FINAL MARK

2024
GIRRAWEEN HIGH SCHOOL
YEAR 11
MATHEMATICS (ADVANCED)
YEARLY EXAMINATION
COVER SHEET

Student Number: ____________________

QUESTION	MARK	MA11-1	MA11-2	MA11-3	MA11-4	MA11-5	MA11-6	MA11-7	MA11-9
1-5	/5	✓	✓	✓			✓	✓	
6	/16	✓							✓
7	/15	✓				✓			✓
8	/16	✓	✓						✓
9	/17	✓		✓					✓
10	/16	✓				✓	✓		✓
11	/14	✓		✓	✓	✓			✓
12	/11	✓						✓	✓
TOTAL									
	/110	/110	/21	/36	/14	/45	/21	/16	/105

Year 11 Mathematics (Advanced) outcomes

A student:

MA11-1	uses algebraic and graphical techniques to solve, and where appropriate, compare alternative solutions to problems
MA11-2	uses the concepts of functions and relations to model, analyse and solve practical problems
MA11-3	uses the concepts and techniques of trigonometry in the solution of equations and problems involving geometric shapes
MA11-4	uses the concepts and techniques of periodic functions in the solutions of trigonometric equations or proof of trigonometric identities
MA11-5	interprets the meaning of the derivative, determines the derivative of functions and applies these to solve simple practical problems
MA11-6	manipulates and solves expressions using the logarithmic and index laws, and uses logarithms and exponential functions to solve practical problems
MA11-7	uses concepts and techniques from probability to present and interpret data and solve problems in a variety of contexts, including the use of probability distributions
MA11-8	uses appropriate technology to investigate, organise, model and interpret information in a range of contexts
MA11-9	provides reasoning to support conclusions which are appropriate to the context

Section I (5 marks)

- **Use the multiple-choice sheet for questions 1-5**

1. What is the solution to the equation $\log_e(x+2) - \log_e x = \log_e 4$?

(A) $\frac{2}{5}$

(B) $\frac{2}{3}$

(C) $\frac{3}{2}$

(D) $\frac{5}{2}$

2. What are the solutions to the equation $2\sin\beta = -\sqrt{3}$ for $0° \leq \beta \leq 360°$?

(A) $\beta = 60°$ or $300°$

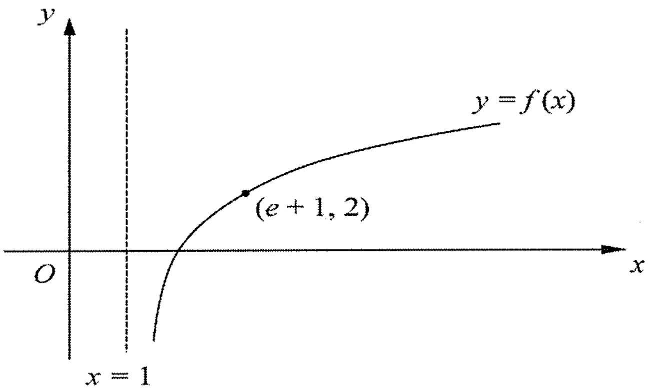
(B) $\beta = 120°$ or $230°$

(C) $\beta = 210°$ or $330°$

(D) $\beta = 240°$ or $300°$

Exam continues on the next page

3. The graph of a function $y = f(x)$ is shown below.



Which of the following best represents the equation of the function?

(A) $y = \ln(x - 1)$

(B) $y = 2\ln(x - 1)$

(C) $y = \ln\big(2(x - 1)\big)$

(D) $y = 2\ln\big(2(x - 1)\big)$

4. In a school, the student population is 45% male and 55% female. Two students are selected at random to represent their school. What is the probability that neither student is female?

(A) 0.2025

(B) 0.2475

(C) 0.3025

(D) 0.45

Exam continues on the next page

5. Which of the following is NOT an even function?

(A) $f(x) = (x-2)^2$

(B) $f(x) = |x|$

(C) $f(x) = \cos x$

(D) $f(x) = e^x + e^{-x}$

Select the alternative A, B, C or D that best answers the question.

1. A ○	B. ○	C. ○	D. ○
2. A ○	B. ○	C. ○	D. ○
3. A ○	B. ○	C. ○	D. ○
4. A ○	B. ○	C. ○	D. ○
5. A ○	B. ○	C. ○	D. ○

Exam continues on the next page

Section II (105 marks)

- **Answer questions in the space provided.**
- **Your responses should include relevant mathematical reasoning and /or calculations.**
- **Extra writing space is provided at the back of this booklet. If you use this space, indicate in the writing space under the question, the page number on which your solution continues, and clearly write the question number on the extra writing space.**

Question 6 (16 marks)

a) Factorise fully: **2**

$$u^2w + vw - u^2x - vx$$

b) Express the following as a single, simplified algebraic fraction: **3**

$$\frac{a+2}{a^2+5a} - \frac{5a}{a^2-25}$$

Exam continues on the next page

c) Solve by completing the square, giving your answer in simplest exact form. **2**

$x^2 + 10x + 7 = 0$

d) Given that x is positive, simplify $\sqrt{x^4 + 2x^3 + x^2}$ **2**

Exam continues on the next page

e) Suppose that $x = \sqrt{7} + \sqrt{6}$.

i. Show that $x + \frac{1}{x} = 2\sqrt{7}$ **2**

ii. Hence, find the value of $x^2 + \frac{1}{x^2}$ **2**

f) Solve the equation $|3 - 2x| = 1$ **3**

Exam continues on the next page

Question 7 (15 marks)

a) Use the definition: $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$, to show that the derivative of

$f(x) = 3x^2 + x$ is $6x + 1$. **3**

Exam continues on the next page

b) Find the derivative of:

i. $f(x) = (x^3 + 2)^5$. **2**

ii. $f(x) = x^7(x^4 + 5x)^6$. **2**

Exam continues on the next page

iii. $f(x) = \frac{x^2+1}{x+1}$. **2**

----

c) By first simplifying and expressing in the form e^{kx}, differentiate **2**

$$y = e^x\sqrt{e^x}$$

Exam continues on the next page

d) Find the x values of the points on the curve $y = \frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x + 1$ at which the tangent:

i. is parallel to the x axis **2**

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ii. What angle does the tangent of the curve at $x = 0$ makes with the positive direction of x axis. Answer to the nearest degree. **2**

Exam continues on the next page

Question 8 (16 marks)

a) Two functions are defined as follows:

$$h(x) = x^3 \quad \text{and}$$
$$j(x) = \frac{2x-1}{3}.$$

Find the value of $h\big(j(5)\big)$. **2**

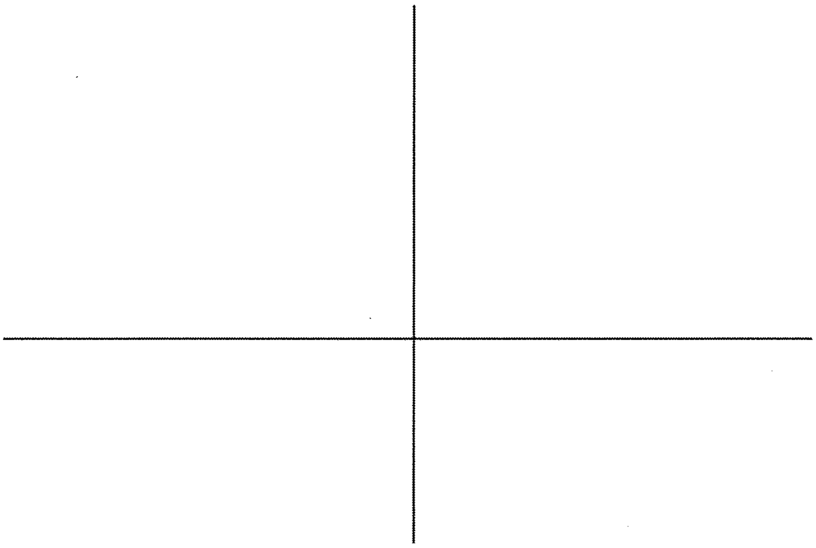
b) Find the domain of the function $f(x) = \frac{x-2}{x^2-5x+6}$ **2**

