GIRRAWEEN HIGH SCHOOL

MATHEMATICS (ADVANCED)

YEAR 11

Yearly Examination 2023

Year 11 Mathematics **September 2023**

Reading time: 10 minutes **Section I** **5** marks
Section II **105** marks

Working time: 120 minutes **TOTAL** **110** marks

Instructions:

- Attempt ALL questions.
- For Questions 1 to 5, colour in the bubble next to the letter corresponding to the correct response on your paper.
- For Questions 6 to 12:
    - All necessary working should be shown in the space provided.
    - Marks will be deducted for careless or badly arranged work.
    - Board-approved calculators may be used.
    - Reference sheet with formulae is attached at the end of the paper.
    - Diagrams are NOT to scale.

Section I (5 marks)

- **Use the multiple-choice answer sheet for questions 1-5.**

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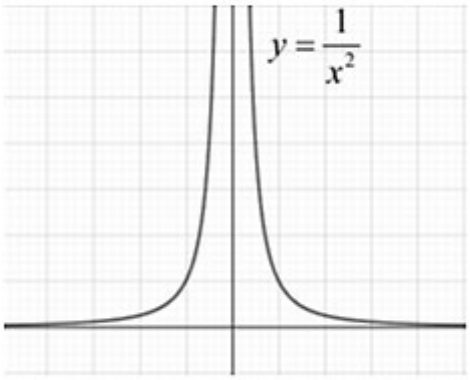
1. What type of relation is shown by the graph on the right?



(A) One-to-one

(B) Many-to-one

(C) One-to-many

(D) Many -to many

2. What is the value of $\cos x$, given that $\tan x = -\frac{3}{4}$, for $0 \le x \le \pi$?

(A) $\frac{4}{5}$

(B) $\frac{3}{5}$

(C) $-\frac{3}{5}$

(D) $-\frac{4}{5}$

3. For what values of k does the equation $x^2 - 6x - 3k = 0$ have real roots?

(A) $k \geq -3$

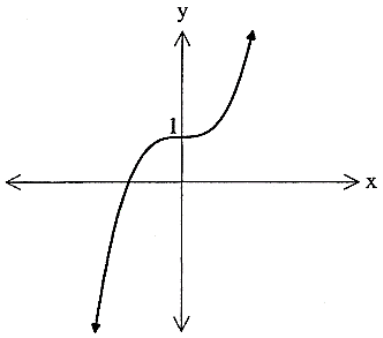
(B) $k \leq -3$

(C) $k \leq 3$

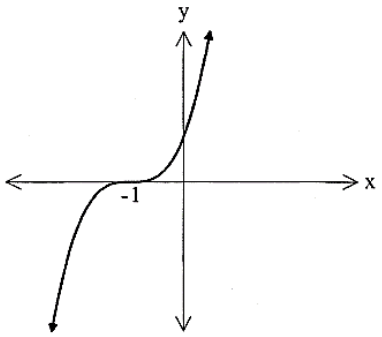
(D) $k \geq 3$

4. The graph of $y = 1 - x^3$ could be

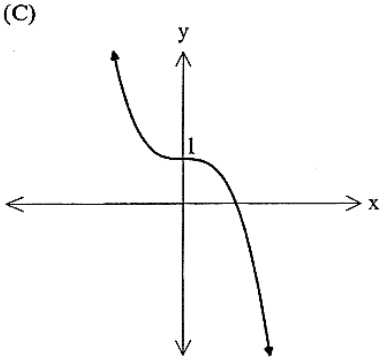
(A)



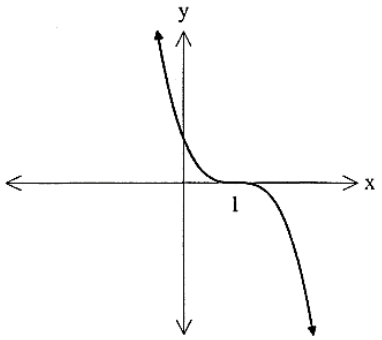
(B)



(C)



(D)



5. The domain of the graph of $y = \log_2 2x - 3$ is:

(A) All real x, $x > \frac{3}{2}$

(B) All real x, $x \neq \frac{3}{2}$

(C) $x > 0$

(D) All real x.

Answers to Multiple Choice Questions

1.	A	○	B	○	C	○	D	○
2.	A	○	B	○	C	○	D	○
3.	A	○	B	○	C	○	D	○
4.	A	○	B	○	C	○	D	○
5.	A	○	B	○	C	○	D	○

Section II (105 marks)

- **Answer questions in the spaces provided.**
- **Your responses should include relevant mathematical reasoning and/or calculations.**
- **Extra writing space is provided at the back of this booklet. If you use this space, indicate in the writing space under the question and clearly write the question number on the extra writing space.**

Question 6 (15 marks)

(a) Evaluate, correct to 3 significant figures: $\dfrac{2.1^3 - 29}{\sqrt{4.01^2 - 0.8^2}}$ 1

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(b) Expand and simplify: $(2x - y)(2x + y) - y(y - x)$ 2

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(c) Factorise the denominators and simplify: $\dfrac{3}{x^2-5x+6}-\dfrac{2}{x^2+2x-8}$ 3

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(d) Find the integers a and b such that $\dfrac{4\sqrt{3}}{\sqrt{7}+\sqrt{3}}=a+b\sqrt{21}$ 3

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(e) Solve: $\frac{1}{2y}-\frac{2}{3}=1-\frac{1}{3y}$ 3

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(f) Use quadratic formula to find the x intercepts of the parabola: $y=-7x^2-10x+6$ .Write your answer in the simplest surd form. 3

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Question 7 (15 marks)

(a) Solve: $|2x-3|=17$ 2

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(b) Find the domain and range of the function $y=-\sqrt{\frac{25}{16}-x^2}$. 2

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(c) Test whether $f(x) = x^5 - 21x$ is an even function, odd function or neither. 2

