# GIRRAWEEN HIGH SCHOOL
# MATHEMATICS ADVANCED
# YEAR 11

## Yearly Examination
## 2022

**Reading time: 10 minutes**

**Working time: 120 minutes**

**Instructions:**

- Attempt ALL questions.
- For Questions 1 to 5, colour in the bubble next to the letter corresponding to the correct response on the MC answer sheet.
- For Questions 6 to 12:
  - All necessary working should be shown in the space provided.
  - Marks will be deducted for careless or badly arranged work.
  - Board-approved calculators may be used.
  - Reference sheet is provided.
  - Diagrams are NOT to scale.

**Section I (5 marks)**

- Use the multiple-choice answer sheet provided (after MC Q5) for questions 1 – 5.

1. Which of the following is true for the function $f(x) = 8x^3 - 7x$?

   A. Odd function

   B. Even function

   C. Neither odd or even

   D. Zero function

2. Which of following is an expression for $\cos^2 x + \sin^2 x + \tan^2 x$ ?

   A. 2

   B. $\operatorname{cosec}^2 x$

   C. $\cot^2 x$

   D. $\sec^2 x$

3. Which of the following is equivalent to $\dfrac{1}{2\sqrt{5} + 3}$ ?

   A. $\dfrac{2\sqrt{5} + 3}{-11}$

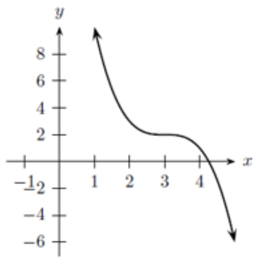
   B. $\dfrac{2\sqrt{5} - 3}{17}$

   C. $\dfrac{2\sqrt{5} - 3}{11}$

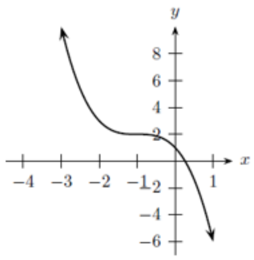
   D. $\dfrac{2\sqrt{5} + 3}{11}$

4. Which diagram is the graph of $y = 2 - (x - 3)^3$ ?

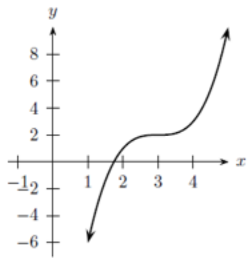
A.



B.



C.



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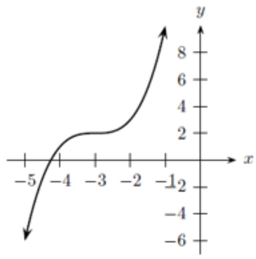

5. What is the solution to the equation $\log_3(2x - 5) = 2$ ?

A. $x = 1$ B. $x = \frac{13}{2}$ C. $x = \frac{11}{2}$ D. $x = 7$

**Select the alternative A, B, C or D that best answers the question.**

1. A ⬭ B. ⬭ C. ⬭ D. ⬭
2. A ⬭ B. ⬭ C. ⬭ D. ⬭
3. A ⬭ B. ⬭ C. ⬭ D. ⬭
4. A ⬭ B. ⬭ C. ⬭ D. ⬭
5. A ⬭ B. ⬭ C. ⬭ D. ⬭

## Section II (98 marks)

- **Answer questions in the spaces provided.**
- **Your responses should include relevant mathematical reasoning and/or calculations.**
- **Extra writing space is provided at the back of this booklet. If you use this space, indicate in the writing space under the question, the page number on which your solution continues, and clearly write the question number on the extra writing space.**

**Question 6** (13 marks)

a. Simplify:

(i) $\dfrac{x}{4} + \dfrac{x-3}{5}$ [2]

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(ii) $\dfrac{3x-4y}{20y-15x}$ [2]

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(iii) $\dfrac{x-3}{x^2+4x+3}-\dfrac{x}{x+3}$ [3]

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(iv) $\dfrac{x^4-x^2}{x-1}$ [3]

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b. Find the integers $a$ and $b$ such that $(2\sqrt{3}-1)^2=a\sqrt{3}+b$ [3]

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**Question 7** (15 marks)

a. Solve:

(i) $|4x - 3| = 5$ [2]

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(ii) $5x - \dfrac{10}{x} = 4$ [2]

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b. Find the values of $k$ for which the equation $x^2 + (k - 6)x + 1 = 0$ has equal roots. [3]

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c. A circle has the equation $x^2 - 2x + y^2 + 6y = 6$.

(i) Find the radius and the coordinates of the centre of the circle. [3]