Exam continues on the next page

c) A function is defined by the rule:

$$f(x) = \begin{cases} 9 - x^2, & x \geq 0 \\ |x + 9|, & x < 0 \end{cases}$$

i. Draw a neat sketch of this function. **2**



ii. Find all possible values of b if $f(b) = 1$. **3**

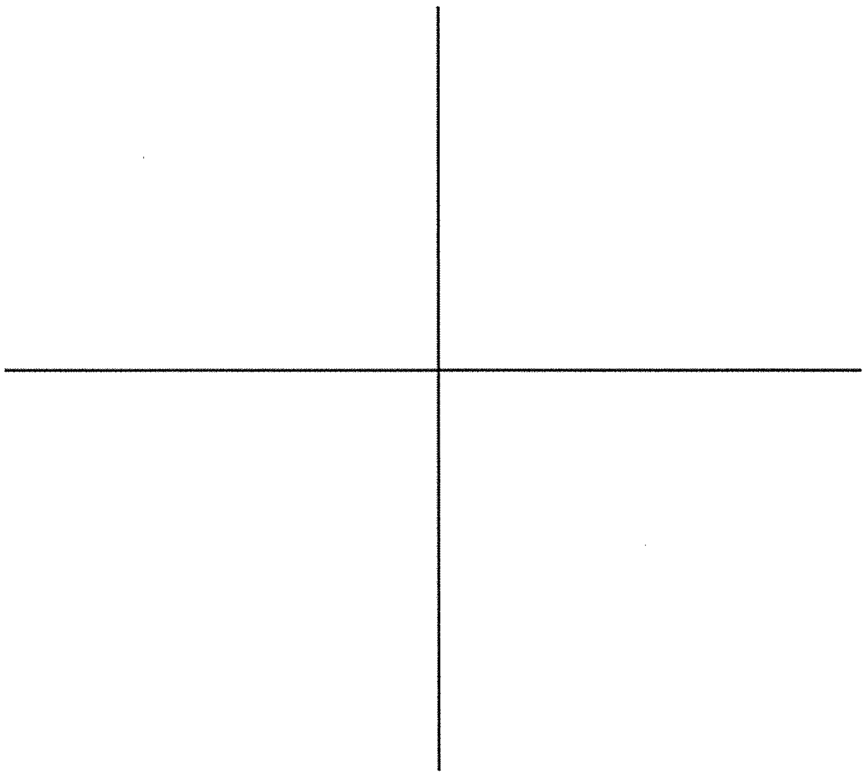
Exam continues on the next page

d) The line $ax + by + 1 = 0$ is perpendicular to $4x + 3y = 7$ and passes through $(9,7)$. Find the values of a and b. **4**

Exam continues on the next page

e) By finding the vertex, x intercepts and y-intercept, sketch the graph of the quadratic function:

$y = -3(x - 2)^2 + 3$ **3**



Exam continues on the next page

Question 9 (17 marks)

a) If $\operatorname{cosec}\alpha = -\frac{13}{5}$ and $\cos\alpha > 0$ find the value of $\cot\alpha$. **2**

b) Show that: $2\sec^2\theta + \frac{\sin^2\theta - 2\cos^2\theta}{\cos^2\theta} = 3\tan^2\theta$. **3**

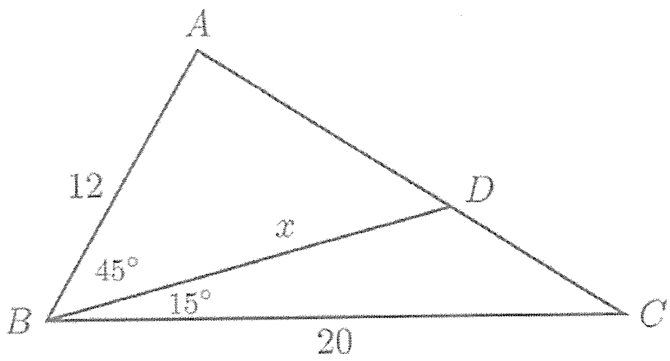
Exam continues on the next page

c) Solve the equation for $\theta\ if\ 0 \leq \theta \leq 360°$

$$cos^2\theta - sin^2\theta + 3\ cos\theta + 2 = 0$$ **4**

Exam continues on the next page

d) Consider the diagram below:

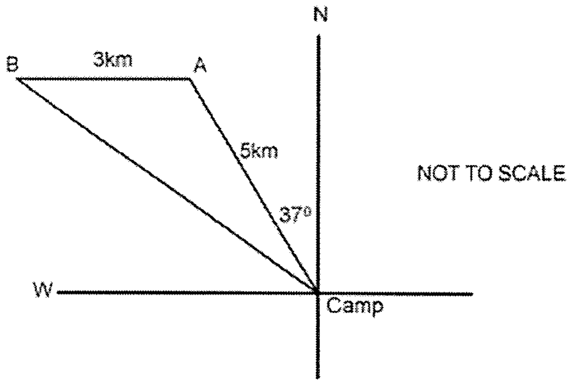


i. Find the exact area of ΔABC. **1**

ii. Hence, find the value of x correct to 1 decimal place. **3**

Exam continues on the next page

e) A hiker walks 5 km from camp to point A in a direction of $N37°W$. They then walk 3km due west to point B, as shown in the diagram below.



i. If the hiker is at point B,what is the shortest distance between the hiker and the camp? (answer to the nearest metre) **2**

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ii. What is the bearing of the hiker at point B from the camp? (answer to the nearest degree) **2**

Exam continues on the next page

Question 10 (16 marks)

a) Let $x = \log_{10}(a)$, $y = \log_{10}(b)$ and $z = \log_{10}(c)$. Express the following in terms of x, y and z.

i. $\log_{10}\left(\frac{a}{b^2}\right)$ **2**

ii. $\log_{10}\left(\sqrt{b^2c}\right)$ **2**

Exam continues on the next page

b) Given that a and b are positive constants, solve the simultaneous equations: **3**

$$a = 3b$$
$$\log_3 a + \log_3 b = 2$$

Express a and b in exact form.

Exam continues on the next page

c) A particle is moving in a straight line. Its displacement, x metres, from the origin, O, at time t seconds, where $t \geq 0$, is given by $x = 1 - \frac{7}{t+4}$.

