at $x = 1$. **2**

(c) Solve $2\cos^2\theta - 5\cos\theta - 3 = 0$ for $0 \leq \theta \leq 2\pi$, giving your solution(s) in terms of π. **3**

(d) A chord of a circle with radius 7 cm subtends an angle of 115° at the centre. Find the area of the minor segment cut off by the chord correct to one decimal place. **2**

(e) For a certain brand of medicine, the amount M in micrograms, in blood after t hours is given by $M = 9e^{-t}$, for $t \geq 0$.

(i) Find the rate of change of the amount of medicine in blood at the start. **2**

(ii) It is found that the medicine is no longer effective if its amount in blood falls below 0.3 micrograms. Find how long the medicine will stay effective to the nearest minute. **3**

Question 3 (13 marks)

(a) There are two bags containing coloured balls. In one of these bags, the chance choosing a blue ball is 65%. In the other bag, the chance choosing a white ball is 40%.

A ball is chosen from each bag at random.

(i) Draw a probability tree diagram. **2**

(ii) Hence, find the probability of choosing a blue and a white ball. **1**

(b) For events A and B from a sample space, $P(B) = \frac{2}{5}$ and $P(A \cap B) = \frac{1}{4}$.

(i) Calculate $P(A|B)$. **1**

(ii) Calculate $P(A)$, if it is known that events A and B are independent. **1**

(iii) Calculate $P(\overline{A \cup B})$. **2**

(c) A coin is tossed and a die is thrown simultaneously.

What is the probability of neither a head nor a five appearing? **2**

(d) In a standard deck of cards, there are 52 cards of four suits. Each suit has 13 cards consisting of 9 number cards, an Ace and 3 court cards; Jack, Queen and King.

From a standard deck of cards, two cards are drawn at random. If it is a number card, it is replaced, and if it is not a number card, it is not replaced.

Find the probabilities of drawing:

(i) two number cards. **1**

(ii) at least one card that is not a number card. **1**

(iii) It is now known that the two cards drawn are not number cards.
What is the probability that both cards are court cards? **2**

End of Examination

NSW Education Standards Authority

2020 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, \ r \neq 1$$

$$S = \frac{a}{1 - r}, \ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab\sin C$$

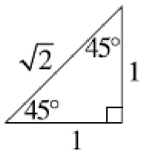
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

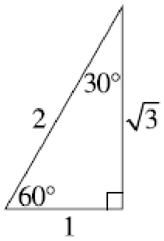
$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$





Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

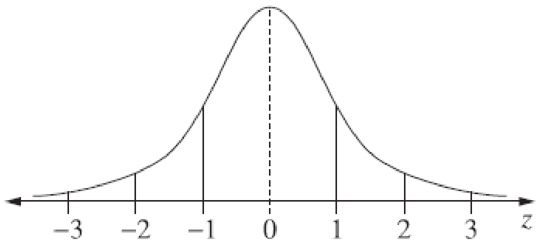
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\text{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \leq x) = \int_a^x f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r(1 - p)^{n-r}$$

$$X \sim \text{Bin}(n, p)$$

$$\Rightarrow \ P(X = x) = \binom{n}{x}p^x(1 - p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\text{Var}(X) = np(1 - p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\dfrac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$
$y = \dfrac{u}{v}$	$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\dfrac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\dfrac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\dfrac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\dfrac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\dfrac{dy}{dx} = -\dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n\,dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}\,dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}\,dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-[f(x)]^2}}\,dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+[f(x)]^2}\,dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \cdots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

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Question 1 (14 marks)

(a) Simplify:

(i) $\left(\frac{64x^{-3}}{y^6}\right)^{\frac{1}{3}} = \frac{64^{\frac{1}{3}}x^{-1}}{y^2} = \frac{4}{xy^2}$ **2**

(ii) $\log_3 6 + \log_3 \frac{1}{486} = \log_3\left(6 \times \frac{1}{486}\right) = \log_3\left(\frac{1}{81}\right) = -4$ **2**

(b) Solve for x:

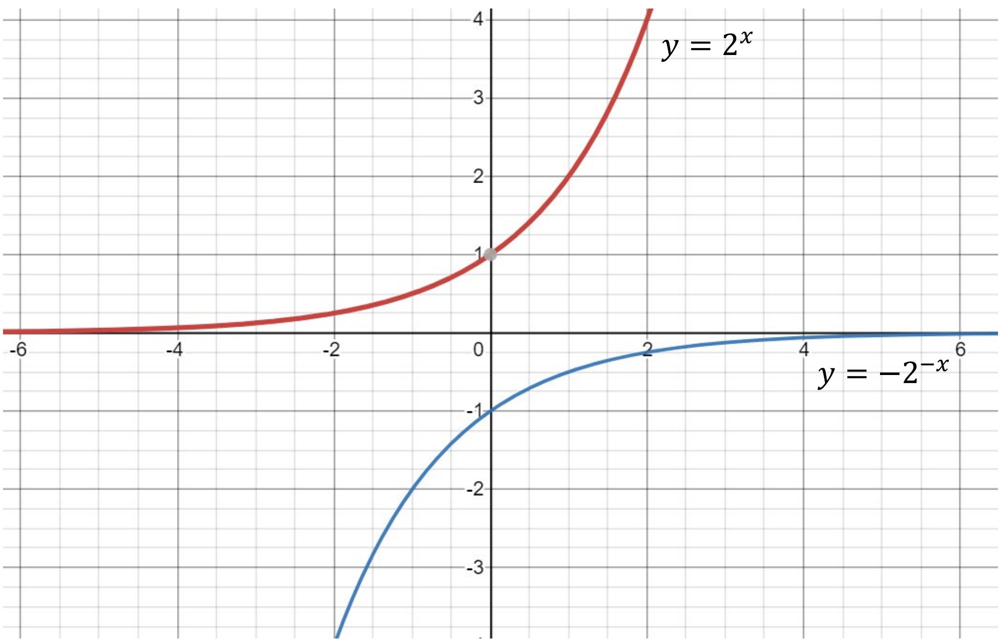
(i) $\left(\frac{2}{3}\right)^x = \frac{27}{8}$ **1**

$\left(\frac{2}{3}\right)^3 = \frac{8}{27} \qquad \therefore x = -3$

(ii) $\frac{1}{3} = \log_{216} x$ **2**

$216^{\frac{1}{3}} = x \qquad \therefore x = 6$

(c) Sketch $y = 2^x$ and $y = -2^{-x}$, on the same set of coordinate axes. **2**



(d) The number of bacteria growing in a Petri dish triples every two hours.

Initially there were 50 bacteria.

(i) Find the value of $\boldsymbol{x}$ so that P is the bacteria population after n hours.

$$P = 50 \times 3^{\frac{n}{x}}$$ **1**

After 2 hours, $P = 50 \times 3 = 50 \times 3^{\frac{2}{2}}$

After 4 hours, $P = 50 \times 3^2 = 50 \times 3^{\frac{4}{2}}$

After n hours, $P = 50 \times 3^{\frac{n}{2}}$

$\therefore x = 2$

(ii) How many bacteria are there after 13 hours? **1**

$$P = 50 \times 3^{\frac{13}{2}} = 63133$$

(iii) How long, to the nearest hour, will it take for the bacteria population to reach 1 000 000? **2**

$$\begin{aligned} 1000000 &= 50 \times 3^{\frac{n}{2}} \\ 20000 &= 3^{\frac{n}{2}} \\ \ln 20000 &= \ln 3^{\frac{n}{2}} \\ \frac{n}{2} &= \frac{\ln 20000}{\ln 3} \\ n &= 18 \end{aligned}$$

Question 2 (18 marks)

(a) Differentiate: $y = e^{5x}(2x+1)$ **2**

$$\frac{dy}{dx} = e^{5x}(2) + (2x+1)5e^{5x} = 2e^{5x} + 5e^{5x}(2x+1) = e^{5x}(2+5(2x+1)) = e^{5x}(10x+7)$$

(b) (i) Find the gradient of the tangent to the curve $y = e^{-2x}$ at $x = 1$. **2**

$\frac{dy}{dx} = -2e^{-2x}$, and at $x = 1$, $\frac{dy}{dx} = -2e^{-2} = -\frac{2}{e^2}$

(ii) Find the equation of the tangent at $x = 1$. **2**

At $x = 1$, $y = e^{-2} = \frac{1}{e^2}$

$$y - \frac{1}{e^2} = -\frac{2}{e^2}(x-1)$$
$$ye^2 - 1 = -2(x-1)$$
$$y = \frac{1}{e^2}(-2x+3) \text{ or } \frac{2}{e^2}x + y - \frac{3}{e^2} = 0 \text{ or } 2x + e^2y - 3 = 0$$

(iii) Find the equation of the normal at $x = 1$. **2**

At $x = 1$, gradient of the normal $= \frac{e^2}{2}$

$$y - \frac{1}{e^2} = \frac{e^2}{2}(x-1)$$
$$y = \frac{e^2}{2}x - \frac{e^2}{2} + \frac{1}{e^2} \text{ or } e^2x - 2y - e^2 + \frac{2}{e^2} = 0 \text{ or } x - 2e^{-2}\mathrm{y} + 2e^{-4} - 1 = 0$$