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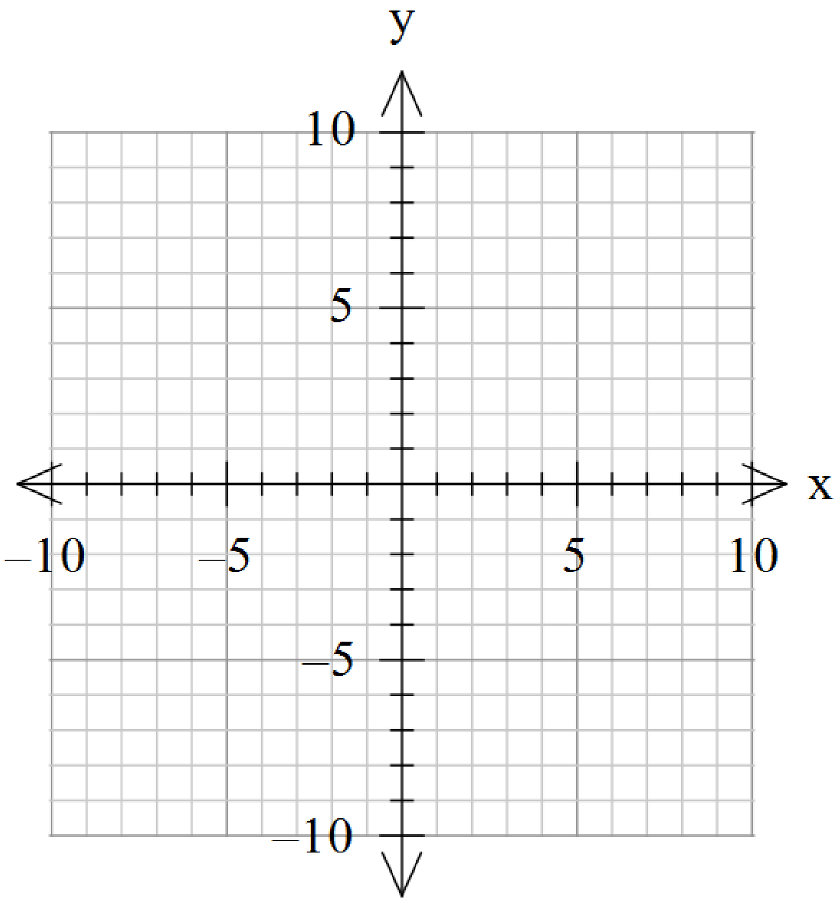
(ii) Sketch the circle, clearly showing the centre and radius. [2]



d. A function $y = f(x)$ is defined as:

$$f(x) = \begin{cases} x - 3 & for\ x \le -3 \\ 2x + 2 & for -3 < x < 0 \\ x^2 & for\ x \ge 0 \end{cases}$$

(i) Evaluate $f(-7)$ [1]

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(ii) Evaluate $f(2) + f(-2) + f(-5)$ [2]

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**Question 8** (14 marks)

a. Differentiate $f(x) = x^2 - 4x$ from first principles. [3]

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b. Differentiate:

(i) $y = 5 - 7x + 13x^4$ [1]

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(ii) $y = 2\sqrt{x}$ [2]

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(iii) $y = (2x - 3)^8$ [2]

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(iv) $y = x\,(1 - x)^6$ [3]

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(v) $y = \dfrac{x^2 + 2}{3x - 4}$ [3]

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**Question 9** (15 marks)

a. The line $l_1$ passes through the point $(9,\ -\ 4)$ and has gradient $\frac{1}{3}$.

(i) Find the equation of $l_1$ in general form. [2]

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(ii) Line $l_2$ passes through the origin and has gradient $-\ 2$.

Find the coordinates of $P$, the point of intersection of lines $l_1$ and $l_2$. [2]

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(iii) Given that $l_1$ crosses the $y$-axis at the point $C$,

calculate the area of $\Delta\, OPC$. [3]

*(You may draw a diagram in the space below to help you answer the question)*

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b. Given that $\sin\ \theta = \frac{5}{6}$ and $90° \leq \theta \leq 180°$, find the value of $\cos\ \theta$. [2]

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c. Solve: $\tan x = \dfrac{1}{\sqrt{3}}$ for $0 \leq x \leq 360°$ [2]

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d. Prove that $\dfrac{\sec \theta}{\cot \theta + \tan \theta} - \dfrac{1 + \cot \theta}{\operatorname{cosec} \theta} = -\cos \theta$ [4]

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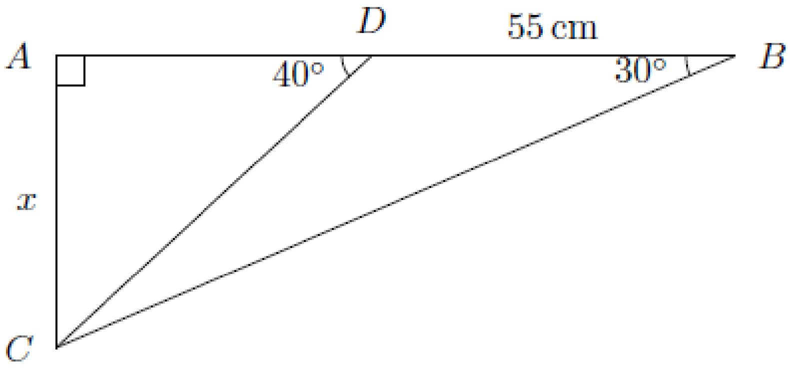
**Question 10**(13 marks)

a. Find the value of $x$ where $BD = 55\,\text{cm}$. [3]

Answer correct to 3 significant figures.