i. Find the initial displacement of the particle. **1**

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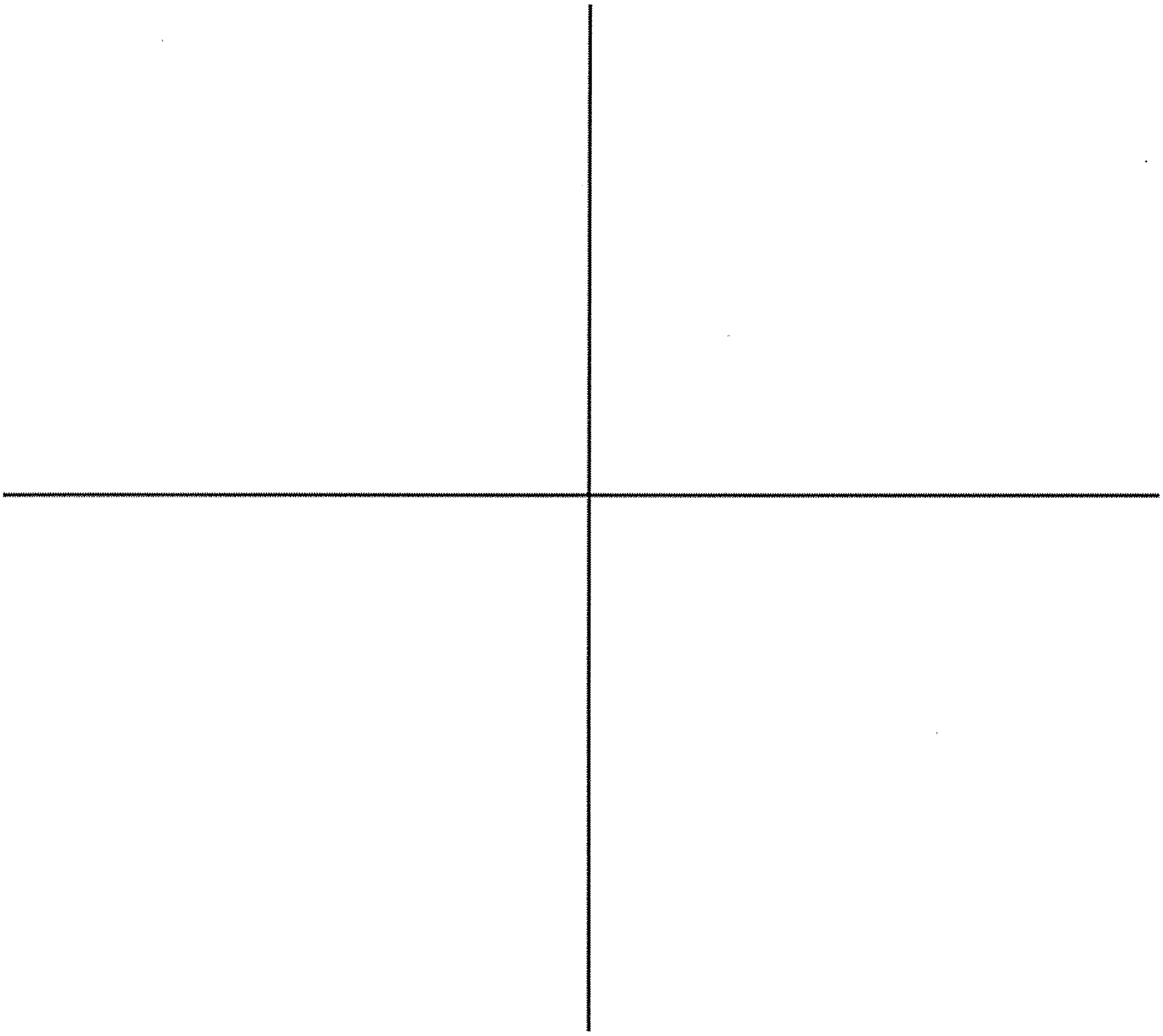
ii. Velocity is rate of change of displacement with time. Find the velocity of the particle as it passes through the origin. **3**

---

iii. Acceleration is rate of change of velocity with time. Show that acceleration of the particle is always negative. **2**

Exam continues on the next page

iv. Sketch the graph of the displacement of the particle as a function of time. **3**



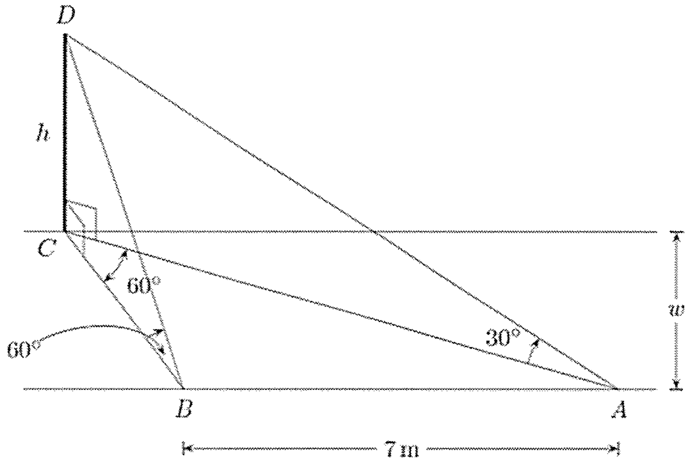
Exam continues on the next page

Question 11 (14 marks)

a) Find the values of p for which $y = px - 3p - 5$ is tangent to $x^2 = 8y$. **3**

Exam continues on the next page

b) A footpath on horizontal ground has two parallel edges. CD is a vertical street sign pole of height h metres which stands with its base C on one edge of the footpath. A and B are two points on the other edge of the footpath such that $AB = 7$ metres and $\angle ACB = 60°$. From A and B, the angles of elevation to the top of the pole at D are $30°$ and $60°$ respectively.



i. Show that the exact height of the flagpole is $h = \sqrt{21}$ metres. **3**

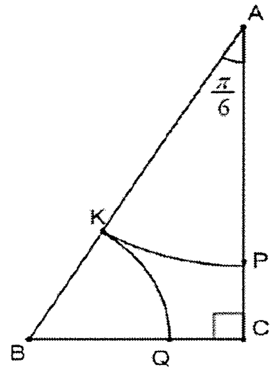
Exam continues on the next page

ii. By considering the area of ΔABC, find the exact width w of the footpath. **3**

Exam continues on the next page

c) In the diagram, ΔABC is a right angled at $C, \angle BAC = \frac{\pi}{6}$ and $AB = 5$ cm. **5**

KP is a circular arc with centre A and KQ is a circular arc with centre B.



NOT TO SCALE

If AK has length x cm, find the value of x, correct to one decimal place, for which the areas of sectors AKP and BKQ are equal.

Exam continues on the next page

Question 12 (11 marks)

a) Sixty tickets are sold in a raffle with two prizes. Anthony buys 5 tickets. Find the probability that Anthony:

i. wins both the prizes **1**

ii. wins one prize only **1**

iii. wins at least one prize **1**

Exam continues on the next page

b) In a lottery, the probability of the jackpot being won in any draw is $\frac{1}{100}$.

i. What is the probability that the jackpot prize will be won in each of three consecutive draws? **1**

ii. How many consecutive draws need to be made for there to be a greater than 99% chance that at least one jackpot prize will have been won? **3**

Exam continues on the next page

c) For events A and B from a sample space, $P(A|B) = \frac{3}{4}$ and $P(B) = \frac{1}{3}$.

i. Calculate $P(A \cap B)$. **1**

ii. Calculate $P(A' \cap B)$, where A' denotes the complement of A. **1**

iii. If events A and B are independent, calculate $P(A \cup B)$. **2**

END OF EXAM

Spare working out paper

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8	/16	✓	✓						✓
9	/17	✓		✓					✓
10	/16	✓				✓	✓		✓
11	/14	✓		✓	✓	✓			✓
12	/11	✓						✓	✓
TOTAL									
	/110	/110	/21	/36	/14	/45	/21	/16	/105

Year 11 Mathematics (Advanced) outcomes

A student:

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MA11-9	provides reasoning to support conclusions which are appropriate to the context

Section I (5 marks)

- **Use the multiple-choice sheet for questions 1-5**

1. What is the solution to the equation $\log_e(x+2) - \log_e x = \log_e 4$?

$\log_e \frac{x+2}{x} = \log_e 4$

$\frac{x+2}{x} = 4$

$x+2 = 4x$

$3x = 2$

$x = \frac{2}{3}$

(A) $\frac{2}{5}$

(B) $\frac{2}{3}$

(C) $\frac{3}{2}$

(D) $\frac{5}{2}$

2. What are the solutions to the equation $2\sin\beta = -\sqrt{3}$ for $0° \leq \beta \leq 360°$?

$\sin\beta = -\frac{\sqrt{3}}{2}$

$\beta = 180+60, 360-60$

$= 240°, 300°$

S | A

T | C

(A) $\beta = 60°$ or $300°$

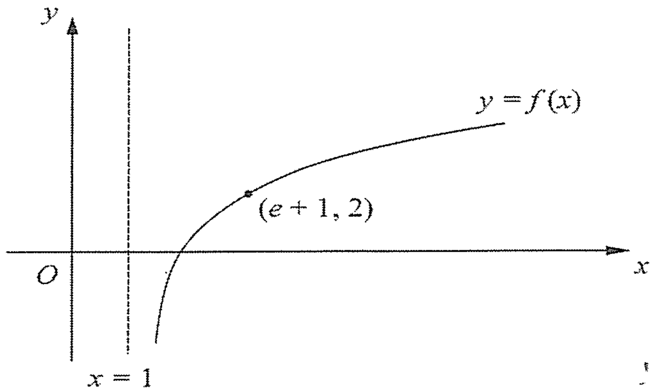
(B) $\beta = 120°$ or $230°$

(C) $\beta = 210°$ or $330°$

(D) $\beta = 240°$ or $300°$

Exam continues on the next page

3. The graph of a function $y = f(x)$ is shown below.



$y = a\ln(x-1)$
sub $(e+1, 2)$
$2 = a\ln_e(e+1-1)$
$2 = a\ln e$
$a = 2$
$\therefore y = 2\ln(x-1)$

Which of the following best represents the equation of the function?