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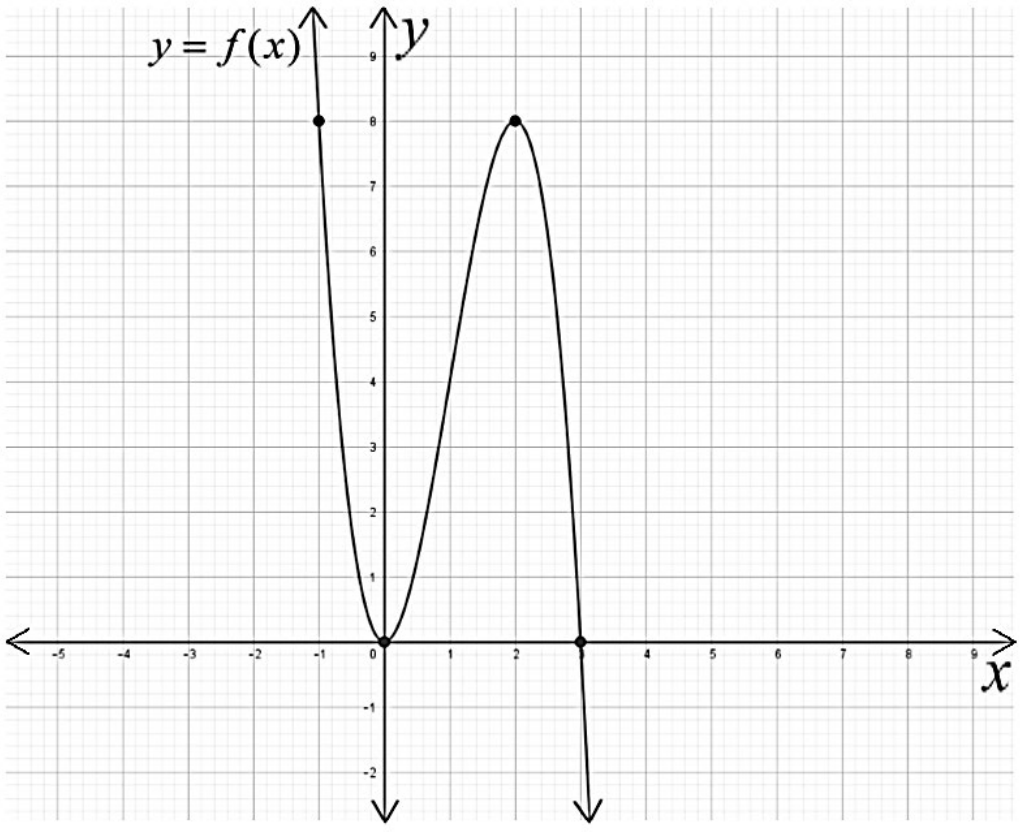
(d) The graph below shows the curve $y = f(x)$. On the same set of axes, sketch $y = f(x-3)$ showing all important features. 3



(e) Given $f(x) = 3x + 2$ and $g(x) = 7x + b$, where b is a constant.

(i) Find expressions for

(I) $g(f(x))$. 2

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(II) $f(g(x))$. 2

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(ii) Hence find the value of b so that $g(f(x)) = f(g(x))$ for all x. 2

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Question 8 (15 marks)

(a) Consider the piece-wise defined function

$$f(x)=\begin{cases} x^2-1, & \text{for} \quad x\le 1 \\ 4-x^2, & \text{for} \quad x>1 \end{cases}$$

(i) Find $f(2)+f(0)$ 1

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(ii) Find the value of $x-$ intercepts of $f(x)$. 2

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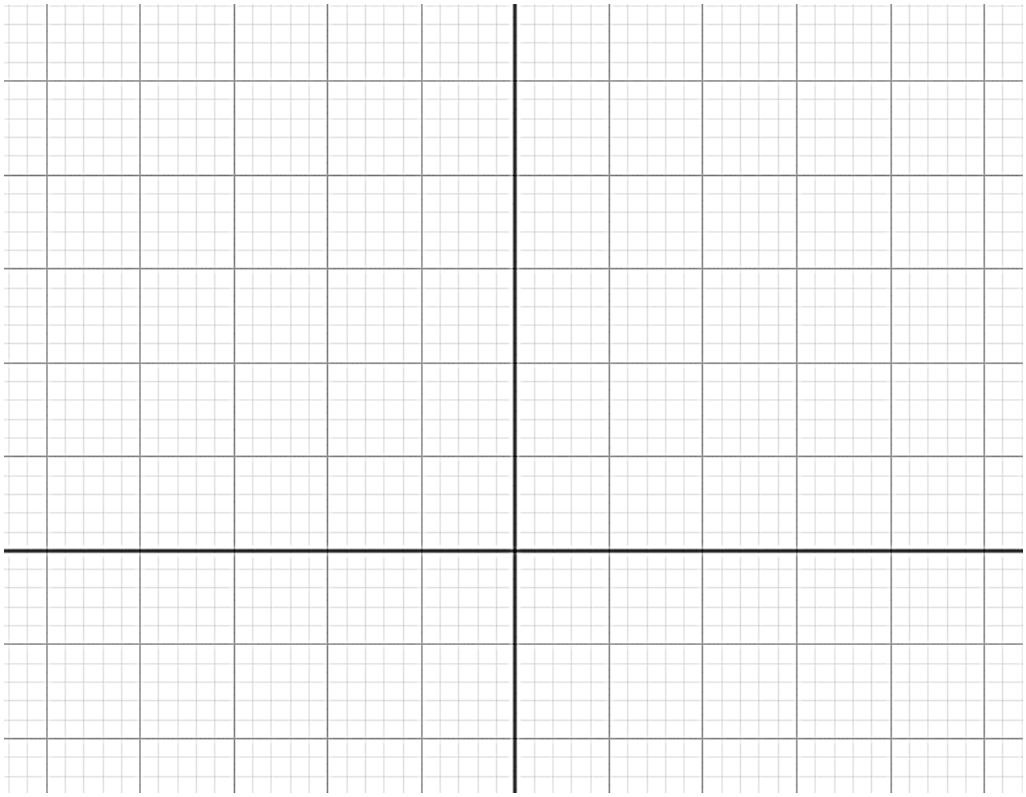
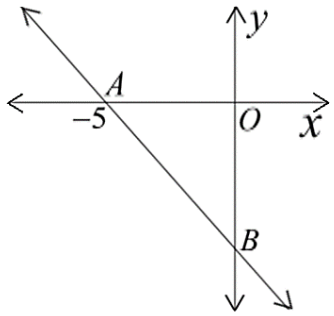
(iii) Sketch the function showing all intercepts with the axes. 3



(b) Find the equation of the line AB, given that $\angle ABO = 60°$. Write your answer in gradient-intercept form. 2



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(c) A circle has the equation $x^2 + y^2 - 5x + 3y - 1 = 0$. Find the radius and centre of the circle. 3

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(d) Show that the three lines given by the equations $3x+5y+7=0,\ x+2y+2=0,$ and $2x-y+9=0$ are concurrent. 4

Question 9 (15 marks)

(a) (i) Differentiate $2x^2 - 8x + 3$ from first principles. 3

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(ii) At what point on the curve is the tangent parallel to the $x-$ axis? 2