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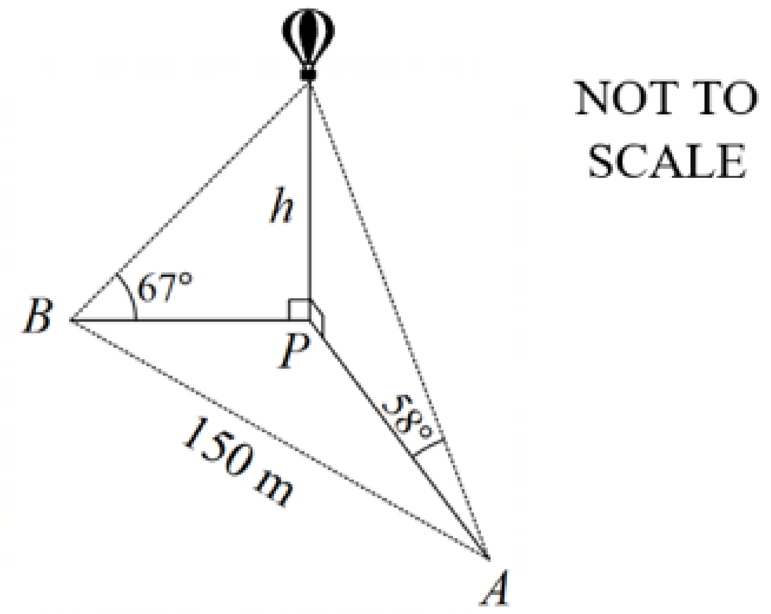
b. A newly launched weather balloon is floating in the air vertically $h$ metres above the point $P$ on the ground. Observer $A$ is due south of the balloon and notes that its angle of elevation is 58º. Observer $B$ is 150 metres from Observer $A$ and due east of the weather balloon. The angle of elevation of the weather balloon from Observer $B$ is 67º.




$PA = h \cot 58°$ and $PB = h \cot 67°$. [2]

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(ii) Use Pythagoras' Theorem to show that $h^2 = \dfrac{150^2}{\cot^2 58° + \cot^2 67°}$ [2]

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(iii) Hence, find $h$ correct to one decimal place. [1]

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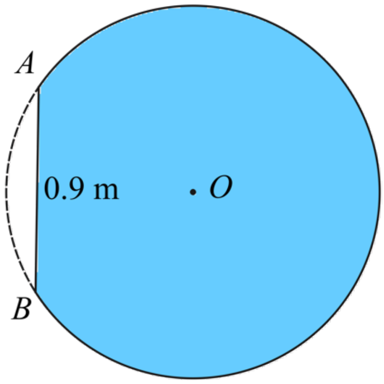
c. A small segment of a circle has been removed as shown below.

The circle has a centre $O$ and radius 0.9 metres.

The length of the straight edge $AB$ is also 0.9 metres.




Not to scale

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(ii) Find the area of $\Delta AOB$. [1]

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(iii) Find the area of the minor sector $AOB$. [1]

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(iv) Hence, find the area of the shaded region. [2]

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**Question 11** (17 marks)

a. Simplify: $$\left(\frac{12x^{-3}y^{2}}{xyz^{\frac{1}{2}}}\right)^{2} \div \frac{6x^{2}y^{3}}{z^{-4}}$$ [3]

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b. Solve:

(i) $5^{1-x} = 25^{x}$ [2]

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(ii) $\log_4(2x) - \log_4(x+1) = 0$ [2]

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c. Given that $\log_a 4 = 1.5$ and $\log_a 5 = 1.7$, find:

(i) $\log_a 1.25$ [1]

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(ii) $\log_a 2$ [1]

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(iii) $\log_a 100$ [2]

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d. Differentiate $y = \dfrac{e^{2x} + 4}{x^3}$ [3]

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e. Find the equation of the tangent to the curve $y = e^{1-2x}$ at the point where $x = 0.5$ [3]

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**Question 12** (11 marks)

a. A tank initially has 3000 litres of water and is leaking according to the formula

$V = V_0 e^{-0.135t}$ where $t$ is in hours.

(i) Find the value of $V_0$. [1]

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(ii) Find the value of $V$ after 5 hours. [1]

Give your answer correct to 1 decimal place.

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(iii) Find the time taken for the volume of water to fall to 250 litres. [3]

Give your answer to the nearest minute.

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b. (i) Factorise the expression $2x^2 - x - 1$ . [2]

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(ii) Hence, solve the equation

$2\sin^2 2x - \sin 2x = 1$ for $0° \leq x \leq 360°$ . [4]

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*End of Examination*

***Extra working out paper***

If you use this space, please clearly number the question you are answering.

Solutions

# GIRRAWEEN HIGH SCHOOL
# MATHEMATICS ADVANCED
# YEAR 11

## Yearly Examination
## 2022

**Reading time: 10 minutes**

**Working time: 120 minutes**

**Instructions:**

- Attempt ALL questions.
- For Questions 1 to 5, colour in the bubble next to the letter corresponding to the correct response on the MC answer sheet.
- For Questions 6 to 12:
    - All necessary working should be shown in the space provided.
    - Marks will be deducted for careless or badly arranged work.
    - Board-approved calculators may be used.
    - Reference sheet is provided.
    - Diagrams are NOT to scale.

**Section I (5 marks)**

- Use the multiple-choice answer sheet provided (after MC Q5) for questions 1 – 5.