(c) Solve $2\cos^2\theta - 5\cos\theta - 3 = 0$ for $0 \le \theta \le 2\pi$, giving your solution(s) in terms of π. **3**

Let $u = \cos\theta$, then the equation to solve will be

$$2u^2 - 5u - 3 = 0$$
$$2u^2 - 6u + u - 3 = 0$$
$$2u(u-3) + (u-3) = 0$$
$$(2u+1)(u-3) = 0$$
$$u = -\frac{1}{2} \quad \text{or} \quad u = 3$$
$$\cos\theta = -\frac{1}{2} \quad \text{or} \quad \cos\theta = 3$$
$$\therefore \theta = \frac{2\pi}{3}, \frac{4\pi}{3} \text{ (as } \cos\theta = 3 \text{ has no solutions)}$$

(d) A chord of a circle with radius 7 cm subtends an angle of 115° at the centre. Find the area of the minor segment cut off by the chord correct to one decimal place. **2**

Area of minor sector = $\frac{1}{2} \times 7^2 \times \frac{115\pi}{180} = 49.2\ \text{cm}^2$

Area of triangle subtended by the radii and the chord = $\frac{1}{2} \times 7^2 \times \sin\frac{115\pi}{180} = 22.2\ \text{cm}^2$

Area of the minor segment = Area of minor sector – Area of triangle

$$= 49.2 - 22.2 = 27.0\ \text{cm}^2$$

(e) For a certain brand of medicine, the amount M in micrograms, in blood after t hours is given by $M = 9e^{-t}$, for $t \geq 0$.

(i) Find the rate of change of the amount of medicine in blood at the start. **2**

Since $\frac{dM}{dt} = -9e^{-t}$,

at $t = 0$, $\frac{dM}{dt} = -9e^{0} = -9$

Therefore the initial rate of change of the amount of medicine in blood is -9 micrograms/h.

(ii) It is found that the medicine is no longer effective if its amount in blood falls below 0.3 micrograms. Find how long the medicine will stay effective to the nearest minute. **3**

Solve for t when $M = 0.3$,

$$\begin{aligned} 0.3 &= 9e^{-t} \\ \frac{1}{30} &= e^{-t} \\ \ln\frac{1}{30} &= \ln e^{-t} \\ -t &= \ln\frac{1}{30} \\ t &= -\ln\frac{1}{30} \\ t &\approx 3.4\ \text{h} \\ \therefore t &\approx 3\text{h}\ 24\text{min} \end{aligned}$$

Question 3 (13 marks)

(a) There are two bags containing coloured balls. In one of these bags, the chance choosing a blue ball is 65%. In the other bag, the chance choosing a white ball is 40%.

A ball is chosen from each bag at random.

(i) Draw a probability tree diagram. **2**

(ii) Hence, find the probability of choosing a blue and a white ball. **1**

$$P(one\ B\ and\ one\ W) = 0.26$$

(b) For events A and B from a sample space, $P(B) = \frac{2}{5}$ and $P(A \cap B) = \frac{1}{4}$.

(i) Calculate $P(A|B)$. **1**

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{2}{5}} = \frac{5}{8}$$

(ii) Calculate $P(A)$, if it is known that events A and B are independent. **1**

$P(A|B) = P(A)$ if events A and B are independent.
$\therefore P(A) = \frac{5}{8}$

(iii) Hence calculate $P(\overline{A \cup B})$. **2**

$$\begin{aligned} P(\overline{A \cup B}) &= 1 - P(A \cup B) \\ &= 1 - [P(A) + P(B) - P(A \cap B)] \\ &= 1 - \left[\frac{5}{8} + \frac{2}{5} - \frac{1}{4}\right] \\ &= \frac{9}{40} \end{aligned}$$

(c) A coin and a die are thrown together.

What is the probability of neither a head nor a five appearing? **2**

$$P\left(\overline{H \cup 5}\right) = P(\overline{H}) \times P(\overline{5}) = \frac{1}{2} \times \frac{5}{6} = \frac{5}{12}$$

(d) In a standard deck of cards, there are 52 cards of four suits. Each suit has 13 cards consisting of 9 number cards, an Ace and 3 court cards; Jack, Queen and King.

From a standard deck of cards, two cards are drawn at random. If it is a number card, it is replaced, and if it is not a number card, it is not replaced.

Find the probabilities of drawing:

(i) two number cards. **1**

There are 36 number cards, 9 number cards for each suit.