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(b) Differentiate:

(i) $y = 20x^5 - 4x^3 + 11x - 11$ 2

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(ii) $\dfrac{7}{2\sqrt{x}}$ 2

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(iii) $y=(3x^2+2x)^{-4}$ 2

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(iv) $y=x^3(4-3x)^5$ 2

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(v) $y=\dfrac{x^2-16}{3x-5}$ 2

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Question 10 (15 marks)

(a) Solve:

(i) $\left(\frac{1}{3}\right)^{x-4} = 243$ 2

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(ii) $\log_e(5x-6) = 2\log_e x$ 2

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(b) Given that $\log_5 3 = 0.68$ and $\log_5 4 = 0.86$. Find: 2

(i) $\log_5 0.75$

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(ii) $\log_5 20$ 2

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(c) Find the equation of the normal to the curve $y = e^{4x+1}$ at the point where $x = -\frac{1}{4}$. Write answer in gradient intercept form. 3

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(d) The population of a city is given by $P = 35000e^{0.024t}$ where t is time in years.

(i) Find the population after 10 years. 2

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(ii) Find when the population will reach 200 000? 2

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Question 11 (15 marks)

(a) Using exact values, show that $\sin 2\theta = 2\sin\theta\cos\theta$ when $\theta = 225°$ 3

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(b) Prove: $\cot\theta + \tan\theta = \cos ec\theta \sec\theta$ 3

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(c) Solve for $0° \le \theta \le 360°$.

(i) $4\cos^2\theta = 1$ 3

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(ii) $\tan 2\theta = \dfrac{1}{\sqrt{3}}$ 3

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(iii) $3\sin^2\theta - 5\sin\theta + 2 = 0$ 3

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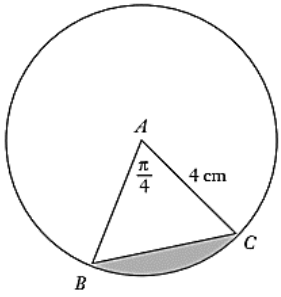
Question 12 (15 marks)

(a) A is the centre of a circle of radius 4 cm and B and C are endpoints of a chord.

Find the exact area of:



(i) the ΔABC. 1

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(ii) the minor sector ABC . 2

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(iii) the shaded minor segment. 2

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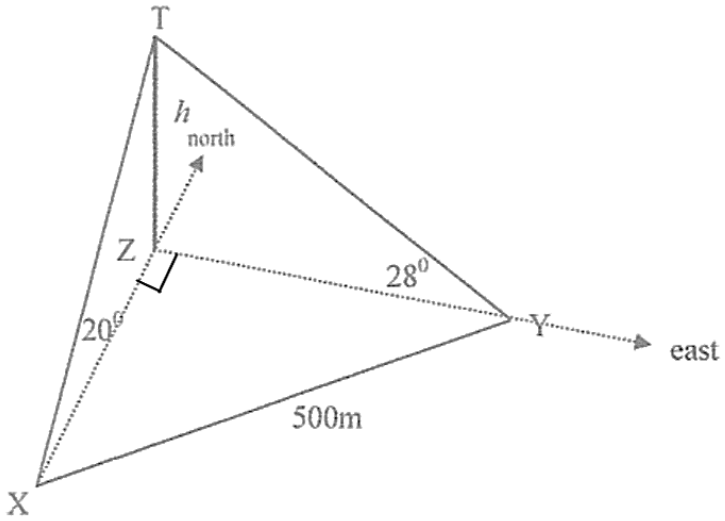
(b) A conservationist observes the angle of elevation of the top of a tree, which is h metres tall, from two positions. From a point X, due south of the tree, it is $20°$ and from point Y, due east of the tree, it is $28°$.The distance XY is 500 m.



(i) Write expressions for XZ and YZ in terms of h. 2

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(ii) Calculate the value of h. 2

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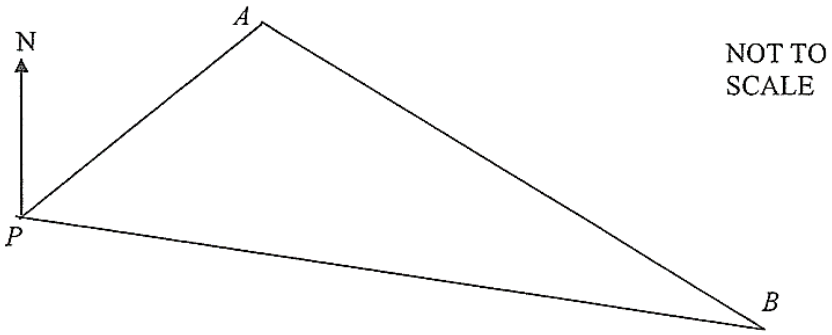
(c) In the diagram a yacht sails 640 metres from point P to point A on a bearing of $050°$. It then sails 960 metres from point A to point B on a bearing of $120°$.



(i) Show that $\angle PAB = 110°$. 2

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(ii) Find the distance of point B from point P correct to the nearest metre. 2

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(iii) Find the bearing of point B from point P correct to the nearest degree. 2

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End of Examination

Extra Writing Space. Clearly write the question number.

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Yearly Examination 2023

Year 11 Mathematics **September 2023**

Reading time: 10 minutes

Working time: 120 minutes

Section I	5 marks
Section II	105 marks
TOTAL	110 marks

Instructions:

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    - Diagrams are NOT to scale.

Section I (5 marks)

- **Use the multiple-choice answer sheet for questions 1-5.**

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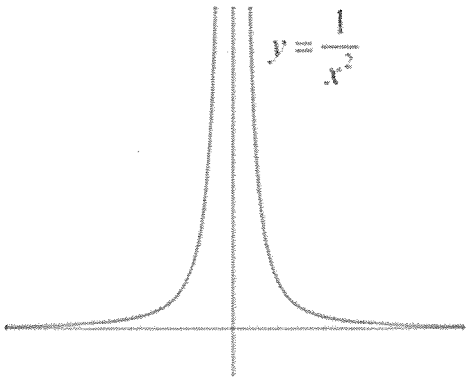
1. What type of relation is shown by the graph on the right?