1. Which of the following is true for the function $f(x) = 8x^3 - 7x$?

A. Odd function

B. Even function

C. Neither odd or even

D. Zero function

$f(-x) = 8(-x)^3 - 7(-x)$
$= -8x^3 + 7x$
$= -(8x^3 - 7x)$
$= -f(x)$
$\therefore$ odd

2. Which of following is an expression for $\underbrace{\cos^2 x + \sin^2 x}_{1} + \tan^2 x$ ?

$1 + \tan^2 x$

A. $2$

B. $\operatorname{cosec}^2 x$

C. $\cot^2 x$

D. $\sec^2 x$

3. Which of the following is equivalent to $\dfrac{1}{2\sqrt{5} + 3}$ ? $\times \dfrac{2\sqrt{5} - 3}{2\sqrt{5} - 3}$

$= \dfrac{2\sqrt{5} - 3}{20 - 9}$

A. $\dfrac{2\sqrt{5} + 3}{-11}$

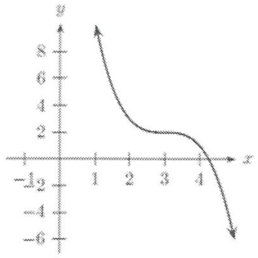
B. $\dfrac{2\sqrt{5} - 3}{17}$

C. $\dfrac{2\sqrt{5} - 3}{11}$

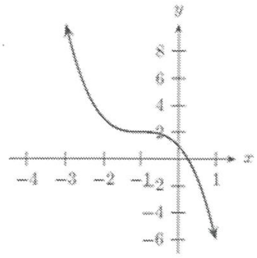
D. $\dfrac{2\sqrt{5} + 3}{11}$

4. Which diagram is the graph of $y = 2 - (x - 3)^3$ ?

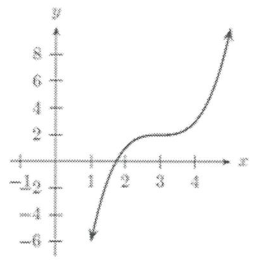
A.



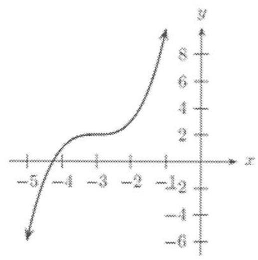

B.




C.




D.




5. What is the solution to the equation $\log_3(2x - 5) = 2$ ?

$2x - 5 = 3^2$

$2x = 14$

A. $x = 1$ B. $x = \dfrac{13}{2}$ C. $x = \dfrac{11}{2}$ D. $x = 7$

**Select the alternative A, B, C or D that best answers the question.**

1. A ● B. ○ C. ○ D. ○

2. A ○ B. ○ C. ○ D. ●

3. A ○ B. ○ C. ● D. ○

4. A ● B. ○ C. ○ D. ○

5. A ○ B. ○ C. ○ D. ●

## Section II (98 marks)

- **Answer questions in the spaces provided.**
- **Your responses should include relevant mathematical reasoning and/or calculations.**
- **Extra writing space is provided at the back of this booklet. If you use this space, indicate in the writing space under the question, the page number on which your solution continues, and clearly write the question number on the extra writing space.**

**Question 6** (13 marks)

a. Simplify:

(i) $\frac{x}{4} + \frac{x-3}{5}$ [2]

$= \frac{5x + 4(x-3)}{20}$

$= \frac{5x + 4x - 12}{20}$

$= \frac{9x - 12}{20}$

(ii) $\frac{3x - 4y}{20y - 15x}$ [2]

$= \frac{3x - 4y}{5(4y - 3x)}$

$= \frac{3x - 4y}{-5(3x - 4y)}$

$= -\frac{1}{5}$

(iii) $\dfrac{x-3}{x^2+4x+3} - \dfrac{x}{x+3}$ [3]

$= \dfrac{x-3}{(x+1)(x+3)} - \dfrac{x}{x+3}$

$= \dfrac{x-3 - x(x+1)}{(x+1)(x+3)}$

$= \dfrac{x-3-x^2-x}{(x+1)(x+3}$

$= \dfrac{-x^2-3}{(x+1)(x+3}$

(iv) $\dfrac{x^4 - x^2}{x-1}$ [3]

$= \dfrac{x^2(x^2-1)}{x-1}$

$= \dfrac{x^2(x+1)(x-1)}{x-1}$

$= x^2(x+1)$

b. Find the integers $a$ and $b$ such that $(2\sqrt{3} - 1)^2 = a\sqrt{3} + b$ [3]

$\Rightarrow (2\sqrt{3})^2 - 2\times 2\sqrt{3} + 1 = a\sqrt{3} + b$

$12 - 4\sqrt{3} + 1 = a\sqrt{3} + b$

$-4\sqrt{3} + 13 = a\sqrt{3} + b$

$\therefore a = -4$

$b = 13$

**Question 7** (15 marks)

a. Solve:

(i) $|4x - 3| = 5$ [2]

$4x - 3 = 5$ or $4x - 3 = -5$

$4x = 8$ $\quad$ $4x = -2$

$x = 2$ $\quad$ $x = -\frac{1}{2}$

$\therefore x = 2, -\frac{1}{2}$

(ii) $5x - \dfrac{10}{x} = 4$ [2]

$5x^2 - 10 = 4x$

$5x^2 - 4x - 10 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{4 \pm \sqrt{216}}{10}$

$= \dfrac{4 \pm 6\sqrt{6}}{10} = \dfrac{2 \pm 3\sqrt{6}}{5}$

b. Find the values of $k$ for which the equation $x^2 + (k - 6)x + 1 = 0$ has equal roots. [3]

equal roots $\Rightarrow b^2 - 4ac = 0$

i.e. $(k-6)^2 - 4 = 0$

$(k-6)^2 = 4$

$k - 6 = \pm 2$

$k = 4$ or $k = 8$

c. A circle has the equation $x^2 - 2x + y^2 + 6y = 6$ .