(A) $y = \ln(x-1)$

(B) $y = 2\ln(x-1)$

(C) $y = \ln\big(2(x-1)\big)$

(D) $y = 2\ln\big(2(x-1)\big)$

4. In a school, the student population is 45% male and 55% female. Two students are selected at random to represent their school. What is the probability that neither student is female?

(A) 0.2025

$0.45 \times 0.45 = 0.2025$

(B) 0.2475

(C) 0.3025

(D) 0.45

Exam continues on the next page

5. Which of the following is NOT an even function?

(A) $f(x) = (x-2)^2$

(B) $f(x) = |x|$

(C) $f(x) = \cos x$

(D) $f(x) = e^x + e^{-x}$

$$f(-x) = (-x-2)^2 = [-(x+2)]^2 = (x+2)^2 \neq f(x)$$

even
$$\begin{cases} f(-x) = |-x| = |x| = f(x) \\ f(-x) = \cos(-x) = \cos x = f(x) \\ f(-x) = e^{-x} + e^{x} = f(x) \end{cases}$$

Select the alternative A, B, C or D that best answers the question.

1.	A ○	B. ●	C. ○	D. ○
2.	A ○	B. ○	C. ○	D. ●
3.	A ○	B. ●	C. ○	D. ○
4.	A ●	B. ○	C. ○	D. ○
5.	A ●	B. ○	C. ○	D. ○

Exam continues on the next page

Section II (105 marks)

- Answer questions in the space provided.
- Your responses should include relevant mathematical reasoning and /or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, indicate in the writing space under the question, the page number on which your solution continues, and clearly write the question number on the extra writing space.

Question 6 (16 marks)

a) Factorise fully: **2**

Generally very well done.

$$u^2w + vw - u^2x - vx$$

$= w(u^2+v) - x(u^2+v)$ 1

$= (u^2+v)(w-x)$ 1

b) Express the following as a single, simplified algebraic fraction: **3**

$$\frac{a+2}{a^2+5a} - \frac{5a}{a^2-25}$$

$= \dfrac{a+2}{a(a+5)} - \dfrac{5a}{(a+5)(a-5)}$ 1

$= \dfrac{(a-5)(a+2) - a \times 5a}{a(a+5)(a-5)}$ 1

$= \dfrac{a^2+2a-5a-10-5a^2}{a(a+5)(a-5)}$

$= \dfrac{-4a^2-3a-10}{a(a+5)(a-5)}$ 1

Generally well done. However, some students used a denominator of $a(a+5)^2(a-5)$ if they "cross multiplied". This led to an answer of $\dfrac{-4a^3-23a^2-25a-50}{a(a+5)^2(a-5)}$ which could be simplified but was hard to see. Lesson is: Do NOT get lowest common denominator by cross multiplying.

Exam continues on the next page

c) Solve by completing the square, giving your answer in simplest exact form. **2**

$x^2 + 10x + 7 = 0$

$x^2 + 10x + 25 = -7 + 25$

$(x+5)^2 = 18$ 1

$x + 5 = \pm\sqrt{18}$

$x = -5 \pm 3\sqrt{2}$ 1

Generally well done.
Students who could complete the square could generally finish the question.

d) Given that x is positive, simplify $\sqrt{x^4 + 2x^3 + x^2}$ **2**

$= \sqrt{x^2(x^2+2x+1)}$

$= \sqrt{x^2(x+1)^2}$ 1

$= x(x+1)$ 1

Generally well done. However, some students made the INCREDIBLY BAD MISTAKE of thinking $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$ WHICH OF COURSE IT DOES NOT, and wrote $x^2 + 2x^{\frac{3}{2}} + x$ which is a VERY bad mistake.

Exam continues on the next page

e) Suppose that $x = \sqrt{7} + \sqrt{6}$.

Students knew what to do BUT LEFT STEPS OUT. In SHOW questions, you need to SHOW EVERYTHING. Students often missed the step where they showed that they multiplied by the conjugate.

i. Show that $x + \frac{1}{x} = 2\sqrt{7}$ **2**

$$\frac{1}{x} = \frac{1}{\sqrt{7}+\sqrt{6}} \times \frac{\sqrt{7}-\sqrt{6}}{\sqrt{7}-\sqrt{6}}$$

$$= \frac{\sqrt{7}-\sqrt{6}}{7-6} = \sqrt{7}-\sqrt{6} \quad (1)$$

$$\cancel{x + \frac{1}{x} = \sqrt{7}+\sqrt{6}+\sqrt{7}-\sqrt{6}}$$

$$= 2\sqrt{7} \quad (1)$$

ii. Hence, find the value of $x^2 + \frac{1}{x^2}$ **2**

Some students didn't use HENCE or had no idea. However, most students could complete this.

$$\left(x+\frac{1}{x}\right)^2 = x^2 + 2 \times x \times \frac{1}{x} + \left(\frac{1}{x}\right)^2$$

$$= x^2 + 2 + \frac{1}{x^2}$$

$$x^2 + \frac{1}{x^2} = \left(x+\frac{1}{x}\right)^2 - 2 \quad (1)$$

$$= (2\sqrt{7})^2 - 2$$

$$= 28 - 2$$

$$= 26 \quad (1)$$

f) Solve the equation $|3-2x| = 1$ **3**

Generally very well done.

$3 - 2x = 1$ $\qquad$ $3 - 2x = -1$

$-2x = -2$ $\qquad$ $-2x = -4$

$x = 1$ $\qquad$ $x = 2$

Exam continues on the next page

Question 7 (15 marks)

a) Use the definition: $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$, to show that the derivative of

$f(x) = 3x^2 + x$ is $6x + 1$. **3**

$$f(x+h) - f(x) = 3(x+h)^2 + (x+h) - (3x^2+x)$$

$$= 3(x^2+2hx+h^2) + x + h - 3x^2 - x$$

Some struggled with expansion

$$= \cancel{3x^2} + 6hx + 3h^2 + \cancel{x} + h - \cancel{3x^2} - \cancel{x}$$

$$= h(6x + 3h + 1) \quad ①$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

Some had a different expression & still simplified to 6x+1

$$f'(x) = \lim_{h\to 0} \frac{\cancel{h}(6x+3h+1)}{\cancel{h}} \quad ①$$

$$f'(x) = 6x + 0 + 1$$

$$f'(x) = 6x + 1 \quad ①$$

Exam continues on the next page

b) Find the derivative of:

i. $f(x) = (x^3 + 2)^5$. **2**

$f'(x) = 5(x^3+2)^4 \times 3x^2$

$= 15x^2(x^3+2)^4$

Well done

ii. $f(x) = x^7(x^4 + 5x)^6$. **2**

$u = x^7$

$u' = 7x^6$

$v = (x^4+5x)^6$

$v' = 6(x^4+5x)^5 \times (4x^3+5)$

$f'(x) = uv' + vu'$

generally well done

$= x^7\left[6(x^4+5x)^5(4x^3+5)\right] + (x^4+5x)^6 \times 7x^6$ 1

Some struggled with factorisations & simplifying

$= x^6(x^4+5x)^5\left[6x(4x^3+5) + 7(x^4+5x)\right]$

$= x^6(x^4+5x)^5(24x^4+30x+7x^4+35x)$

$= x^6(x^4+5x)^5(31x^4+65x)$

$= x^7(x^4+5x)^5(31x^3+65)$ 1

Exam continues on the next page

iii. $f(x) = \frac{x^2+1}{x+1}$. **2**

$u = x^2+1$ $v = x+1$

$u' = 2x$ $v' = 1$

sts did not multiply the +1 with −1

$$f'(x) = \frac{vu' - uv'}{v^2}$$

$$= \frac{(x+1)\times 2x - (x^2+1)\times 1}{(x+1)^2} \quad 1$$

$$= \frac{2x^2+2x-x^2-1}{(x+1)^2}$$

$$= \frac{x^2+2x-1}{(x+1)^2} \quad 1$$

c) By first simplifying and expressing in the form e^{kx}, differentiate **2**

$y = e^x\sqrt{e^x}$

$y = e^x \times e^{\frac{x}{2}}$

some did not simplify to this & did product rule

$y = e^{\frac{3x}{2}} \quad 1$

$\frac{dy}{dx} = e^{\frac{3x}{2}} \times \frac{3}{2}$

$= \frac{3}{2} e^{\frac{3x}{2}} \quad 1$

Exam continues on the next page

d) Find the x values of the points on the curve $y = \frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x + 1$ at which the tangent:

i. is parallel to the x axis **2**

$y = \frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x + 1$

$\frac{dy}{dx} = x^2 - 3x + 2$

$\frac{dy}{dx} = 0$ where tangent is parallel to x-axis

$x^2 - 3x + 2 = 0$

$(x-1)(x-2) = 0$ Well done

$x = 1$ or $x = 2$

ii. What angle does the tangent of the curve at $x = 0$ makes with the positive direction of x axis. Answer to the nearest degree. **2**