Many x values corresponds to one y value

(A) One-to-one

(B) Many-to-one

(C) One-to-many

(D) Many -to many

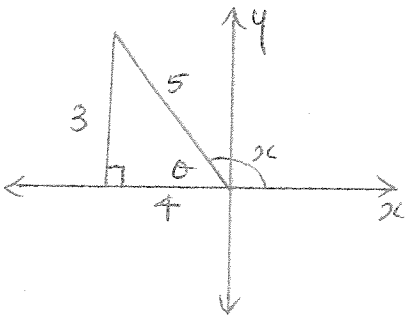
2. What is the value of $\cos x$, given that $\tan x = -\frac{3}{4}$, for $0 \le x \le \pi$?

(A) $\frac{4}{5}$

(B) $\frac{3}{5}$

(C) $-\frac{3}{5}$

(D) $-\frac{4}{5}$



$\cos x = -\frac{4}{5}$

3. For what values of k does the equation $x^2 - 6x - 3k = 0$ have real roots?

(A) $k \geq -3$

(B) $k \leq -3$

(C) $k \leq 3$

(D) $k \geq 3$

$\Delta = b^2 - 4ac$

$= (-6)^2 - 4 \times 1 \times (-3k)$

$= 36 + 12k$

$36 + 12k \geq 0$

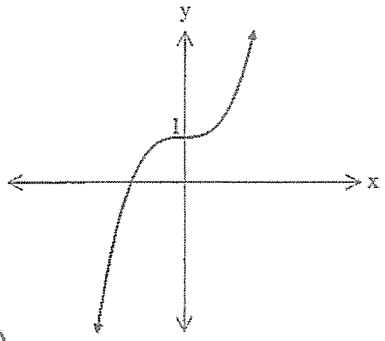
$12k \geq -36$

$k \geq \frac{-36}{12}$

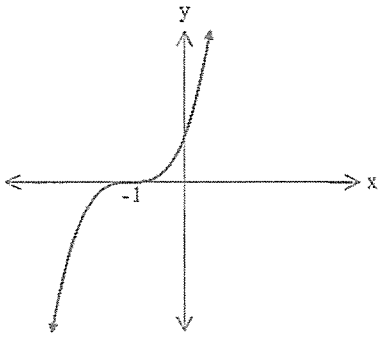
$k \geq -3$

4. The graph of $y = 1 - x^3$ could be

(A)

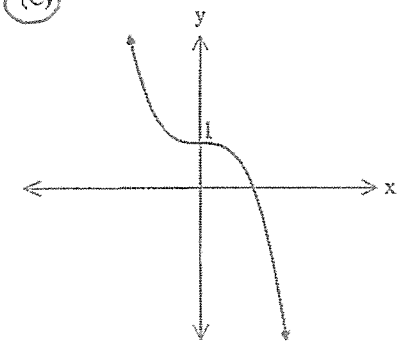


(B)

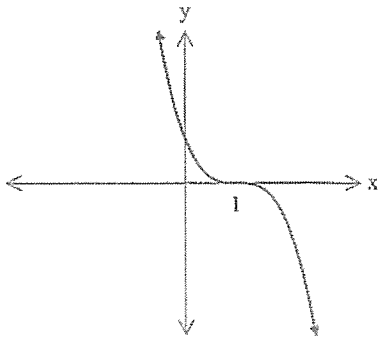


$y = -x^2$ moved up 1 unit

(C)



(D)



5. The domain of the graph of $y = \log_2 2x - 3$ is:

(A) All real x, $x > \frac{3}{2}$

(B) All real x, $x \neq \frac{3}{2}$

(C) $x > 0$

(D) All real x.

$2x - 3 > 0$

$2x > 3$

$x > \frac{3}{2}$

Answers to Multiple Choice Questions

1.	A	○	B	●	C	○	D	○
2.	A	○	B	○	C	○	D	●
3.	A	●	B	○	C	○	D	○
4.	A	○	B	○	C	●	D	○
5.	A	●	B	○	C	○	D	○

Section II (105 marks)

- **Answer questions in the spaces provided.**
- **Your responses should include relevant mathematical reasoning and/or calculations.**
- **Extra writing space is provided at the back of this booklet. If you use this space, indicate in the writing space under the question and clearly write the question number on the extra writing space.**

Question 6 (15 marks)

(a) Evaluate, correct to 3 significant figures: $\dfrac{2.1^3-29}{\sqrt{4.01^2-0.8^2}}$ 1

-5.02

Students who wrote -5.023 lost a mark

(b) Expand and simplify: $(2x-y)(2x+y)-y(y-x)$ 2

$= (2x)^2 - y^2 - y^2 + xy$

$= 4x^2 - 2y^2 + xy$

Some Students had trouble with multiplying in the negative numbers

(c) Factorise the denominators and simplify: $\dfrac{3}{x^2-5x+6}-\dfrac{2}{x^2+2x-8}$ 3

$= \dfrac{3}{(x-3)(x-2)} - \dfrac{2}{(x-2)(x+4)}$

$= \dfrac{3(x+4)-2(x-3)}{(x-3)(x-2)(x+4)}$

$= \dfrac{3x+12-2x+6}{(x-3)(x-2)(x+4)}$

$= \dfrac{x+18}{(x-3)(x-2)(x+4)}$

Mostly done well but some students had trouble with find the LCD.

(d) Find the integers a and b such that $\dfrac{4\sqrt{3}}{\sqrt{7}+\sqrt{3}} = a+b\sqrt{21}$ 3

$\dfrac{4\sqrt{3}}{\sqrt{7}+\sqrt{3}} \times \dfrac{\sqrt{7}-\sqrt{3}}{\sqrt{7}-\sqrt{3}}$

$= \dfrac{4\sqrt{21}-12}{4}$

$= \dfrac{4(\sqrt{21}-3)}{4}$

$= -3+\sqrt{21}$

$-3+\sqrt{21} = a+b\sqrt{21}$

$a=-3 \quad b=1$

Students lost marks if they wrote $(\sqrt{7}+\sqrt{3})(\sqrt{7}-\sqrt{3}) = 7+3$

(e) Solve: $\frac{1}{2y}-\frac{2}{3}=1-\frac{1}{3y}$ 3

$6y \times \frac{1}{2y} - 6y \times \frac{2}{3} = 6y - 6y \times \frac{1}{3y}$

$3 - 4y = 6y - 2$

Mostly well done!

$10y = 5$

$y = \frac{1}{2}$

(f) Use quadratic formula to find the x intercepts of the parabola: $y=-7x^2-10x+6$. Write your answer in the simplest surd form. 3

$x = \frac{10 \pm \sqrt{100 - 4 \times -7 \times 6}}{-14}$

$= \frac{10 \pm \sqrt{268}}{-14}$

$= \frac{10 \pm \sqrt{67 \times 4}}{-14}$

$= \frac{10 \pm 2\sqrt{67}}{-14}$

$= \frac{5 \pm \sqrt{67}}{-7}$

$= \frac{-5 \pm \sqrt{67}}{7}$

Mostly done well but Students who left their answer in unsimplified form lost a mark.