(i) Find the radius and the coordinates of the centre of the circle. [3]

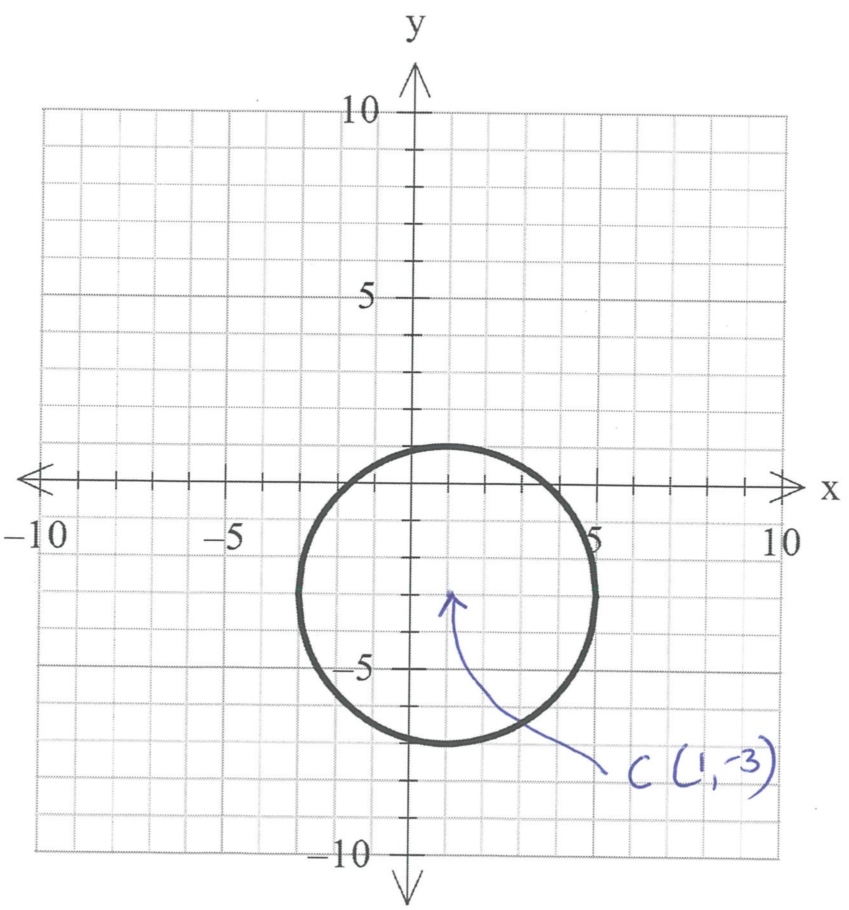
$x^2 - 2x + 1 + y^2 + 6y + 9 = 6 + 1 + 9$

$(x-1)^2 + (y+3)^2 = 16$

centre $= (1, -3)$

radius $= 4$

(ii) Sketch the circle, clearly showing the centre and radius. [2]



d. A function $y = f(x)$ is defined as:

$$f(x) = \begin{cases} x - 3 & for\ x \le -3 \\ 2x + 2 & for - 3 < x < 0 \\ x^2 & for\ x \ge 0 \end{cases}$$

(i) Evaluate $f(-7)$ [1]

$= -7 - 3$

$= -10$

(ii) Evaluate $f(2) + f(-2) + f(-5)$ [2]

$= 2^2 + (2(-2) + 2) + (-5 - 3)$

$= 4 + (-2) + (-8)$

$= -6$

**Question 8** (14 marks)

a. Differentiate $f(x) = x^2 - 4x$ from first principles. [3]

$f(x+h) = (x+h)^2 - 4(x+h)$

$= x^2 + 2xh + h^2 - 4x - 4h$

$f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \dfrac{x^2 + 2xh + h^2 - 4x - 4h - x^2 + 4x}{h}$

$= \lim_{h \to 0} \dfrac{2xh + h^2 - 4h}{h}$

$= \lim_{h \to 0} 2x + h - 4$

$f'(x) = 2x - 4$

b. Differentiate:

(i) $y = 5 - 7x + 13x^4$ [1]

$\frac{dy}{dx} = -7 + 52x^3$

(ii) $y = 2\sqrt{x} = 2x^{\frac{1}{2}}$ [2]

$\frac{dy}{dx} = 2 \times \frac{1}{2} x^{-\frac{1}{2}}$

$= \frac{1}{\sqrt{x}}$

(iii) $y = (2x - 3)^8$ [2]

$\frac{dy}{dx} = 8(2x-3)^7 \cdot 2$

$= 16(2x-3)^7$

(iv) $y = x(1 - x)^6$ [3]

$\frac{dy}{dx} = vu' + uv'$

$u = x$

$u' = 1$

$v = (1-x)^6$

$v' = -6(1-x)^5$

$= (1-x)^6 + x(-6(1-x)^5)$

$= (1-x)^5 [(1-x) - 6x]$

$= (1-x)^5 (1-7x)$

(v) $y = \dfrac{x^2 + 2}{3x - 4}$ [3]

$\dfrac{dy}{dx} = \dfrac{vu' - uv'}{v^2}$

$u = x^2 + 2$

$u' = 2x$

$v = 3x - 4$

$v' = 3$

$= \dfrac{(3x-4)(2x) - (x^2+2)(3)}{(3x-4)^2}$

$= \dfrac{6x^2 - 8x - 3x^2 - 6}{(3x-4)^2}$

$= \dfrac{3x^2 - 8x - 6}{(3x-4)^2}$

**Question 9** (15 marks)

a. The line $l_1$ passes through the point $(9, -4)$ and has gradient $\frac{1}{3}$.