$\left.\frac{dy}{dx}\right|_{x=0} = 0 - 0 + 2 = 2$ 1

$\tan\theta = 2$ Well done

$\theta = 63.43^0$

$\theta \simeq 63^0$ 1

Exam continues on the next page

Question 8 (16 marks)

a) Two functions are defined as follows:

$$h(x) = x^3 \quad \text{and}$$
$$j(x) = \frac{2x-1}{3}.$$

Find the value of $h(j(5))$. **2**

$j(x) = \frac{2x-1}{3}$

$j(5) = \frac{2\times 5-1}{3}$

$= 3$ 1

$h(x) = x^3$

$h(j(5)) = h(3)$ Mostly done well.

$= 3^3$

$= 27$ 1

b) Find the domain of the function $f(x) = \frac{x-2}{x^2-5x+6}$ **2**

$f(x) = \frac{x-2}{(x-2)(x-3)}$

Some wrote $x > 3$, or $x < 2$ which is wrong.

Domain: All real x but $x \neq 2$, $x \neq 3$ 2

Lot of students got this question wrong.

* Some factorised the denominator wrong.

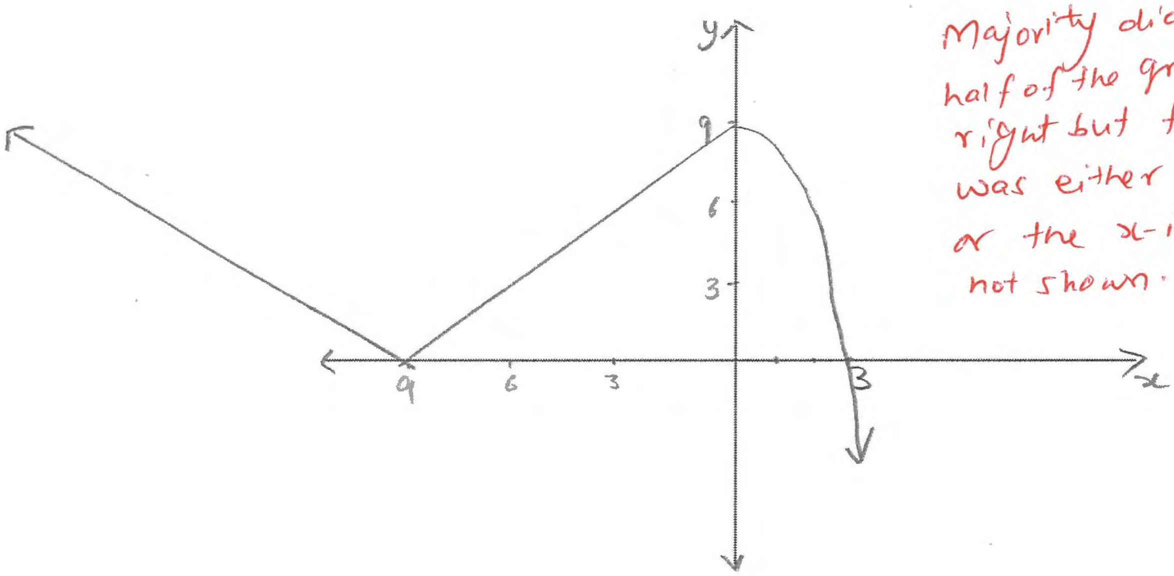
* Majority canceled $\frac{\cancel{(x-2)}}{\cancel{(x-2)}(x-3)}$, so only gave $x \neq 3$ as answer.

Exam continues on the next page

c) A function is defined by the rule:

$$f(x) = \begin{cases} 9 - x^2, & x \geq 0 \\ |x + 9|, & x < 0 \end{cases}$$

i. Draw a neat sketch of this function. **2**



Majority did half of the graph right but $f(x) = |x+9|$ $x<0$ was either done wrong or the x-intercept was not shown.

ii. Find all possible values of b if $f(b) = 1$. **3**

when $x \geq 0$, $9 - b^2 = 1$

$b^2 = 8$

$b = \pm 2\sqrt{2}$

Some left answer as $b = \pm 2\sqrt{2}$

$b = 2\sqrt{2}$ as $x \geq 0$ 1

when $x < 0$

Some students did not find these two values.

$|b+9| = 1$

$b+9 = 1$ or $b+9 = -1$

$b = -8$ or $b = -10$

1 1

Exam continues on the next page

d) The line $ax + by + 1 = 0$ is perpendicular to $4x + 3y = 7$ and passes through (9,7). Find the values of a and b. **4**

$4x+3y=7$ —— ①

$3y=-4x+7$

$y=-\frac{4}{3}x+\frac{7}{3}$

$m_1=-\frac{4}{3}$

$m_2=\frac{3}{4}$

$ax+by+1=0$ —— ②

$by=-ax-1$

$y=-\frac{a}{b}x-1$

As ① $\perp$ ②

$-\frac{a}{b}=\frac{3}{4}$

$4a=-3b$

$4a+3b=0$ —— ③

(9,7) lies on $ax+by+1=0$

$9a+7b=-1$ —— ④

③ ×9 $36a+27b=0$ —— ⑤

④ × 4 $36a+28b=-4$ —— ⑥

⑥ − ⑤ $b=-4$

from ③ $4a+3\times-4=0$

$4a=12$

$a=3$

2 marks for making up the equations + 2 marks for solving

Method 2: Eqn. of the line with $m=3/4$, passing through (9,7)

$y-7=\frac{3}{4}(x-9)$

$4y-28=3x-27$

$3x-4y+1=0$

$ax+by+1=0$

$\therefore a=3, b=-4$

Some students did $-3x+4y-1=0$ and said $a=-3, b=4$ which is wrong

Exam continues on the next page

e) By finding the vertex, x intercepts and y-intercept, sketch the graph of the quadratic function:

$y = -3(x-2)^2 + 3$ **3**

y int $= -3(0-2)^2 + 3$

$= -3 \times 4 + 3 = -9$ 1m

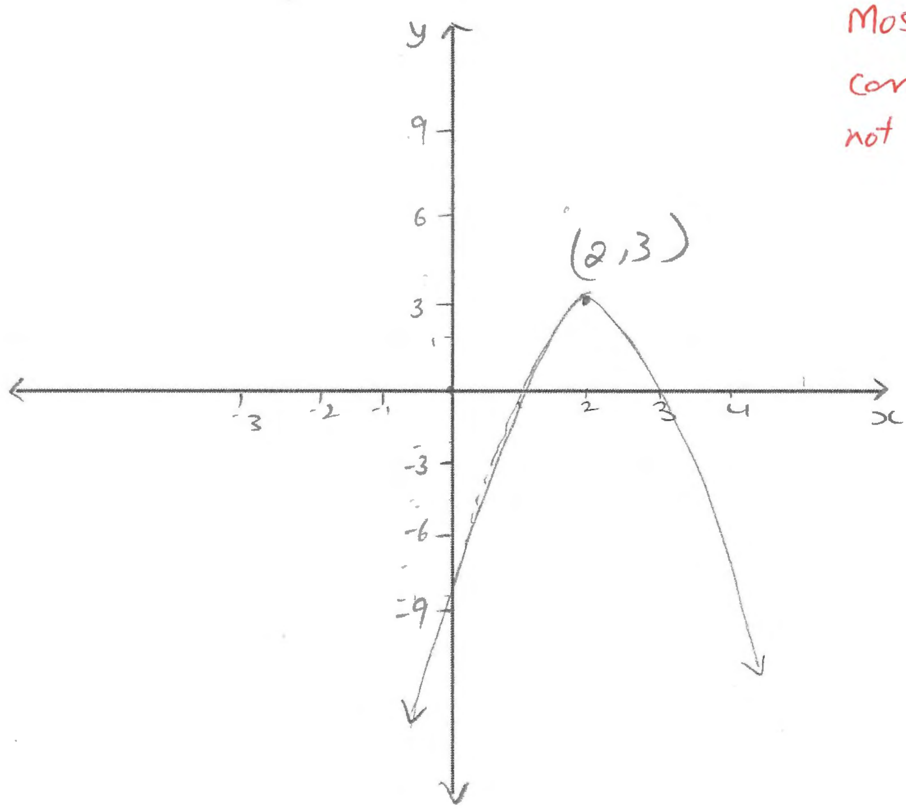
Vertex $= (2,3)$ 1m

x ints: $-3(x-2)^2 + 3 = 0$

$(x-2)^2 = 1$

$x - 2 = \pm 1$

$x = 3, 1$ 1m



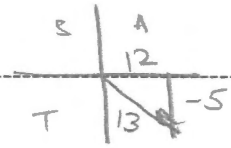
Mostly done correctly but not neatly.

Exam continues on the next page

Question 9 (17 marks)

a) If $\operatorname{cosec} \alpha = -\frac{13}{5}$ and $\cos \alpha > 0$ find the value of $\cot \alpha$. **2**

1m $\sin \alpha = -\frac{5}{13}$



1m $\cot \alpha = -\frac{12}{5}$

Mostly done well

b) Show that: $2\sec^2\theta + \frac{\sin^2\theta - 2\cos^2\theta}{\cos^2\theta} = 3\tan^2\theta.$ **3**

LHS: $\frac{2}{\cos^2\theta} + \frac{\sin^2\theta - 2\cos^2\theta}{\cos^2\theta}$

$= \frac{2 + \sin^2\theta - 2\cos^2\theta}{\cos^2\theta}$ (1)

$= \frac{2(1 - \cos^2\theta) + \sin^2\theta}{\cos^2\theta}$

$= \frac{2\sin^2\theta + \sin^2\theta}{\cos^2\theta}$ (1)

$= \frac{3\sin^2\theta}{\cos^2\theta}$

Done well.

$= 3\tan^2\theta = \text{RHS}$ (1)

Exam continues on the next page

c) Solve the equation for θ if $0 \leq \theta \leq 360°$

$$cos^2\theta - sin^2\theta + 3\,cos\theta + 2 = 0$$ **4**

$\cos^2\theta - (1 - \cos^2\theta) + 3\cos\theta + 2 = 0$

$\cos^2\theta - 1 + \cos^2\theta + 3\cos\theta + 2 = 0$

$2\cos^2\theta + 3\cos\theta + 1 = 0$

Let $\cos\theta = x$

$2x^2 + 3x + 1 = 0$ (1)

$2x^2 + 2x + x + 1 = 0$

$2x(x+1) + (x+1) = 0$

$(x+1)(2x+1) = 0$

$x = -1$ or $x = -\frac{1}{2}$ (1)

$\cos\theta = -1$ $\quad$ $\cos\theta = -\frac{1}{2}$

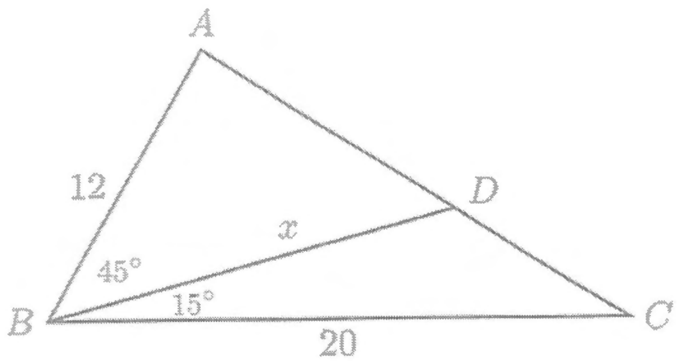
$\theta = 180^0$ $\quad$ $\theta = 120^0, 240^0$

(1) $\quad$ (1)

Mostly done well.

Exam continues on the next page

d) Consider the diagram below:



i. Find the exact area of ΔABC. **1**

$A = \frac{1}{2} \times 12 \times 20 \times \sin 60$

$= 120 \times \frac{\sqrt{3}}{2}$

$= 60\sqrt{3}\ u^2$

Done well

ii. Hence, find the value of x correct to 1 decimal place. **3**

Area of ΔABD + area of ΔDBC = Area of ΔABC

$\frac{1}{2} \times 12 \times x \times \sin 45 + \frac{1}{2} \times x \times 20 \times \sin 15^{\circ} = 60\sqrt{3}$ 1

$x\left[6 \sin 45 + 10 \sin 15\right] = 60\sqrt{3}$

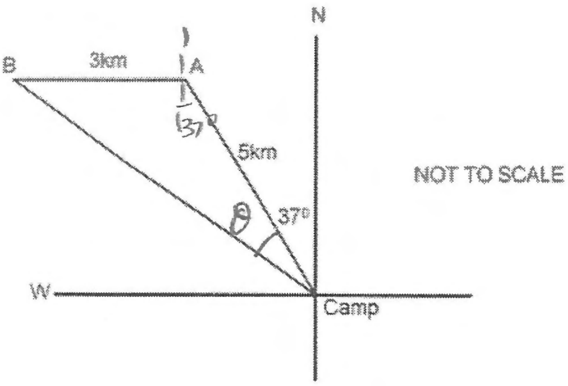
$x = \dfrac{60\sqrt{3}}{6 \sin 45 + 10 \sin 15}$ 1

$x = 15.2$ 1

Some students had trouble with this question.

Exam continues on the next page

e) A hiker walks 5 km from camp to point A in a direction of $N37°W$. They then walk 3km due west to point B, as shown in the diagram below.



i. If the hiker is at point B,what is the shortest distance between the hiker and the camp? (answer to the nearest metre) **2**

$\angle BAC = 90 + 37 = 127^0$

$a^2 = 3^2 + 5^2 - 2 \times 3 \times 5 \times \cos 127^0$ (1)

$a^2 = 52.05445$

$a = 7.214877...$

$a = 7.215$ km OR 7215 m (1)

'1' mark deducted if the student didn't give the answer to the nearest metre.

ii. What is the bearing of the hiker at point B from the camp?
(answer to the nearest degree) **2**

$\frac{\sin\theta}{3} = \frac{\sin 127}{7.215}$

Generally well done.

$\sin\theta = \frac{3\sin 127}{7.215}$

$\theta = 19^0 24' \simeq 19^0$ (1)

Bearing of B from the camp $= 360 - 37 - 19$
$= 304^0$ or $N 56^0 W$ (1)

Exam continues on the next page

Question 10 (16 marks)

a) Let $x = \log_{10}(a)$, $y = \log_{10}(b)$ and $z = \log_{10}(c)$. Express the following in terms of x, y and z.

i. $\log_{10}\left(\frac{a}{b^2}\right)$ **2**

$= \log_{10} a - \log_{10} b^2$ ①

$= \log_{10} a - 2\log_{10} b$

$= x - 2y$ ①

well done

ii. $\log_{10}\left(\sqrt{b^2 c}\right)$ **2**

$= \log_{10} (b^2 c)^{\frac{1}{2}}$

$= \frac{1}{2} \log_{10} (b^2 c)$

$= \frac{1}{2} \{ \log_{10} b^2 + \log_{10} c \}$ ①

$= \frac{1}{2} [2\log_{10} b + \log_{10} c]$

$= \log_{10} b + \frac{1}{2} \log_{10} c$

$= y + \frac{z}{2}$ ①

well done

Exam continues on the next page

b) Given that a and b are positive constants, solve the simultaneous equations: **3**

$$a = 3b$$
$$\log_3 a + \log_3 b = 2$$

Express a and b in exact form.