Question 7 (15 marks)

(a) Solve: $|2x-3|=17$ 2

$2x-3=17$ or $-(2x-3)=17$

Well done.

$2x=20$ $3-2x=17$

$-2x=-14$

$x=10$ $x=7$

$\therefore x=10$ or -7

(b) Find the domain and range of the function $y=-\sqrt{\frac{25}{16}-x^2}$. 2

$D: -\frac{5}{4} \leq x \leq \frac{5}{4}$

Many students had trouble with this question.

$R: -\frac{5}{4} \leq y \leq 0$

(c) Test whether $f(x) = x^5 - 21x$ is an even function, odd function or neither. 2

$f(-x) = (-x)^5 - 21(-x)$

Mostly well done.

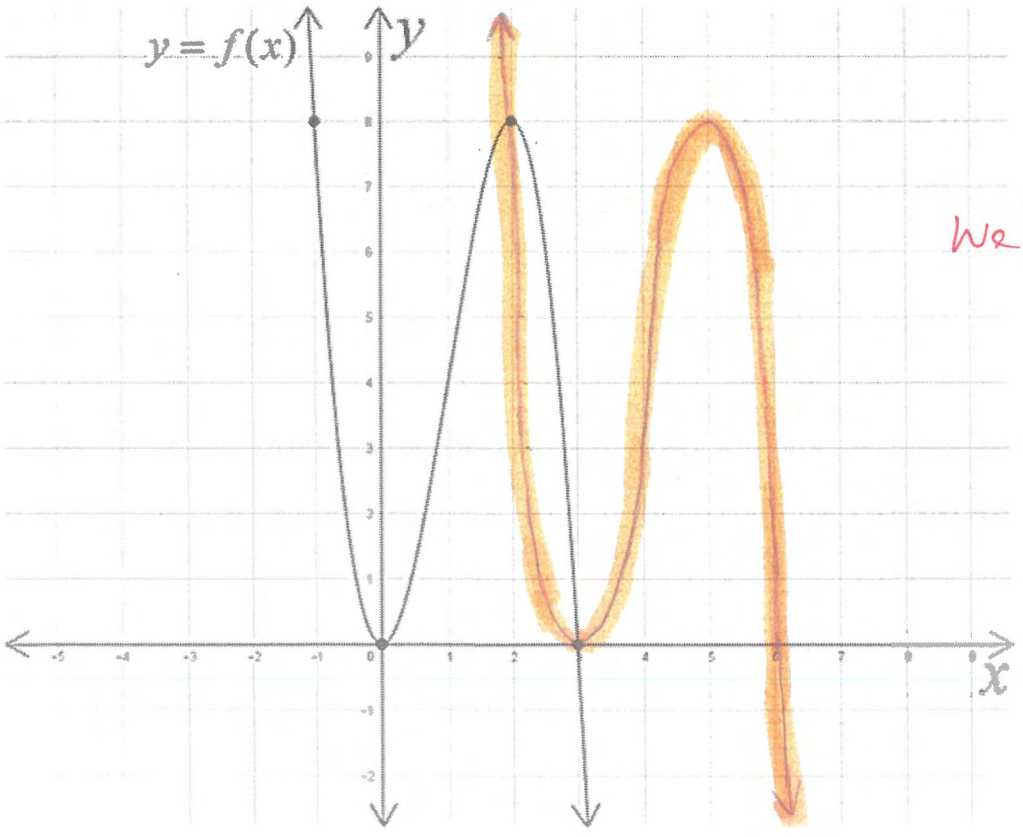
$= -x^5 + 21x$

$= -(x^5 - 21x)$

$= -f(x)$

$f(-x) = -f(x)$. $\therefore f(x)$ is an odd function

(d) The graph below shows the curve $y = f(x)$. On the same set of axes, sketch $y = f(x-3)$ showing all important features. 3



Well done.

(e) Given $f(x)=3x+2$ and $g(x)=7x+b$, where b is a constant.

(i) Find expressions for

(I) $g(f(x))$. 2

$= g(3x+2)$

$= 7(3x+2)+b$

$= 21x+14+b$

Mostly well done

(II) $f(g(x))$. 2

$= f(7x+b)$

$= 3(7x+b)+2$

$= 21x+3b+2$

(ii) Hence find the value of b so that $g(f(x))=f(g(x))$ for all x. 2

$21x+14+b = 21x+3b+2$

$14+b = 3b+2$

Well done.

$2b = 12$

$b = 6$

Question 8 (15 marks)

(a) Consider the piece-wise defined function

$$f(x)=\begin{cases} x^2-1, & \text{for } x\le 1 \\ 4-x^2, & \text{for } x>1 \end{cases}$$