(i) Find the equation of $l_1$ in general form. [2]

$y - y_1 = m(x - x_1)$

$y + 4 = \frac{1}{3}(x - 9)$

$3y + 12 = x - 9$

$l_1 : x - 3y - 21 = 0$

(ii) Line $l_2$ passes through the origin and has gradient $-2$.

Find the coordinates of $P$, the point of intersection of lines $l_1$ and $l_2$. [2]

$l_2 : y = -2x$ ; $l_1 = x - 3y - 21 = 0$

Solving $l_1$ and $l_2$ simultaneously

$x - 3(-2x) - 21 = 0$

$7x - 21 = 0$

$7x = 21$

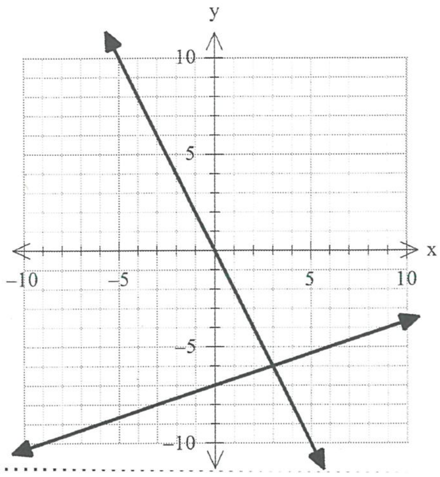
$x = 3$

$\therefore P = (3, -6)$

(iii) Given that $l_1$ crosses the $y$-axis at the point $C$,

calculate the area of $\Delta\ OPC$. [3]

*(You may draw a diagram in the space below to help you answer the question)*




At $C$, $x = 0 \Rightarrow -3y - 21 = 0$

$y = -7 \quad \therefore C = (0, -7)$

Area of $\Delta$ OPC

$= \frac{1}{2} \times 7 \times 3$

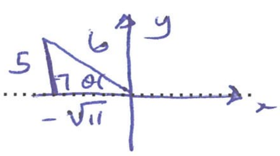
$= \frac{21}{2}$ square units

Q2 $\Rightarrow$ cosine is negative

b. Given that $\sin\theta = \dfrac{5}{6}$ and $90° \leq \theta \leq 180°$, find the value of $\cos\theta$. [2]

Adjacent side $= \sqrt{6^2 - 5^2}$

$= \sqrt{11}$




$\cos\theta = \dfrac{-\sqrt{11}}{6}$

c. Solve: $\tan x = \dfrac{1}{\sqrt{3}}$ for $0 \leq x \leq 360°$ [2]

tan is positive in Q1, Q3

$\therefore x = 30°, (180 + 30°)$

$= 30°, 210°$

d. Prove that $\dfrac{\sec\theta}{\cot\theta + \tan\theta} - \dfrac{1+\cot\theta}{\csc\theta} = -\cos\theta$ [4]

$$LHS = \frac{\sec\theta}{\cot\theta + \tan\theta} - \frac{1+\cot\theta}{\csc\theta}$$

$$= \frac{\frac{1}{\cos\theta}}{\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}} - \frac{1 + \frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}} \quad \frac{\times \sin\theta\cos\theta}{\times \sin\theta\cos\theta}$$

$$= \frac{\sin\theta}{\cos^2\theta + \sin^2\theta} - \left(\frac{\sin\theta\cos\theta + \cos^2\theta}{\cos\theta}\right)$$

$$= \frac{\sin\theta}{1} - \left(\frac{\cancel{\cos\theta}(\sin\theta + \cos\theta)}{\cancel{\cos\theta}}\right)$$

$$= \sin\theta - \sin\theta - \cos\theta$$

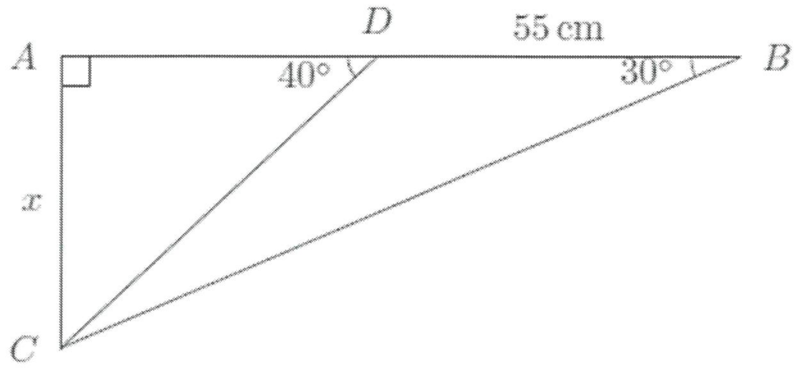
$$= -\cos\theta$$

$= RHS.$

**Question 10**(13 marks)

a. Find the value of $x$ where $BD = 55\text{cm}$. [3]

Answer correct to 3 significant figures.