$a = 3b$ — 1

$\log_3 a + \log_3 b = 2$

$\log_3 ab = 2$ — 2 (using log laws)

Sub 1 into 2

$\log_3 3b \times b = 2$

$\log_3 3b^2 = 2$

$\log_3 3 + \log_3 b^2 = 2$

$1 + 2\log_3 b = 2$

$2\log_3 b = 1$

$\log_3 b = \frac{1}{2}$ 1m

$b = 3^{1/2} = \sqrt{3}$

$a = 3\sqrt{3}$ 1m

common errors: $\log_{(3)} 4b = 2$

$\log_3 3b^2 \neq 2\log_3 3b$

Reasonably well done.

Exam continues on the next page

c) A particle is moving in a straight line. Its displacement, x metres, from the origin, O, at time t seconds, where $t \geq 0$, is given by $x = 1 - \frac{7}{t+4}$.

i. Find the initial displacement of the particle. **1**

Sub $t = 0$

$x = 1 - \frac{7}{4}$

$= -\frac{3}{4}$ m

well done

ii. Velocity is rate of change of displacement with time. Find the velocity of the particle as it passes through the origin. **3**

$x = 1 - 7(t+4)^{-1}$

very poorly done

$v = \frac{dx}{dt} = 0 + 7(t+4)^{-2}$

$= \frac{7}{(t+4)^2}$ ①

when $x = 0$, $0 = 1 - \frac{7}{t+4}$

many did not find $t = 3$.

$t + 4 = 7$, $t = 3$ ①

$\therefore \left.\frac{dx}{dt}\right|_{\substack{x=0 \\ t=3}} = \frac{7}{(3+4)^2} = \frac{1}{7}$ m/s ①

iii. Acceleration is rate of change of velocity with time. Show that acceleration of the particle is always negative. **2**

many used quotient rule but this was not necessary.

$a = \frac{dv}{dt} = \frac{d}{dt}\left(7(t+4)^{-2}\right)$

1m

$= -14(t+4)^{-3}$

must have reasoning for 2nd mark

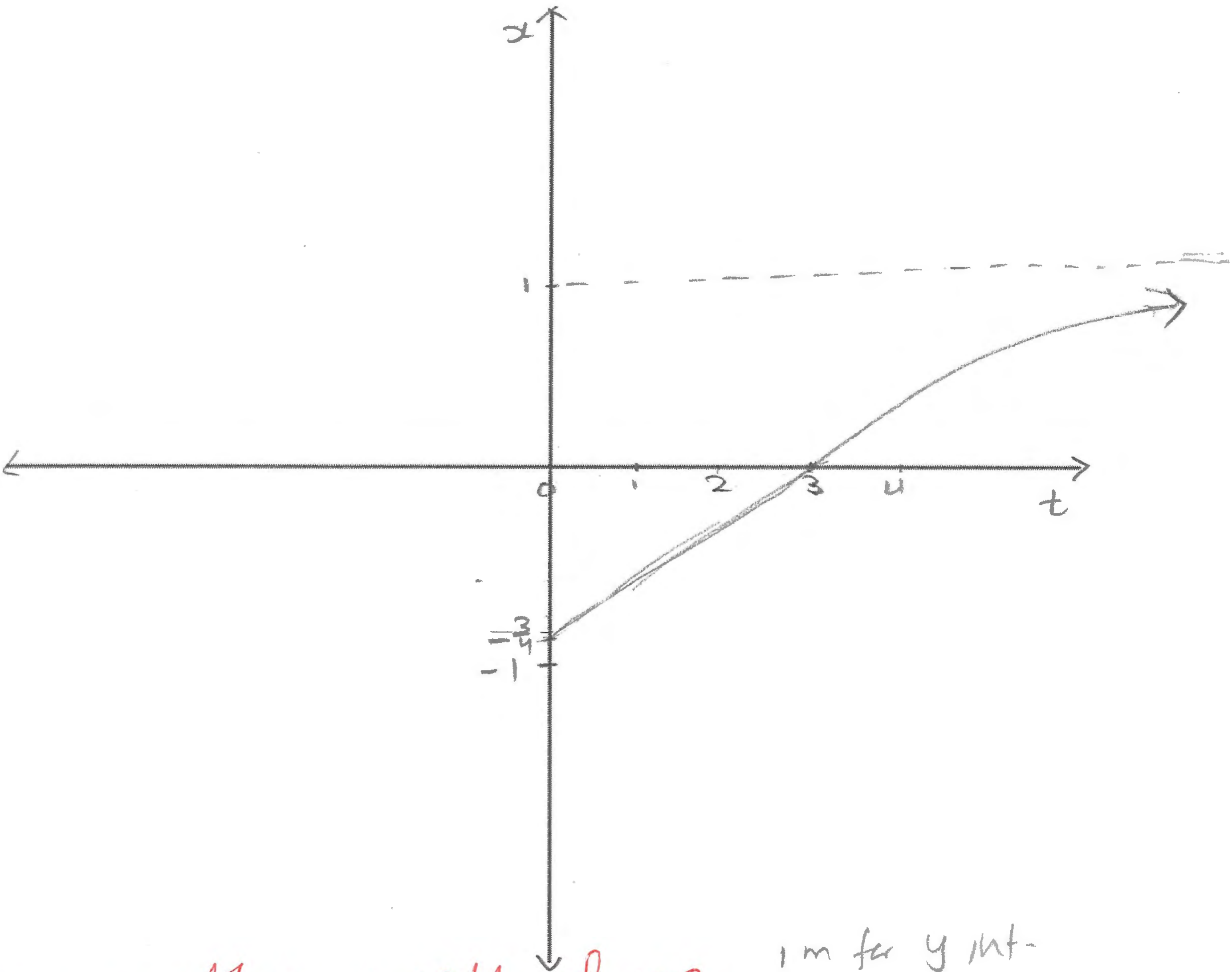
$= \frac{-14}{(t+4)^3}$ as $t > 0$, $(t+4)^3 > 0$ always

$\therefore \frac{-14}{(t+4)^3}$ is always negative

1m

Exam continues on the next page

iv. Sketch the graph of the displacement of the particle as a function of time. **3**



Reasonably well done.
common errors:
- missing asymptote
- intercepts

1 m for y int.
1 m for x int.
1 m for asymptote

Exam continues on the next page

Question 11 (14 marks)

a) Find the values of p for which $y = px - 3p - 5$ is tangent to $x^2 = 8y$. **3**

$x^2 = 8y$ — ①

$y = px - 3p - 5$ — ②

Solve ① + ② simultaneously

$x^2 = 8(px - 3p - 5)$

$x^2 = 8px - 24p - 40$

$x^2 - 8px + 24p + 40 = 0$ 1m

$\Delta = b^2 - 4ac = 0$ for the line to be tangent

$= (-8p)^2 - 4 \times 1 \times (24p + 40) = 0$

$64p^2 - 96p - 160 = 0$

$2p^2 - 3p - 5 = 0$ 1m

$p = \frac{3 \pm \sqrt{9 + 40}}{4}$

$p = \frac{3 \pm 7}{4}$

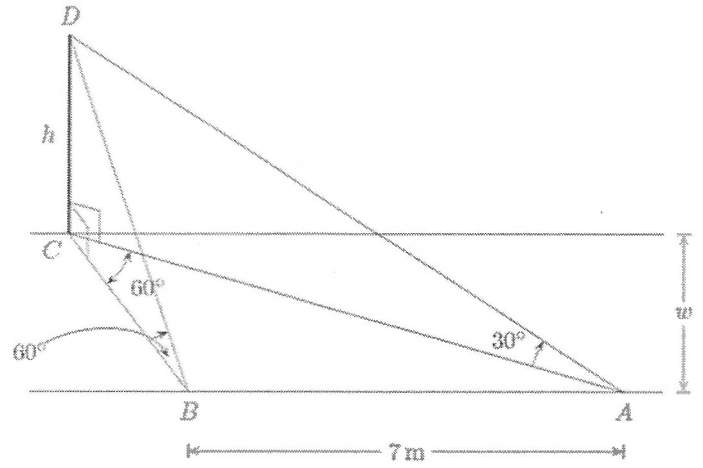
$p = -1$ or $\frac{5}{2}$ 1m

Students had trouble with application of the Quadratic formula and some students did know how to approach a solution.