(i) Find $f(2)+f(0)$ 1

$= 4-2^2+0^2-1$

$= -1$

well done

(ii) Find the value of x – intercepts of $f(x)$. 2

$x^2-1=0$

$x^2=1$

$x=\pm 1$

$4-x^2=0$

$x^2=4$

$x=\pm 2$

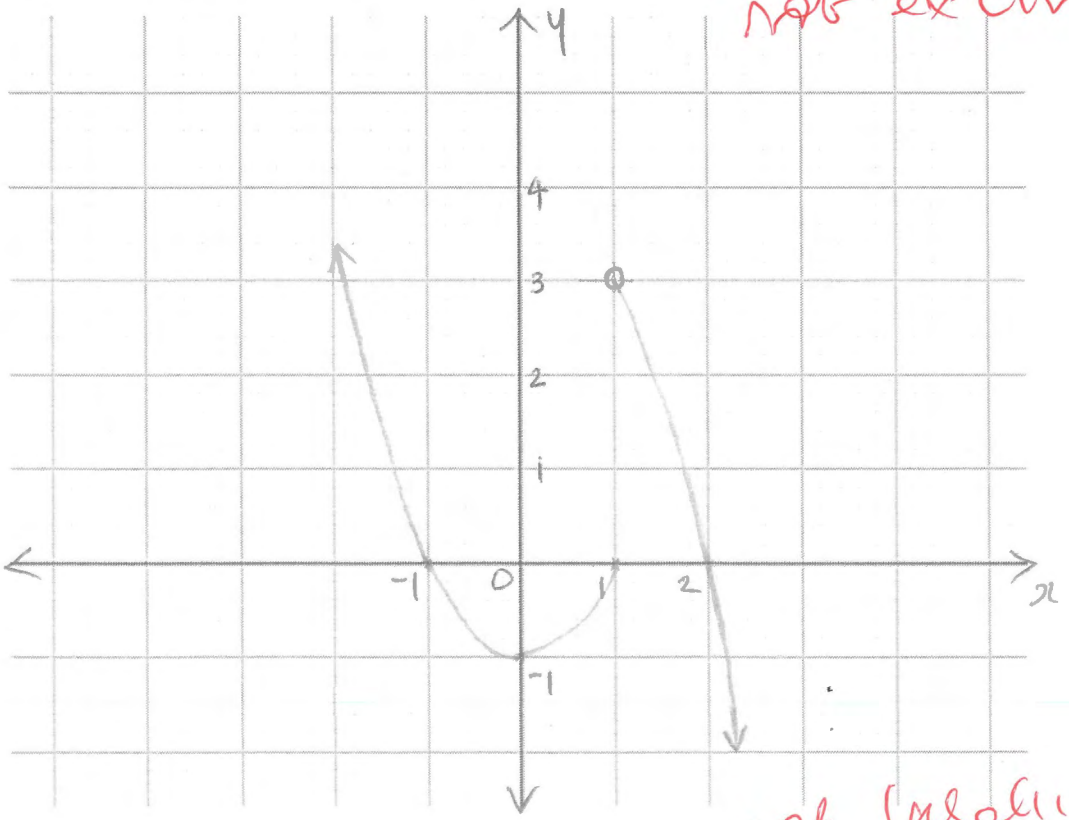
$x=-2$ is not in the domain

$\therefore$ x intercepts are

± 1 and 2

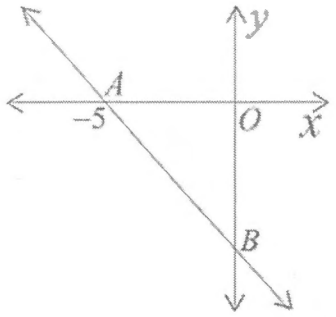
Some students did not exclude $x=-2$

(iii) Sketch the function showing all intercepts with the axes. 3



common errors = not labelling scale
= using straight lines for curves

(b) Find the equation of the line AB, given that $\angle ABO = 60°$. Write your answer in gradient-intercept form. 2



poorly done.
common error
$m = \tan 60\theta = \sqrt{3}$

$m = \tan 150$

$= \frac{-1}{\sqrt{3}}$

$m = \frac{-1}{\sqrt{3}}$ $(-5, 0)$

Equation of AB

$y - 0 = \frac{-1}{\sqrt{3}}(x + 5)$

$y = \frac{-1}{\sqrt{3}}x - \frac{5}{\sqrt{3}}$

or $y = \frac{-\sqrt{3}}{3} - \frac{5\sqrt{3}}{3}$

(c) A circle has the equation $x^2 + y^2 - 5x + 3y - 1 = 0$. Find the radius and centre of the circle. 3

$x^2 - 5x + y^2 + 3y = 1$

$x^2 - 5x + \left(\frac{5}{2}\right)^2 + y^2 + 3y + \left(\frac{3}{2}\right)^2 = 1 + \frac{25}{4} + \frac{9}{4}$

$\left(x - \frac{5}{2}\right)^2 + \left(y + \frac{3}{2}\right)^2 = \frac{38}{4} = \frac{19}{2}$

centre $\left(\frac{5}{2}, \frac{-3}{2}\right)$

Generally well done

radius $\frac{\sqrt{38}}{2}$ units

or $\sqrt{\frac{19}{2}}$

(d) Show that the three lines given by the equations $3x+5y+7=0$, $x+2y+2=0$, and $2x-y+9=0$ are concurrent. 4

$3x+5y=-7$ —— (1)

$x+2y=-2$ —— (2)

$2x-y=-9$ —— (3)

(2) × 3 $4x-2y=-18$ —— (4)

(2) + (4)

$x+2y=-2$

$4x-2y=-18$

$5x=-20$

$x=-4$

$2y=-2-x$

$=-2-(-4)$

$=2$

$y=1$

Substitute $x=-4, y=1$ in (1)

LHS $= 3\times-4+5\times1$

$=-12+5=-7$

RHS $=-7$

LHS = RHS

The three lines passes through the point $(-4,1)$

$\therefore$ the given lines are concurrent.

Poorly done
- Many students did not know the meaning of 'concurrent'
- Many students used inefficient methods

Question 9 (15 marks)

(a) (i) Differentiate $2x^2 - 8x + 3$ from first principles. 3

$f(x+h) = 2(x+h)^2 - 8(x+h) + 3$

$= 2(x^2 + 2xh + h^2 - 8x - 8h + 3$

$= 2x^2 + 4xh + 2h^2 - 8x - 8h + 3$

$f(x+h) - f(x)$

$= 2x^2 + 4xh + 2h^2 - 8x - 8h + 3 - 2x^2 + 8x - 3$

$= 4xh + 2h^2 - 8h$

$\frac{f(x+h) - f(x)}{h} = 4x + 2h - 8$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} 4x + 2h - 8$

$= 4x - 8$

Some students not quoting formula and/or not setting out solution properly.

(ii) At what point on the curve is the tangent parallel to the x – axis? 2

$4x - 8 = 0$

$4x = 8$

$x = 2$

when $x = 2$,

$y = 2 \times 2^2 - 8 \times 2 + 3$

$= 8 - 16 + 3$

$= -5$

The tangent is parallel to the x axis at $(2, -5)$

Done quite well. Some students did not find the y-value.

(b) Differentiate:

(i) $y = 20x^5 - 4x^3 + 11x - 11$ 2

$y' = 100x^4 - 12x^2 + 11$

For all parts of (b), a significant number of students did not say y' or $\frac{dy}{dx}$ when starting to differentiate

(ii) $\frac{7}{2\sqrt{x}}$ 2

$y = \frac{7}{2} x^{-\frac{1}{2}}$

$y' = \frac{7}{2} \times -\frac{1}{2} x^{-\frac{3}{2}}$

$= -\frac{7}{4} x^{-\frac{3}{2}}$

$= -\frac{7}{4} \times \frac{1}{x^{\frac{3}{2}}}$

$= \frac{-7}{4\sqrt{x^3}}$

Some students had trouble with negative indices.