$\angle ADC = \angle DBC + \angle DCB$ (exterior $\angle$ of $\Delta$ = sum of interior opposi $\angle$s)

$\therefore \angle DCB = 40° - 30° = 10°$

In $\Delta$ BCD

$$\frac{DC}{\sin 30°} = \frac{55}{\sin 10°}$$

$$DC = \frac{55}{2\sin 10°}$$

In $\Delta$ ADC,

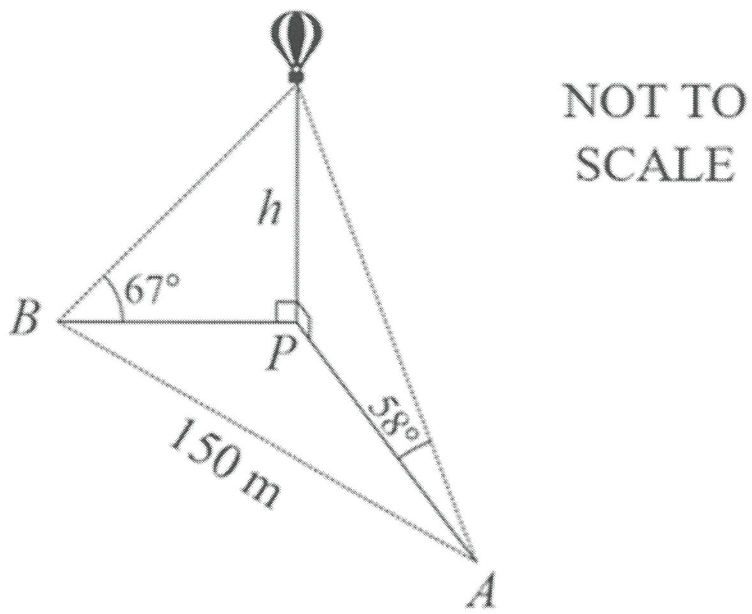
$$\sin 40° = \frac{x}{DC}$$

$$x = DC \sin 40°$$

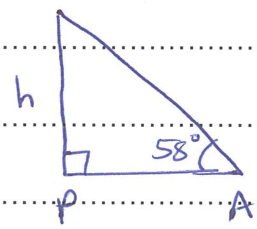
$$= \frac{55 \sin 40°}{2 \sin 10°}$$

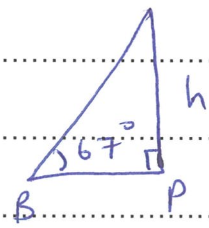
$x = 102$ cm (to 3 sig figs)

b. A newly launched weather balloon is floating in the air vertically $h$ metres above the point $P$ on the ground. Observer $A$ is due south of the balloon and notes that its angle of elevation is 58°. Observer $B$ is 150 metres from Observer $A$ and due east of the weather balloon. The angle of elevation of the weather balloon from Observer $B$ is 67°.




(i) Show that $PA = h \cot 58°$ and $PB = h \cot 67°$. [2]



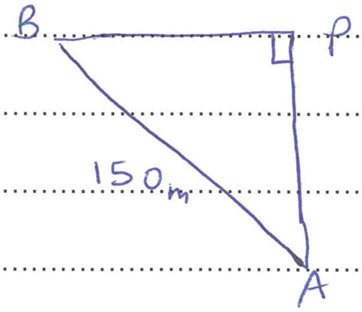




$\tan 58° = \dfrac{h}{PA}$ $\tan 67° = \dfrac{h}{BP}$

$PA = \dfrac{h}{\tan 58°}$ $BP = \dfrac{h}{\tan 67°}$

$= h \cot 58°$ $= h \cot 67°$

(ii) Use Pythagoras' Theorem to show that $h^2 = \dfrac{150^2}{\cot^2 58° + \cot^2 67°}$ [2]




$PA^2 + PB^2 = 150^2$

$(h \cot 58°)^2 + (h \cot 67°)^2 = 150^2$

$h^2(\cot^2 58° + \cot^2 67°) = 150^2$

$h^2 = \dfrac{150^2}{\cot^2 58° + \cot^2 67°}$

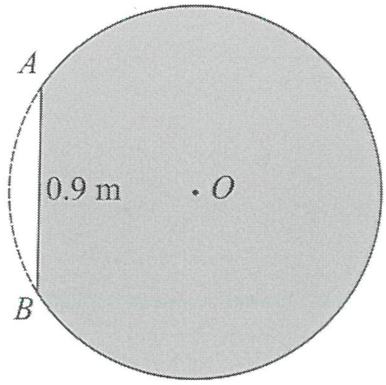
(iii) Hence, find $h$ correct to one decimal place. [1]

$h = 198 \cdot 6\text{m}$

c. A small segment of a circle has been removed as shown below.

The circle has a centre $O$ and radius 0.9 metres.

The length of the straight edge $AB$ is also 0.9 metres.




Not to scale

(i) Explain why $\angle AOB = \frac{\pi}{3}$ [1]

$AB = 0.9\text{m}$ (given)

$OA = OB = 0.9\text{m}$ (radii)

$\therefore \Delta AOB$ is equilateral (all sides =)

$\therefore \angle AOB = \frac{\pi}{3}$ (all <s of equilateral Δ =)

(ii) Find the area of $\Delta AOB$. [1]

$A_{\Delta AOB} = \frac{1}{2} r^2 \sin\theta$

$= \frac{1}{2} \times (0.9)^2 \times \sin\frac{\pi}{3}$

$= 0.351\text{ m}^2$ (to 3dp)

(iii) Find the area of the minor sector $AOB$. [1]

$A = \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \times (0.9)^2 \times \frac{\pi}{3}$

$= 0.424\text{ m}^2$ (to 3dp)

(iv) Hence, find the area of the shaded region. [2]