$P(two\ number\ cards) = \frac{36}{52} \times \frac{36}{52} = \frac{81}{169}$

(ii) at least one card that is not a number card. **1**

$P(at\ least\ one\ number\ card) = 1 - P(both\ are\ number\ cards) = 1 - \frac{81}{169} = \frac{88}{169}$

(iii) It is now known that the two cards drawn are not number cards. What is the probability that both cards are court cards? **2**

$|A| = \{two\ court\ cards\} = 12 \times 11 = 132$
$|B| = \{two\ non\ number\ cards\} = 16 \times 15 = 240$

$|A \cap B| = |A|$ since all of court cards are non-number cards

$P(A|B) = \frac{|A \cap B|}{|B|} = \frac{132}{240} = \frac{11}{20}$

End of Examination

NSW Education Standards Authority

2020 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

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Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a+b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x-h)^2 + (y-k)^2 = r^2$$

Financial Mathematics

$$A = P(1+r)^n$$

Sequences and series

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}(a+l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{a(r^n-1)}{r-1},\ r \neq 1$$

$$S = \frac{a}{1-r},\ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$\sin A = \dfrac{\text{opp}}{\text{hyp}}, \quad \cos A = \dfrac{\text{adj}}{\text{hyp}}, \quad \tan A = \dfrac{\text{opp}}{\text{adj}}$

$A = \dfrac{1}{2}ab\sin C$

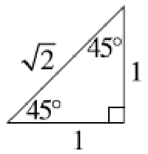
$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

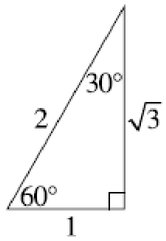
$c^2 = a^2 + b^2 - 2ab\cos C$

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

$l = r\theta$

$A = \dfrac{1}{2}r^2\theta$





Trigonometric identities

$\sec A = \dfrac{1}{\cos A}, \ \cos A \neq 0$

$\operatorname{cosec} A = \dfrac{1}{\sin A}, \ \sin A \neq 0$

$\cot A = \dfrac{\cos A}{\sin A}, \ \sin A \neq 0$

$\cos^2 x + \sin^2 x = 1$

Compound angles

$\sin(A + B) = \sin A\cos B + \cos A\sin B$

$\cos(A + B) = \cos A\cos B - \sin A\sin B$

$\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$

If $t = \tan\dfrac{A}{2}$ then $\sin A = \dfrac{2t}{1 + t^2}$

$\cos A = \dfrac{1 - t^2}{1 + t^2}$

$\tan A = \dfrac{2t}{1 - t^2}$

$\cos A\cos B = \dfrac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$

$\sin A\sin B = \dfrac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$

$\sin A\cos B = \dfrac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$

$\cos A\sin B = \dfrac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$

$\sin^2 nx = \dfrac{1}{2}(1 - \cos 2nx)$

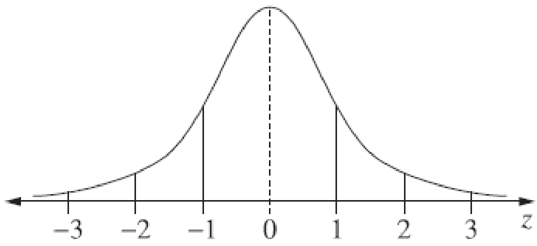
$\cos^2 nx = \dfrac{1}{2}(1 + \cos 2nx)$

Statistical Analysis

$z = \dfrac{x - \mu}{\sigma}$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$E(X) = \mu$

$\text{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$

Probability

$P(A \cap B) = P(A)P(B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A|B) = \dfrac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$

Continuous random variables

$P(X \leq x) = \displaystyle\int_a^x f(x)\,dx$

$P(a < X < b) = \displaystyle\int_a^b f(x)\,dx$

Binomial distribution

$P(X = r) = {}^nC_r p^r(1 - p)^{n-r}$

$X \sim \text{Bin}(n, p)$

$\Rightarrow \ P(X = x)$

$= \dbinom{n}{x}p^x(1 - p)^{n-x}, \ x = 0, 1, \ldots, n$

$E(X) = np$

$\text{Var}(X) = np(1 - p)$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)\left[f(x)\right]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+\left[f(x)\right]^2}$

Integral Calculus

$$\int f'(x)\left[f(x)\right]^n dx = \frac{1}{n+1}\left[f(x)\right]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}\,dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln\left|f(x)\right| + c$$

$$\int f'(x)a^{f(x)}\,dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-\left[f(x)\right]^2}}\,dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+\left[f(x)\right]^2}\,dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f\left(x_1\right) + \cdots + f\left(x_{n-1}\right)\right]\right\}$$

where $a = x_0$ and $b = x_n$