(iii) $y = (3x^2 + 2x)^{-4}$ 2

$y' = -4(3x^2+2x)^{-5} \times (3 \times 2x + 2)$

$= -4(6x+2)(3x^2+2x)^{-5}$

done quite well

(iv) $y = x^3(4-3x)^5$ 2

$y' = x^3 \times 5(4-3x)^4 \times -3 + (4-3x)^5 \times 3x^2$

$= -15x^3(4-3x)^4 + 3x^2(4-3x)^5$

$= (4-3x)^4(-15x^3 + 3x^2(4-3x))$

$= (4-3x)^4(-15x^3 + 12x^2 - 9x^3)$

$= (4-3x)^4(-24x^3 + 12x^2)$

Some students did not know to use product rule.

(v) $y = \frac{x^2-16}{3x-5}$ 2

$y' = \frac{(3x-5) \times 2x - (x^2-16) \times 3}{(3x-5)^2}$

$= \frac{6x^2 - 10x - 3x^2 + 48}{(3x-5)^2} = \frac{3x^2 - 10x + 48}{(3x-5)^2}$

done quite well.

Question 10 (15 marks)

(a) Solve:

(i) $\left(\frac{1}{3}\right)^{x-4} = 243$ 2

$(3^{-1})^{x-4} = 3^5$

$3^{-x+4} = 3^5$

$-x+4 = 5$

$4-5 = x$

$x = -1$

done well

(ii) $\log_e(5x-6) = 2\log_e x$ 2

$\log_e(5x-6) = \log_e x^2$

$5x-6 = x^2$ ($\because$ log is one-one function)

$x^2-5x+6 = 0$

$(x-3)(x-2) = 0$

$x = 3$ or $x = 2$

done well

(b) Given that $\log_5 3 = 0.68$ and $\log_5 4 = 0.86$. Find: 2

(i) $\log_5 0.75$

$= \log_5 \frac{3}{4}$

$= \log_5 3 - \log_5 4$

$= 0.68 - 0.86$

$= -0.18$

Some students wrote $\log_5 0.75 = \frac{\log_5 3}{\log_5 4}$

(ii) $\log_5 20$ 2

$= \log_5 (5 \times 4)$

$= \log_5 5 + \log_5 4$

$= 1 + 0.86 = 1.86$

Many didn't get this

(c) Find the equation of the normal to the curve $y = e^{4x+1}$ at the point where $x = -\frac{1}{4}$. Write answer in gradient intercept form. 3

$y' = 4e^{4x+1}$

when $x = -\frac{1}{4}$, $y' = 4e^{4\times -\frac{1}{4}+1}$

$= 4e^{-1+1} = 4$

when $x = -\frac{1}{4}$, $y = e^0 = 1$ $\left(-\frac{1}{4}, 1\right)$

Gradient of the normal $= -\frac{1}{4}$

Equation of the normal

$y - 1 = -\frac{1}{4}\left(x + \frac{1}{4}\right)$

$y - 1 = -\frac{1}{4}x - \frac{1}{16}$

$y = -\frac{1}{4}x + \frac{15}{16}$

Some are taking 4 as the gradient of the normal.

Few students failed to differentiate correctly.

(d) The population of a city is given by $P = 35000e^{0.024t}$ where t is time in years.

(i) Find the population after 10 years. 2

$P = 35000 \times e^{0.24 \times 10}$

$= 44493$

done well

(ii) Find when the population will reach 200 000? 2

$200000 = 35000e^{0.024t}$

$e^{0.024t} = \frac{200}{35}$

$0.024t = \log_e\left(\frac{200}{35}\right)$

$t = \frac{1}{0.024}\log_e\left(\frac{200}{35}\right)$

$= 72.6$ years.

Mostly done well

Question 11 (15 marks)

(a) Using exact values, show that $\sin 2\theta = 2\sin\theta\cos\theta$ when $\theta = 225°$ 3

LHS $= \sin 2\theta$

$= \sin 2 \times 225$

$= \sin 450$

$= \sin 90$

$= 1$

RHS $= 2 \times \sin 225 \times \cos 225$

$= 2 \times -\sin 45 \times -\cos 45$

$= 2 \times \frac{-1}{\sqrt{2}} \times \frac{-1}{\sqrt{2}}$

$= 1$

LHS = RHS

$\therefore \sin 2\theta = 2\sin\theta\cos\theta$ when $\theta = 225°$

* Working out not shown properly.

* Some students wrote $\sin(225) = \sin(-45)$ which is wrong.

They must show every step to get full marks

(b) Prove: $\cot\theta + \tan\theta = \operatorname{cosec}\theta \sec\theta$ 3

LHS $= \cot\theta + \tan\theta$

$= \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}$

$= \frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta}$

$= \frac{1}{\sin\theta\cos\theta}$

$= \frac{1}{\sin\theta} \times \frac{1}{\cos\theta}$

$= \operatorname{cosec}\theta \sec\theta$

$= $ RHS

Mostly done well

(c) Solve for $0° \leq \theta \leq 360°$.

(i) $4\cos^2\theta = 1$ 3

$\cos^2\theta = \frac{1}{4}$

$\cos\theta = \pm\frac{1}{2}$

$\cos\theta = \frac{1}{2}$

$\theta = 60°, 300°$

$\cos\theta = \frac{-1}{2}$

$\theta = 120°, 240°$

$\theta = 60°, 120°, 240°, 300°$

* some students forgot ± sign and only got two solutions and given 1 mark only.
* Few students did $\sqrt{\frac{1}{4}} = \pm\frac{1}{\sqrt{2}}$ which is wrong and lost 1 mark.

(ii) $\tan 2\theta = \frac{1}{\sqrt{3}}$ 3

Let $u = 2\theta$

$\tan u = \frac{1}{\sqrt{3}}$, $0 \leq u \leq 720$

$u = 30°, 210°, 390°, 570°$

$\theta = 15°, 105°, 195°, 285°$

* Some did $2\theta = 30$, then $\theta = 15$ and then did the other 3 solutions.

* Mostly done well but some only gave two solutions as they did not change the domain.

(iii) $3\sin^2\theta - 5\sin\theta + 2 = 0$ 3

Let $u = \sin\theta$

$3u^2 - 5u + 2 = 0$

$3u^2 - 3u - 2u + 2 = 0$

$3u(u-1) - 2(u-1) = 0$

$(u-1)(3u-2) = 0$

Mostly done well.

$u = 1$ or $u = \frac{2}{3}$

$\sin\theta = 1$ or $\sin\theta = \frac{2}{3}$

$\sin\theta = 1$ $\theta = 90°$

$\sin\theta = \frac{2}{3}$

acute $\angle = 41°49'$

II nd quadrant $\angle = 180 - 41°49'$

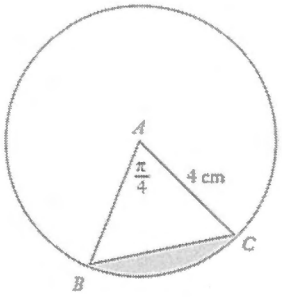
$= 138°11'$ Some forgot this solution.