A of minor segment (unshaded)

= A of minor sector − A of $\Delta AOB$

$= 0.073 \text{ m}^2$

Shaded Area = A of circle − A of minor segment

$= \pi (0.9)^2 - 0.073$

$= 2.472 \text{ m}^2$ (to 3 dp)

**Question 11** (17 marks)

a. Simplify: $\left(\dfrac{12x^{-3}y^{2}}{xyz^{\frac{1}{2}}}\right)^{2} \div \dfrac{6x^{2}y^{3}}{z^{-4}}$ [3]

$= \dfrac{144x^{-6}y^{4}}{x^{2}y^{2}z} \times \dfrac{z^{-4}}{6x^{2}y^{3}}$

$= \dfrac{144y^{4}}{x^{8}y^{2}z} \times \dfrac{1}{6x^{2}y^{3}z^{4}}$

$= \dfrac{24}{x^{10}yz^{5}}$

b. Solve:

(i) $5^{1-x} = 25^{x}$ [2]

$5^{1-x} = 5^{2x}$

$1 - x = 2x$

$3x = 1$

$x = \frac{1}{3}$

(ii) $\log_4(2x) - \log_4(x+1) = 0$ [2]

$\log_4\left(\frac{2x}{x+1}\right) = 0$

$\frac{2x}{x+1} = 4^0$

$2x = x+1$

$x = 1$

c. Given that $\log_a 4 = 1.5$ and $\log_a 5 = 1.7$, find:

(i) $\log_a 1.25$ [1]

$= \log_a\left(\frac{5}{4}\right)$

$= \log_a 5 - \log_a 4$

$= 1.7 - 1.5$

$= 0.2$

(ii) $\log_a 2$ [1]

$= \log_a 4^{\frac{1}{2}}$

$= \frac{1}{2}\log_a 4$

$= \frac{1}{2} \times 1.5$

$= 0.75$

(iii) $\log_a 100$ [2]

$= \log_a (5^2 \times 4)$

$= 2\log_a 5 + \log_a 4$

$= 2(1.7) + 1.5$

$= 4.9$

d. Differentiate $y = \dfrac{e^{2x} + 4}{x^3}$ [3]

$\dfrac{dy}{dx} = \dfrac{vu' - uv'}{v^2}$

$u = e^{2x} + 4$

$u' = 2e^{2x}$

$v = x^3$

$v' = 3x^2$

$= \dfrac{x^3(2e^{2x}) - (e^{2x}+4)(3x^2)}{(x^3)^2}$

$= \dfrac{x^2\left[2xe^{2x} - 3(e^{2x}+4)\right]}{x^6}$

$= \dfrac{2xe^{2x} - 3e^{2x} - 12}{x^4}$

e. Find the equation of the tangent to the curve $y = e^{1-2x}$ at the point where $x = 0.5$ [3]

$\frac{dy}{dx} = -2e^{1-2x}$

when $x = 0.5$ ;

$\frac{dy}{dx} = -2e^{1-2(0.5)}$ and $y = e^{1-2(0.5)}$

$= -2$ $= 1$

pt $(0.5, 1)$

Equation of tangent :

$y - y_1 = m(x - x_1)$

$y - 1 = -2(x - 0.5)$

$y - 1 = -2x + 1$

$2x + y - 2 = 0$

**Question 12** (11 marks)

a. A tank initially has 3000 litres of water and is leaking according to the formula

$V = V_0 e^{-0.135t}$ where $t$ is in hours.

(i) Find the value of $V_0$. [1]

Initially => $t = 0$, $V = 3000$

$\therefore 3000 = V_0 e^0$

$V_0 = 3000$ litres

(ii) Find the value of $V$ after 5 hours. [1]

Give your answer correct to 1 decimal place.

when $t = 5$, $V = 3000 e^{-0.135(5)}$

$= 1527.5$ litres

(iii) Find the time taken for the volume of water to fall to 250 litres. [3]

Give your answer to the nearest minute.

$V = 250$, $t = ?$

$250 = 3000 \times e^{-0.135t}$

$e^{-0.135t} = \frac{1}{12}$

$-0.135t = \log_e\left(\frac{1}{12}\right)$

$t = \frac{\log_e\left(\frac{1}{12}\right)}{-0.135}$

$= 18.4067...$

$= 18h\ 24\ mins\ 24\ secs$

It takes 18h 25 mins for the volume to fall to 250 litres

b. (i) Factorise the expression $2x^2 - x - 1$. [2]

$x - 2$
$+ -1$

$= 2x^2 + x - 2x - 1$

$= x(2x+1) - 1(2x+1)$

$= (x-1)(2x+1)$

(ii) Hence, solve the equation

$2\sin^2 2x - \sin 2x = 1$ for $0° \leq x \leq 360°$. [4]

$2\sin^2 2x - \sin 2x - 1 = 0$

$(\sin 2x - 1)(2\sin 2x + 1) = 0$ (using (i))

$\sin 2x - 1 = 0$ or $2\sin 2x + 1 = 0$

$\sin 2x = 1$ $\quad$ $\sin 2x = -\frac{1}{2}$ ; $0° \leq 2x \leq 720°$

$x = 90°, 450°$

$x = 45°, 225°$ $\quad$ $2x = 210°, 330°, 570°, 690°$

$x = 105°, 165°, 285°, 345°$

$\therefore x = 45°, 105°, 165°, 225°, 285°, 345°$

End of Examination