Students to revise and understand the properties of derivative functions, their purpose and equations of tangents and normals.

Exam continues on the next page

b) A footpath on horizontal ground has two parallel edges. CD is a vertical street sign pole of height h metres which stands with its base C on one edge of the footpath. A and B are two points on the other edge of the footpath such that $AB = 7$ metres and $\angle ACB = 60°$. From A and B, the angles of elevation to the top of the pole at D are $30°$ and $60°$ respectively.



i. Show that the exact height of the flagpole is $h = \sqrt{21}$ metres. **3**

In $\triangle DCB$ $\tan 60^0 = \frac{h}{CB}$

$CB = \frac{h}{\tan 60} = \frac{h}{\sqrt{3}}$

In $\triangle DCA$ $\tan 30^0 = \frac{h}{CA}$

$CA = \frac{h}{\tan 30} = \frac{h}{1/\sqrt{3}}$

$= \sqrt{3}h$ (1)

In $\triangle CBA$, using cosine rule

$\left(\frac{h}{\sqrt{3}}\right)^2 + (\sqrt{3}h)^2 - 2\left(\frac{h}{\sqrt{3}}\right)(\sqrt{3}h) \times \frac{1}{2} = 7^2$ (1)

$\frac{h^2}{3} + 3h^2 - h^2 = 49$

$\frac{h^2}{3} + 2h^2 = 49$

$\frac{7h^2}{3} = 49$

$h^2 = 21$

$h = \sqrt{21}$ $h > 0$ (1)

Most students have shown good understanding of Trig ratios and cosine rule.

Exam continues on the next page

ii. By considering the area of ΔABC, find the exact width w of the footpath. **3**

Area of $\Delta ABC = \frac{1}{2} \times \frac{h}{\sqrt{3}} \times \sqrt{3}h \times \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{4} h^2$

$= \frac{\sqrt{3}}{4} \times 21$

$= \frac{21}{4}\sqrt{3}$ 1m

Area of $\Delta ABC = \frac{1}{2} \times 7 \times w$

$\frac{21}{4}\sqrt{3} = \frac{1}{2} \times 7 \times w$ 1m

$\frac{7}{2} w = \frac{21}{4}\sqrt{3}$

$w = \frac{2}{\cancel{7}} \times \frac{\overset{3}{\cancel{21}}}{\underset{2}{\cancel{4}}}\sqrt{3}$

$w = \frac{3\sqrt{3}}{2}$

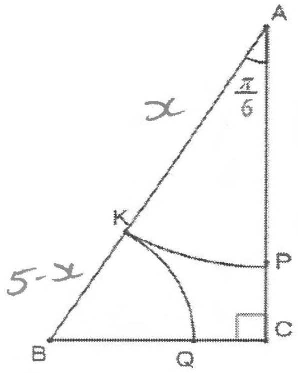
$\therefore$ width of the path $= \frac{3\sqrt{3}}{2}$ m 1m

Most students could find area of ΔABC using $\frac{1}{2}ab\sin C$ however failed to find the area of using $\frac{1}{2}b(w)$ and equate the two areas to find w.

Exam continues on the next page

c) In the diagram, ΔABC is a right angled at $C, \angle BAC = \frac{\pi}{6}$ and $AB = 5$ cm. **5**

KP is a circular arc with centre A and KQ is a circular arc with centre B.



NOT TO SCALE

If AK has length x cm, find the value of x, correct to one decimal place, for which the areas of sectors AKP and BKQ are equal.

Area AKP $= \frac{1}{2}r^2\theta$

$= \frac{1}{2}x^2 \times \frac{\pi}{6} = \frac{\pi x^2}{12}$ 1m

Area BKQ $= \frac{1}{2}(5-x)^2 \times \frac{\pi}{3}$

$= \frac{\pi}{6}(5-x)^2$ 1m

for the areas to be equal

$\frac{\pi x^2}{12} = \frac{\pi}{6}(5-x)^2$

$x^2 = 2(5-x)^2$

$x^2 = 2(25-10x+x^2)$

$x^2 - 20x + 50 = 0$ 1m

$x = \frac{20 \pm \sqrt{400-200}}{2} = \frac{20 + 10\sqrt{2}}{2}$

$= 2.92$ or 17.01 1m

but $0 < x < 5$

$\therefore x = 2.9$ cm 1m

Mostly well done. Either students made a mistake expanding and solving $x^2 = 2(5-x)^2$ or had trouble justifying that $x = 2.9$cm was the only solution. ($\because 0 < x < 5$)

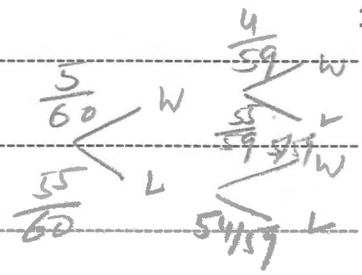
Exam continues on the next page

Question 12 (11 marks)

a) Sixty tickets are sold in a raffle with two prizes. Anthony buys 5 tickets. Find the probability that Anthony:

i. wins both the prizes **1**

$\frac{5}{60} \times \frac{4}{59} = \frac{1}{177}$



ii. wins one prize only **1**

$\frac{5}{60} \times \frac{55}{59} + \frac{55}{60} \times \frac{5}{59}$

$= \frac{55}{354}$

iii. wins at least one prize **1**

$1 - P(LL)$

$= 1 - \frac{55}{60} \times \frac{54}{59}$

$= \frac{19}{118}$

Many students got these questions wrong.

Exam continues on the next page

b) In a lottery, the probability of the jackpot being won in any draw is $\frac{1}{100}$.

i. What is the probability that the jackpot prize will be won in each of three consecutive draws? **1**

done well

$\frac{1}{100} \times \frac{1}{100} \times \frac{1}{100} = \frac{1}{100000} = \frac{1}{(100)^3}$

ii. How many consecutive draws need to be made for there to be a greater than 99% chance that at least one jackpot prize will have been won? **3**

P(not winning in one draw) $= \frac{99}{100}$

P(not winning in n draws) $= \left(\frac{99}{100}\right)^n$

P(at least winning one) $= 1 - \left(\frac{99}{100}\right)^n$

$1 - \left(\frac{99}{100}\right)^n > 0.99$ 1m

$-(0.99)^n > -0.01$

$(0.99)^n < 0.01$

$n > \frac{\ln 0.01}{\ln 0.99}$ (as $\ln 0.99$ is -ve) 1m

$n > 458.2$

$n = 459$

∴ 459 consecutive draws need to be made. 1m

Students struggled to get the answer

Exam continues on the next page

c) For events A and B from a sample space, $P(A|B) = \frac{3}{4}$ and $P(B) = \frac{1}{3}$.

i. Calculate $P(A \cap B)$. **1**

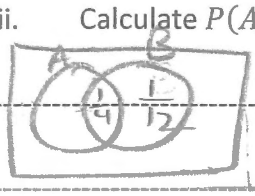
$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\frac{3}{4} = \frac{P(A \cap B)}{1/3}$$

$$P(A \cap B) = \frac{3}{4} \times \frac{1}{3} = \frac{1}{4}$$

Students seems to be not familiar with the formula.

ii. Calculate $P(A' \cap B)$, where A' denotes the complement of A. **1**



$$P(A' \cap B) = P(B) - P(A \cap B)$$

$$= \frac{1}{3} - \frac{1}{4}$$

$$= \frac{1}{12}$$

Only few got correct

iii. If events A and B are independent, calculate $P(A \cup B)$. **2**

If A and B are independent

$$P(A \cap B) = P(A) \times P(B)$$

$$\frac{1}{4} = P(A) \times \frac{1}{3}$$

$$P(A) = \tfrac{1}{4} \times 3 = \frac{3}{4}$$ 1m

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{3}{4} + \frac{1}{3} - \frac{1}{4} = \frac{5}{6}$$ 1m

END OF EXAM

Making mistakes in formulas.