$\theta = 90°, 41°49', 138°11'$

Question 12 (15 marks)

(a) A is the centre of a circle of radius 4 cm and B and C are endpoints of a chord.

Find the exact area of:



(a) Generally well done although a few students forgot formulae for sector & triangle area.

(i) the ΔABC. 1

$= \frac{1}{2} \times 4 \times 4 \times \frac{1}{\sqrt{2}}$

$= 4\sqrt{2}\ cm^2$

The rounding was penalised here, so if the answer wasn't $4\sqrt{2}$ or $\frac{8}{\sqrt{2}}$ students didn't get the mark in (i).

(ii) the minor sector ABC. 2

$= \frac{1}{2} r^2\theta$

$= \frac{1}{2} \times 4 \times 4 \times \frac{\pi}{4}$

$= 2\pi\ cm^2$

(iii) the shaded minor segment. 2

= area of sector − area of Δ

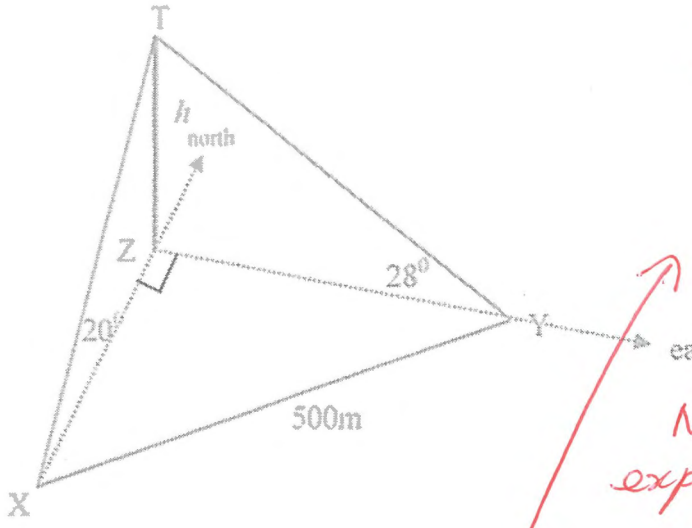
$= (2\pi - 4\sqrt{2})\ cm^2$

Just about everyone knew to subtract triangle area from sector area.

Students DID get carry over marks for this.

Some students used the segment area $A = \frac{1}{2}r^2(\theta - \sin\theta)$ formula → also correct.

(b) A conservationist observes the angle of elevation of the top of a tree, which is h metres tall, from two positions. From a point X, due south of the tree, it is $20°$ and from point Y, due east of the tree, it is $28°$. The distance XY is 500 m.



A small minority got to $h = XZ \tan 20°$ but then didn't go on to get XZ in terms of h.

Most students could get expressions for XZ & YZ $XZ = \frac{h}{\tan 20°}$; $h \cot 20° = h \tan 70°$ etc.

(i) Write expressions for XZ and YZ in terms of h. 2

$\tan 20 = \frac{h}{XZ}$ | $\tan 28 = \frac{h}{YZ}$

$XZ = \frac{h}{\tan 20}$ | $YZ = \frac{h}{\tan 28°}$

(ii) Calculate the value of h. 2

$XZ^2 + YZ^2 = 500^2$ by Pythagoras' theorem

$\frac{h^2}{\tan^2 20} + \frac{h^2}{\tan^2 28} = 250000$

$\frac{h^2 \tan^2 28 + h^2 \tan^2 20}{\tan^2 20 \tan^2 28} = 250000$

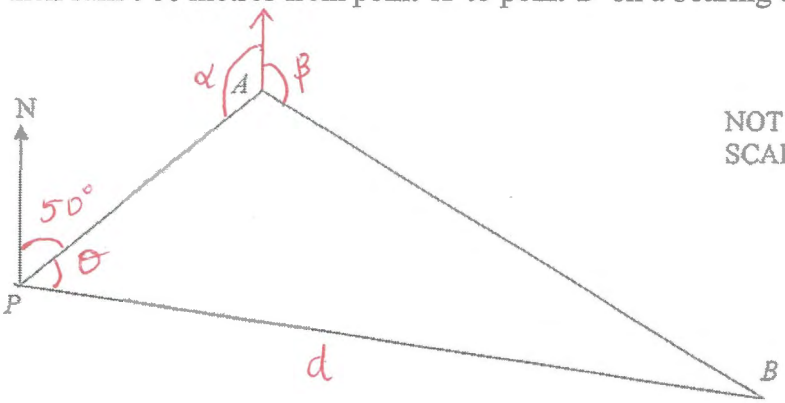
$\frac{h^2(\tan^2 28 + \tan^2 20)}{\tan^2 20 \tan^2 28} = 250000$

$h^2 = \frac{250000 \tan^2 20 \tan^2 28}{\tan^2 28 + \tan^2 20}$

* Some students used the cosine rule here (will still work).

$h = 150$ m

(c) In the diagram a yacht sails 640 metres from point P to point A on a bearing of 050°. It then sails 960 metres from point A to point B on a bearing of 120°.



Badly done.

Geometric reasons NOT given in a majority of cases!

(i) Show that $\angle PAB = 110°$. 2

$\alpha + 50 = 180$ (cointerior angles on parallel lines)

$\alpha = 130°$

$\beta = 120°$ (given) → Have to give REASONS!

$\angle PAB = 360 - 120 - 130$ (angle sum at a point)

$= 110$

(ii) Find the distance of point B from point P correct to the nearest metre. 2

$d^2 = 640^2 + 960^2 - 2 \times 640 \times 960 \cos 110°$

$d = 1323$ m

Generally well done although a small minority of students got the wrong answer through being in radians mode.

(iii) Find the bearing of point B from point P correct to the nearest degree. 2

$\frac{960}{\sin\theta} = \frac{1323}{\sin 110^\circ}$ (By sine rule)

$\sin\theta = \frac{960 \times \sin 110}{1323}$

$\theta = 43^\circ$ ← Most students got either $\angle A\theta = 43^\circ$ or $\angle B = 27^\circ$ (1 mark) but then many didn't go on to give the bearing.

Bearing of B from P

$= 50 + 43$

$= 93^\circ$ ← Was paid full marks.

$= 093^\circ T$ → But should have been this!

End of Examination